## VI. Conclusion

In this communication, the robustness of adaptive manipulator control was investigated. A simple example demonstrated the nonrobustness of a recent adaptive result to bounded disturbances. A new robust adaptive controller was then analyzed to show the existence of a "large" region of attraction from which all signals remain bounded. The mean of the tracking error is shown to be proportional to the bound on the disturbances, the bounds on the speed of the smooth plant parameter variations and size of the jump plant parameter changes, and the strength of the unmodeled dynamics. The parameters are not required to be slowly varying in order to guarantee boundedness; but tracking performance may be affected by fast variations. Furthermore, for jump parameter changes, the minimum time between jumps is assumed to be bounded from below. In the analysis herein, the designer is not forced to select controller parameters that depend directly upon the particular plant dynamics in order to maintain stability. The only *a priori* knowledge necessary is an upper bound on the magnitude of the unknown parameter vector; and this information is not required to show boundedness of all closed-loop signals. Robustness is obtained through the adaptation strucure; and "persistently exciting" signals are not required for stability or robustness.

## References


[1] T. C. Hsia, "Adaptive control of robotic manipulators—A review," in *Proc. IEEE Int. Conf. on Robotics and Automation* (San Francisco, CA, 1986).

[2] R. Horowitz and M. Tomizuka, "An adaptive control scheme for mechanical manipulators—Compensation of nonlinearity and decoupling control," *ASME J. Dyn. Syst. Meas. Contr.*, vol. 108, pp. 127–135, Jan. 1986. Originally presented at the 1980 ASME Winter Annual Meeting as Paper 80-WA/DSC-6.

[3] B. K. Kim and K. G. Shin, "An adaptive model following control of industrial manipulators," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-19, pp. 805–813, Nov. 1983.

[4] M. K. Sundareshan and M. A. Koeing, "Decentralized model reference adaptive control of robot manipulators," in *Proc. 1985 Automatic Control Conf.*, pp. 44-49, 1985.

[5] S. Dubowsky and D. T. DesForges, "The application of model reference adaptive control to robotic manipulators," *ASME J. Dyn. Syst. Meas. Contr.*, vol. 101, pp. 193–200, 1979.

[6] M. Takegaki and S. Arimoto, "An adaptive trajectory control of manipulators," *Int. J. Contr.*, vol. 34, pp. 219–230, 1981.

[7] C. S. G. Lee and M. J. Chung, "An adaptive control strategy for mechanical manipulators," *IEEE Trans. Automat. Contr.*, vol. AC-29, pp. 837–840, 1984.

[8] M. Vukobratovic and N. Kircanski, "An approach to adaptive control of robotic manipulators," *Automatica*, vol. 21, pp. 639–647, 1985.

[9] Y. K. Choi, M. J. Chung, and Z. Bien, "An adaptive control scheme for robotic manipulators," *Int. J. Contr.*, vol. 44, pp. 1185–1191, 1986.

[10] H. Seraji, "Adaptive control of robotic manipulators," in *Proc. 26th Conf. on Decision and Control* (Los Angeles, CA, 1987), pp. 599–601.

[11] M. Tomizuka and R. Horowitz, "Model reference adaptive control of mechanical manipulators," in *Proc. IFAC Symp. on Adaptive Systems in Control and Signal Processing* (San Francisco, CA, 1983), pp. 27–32.

[12] H. Elliot *et al.*, "Nonlinear adaptive control of mechanical linkage systems with application to robotics," in *Proc. 1983 Automatic Control Conf.*, pp. 1050–1055, 1983.

[13] S. Nicosia and P. Tomei, "Model reference adaptive control algorithms for industrial robots," *Automatica*, vol. 20, pp. 635–644, Sept. 1984.

[14] A. Balestrino, G. DeMaria, and L. Sciavicco, "An adaptive model following control for robot manipulators," *ASME J. Dyn. Syst. Meas. Contr.*, vol. 105, pp. 143–151, Sept. 1984.

[15] S. N. Singh, "Adaptive model following control of nonlinear robotic systems," *IEEE Trans. Automat. Contr.*, vol. AC-30, pp. 1099–1100, Nov. 1985.

[16] K. Y. Lim and M. Eslami, "Robust adaptive controller designs for robot manipulator systems," *IEEE J. Robotics Automat.*, vol. RA-3, pp. 54–66, Feb. 1987.

[17] J.-J. E. Slotine, "On modeling and adaptation in robot control," in *Proc. IEEE Int. Conf. on Robotics and Automation* (San Francisco, CA, 1986), pp. 1387–1392.

