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A COMPARISON OF FILTERED MODELS FOR DYNAMIC IDENTIFICATION OF ROBOTS

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ABSTRACT

This paper presents a comparison of three models which can be used to identify the minimum dynamic parameters of robots :

- a dynamic model which depends on the joint acceleration and needs an explicit derivation of the velocity, it will be named the explicit dynamic model.
- a dynamic model which avoids using explicit acceleration but needs the derivation of a function of the velocity, it will be named the implicit dynamic model.
- the energy model, which doesn't need neither acceleration nor implicit velocity derivation. A power model, which is the differential expression of the energy model, is introduced to enlighten the comparison between dynamic and energy models and to improve the filtering of the energy model.

Theoretical analysis is carried out from a filtering point of view and clearly shows the differences between the 3 identification models.

These results are checked from comparing simulated and experimental identifications of the dynamic parameters of a planar scara prototype robot.

1. INTRODUCTION

Accurate dynamic models of robots are required to control or simulate their motions. These models are functions of the geometric parameters of robots (length of links, angle between joint axis,...) and the dynamic parameters (inertia, first moments, masses, friction). This paper is focused on comparing the estimation of the dynamic parameters using 3 identification models which are linear in relation to these parameters, and least squares techniques, while the geometric parameters are supposed known.

The first model is the explicit dynamic model which depends on the joint acceleration. In order to carry out with success practical identification using this model, low-pass parallel filtering of the vector of measurements and of the columns of the observation matrix must be used [1].

The second model comes from the explicit dynamic model which has been rewritten in order to move the derivation of the velocity to the derivation of a function of the velocity. Then it has been used by many authors for the adaptive control of robots [2,3] and the identification [1,4,5] because it doesn't depend explicitly on the acceleration. However, practical identification using this model also needs filtering [1]. This is the reason why it has been called the filtered dynamic model, but in fact the first model is also a filtered dynamic model. This is the reason to change it's name to the implicit dynamic model.

The third model is the energy model which avoids any derivation of velocity or function of velocity. Rewriting this model in a differential form to get a power model allows to compare it easily to the dynamic models.

Many experimental works have used one or several of the 3 models an some of them have compared the dynamic and the energy models [6]. But no work allows a theoretical comparison. In this paper we'll show how to obtain one model from the other ones with a unified approach based on the use of filters, which allows to analyze the advantages and drawbacks of each model. Some practical features will be given to choose the models and the filters in order to get a L.S. estimation of parameters without neither distortion nor bias effect. Simulated and experimental identification of a 2 d.o.f. scara robot will illustrate theoretical results.

2. THE EXPLICIT DYNAMIC MODEL

The inverse dynamic model of a rigid robot composed of n moving links calculates the motor torque vector Γ_m (the control input) as a function of the generalized coordinates (the state vector and it's derivative). It can be obtained from the Lagrangian equation as recalled here :

$$\Gamma_m = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} + \Gamma_f \qquad (1)$$

q, $\dot{q}$ are the (1xn) vectors of generalized joint positions and velocities.

L is the Lagrangian of the system, equal to $E(q,\dot{q}) - U(q)$.

E and U are the kinetic and the potential energy of the system with :

$$E = \frac{1}{2}\dot{q}^T M(q)\,\dot{q}$$

M(q) is the (nxn) robot inertia matrix.

Γ_f is the friction torque which is usually modelized at non zero velocity as following :

$$\Gamma_{fj} = F_{sj}\,\mathrm{Sign}(\dot{q}_j) + F_{vj}\,\dot{q}_j$$

$\dot{q}_j$ is the joint j velocity.

Sign(x) denotes the sign function.

F_{vj}, F_{sj}, are the viscous and Coulomb friction coefficients of joint j. (Eq.1) can be written as :

$$\Gamma_m = \frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} + \Gamma_f = \frac{d}{dt}(M(q)\,\dot{q}) - \frac{\partial L}{\partial q} + \Gamma_f \qquad (2)$$

with :

$$\frac{d}{dt}(M(q)\,\dot{q}) = M(q)\,\ddot{q} + \dot{M}(q)\,\dot{q}$$

$$N(q,\,\dot{q}) = \dot{M}(q)\,\dot{q} - \frac{\partial E}{\partial q} + \frac{\partial U}{\partial q} + \Gamma_f$$

Comes the classical inverse dynamic model which explicitly depends on the joint acceleration :

$$\Gamma_m = M(q)\,\ddot{q} + N(q,\,\dot{q}) \qquad (3)$$

$\ddot{q}$ is the (1xn) vector of joint accelerations.