[18] J. J. Craig, P. Hsu, and S. S. Sastry, "Adaptive control of mechanical manipulators," in *Proc. IEEE Int. Conf. on Robotics and Automation.* (San Francisco, CA, 1986).

[19] R. H. Middleton and G. C. Goodwin, "Adaptive computed torque control for rigid-link manipulators," in *Proc. 25th Conf. on Decision and Control* (Athens, Greece, 1986), pp. 68–73.

[20] J.-J. E. Slotine and W. Li, "On the adaptive control of robot manipulators," presented at the ASME Winter Annual Meeting, Anaheim, CA, 1986.

[21] N. Sadegh and R. Horowitz, "Stability analysis of an adaptive controller for robotic manipulators," in *Proc. IEEE Int. Conf. on Robotics and Automation* (Raleigh, NC, Apr. 1987).

[22] D. S. Bayard and J. T. Wen, "Robust control for robotic manipulators, Part II: Adaptive case," in *Proc. Space Telerobotics Workshop* (Jet Propulsion Lab., Pasadena, CA, 1987).

[23] P. A. Ioannou and P. V. Kokotovic," Instability analysis and improvement of robustness of adaptive control," *Automatica*, vol. 20, pp. 583–594, Sept. 1984.

[24] J.-J. E. Slotine and W. Li, "Theoretical issues in adaptive manipulator control," in *Proc. 5th Yale Workshop on Applications of Adaptive Systems Theory*, pp. 252–258, May 1987.

[25] P. A. Ioannou, "Decentralized adaptive control of interconnected systems," *IEEE Trans. Automat. Contr.*, vol. AC-31, pp. 291–298, Apr. 1986.

[26] P. A. Ioannou and K. Tsakalis, "A robust direct adaptive controller," *IEEE Trans. Automat. Contr.*, vol. AC-313, pp. 1033–1043, Nov. 1986.

[27] P. A. Ioannou and J. Sun, "The theory and design of robust direct and indirect adaptive control schemes," Tech. Rep. 86-06-1, Univ. of Southern Calif., Dep. of EE–Syst., May, 1986. To appear in *Int. J. Contr.*

[28] P. A. Ioannou, "Robust adaptive control with zero residual tracking errors," *IEEE Trans. Automat. Contr.*, vol. AC-31, pp. 773–776, Aug. 1986.

[29] K. Tsakalis and P. A. Ioannou, "Adaptive control of linear time-varying plants: A new controller," in *Proc. American Control Conf.* (Minneapolis, MN, 1987).


# An Efficient Estimation Algorithm for the Model Parameters of Robotic Manipulators

IN-JOONG HA, MYOUNG-SAM KO, AND SEOK KI KWON

***Abstract*—The dynamic equations of robotic manipulators can be derived from either the Newton–Euler equation or the Lagrangian equation. The model parameters which appear in the resulting dynamic equations are the nonlinear functions of both the inertial parameters and the geometric parameters of robotic manipulators. In this communication, we propose an approach to identification of the model parameters, in which neither prior knowledge of the geometric parameters nor restrictive robot motions are required. The number of the model parameters to be identified is minimized through a regrouping procedure for the Lagrangian functions of robotic manipulators. The identification model for the model parameters is formulated in an upper block**

Manuscript received November 4, 1987; revised August 22, 1988. Part of the material in this communication was presented at the 1987 Korean Automatic Control Conference, Seoul, Korea, Oct. 1987. This work was supported by the Korea Science and Engineering Foundation.

The authors are with the Department of Control and Instrumentation Engineering, Seoul National University, Seoul, Korea.

IEEE Log Number 8825236.

1042-296X/89/0600-0386$01.00 

**triangular form. Based on these results, a computationally efficient estimation algorithm for the model parameters is obtained. To illustrate the practical use of our method, a 4-DOF SCARA robot is examined.**

## I. Introduction

Dynamic modeling of a given plant is important for the control system design. Dynamic equations of robotic manipulators with serial links can be obtained by the Newton–Euler method [16] or the Lagrangian method [12], [22]. Efficient programs for the symbolic generation of the dynamic equations of robotic manipulators are now available [4], [5], [19]. Various coefficients appear in the resultant dynamic equations of robotic manipulators. Identification of these coefficients is important for advanced robot control. In fact, many advanced control schemes for robotic manipulators which have been presented in recent literature require the information on either all or some of these coefficients [7], [10], [11], [14], [15], [23]–[27]. These coefficients, which will be called model parameters from now on, are the nonlinear functions of geometric parameters (constant parameters of the homogeneous transformation matrices), inertial parameters (mass, first moments, cross products of inertia, moments of inertia of links), and friction coefficients. In this communication, we present some efficient estimation algorithms for the model parameters.

Most of the earlier work [3], [13], [18], [20], [21] addresses the identification problem of the inertial parameters under the condition that the geometric parameters are known *a priori*. In fact, the geometric parameters can be separately identified with high accuracy through some calibration procedure [28]. In many aspects, it would be most desirable if each of the inertial parameters could be identified. However, it seems difficult to identify all inertial parameters one by one without disassemblying robotic manipulators into components [2]. When the geometric parameters are known *a priori*, the model parameters are reduced to linear functions of the inertial parameters. Hence, these methods for identification of the inertial parameters can be used to estimate the model parameters when the geometric parameters are known *a priori*. In [6], [17], the geometric parameters need not be known in advance. In [17], robotic manipulators are required to follow some predetermined trajectories. In [6], the model parameters can be identified if the desired trajectories satisfy the persistent excitation condition.

Our estimation method for the model parameters does not require prior knowledge of the geometric parameters. The robotic manipulators need not follow some predetermined trajectories. The number of model parameters to be identified is minimized through a regrouping procedure for the Lagrangian function of a robotic manipulator. Two forms of identification models of the model parameters are proposed. The data of joint accelerations are required in one form but not in the other. Due to the regrouping procedure, the estimation model for the model parameters can be formulated in an upper block triangular form. This special structure can be utilized to get a computationally efficient estimation algorithm for the model parameters. It is shown that our estimation algorithm can estimate the model parameters accurately in the absence of measurement and computation errors if the robotic manipulator is excited so as to take diversified dynamic motions. Some simulation results for the case of a 4-DOF SCARA robot are presented to illustrate the practical use of our estimation methods. The earlier work related to ours is discussed in detail.

Finally, we introduce some notations needed in Sections II and III. For a function $f:R^r \to R^s$, $D_j f(x)$ denotes the first partial derivative of $f$ at $x \in R^r$ with respect to the $j$th argument. A column vector $x$ with scalar components $x_i$, $i = 1, \cdots, r$ is denoted by $x \triangleq (x_1, \cdots, x_r)$. A row vector $x$ with scalar components $x_i$, $x = 1, \cdots, r$ is denoted by $x \triangleq [x_1 \cdots x_r]$. The transpose of a vector $x$ is denoted by $x^T$. And $\delta_{ij}$ denotes the Kronecker index function such that $\delta_{ii} = 1$ and $\delta_{ij} = 0$ if $i \neq j$. The functions $f_i:R^r \to R^s$, $i = 1, \cdots, n$ are linearly dependent over $R^r$ if there are constants $\alpha_i$, $i = 1, \cdots, n$ (not all zero) such that

$$\sum_{i=1}^{n} \alpha_i f_i(x) = 0, \qquad x \in R^r.$$

The functions $f_i:R^r \to R^s$, $i = 1, \cdots, n$ are linearly dependent at each $x \in R^r$ if, for each $x \in R^r$, there are constants $\alpha_i(x)$, $i = 1, \cdots, n$ (not all zero) such that

$$\sum_{i=1}^{n} \alpha_i(x) f_i(x) = 0.$$

It should be clear from these definitions that the functions which are linearly independent at each point are not necessarily linearly independent over the whole domain. For a function $h:R^r \to R$, $h|_{t_1}^{t_2}$ denotes $h(t_2) - h(t_1)$.

## II. Main Result

The Lagrangian formulation of the dynamic equations of a system with $n$ degrees of freedom is given [9] by

$$\left[\frac{d}{dt} D_2 L(q, \dot{q}) - D_1 L(q, \dot{q})\right]^T = \tau - Q(\dot{q}) \tag{1}$$

where

| | |
|---|---|
| $q(t) \in R^n$ | generalized coordinate vector of the system, |
| $\dot{q}(t) \in R^n$ | time derivative of $q(t)$, |
| $\tau(t) \in R^n$ | generalized force (or torque) vector applied to the system, |
| $L(q, \dot{q})$ | Lagrangian function of the system (= kinetic energy − potential energy), |
| $Q(\dot{q})$ | vector representing the friction and the back EMF effects. |