$N(q,\,\dot{q})$ is the (nx1) vector of centrifugal, Coriolis, gravitational and friction torques.

The choice of modified Denavit and Hartenberg frames attached to each link allows to obtain a dynamic model (Eq.3) linear in relation to a set of standard dynamic parameters X_S [1,10] :

$$\Gamma_m = D_S(q,\,\dot{q},\,\ddot{q})\,X_S \qquad (4)$$

The (13nx1) vector X_S is composed for each link of the 6 components of the inertia tensor, the 3 components of the first moment and the mass, a total inertia moment for rotor actuator and gears, Coulomb and viscous friction parameters. It has been shown that the set of standard dynamic parameters can be simplified to obtain the base inertial parameters. The base inertial parameters are defined as the minimum parameters which can be used to get the dynamic model. They represent the set of Np parameters which can be identified using the dynamic model [7,8,9]. These parameters can be obtained from the standard inertial parameters by eliminating those which have no effect on the dynamic model and by regrouping some others in linear relations. Symbolic and numerical solutions have been proposed for any open or closed chain manipulator [10,11,12] to get a minimal dynamic model :

$$\Gamma_m = D(q,\,\dot{q},\,\ddot{q})\,X = \sum_{i=1,Np} D_{:,i}\,X_i \qquad (5)$$

The i^{th} column $D_{:,i}$ of D is given as following :

$$D_{:,i}(q,\,\dot{q},\,\ddot{q}) = M_i(q)\,\ddot{q} + N_i(q,\,\dot{q}),\; M_i = \frac{\partial M}{\partial X_i},\; N_i = \frac{\partial N}{\partial X_i} \qquad (6)$$

3. THE IMPLICIT DYNAMIC MODEL

The implicit dynamic model is given by (Eq.2), where the derivation of the velocity to get the joint acceleration has been replaced by the derivation of the function $(M(q)\,\dot{q})$ of the velocity :

$$\Gamma_m = \frac{d}{dt}(M(q)\,\dot{q}) + R(q,\,\dot{q}) = F(q,\,\dot{q})\,X = \sum_{i=1,Np} F_{:,i}\,X_i \qquad (7)$$

with :

$$R(q,\,\dot{q}) = -\frac{\partial L}{\partial q} + \Gamma_f \qquad (8)$$

$$F_{:,i} = \frac{d}{dt}(M_i(q)\,\dot{q}) + R_i(q,\,\dot{q}),\quad R_i = \frac{\partial R}{\partial X_i} \qquad (9)$$

A new efficient algorithm has been proposed to calculate the symbolic expressions of $M_i(q)\,\dot{q}$ and $R_i(q,\,\dot{q})$ [17], but it's computational burden remains higher than that of (Eq.5,6), calculated with a customized Newton Euler formulation [13]. In spite it has been extensively used in control [2,3] and sometimes in identification [1,4,5], it's superiority compared to the explicit dynamic model has not yet been proved [1].

In order to eliminate the derivative of $(M(q)\,\dot{q})$, an integral form of this model can be used, integrating both sides of (Eq.7) between 2 times t_a and t_b :

$$\int_{ta}^{tb}\Gamma_m\,dt = \left(\int_{ta}^{tb}F(q,\,\dot{q})dt\right)X = \sum_{i=1,Np} X_i \int_{ta}^{tb}F_{:,i}\,dt$$

$$\int_{ta}^{tb}F_{:,i}\,dt = \left(M_i(q)\,\dot{q}\right)(tb) - \left(M_i(q)\,\dot{q}\right)(ta) + \int_{ta}^{tb}R_i\,dt$$

4. THE ENERGY MODEL

In order to eliminate any derivation of velocity in the identification process, a model based on the energy theorem has been proposed [9,14]. This model can be obtained in a differential form calculating the power of the system with the Lagrange equation (Eq.1) :

$$\dot{q}^T\,\Gamma_m = \dot{q}^T\left(\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q}\right) + \dot{q}^T\,\Gamma_f \qquad (10)$$

$$\dot{q}^T\,\Gamma_m = \frac{d}{dt}(H(q,\dot{q})) + \dot{q}^T\,\Gamma_f = \left(\frac{d}{dt}h(q,\dot{q})\right)X = dh\,X$$

$H(q,\dot{q}) = E(q,\dot{q}) + U(q)$ is the total energy of the system.