The Lagrangian function $L$ of a robotic manipulator with serial links can be written [22] as

$$L(q, \dot{q}) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{i} \mathrm{Tr}\left(\left(\frac{\partial T_i}{\partial q_j}\right) \cdot J_i \left(\frac{\partial T_i}{\partial q_k}\right)^T\right) \dot{q}_j \dot{q}_k + \sum_{i=1}^{n} m_i \bar{g}^T T_i \bar{r}_i \tag{2}$$

where

| | |
|---|---|
| $T_i \in R^{4\times 4}$ | homogeneous transformation matrix relating the coordinate frame of the $i$th link to that of the 0th link (the base coordinate frame), |
| $J_i \in R^{4\times 4}$ | pseudo inertia matrix of the $i$th link, |
| $\bar{g} \triangleq (g_x, g_y, g_z, 0)$ | gravity vector with respect to the base coordinate frame, |
| $\bar{r}_i \triangleq (\bar{x}_i, \bar{y}_i, \bar{z}_i, 1)$ | position vector of the center of the $i$th link mass with respect to the $i$th coordinate frame, |
| $m_i$ | mass of the $i$th link. |

Here, the inertial parameters are $m_i$, $m_i\bar{x}_i$, $m_i\bar{y}_i$, $m_i\bar{z}_i$, $I_{ixx}$, $I_{iyy}$, $I_{izz}$, $I_{ixy}$, $I_{iyz}$, $I_{ixz}$, $i = 1, \cdots, n$. Hence, the total number of the inertial parameters of a robotic manipulator with $n$ DOF is $10n$. We define the geometric parameters $d_i$, $a_i$, $\alpha_i$ of the $i$th links as shown in Fig. 1. Hence, the total number of the geometric parameters of a robotic manipulator with $n$ DOF is $3n$. Let $V_i \triangleq (m_i, m_i\bar{x}_i, m_i\bar{y}_i, m_i\bar{z}_i, I_{ixx}, I_{iyy}, I_{izz}, I_{ixy}, I_{iyz}, I_{ixz})$, $W_i \triangleq (d_i, a_i, \alpha_i)$, $i = 1, \cdots, n$. Let $V \triangleq (V_1^T, \cdots, V_n^T)$ and $W \triangleq (W_1^T, \cdots, W_n^T)$. For each $i = 1, \cdots, n$, let

$$L_i(q, \dot{q}) \equiv \frac{1}{2} \sum_{j=1}^{i} \sum_{k=1}^{i} \mathrm{Tr}\left(\left(\frac{\partial T_i}{\partial q_j}\right) \cdot J_i \left(\frac{\partial T_i}{\partial q_k}\right)^T\right) \dot{q}_j \dot{q}_k + m_i \bar{g}^T T_i \bar{r}_i. \tag{3}$$

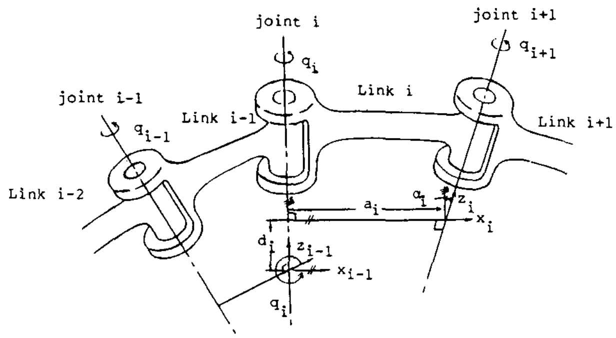



Fig. 1. Kinematic parameters $q_i$, $a_i$, $d_i$, and $\alpha_i$.

Then, the Lagrangian function $L$ in (2) can be written as

$$L(q, \dot{q})=\sum_{i=1}^{n} L_i(q, \dot{q}). \tag{4}$$

For notational convenience we assume that all joints are rotational. However, it will be clear from the developments that such an assumption entails no loss of generality.

To derive the identification model for the model parameters, we first reformulate $L$ in a special form. The homogeneous transformation matrix $A_i$ relating to the coordinate frame of the $i$th link to that of the $(i-1)$th link is given [22] by

$$A_i=\hat{A}_i C_i \tag{5}$$

where

$$\hat{A}_i \triangleq \text{Rot}\,(z_{i-1}, q_i) \tag{6}$$

$$C_i \triangleq \text{Trans}\,(0, 0, d_i)\ \text{Trans}\,(a_i, 0, 0)\ \text{Rot}\,(x_i, \alpha_i). \tag{7}$$

Namely, each $A_i$ can be factorized into two matrices such that 1) $\hat{A}_i$ depends only on $q_i$ and 2) $C_i$ depends only on the geometric parameters $d_i, a_i, \alpha_i$ of the $i$th link. By (5), $T_i$ can be expressed in the form

$$T_i=A_1A_2 \cdots A_i=\hat{A}_1C_1\hat{A}_2C_2 \cdots \hat{A}_iC_i, \qquad i=1, \cdots, n. \tag{8}$$

By (8), we can always find an integer $k_i$ and matrix functions $\hat{T}_i$: $R^i \to R^{4\times k_i}$ and $\hat{C}_i$:$R^{3i} \to R^{k_i \times 4}$ such that $T_i$ can be factorized in the form

$$T_i=\hat{T}_i(q_1, \cdots, q_i)\hat{C}_i(W_1, \cdots, W_i). \tag{9}$$

Then $L_i$ in (3) can be written as

$$L_i(q, \dot{q})=\frac{1}{2}\sum_{j=1}^{i}\sum_{k=1}^{i} \dot{q}_j\dot{q}_k \,\text{Tr}\left(\left(\frac{\partial \hat{T}_i}{\partial q_j}\right) \cdot \hat{J}_i \left(\frac{\partial \hat{T}_i}{\partial q_k}\right)^T\right)+\bar{g}^T\hat{T}_i\hat{r}_i \tag{10}$$

where

$$\hat{J}_i \triangleq \hat{C}_iJ_i\hat{C}_i^T \quad \text{and} \quad \hat{r}_i \triangleq m_i\hat{C}_i\bar{r}_i. \tag{11}$$

From (9)–(11), we see that 1) $L_i$ does not depend on $q_k, \dot{q}_k$, $k=i+1, \cdots, n$ and 2) $L_i$ is linear with respect to $\hat{J}_i$ and $\hat{r}_i$. By these facts, there exist an integer $n_i$, functions $F_{ij}$:$R^{2i} \to R$, and $X_{ij}$:$R^{3i+10} \to R$, $j=1, \cdots, n_i$ such that $L_i$ can be formulated in the form

$$L_i(q, \dot{q})=\sum_{j=1}^{n_i} F_{ij}(q_1, \cdots, q_i, \dot{q}_1, \cdots, \dot{q}_i)X_{ij}(V_i, W_1, \cdots, W_i). \tag{12}$$

In particular, the $X_{ij}$ are linear with respect to $V_i$. By (12), $L$ in (4) can be written as

$$L(q, \dot{q})=\sum_{i=1}^{n}\sum_{j=1}^{n_i} F_{ij}(q_1, \cdots, q_i, \dot{q}_1, \cdots, \dot{q}_i)X_{ij}(V_i, W_1, \cdots, W_i). \tag{13}$$

Next, we group the terms in (13) as follows. For each $i=1, \cdots, n$, define $\hat{L}_i$ by the sum of all terms in (13) that depend on at least one of $q_i, \dot{q}_i$ but not on any of $q_k, \dot{q}_k$, $k=i+1, \cdots, n$. Let $p_i$ be the total number of the terms in $\hat{L}_i$. We denote the terms in $\hat{L}_i$ by $\hat{F}_{ij}\hat{X}_{ij}$, $j=1, \cdots, p_i$. Then

$$L(q, \dot{q})=\sum_{i=1}^{n} \hat{L}_i =\sum_{i=1}^{n}\sum_{j=1}^{p_i} \hat{F}_{ij}(q_1, \cdots, q_i, \dot{q}_1, \cdots, \dot{q}_i) \cdot \hat{X}_{ij}(V_i, \cdots, V_n, W_1, \cdots, W_n). \tag{14}$$

The representation of each $\hat{L}_i$ can be further simplified by the following two-step regrouping procedure.

*Step 1:* Suppose that there is a term $\hat{F}_{ij}\hat{X}_{ij}$ in $\hat{L}_i$ such that for some constants $\alpha_{ik}$ (not all zero)

$$\hat{F}_{ij}=\sum_{k\neq j} \alpha_{ik}\hat{F}_{ik}.$$

Then the term $\hat{F}_{ij}\hat{X}_{ij}$ is removed while the $\hat{F}_{ik}\hat{X}_{ik}$ are replaced by the $\hat{F}_{ik}(\hat{X}_{ik}+\alpha_{ik}\hat{X}_{ij})$, respectively. Such operations are repeated for each $\hat{L}_i$, $i=1, \cdots, n$.

*Step 2:* Suppose that there is a term $\hat{F}_{ij}\hat{X}_{ij}$ in $\hat{L}_i$ such that for some constants $\beta_{lk}$ (not all zero)

$$\hat{X}_{ij}=\sum_{l=i}^{n}\sum_{k=1}^{p_l} \beta_{lk}\hat{X}_{lk}$$

where $\beta_{ij}=0$. Then, the term $\hat{F}_{ij}\hat{X}_{ij}$ is removed while the $\hat{F}_{lk}\hat{X}_{lk}$ are replaced by the $(\hat{F}_{lk}+\beta_{lk}\hat{F}_{ij})\hat{X}_{lk}$, respectively. Repeat such operations for $i=1, \cdots, n$ in order.