$h(q,\dot{q})$ is the (1xNp) row matrix of the energy functions $h_i(q,\dot{q})$, including friction effect :

$$h_i(q,\dot{q}) = \frac{\partial(H + \int \dot{q}^T\,\Gamma_f\,dt)}{\partial X_i}$$

Integrating both sides of (Eq.10) between 2 times t_a and t_b yields the energy identification model :

$$\int_{t_a}^{t_b}\dot{q}^T\Gamma_m dt = H(q,\dot{q})(t_b) - H(q,\dot{q})(t_a) + \int_{t_a}^{t_b}\dot{q}^T\Gamma_f dt = \Delta h\,X \qquad (11)$$

Δh is the (1xNp) regressor row matrix defined from the energy function row matrix $h(q,\dot{q})$:

$$\Delta h(q,\dot{q}) = h(q,\dot{q})(t_b) - h(q,\dot{q})(t_a)$$

The energy model is a scalar equation whose symbolic equations are easier to calculate and manipulate than the vector equations of the dynamic model [13].

5. THE IDENTIFICATION METHOD

Generally, least squares (L.S.) techniques are used to estimate the minimum dynamic parameters solving an overdetermined linear system obtained from a sampling of the dynamic models (Eq.5, Eq.7) or the energy model (Eq.11) along a trajectory $(q,\;\dot{q}$ or $\ddot{q})$ [1,5,6,7,8,9] :

$$Y = W\,X + \rho \qquad (12)$$

The L.S. solution $\hat{X}$ minimizes the 2 norm $\|\rho\|^2$ of the vector of errors ρ. The unicity of $\hat{X}$ depends on the rank of the observation matrix W.

W numerical rank deficiency can come from two origins :

– structural rank deficiency which stands for any samples of (q, $\dot{q}$ or $\ddot{q}$) in W.

This is the structural parameters identifiability problem which is solved using base parameters.

– data rank deficiency due to a bad choice of noisy (q, $\dot{q}$ or $\ddot{q}$) samples in W. This is the problem of optimal measurement strategies which is solved using closed loop identification to track exciting trajectories.

In order to decrease the sensitivity of the L.S. solution of system (Eq.12) to errors in Y and W, the condition number of the observation matrix W, Cond(W), must be close to one before computing $\hat{X}$. Exciting trajectories can be obtained by non linear optimization of a criterion function of the condition number of W, under constraints of the equation of an interpolator and the joints positions and velocities limits [19]. In the following, we suppose that this stage has been reached, that is W is a (rxNp) full rank and well conditioned matrix obtained on an exciting trajectory [15]. Since the analytical expressions of the dynamic or energy identification models are equivalent, it is proposed to compare the efficiency of the different models from a filtering point of view.

6. COMPARISON OF IDENTIFICATION MODELS

6.1. Data acquisition

Torques are calculated using the relation :

$$\Gamma_{mj} = G_{Tj} V_{Tj} \tag{13}$$

V_{Tj} is the current reference of the amplifier current loop which is directly the control data at a sample rate w_c in the case of using a numerical controller. In the case of analog control it must be analog lowpass filtered to prevent aliasing before to be sampled at w_c with an analog to digital converter.

G_{Tj} is the gain of the joint j drive chain, which is taken as a constant in the frequency range [0 w_{dyn}] of the robot. Accurate determination of G_{Tj} using methods described in [16] is essential for the success of the identification.

Usually, robot sensors provide discrete joint position measurements from encoders or resolvers. Then the use of an analog antialiasing filter is not possible and the sample rate w_c must be large enough to avoid high frequency noise aliasing in the bandwidth [0 w_{dyn}] of the joint position closed loop. The rule w_c=100*w_{dyn} is generally used to get an acceptable sample rate for the control input V_T.