Note that the above regrouping procedure helps to reduce the $p_i$ with the special form of the $\hat{L}_i$ in (14) reserved. We assume that $L$ in (14) is obtained through the above regrouping procedure. Then it can be easily verified that

$$\hat{X}_{ij}, \quad j=1, \cdots, p_i, \quad i=1, \cdots, n \quad \text{are linearly independent over } R^{13n} \tag{15}$$

$$\hat{F}_{ij}, \quad j=1, \cdots, p_i, \quad i=1, \cdots, n \quad \text{are linearly independent over } R^{2n}. \tag{16}$$

What remains now is to show

$$p_i \geq 1, \qquad i=1, \cdots, n. \tag{17}$$

Let $i$ be an integer such that $1 \leq i \leq n$. It can be easily shown that if $\dot{q}_i \neq 0$

$$a_{ki} \triangleq (\dot{q}_i)^2 \,\text{Tr}\left(\left(\frac{\partial \hat{T}_k}{\partial q_i}\right) \hat{J}_k \left(\frac{\partial \hat{T}_k}{\partial q_i}\right)^T\right)>0, \qquad k=i, \cdots, n. \tag{18}$$

This implies that in $L$, any term containing $(\dot{q}_i)^2$ cannot be canceled out by any other terms. In particular, there are functions $f_i: R^i \to R$, $x_i: R^{10} \to R$ such that $a_{ii}$ can be written as

$$a_{ii} = (\dot{q}_i)^2 f_i(q_1, \cdots, q_i) x_i(V_i). \tag{19}$$

Consequently, $\hat{L}_i$ obtained through the above regrouping procedure must contain at least $a_{ii}$. This proves (17). Let $\hat{F}_i \triangleq (\hat{F}_{i1}, \cdots, \hat{F}_{ip_i})$, and $\hat{X}_i \triangleq (\hat{X}_{i1}, \cdots, \hat{X}_{ip_i})$. Then (14) can be expressed in a simpler form

$$L(q, \dot{q}) = \sum_{i=1}^{n} [\hat{F}_i(q_1, \cdots, q_i, \dot{q}_1, \cdots, \dot{q}_i)]^T \cdot \hat{X}_i(V_i, \cdots, V_n, W_1, \cdots, W_n). \tag{20}$$

We state the above result as Theorem 1.

*Theorem 1:* The Lagrangian function $L$ of an $n$ DOF robotic manipulator with serial links can be arranged in the form (20) with the properties (15)–(17) through the regrouping procedure described above.

As will be seen soon, rearrangement of the Lagrangian function in the form (20) has several advantages in simplifying the estimation method of the model parameters.

Now, we formulate two kinds of identification models for the model parameters. First, note that $Q$ in (1) can be represented as

$$Q(\dot{q}) = \text{diag}\ (b_i \dot{q}_i + c_i\ \text{sgn}\ (\dot{q}_i)) \tag{21}$$

where the $b_i$ and $c_i$ are constants. Let $\tilde{X} \triangleq (\tilde{X}_1^T, \cdots, \tilde{X}_n^T)$, where

$$\tilde{X}_i \triangleq ((\hat{X}_i)^T, b_i, c_i), \qquad i = 1, \cdots, n.$$

We call $\tilde{X}$ the vector of the model parameters. The total number of the model parameters is

$$p \triangleq \sum_{i=1}^{n} (p_i + 2).$$

By (20) and (21), (1) can be written in the form

$$U(q, \dot{q}, \ddot{q}) \tilde{X} = \tau. \tag{22}$$

By the definition of the $\hat{F}_i$

$$\frac{\partial \hat{F}_i}{\partial q_k} \equiv 0 \quad \text{and} \quad \frac{\partial \hat{F}_i}{\partial \dot{q}_k} \equiv 0, \qquad k = i+1, \cdots, n, \quad i = 1, \cdots, n. \tag{23}$$

Consequently, $U$ in (22) has the following upper block triangular form:

$$\begin{bmatrix} U_{11} & U_{12} & \cdots & U_{1n} \\ 0 & U_{22} & \cdots & U_{2n} \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & & U_{nn} \end{bmatrix} \begin{bmatrix} \tilde{X}_1 \\ \tilde{X}_2 \\ \vdots \\ \tilde{X}_n \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} \tag{24}$$

where $Y_i \triangleq \tau_i$ and

$$U_{ij} \triangleq \left[ \left\{ \frac{d}{dt}\left(\frac{\partial \hat{F}_j}{\partial \dot{q}_i}\right) - \left(\frac{\partial \hat{F}_j}{\partial q_i}\right) \right\}^T \vdots\ \dot{q}_i \delta_{ij} \ \vdots\ \text{sgn}\ (\dot{q}_i) \delta_{ij} \right] \in R^{1 \times (p_j + 2)}. \tag{25}$$

Theorem 2 further characterizes the structure of our identification model as follows.

*Theorem 2.* Suppose that Theorem 1 holds. Then, the identification model for the model parameters can be formulated in an upper block triangular form (22), (24), (25) with the following properties:

$$\text{the columns of } U \text{ are linearly independent over } R^{3n} \tag{26}$$

$$\text{the components of } \tilde{X} \text{ are linearly independent over } R^{15n}. \tag{27}$$

*Proof:* For the proof of our claim, it suffices only to show that if for a constant vector $\alpha \in R^{p_i+2}$,

$$U_{ii}\alpha = 0, \ (q_1, \cdots, q_i, \dot{q}_1, \cdots, \dot{q}_i, \ddot{q}_1, \cdots, \ddot{q}_i) \in R^{3i} \tag{28}$$

then $\alpha = 0$. Suppose that (28) holds. Let

$$h \triangleq \sum_{j=1}^{p_i} \alpha_j \hat{F}_{ij}.$$

By (25), (28) implies that

$$\frac{d}{dt}\left(\frac{\partial h}{\partial \dot{q}_i}\right) = \frac{\partial h}{\partial q_i} - \alpha_{(p_i+1)} \dot{q}_i - \alpha_{(p_i+2)}\ \text{sgn}\ (\dot{q}_i). \tag{29}$$

By considering the terms in (3) and the definition of $\hat{F}_i$, we see that $h$ must have the form

$$h = \sum_{k=1}^{i} \sum_{j=k}^{i} f_{ki}(q_1, \cdots, q_i) \dot{q}_k \dot{q}_j + f_i(q_1, \cdots, q_i) \tag{30}$$

where $f_i$ and $f_{kj}$ are appropriate functions such that

$$D_i f_i \not\equiv 0 \quad \text{and} \quad D_i f_{ki} \not\equiv 0, \qquad \text{if } j \neq i \text{ and } k \neq i. \tag{31}$$

By (29) and (30)

$$\sum_{k=1}^{i} (1 + \delta_{ki}) \cdot f_{ki} \ddot{q}_k + \sum_{k=1}^{i} (1 + \delta_{ki}) \cdot \left( \sum_{j=1}^{i} D_j f_{ki} \dot{q}_j \right) \dot{q}_k - \sum_{k=1}^{i} \sum_{j=k}^{i} D_i f_{kj} \dot{q}_k \dot{q}_j - D_i f_i + \alpha_{(p_i+1)} \dot{q}_i + \alpha_{(p_i+2)}\ \text{sgn}\ (\dot{q}_i) = 0 \qquad \text{over } R^{3i}. \tag{32}$$

Then it is not difficult to see that (31) and (32) imply that $\alpha_{(p_i+1)} = \alpha_{(p_i+2)} = 0$, $f_i \equiv 0$, and all $f_{kj} \equiv 0$ over $R^{3i}$. By (30), this also implies that

$$\sum_{j=1}^{p_i} \alpha_j \hat{F}_{ij} = 0 \qquad \text{over } R^{3i}. \tag{33}$$

By (16), this implies: $\alpha_j = 0$, $j = 1, \cdots, p_i$. Hence, our claim has been proven.

In the proof of Theorem 2, we have shown that (15) and (16) imply (26) and (27). In other words, our regrouping procedure yields the minimal number (in the sense of linear independence) of the model parameters which are required to be identified for the determination of the dynamic equations of a robotic manipulator.

The $\tilde{X}_{ij}$ in (24) can be estimated by using one of the well-known off-line or on-line least squares methods [8]. We discuss only a simple off-line nonrecursive least squares method. The required measurement data are the trajectories of $q$, $\dot{q}$, $\ddot{q}$, and $\tau$. Suppose that we choose data points up to some $N$ from a set of the test trajectories of $q$, $\dot{q}$, $\ddot{q}$, and $\tau$. Normally, $N >> \max\ (p_i + 2, i = 1, \cdots, n)$ in the case of noisy environment. Each data point determines numerically $U$ and $\tau$ in (22), which will be denoted by $U(i)$, $\tau(i)$, $i = 1, \cdots, N$. Then, $\tilde{X}$ can be estimated by

$$\tilde{X} = (\hat{U}^T \hat{U})^{-1} \hat{U}^T \hat{Y} \tag{34}$$

where

$$\hat{U} \triangleq \begin{bmatrix} U(1) \\ \vdots \\ U(N) \end{bmatrix} \quad \text{and} \quad \hat{Y} \triangleq \begin{bmatrix} Y(1) \\ \vdots \\ Y(N) \end{bmatrix}. \tag{35}$$