Calculating the L.S. solution of (Eq.12) from noisy discrete measurements or estimations of (q, $\dot{q}$, $\ddot{q}$, Γ_m) may lead to bias because W and Y are non independent random matrices [1]. Then it is essential to filter data in Y and W, before computing the L.S. solution.

6.2. Filtering the explicit dynamic model

Samples of V_T and q at rate w_c allow to calculate samples of (Eq.5) at times t_i, i=1,...,n_e, to get the (1xr) measurement vector $Y(\Gamma_m)$ and the (rxNp) observation matrix $W(q, \dot{q}, \ddot{q})$, with $r=nxn_e>Np$:

$$Y = \begin{bmatrix} \Gamma_m(1) \\ \ldots \\ \Gamma_m(n_e) \end{bmatrix}, \quad W = \begin{bmatrix} D(1) \\ \ldots \\ D(n_e) \end{bmatrix} \tag{14}$$

$\Gamma_m(i) = \Gamma_m(t_i)$ is calculated with (Eq.13)

$D(i) = D(q(t_i), \dot{q}(t_i), \ddot{q}(t_i))$, is calculated with (Eq.6) or with it's customized Newton Euler formulation [13]. In order to avoid distortion in the dynamic regressor which is composed of non linear functions of (q, $\dot{q}$, $\ddot{q}$) (Eq.6), the joint velocity and acceleration are calculated without phase shift by central difference of q. The drawback of this derivation is a dramatic increase of high frequency noise effect in ($\dot{q}$, $\ddot{q}$) estimations [17]. Then the matrix W is very perturbed and has to be lowpass filtered. The torque Γ_m is perturbed by the rejection of perturbations (high frequency torque ripple of the joint drive chain) of the closed loop control and has also to be filtered. Then Γ_m and $D(q, \dot{q}, \ddot{q})$ in (Eq.14) are both filtered by a lowpass filter fp(s), with s a derivative operator, to get a new filtered linear system :

$$Y_{fp} = W_{fp}(q, \dot{q}, \ddot{q})\, X + \rho_{fp} \tag{15}$$

It is to be noted that no error is introduced by this filtering process in the linear relation (Eq.15) compared with (Eq.12). The only point is to approximately choose the cut-off frequency w_{fp} around 5*w_{dyn} in order to keep useful signal of the dynamic behavior of the robot in the filter bandwidth. Because there is no more signal in the range [w_{fp} w_c/2], Y_{fp} and W_{fp} are resampled at a lower rate, keeping one sample over n_d. The decimate procedure of Matlab is used to easily calculate a filtered and decimated linear system :

$$Y_{fpd} = W_{fpd}(q, \dot{q}, \ddot{q})\, X + \rho_{fpd} \tag{16}$$

with :

n_d=(w_c/2)/w_{fp} for a FIR filter, and n_d=0.8*(w_c/2)/w_{fp} for an IIR filter.

Taking w_c=100*w_{dyn} and w_{fp}=5*w_{dyn} gives a value of $n_d \approx$ 10.

In order to decrease noise in W, it is better to prefilter the joint position q before calculating the derivatives, but it is to be noted that this prefiltering is sensitive to the choice of the cut-off frequency w_{fq}. The filtered data $(q_{fq}, \dot{q}_{fq}, \ddot{q}_{fq})$ must be equal to (q, $\dot{q}$, $\ddot{q}$) in the range [0, w_{fq}] in order to avoid distortion in the dynamic regressor :

$$D_{:,i}(q,\dot{q},\ddot{q}) = M_i(q)\,\ddot{q} + N_i(q,\dot{q}) = M_i(q_{fq})\,\ddot{q}_{fq} + N_i(q_{fq},\dot{q}_{fq})$$

$$M_i(q_{fq})\,\ddot{q}_{fq} + N_i(q_{fq},\dot{q}_{fq}) = D_{:,i}(q_{fq},\dot{q}_{fq},\ddot{q}_{fq}) = D_{fq:,i}$$

The derivatives $\dot{q}_{fq}$, $\ddot{q}_{fq}$ are obtained without phase shift using a central difference algorithm of the filtered position q_{fq}. In order to eliminate high frequency noise differentiation, the order of the lowpass filter fq(s) must be greater than 2 to get a passband filter s*fq(s) to calculate the velocity and s^2*fq(s) to calculate the acceleration. The filter fq(s) must have a flat amplitude characteristic without phase shift in the range $[0\ w_{fq}]$, with the rule of thumb $w_{fq}>10*w_{dyn}$. Considering an off-line identification, this is easily obtained with a non causal zero-phase digital filtering by processing the input data through an IIR lowpass butterworth filter in both the forward and reverse direction using a filtfilt procedure from Matlab. Then the parallel filtering process is carried out to get the linear system :

$$Y_{fpd} = W_{fpd}(q_{fq}, \dot{q}_{fq}, \ddot{q}_{fq})\,X + \rho_{fpd} \tag{17}$$

6.3. Filtering the implicit dynamic model

The same decimate procedure fpd as that of previous section is applied to get a parallel filtered and decimated linear system :

$$Y_{fpd} = W_{fpd}(q_{fq}, \dot{q}_{fq})\,X + \rho_{fpd} \tag{18}$$

It is composed of a sampling of the filtered torque Γ_m (Eq.13) and regressor F (Eq.9) as following :

$$(\Gamma_m)_{fpd} = F_{fpd}(q_{fq}, \dot{q}_{fq})\,X = \sum_{i=1,Np}(F_{:,i})_{fpd}\,X_i \tag{19}$$

$$(F_{:,i})_{fpd}(q_{fq}, \dot{q}_{fq}) = \left(sM_i(q_{fq})\,\dot{q}_{fq}\right)_{fpd} + (P_i)_{fpd}(q_{fq}, \dot{q}_{fq})$$

$$(F_{:,i})_{fpd}(q_{fq}, \dot{q}_{fq}) = s\left(M_i(q_{fq})\,\dot{q}_{fq}\right)_{fpd} + (P_i)_{fpd}$$

This expression shows an advantage of the implicit dynamic model over the explicit one, because it allows to proceed to the second derivative $s\left(M_i(q_{fq})\,\dot{q}_{fq}\right)_{fpd}$ (using a central difference algorithm) without any distortion compared with the expression $\left(M_i(q_{fq})\,\ddot{q}_{fq}\right)_{fpd}$ in the regressor of the explicit dynamic model, where the second derivative of filtered position can introduce distortion depending on the choice of fq(s). In order to eliminate high frequency noise differentiation, the order of the lowpass filter fp(s) in the decimate procedure fpd must be greater than 1 to get a passband filter s*fp(s) to calculate the derivative $s\left(M_i(q_{fq})\,\dot{q}_{fq}\right)_{fpd}$.

6.4. Filtering the energy model

The energy model has been used by sampling (Eq.11) at different times $t_a(i)$, $t_b(i)=t_a(i+1)$, $i=1,\dots,r$, $r>Np$, as following [6,9,18] :

$$Y = \begin{bmatrix} y(1) \\ \dots \\ y(r) \end{bmatrix},\ y(i) = \int_{t_a(i)}^{t_b(i)} \dot{q}_{fq}{}^T \Gamma_{m_{fq}}\,dt \ ,\ W = \begin{bmatrix} \Delta h(1) \\ \dots \\ \Delta h(r) \end{bmatrix} \tag{20}$$

$$\Delta h(i) = h(q_{fq}, \dot{q}_{fq})(t_b(i)) - h(q_{fq}, \dot{q}_{fq})(t_a(i)) \tag{21}$$

One point is how to choose the times $t_a(i)$. In order to avoid offset in y(i) due to constant perturbation in $(\dot{q}_{fq}{}^T \Gamma_{m_{fq}})$, $t_b(i)-t_a(i)$ must be bounded. In [15,18], sampling times are optimized to get a well conditioned observation matrix W (Eq.20). This formulation needs a lower resampling of the energy function h at times $t_a(i)$, $t_b(i)$. Then the function h calculated at the acquisition rate w_c must be lowpass filtered with fp(s) in order to avoid aliasing. So it is easier and natural to associate the sampling times with the decimate procedure with a ratio n_d, which results in choosing $t_b(i)$ and $t_a(i)$ with a constant value $t_b(i)-t_a(i)=n_d*2*\pi/w_c$. As a result, the parallel decimate procedure fpd must be applied to (Eq. 20,21) as following :

$$\int_{t_a}^{t_b}\left(\dot{q}_{fq}{}^T \Gamma_{m_{fq}}\right)_{fpd} dt = \left[h_{fpd}(t_b) - h_{fpd}(t_a)\right]X = \Delta h_{fpd} X \tag{22}$$

This is equivalent to parallel filter and decimate the differential expression of the energy model (Eq.10), which is the power model :

$$P_{fpd} = \left(\dot{q}_{fq}{}^T \Gamma_{m_{fq}}\right)_{fpd} = s\,h_{fpd}\,X = dh_{fpd}\,X \tag{23}$$

The derivative dh_{fpd} is calculated without phase shift using a central difference algorithm.

This formulation clearly defines the choice of the sampling times $t_a(i)$ which depends on $n_d \approx 10$ and shows that the power model is the scalar formulation of the filtered implicit dynamic model (Eq.19). The main advantage of this model is the simplicity of the energy functions h.

7. VALIDATION

7.1. Description of the robot

The comparison is carried out on a 2 joints planar direct drive prototype robot (Fig. 1), without gravity effect. The description of the geometry of the robot uses the modified Denavit and Hartenberg notation [9,13].

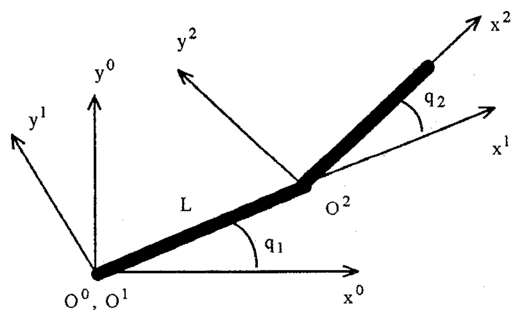

Fig. 1. SCARA robot : frames and joint variables.

The robot is direct drived by 2 DC permanent magnet motors supplied by PWM choppers.

The dynamic model depends on 8 minimal dynamic parameters, considering 4 friction parameters :

$$X = [ZZR_1\ \ Fv_1\ \ Fs_1\ \ ZZ_2\ \ LMX_2\ \ LMY_2\ \ Fv_2\ \ Fs_2]^T$$

$$ZZR_1 = ZZ_1 + M_2 L^2$$

L is the length of the first link,

M_2 is the mass of the link 2,

ZZ_1 and ZZ_2 are the drive side moment of inertia of link 1 and 2 respectively,

MX_2 and MY_2 are the first moments of link 2.
The simulation is carried out with the supposed true values (SI Units) :

$$X=\begin{bmatrix}3.5 & 0.05 & 0.5 & 0.06 & 0.12 & 0.005 & 0.01 & 0.1\end{bmatrix}^T$$

The columns of the regressors are the following :

—explicit dynamic model (Eq.5, 6) :

$$D_{:,1}=D_{ZZR_1}=\begin{bmatrix}\ddot{q}_1\\0\end{bmatrix},\ D_{:,2}=D_{Fv_1}=\begin{bmatrix}\dot{q}_1\\0\end{bmatrix}$$

$$D_{:,3}=D_{Fs_1}=\begin{bmatrix}\mathrm{sign}(\dot{q}_1)\\0\end{bmatrix},\ D_{:,4}=D_{ZZ_2}=\begin{bmatrix}\ddot{q}_1+\ddot{q}_2\\\ddot{q}_1+\ddot{q}_2\end{bmatrix}$$

$$D_{:,5}=D_{LMX_2}=\begin{bmatrix}(2\ddot{q}_1+\ddot{q}_2)\cos q_2-\dot{q}_2\,(2\dot{q}_1+\dot{q}_2)\sin q_2\\\ddot{q}_1\cos q_2+\dot{q}_1^2\sin q_2\end{bmatrix}$$

$$D_{:,6}=D_{LMY_2}=\begin{bmatrix}-(2\ddot{q}_1+\ddot{q}_2)\sin q_2-\dot{q}_2\,(2\dot{q}_1+\dot{q}_2)\cos q_2\\\dot{q}_1^2\cos q_2-\ddot{q}_1\sin q_2\end{bmatrix}$$

$$D_{:,7}=D_{Fv_2}=\begin{bmatrix}0\\\dot{q}_2\end{bmatrix},\ D_{:,8}=D_{Fs_2}=\begin{bmatrix}0\\\mathrm{sign}(\dot{q}_2)\end{bmatrix}$$

—implicit dynamic model (Eq.7, 8, 9) :

$$F_{:,1}=F_{ZZR_1}=\frac{d}{dt}\begin{bmatrix}\dot{q}_1\\0\end{bmatrix},\ F_{:,2}=F_{Fv_1}=\begin{bmatrix}\dot{q}_1\\0\end{bmatrix}$$

$$F_{:,3}=F_{Fs_1}=\begin{bmatrix}\mathrm{sign}(\dot{q}_1)\\0\end{bmatrix},\ F_{:,4}=F_{ZZ_2}=\frac{d}{dt}\begin{bmatrix}\dot{q}_1+\dot{q}_2\\\dot{q}_1+\dot{q}_2\end{bmatrix}$$

$$F_{:,5}=F_{LMX_2}=\frac{d}{dt}\begin{bmatrix}(2\,\dot{q}_1+\dot{q}_2)\cos q_2\\\dot{q}_1\cos q_2\end{bmatrix}+\begin{bmatrix}0\\\dot{q}_1(\dot{q}_1+\dot{q}_2)\sin q_2\end{bmatrix}$$

$$F_{:,6}=F_{LMY_2}=\frac{d}{dt}\begin{bmatrix}-(2\,\dot{q}_1+\dot{q}_2)\sin q_2\\-\dot{q}_1\sin q_2\end{bmatrix}+\begin{bmatrix}0\\\dot{q}_1(\dot{q}_1+\dot{q}_2)\cos q_2\end{bmatrix}$$

$$F_{:,7}=F_{Fv_2}=\begin{bmatrix}0\\\dot{q}_2\end{bmatrix},\ F_{:,8}=F_{Fs_2}=\begin{bmatrix}0\\\mathrm{sign}(\dot{q}_2)\end{bmatrix}$$

— energy model (Eq. 10) :

$$h_1=h_{ZZR_1}=1/2\,\dot{q}_1^2,$$

$$dh_2=dh_{Fv_1}=\dot{q}_1^2,$$

$$dh_3=dh_{Fs_1}=|\dot{q}_1|,$$

$$h_4=h_{ZZ_2}=1/2\,(\dot{q}_1^2+\dot{q}_2^2),$$

$$h_5=h_{LMX_2}=\dot{q}_1(\dot{q}_1+\dot{q}_2)\cos q_2,$$

$$h_6=h_{LMY_2}=-\dot{q}_1(\dot{q}_1+\dot{q}_2)\sin q_2,$$

$$dh_7=dh_{Fv_2}=\dot{q}_2^2,$$

$$dh_8=dh_{Fs_2}=|\dot{q}_2|$$

7.2. Comparison by simulation

The robustness of the 3 methods with respect to the systematic errors introduced by the filtering and derivative processes, without any noise, is investigated using the 3 models to estimate the dynamic parameters of the robot and compare them to the true values. This stage is important to check that filtering doesn't introduce distortion in the identification process. The sampled torque is calculated with the dynamic model and a sampling of a successive point to point trajectories using a classical 5th order polynomial trajectory generator, with a sample rate $w_e=2*\pi*100$rd/s, (Nyquist frequency = $w_e/2=2*\pi*50$rd/s). Starting with $n_e=8000$ samples, and $n_d=10$ ($w_{fp}=0.8*(w_e/2)/10$), we get $r=800$ equations and a condition number of W Cond(W)=28, for the dynamic model and r=400 with Cond(W)=93, for the energy model. Joint positions and torques are prefiltered using a butterworth filter with a cut-off frequency $w_{fq}=0.8*(w_e/2)/5$.
Results given in Tab.1 show that errors are close for the 3 models which can be considered equivalent from the filtering systematic error point of view, with a little advantage for the dynamic model with explicit acceleration.

Table 1 : Comparison of the systematic errors of the filtering process.

		Dy. explicit		Dy. implicit		Power	
Name	X	$\hat{X}$	$\%e_{\hat{X}r}$	$\hat{X}$	$\%e_{\hat{X}r}$	$\hat{X}$	$\%e_{\hat{X}r}$
ZZR_1	3.5	3.501	0.040	3.501	0.042	3.502	0.057
Fv_1	0.05	0.049	0.207	0.049	0.649	0.049	1.064
Fs_1	0.5	0.500	0.008	0.500	0.011	0.501	0.114
ZZ_2	0.06	0.060	0. 003	0.060	0.008	0.060	0.001
LMX_2	0.12	0.120	0.014	0.120	0.072	0.120	0.089
LMY_2	0.005	0.005	0.040	0.005	0.369	0.005	0.613
Fv_2	0.01	0.010	0.024	0.010	0.049	0.010	0.622
Fs_2	0.1	0.100	0.006	0.100	0.015	0.099	0.346
		Cond(W)=28		Cond(W)=28		Cond(W)=93	

7.3. Experimental comparison

The 3 models and the same filtering process as defined for simulation have been used to calculate the L.S estimation $\hat{X}$ of the dynamic parameters of the prototype robot. In order to get the same number of equations r=1600, 2 independent realizations of the trajectory have been used for the dynamic models and 4 realizations for the energy model.
Standard deviations $\sigma_{\hat{X}i}$ are estimated using classical and simple results from statistics, considering the matrix W to be a deterministic one, and ρ to be a zero mean additive independent noise, with standard deviation σ_ρ such that $C_{\rho\rho}=E(\rho^T\rho)=\sigma_\rho^2\,I_r$, where E is the expectation operator. The variance-covariance matrix of the estimation error and standard deviations can be calculated by :

$$C_{\hat{X}\hat{X}}=E\left[(X-\hat{X})(X-\hat{X})^T\right]=\sigma_\rho^2(W^T\,W)^{-1}$$

$\sigma_{\hat{X}i}{}^2=C_{\hat{X}\hat{X}ii}$, the diagonal coefficient of $C_{\hat{X}\hat{X}}$

An unbiased estimation of σ_ρ is used to get the relative standard deviation $\sigma_{\hat{X}ri}$ by the expression :

$$\hat{\sigma}_\rho^2 = \frac{\left\| Y - W\hat{X} \right\|^2}{r - Np} \quad , \quad \%\sigma_{\hat{X}ri} = 100 * \frac{\sigma_{\hat{X}i}}{\hat{X}i}$$

Table 2 : Comparison of experimental estimation

	Dy. explicit & implicit			Power		
Name	$\hat{X}$	$2\sigma_{\hat{X}}$	$\%\sigma_{\hat{X}r}$	$\hat{X}$	$2\sigma_{\hat{X}}$	$\%\sigma_{\hat{X}r}$
ZZR_1	3.38	0.025	0.37	3.39	0.025	0.37
Fv_1	-0.18	0.046	13	-0.21	0.04	8.6
Fs_1	0.59	0.024	2.1	0.61	0.03	2.8
ZZ_2	0.060	0.002	2	0.0633	0.0005	0.4
LMX_2	0.119	0.002	0.63	0.121	0.001	0.5
LMY_2	0.001	0.001	50	-0.001	0.001	53
Fv_2	0.012	0.007	30	0.012	0.002	7
Fs_2	0.12	0.02	6.6	0.12	0.01	4
	Cond(W)=37			Cond(W)=111		

Results given in Tab.2 are exactly the same for both dynamic models. This study confirms that there is no advantage in using the dynamic identification model with implicit acceleration, because it's symbolic expression is more complicated than that of the dynamic model with explicit acceleration, and because experimental results are the same [1]. Another result is to show that the new formulation of the energy model, with a filtered power model, gives the same accuracy than that obtained with the dynamic models, using the same number r of rows in W and the same filters (w_{fp}, w_{fq}). So, it is possible to take advantage of a very simple identification model which is less sensitive to the use of exciting trajectories compared to it's integral formulation [18].

8. CONCLUSION

This paper presents a unified approach of 3 identification models which are used to estimate the dynamic parameters of robots. A new formulation of the energy model, based on a filtered power model, is proposed. Theoretical, simulation and experimental studies show that the 3 models give close estimations and accuracy, providing to use the same filters and the same number of equations of the overdetermined linear systems. Our conclusion is that the filtered power model is very attractive for it's simplicity and accuracy. The 2 dynamic models give the same results, then there is no advantage in using the dynamic model with implicit acceleration because it's expression is more complicated than that of the dynamic model with explicit acceleration.

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