While $q$, $\dot{q}$ can be directly measured, $\ddot{q}$, $\tau$ should be estimated in indirect ways. The data of $\tau$ can be easily estimated from joint motor currents. The data of $\ddot{q}$ are usually obtained by passing the data of $\dot{q}$ through a band-limited differentiator [3]. When we desire to avoid the differentiation of $\dot{q}$, the following approach can be taken. Integrating both sides of (24) from the initial time $t_0$ to a time $t$, we see that (24) still holds with

$$Y_i \triangleq \int_{t_0}^{t} \tau_i \, dt$$

and

$$U_{ij} \triangleq \left[ \left\{ \frac{\partial \hat{F}_j}{\partial \dot{q}_i} \bigg|_{t_0}^{t} - \int_{t_0}^{t} \left( \frac{\partial \hat{F}_j}{\partial q_i} \right) dt \right\}^T \vdots \; \delta_{ij} q_i |_{t_0}^{t} \; \vdots \; \cdot \int_{t_0}^{t} \operatorname{sgn}(\dot{q}_i) \delta_{ij} \, dt \right]. \tag{36}$$

In this case, the data of $\ddot{q}$ are not necessary and the $U_{ij}$ in (24) are functions of $t$. Instead, integration of various functions along the test trajectories of $q$, $\dot{q}$, $\tau$ are required. This integration approach was first considered in [3] for load estimation. Choosing $N$ different points of $t$, we can construct $\hat{U}$ and $\hat{Y}$ required in (34). The identification model in (24), (25) shall be called the differential form while the one in (24), (36) shall be called the integral form.

Our regrouping procedure helps to reduce the total number ($p$) of the model parameters. When $p$ is still large, the inverse of $\hat{U}^T \hat{U} \in R^{p \times p}$ may be computationally difficult. Since $U$ in (24) has an upper block triangular form, it can be written as

$$U_{ii} \tilde{X}_i = \hat{Y}_i - \sum_{j=i+1}^{n} U_{ij} \tilde{X}_j, \qquad i = n, (n-1), \cdots, 1. \tag{37}$$

This implies that $\tilde{X}_i$, $i = 1, \cdots, n$ can be estimated one by one in the reverse order. Furthermore, different data points can be selected at each step. Let $N_i$ be the number of data points selected for the estimation of $\tilde{X}_i$. Normally, $N_i >> p_i + 2$ in the case of noisy environment. Then, we can construct the following sequential estimation algorithm which may be useful for the reduction of computational load:

$$\tilde{X}_i = (\hat{U}_{ii}^T \hat{U}_{ii})^{-1} \hat{U}_{ii}^T \left( \hat{Y}_i - \sum_{j=i+1}^{n} \hat{U}_{ij} \tilde{X}_j \right), \qquad i = n, (n-1), \cdots, 1 \tag{38}$$

where

$$\hat{U}_{ij} \triangleq \begin{bmatrix} U_{ij}(1) \\ \vdots \\ U_{ij}(N_i) \end{bmatrix}$$

$$\hat{Y}_i \triangleq \begin{bmatrix} Y_i(1) \\ \vdots \\ Y_i(N_i) \end{bmatrix}. \tag{39}$$

In (38), only the inverses of $\hat{U}_{ii}^T \hat{U}_{ii} \in R^{(p_i+2) \times (p_i+2)}$ need to be computed. Further reduction of computational load can be gained by replacing the nonrecursive least squares method in (38) by a recursive least squares method [8].

The selection of data points is important for the success of these algorithms. In both algorithms (34) and (38), the condition

$$\operatorname{rank} \hat{U}_{ii} = p_i + 2, \qquad i = 1, \cdots, n \tag{40}$$

is necessary and sufficient for the existence of the required inverses. If (40) is satisfied, and if no measurement and computation errors are involved, then these algorithms can estimate precisely the model parameters. From Lemma 1 to be presented later, it can be easily shown that the test trajectories of $q$, $\dot{q}$, $\ddot{q}$ can provide the data points satisfying (40) if and only if

$$\text{the components of } U_{ii} \text{ determined by the test trajectories of } q, \dot{q}, \ddot{q} \text{ are linearly independent over } [0, \infty]. \tag{41}$$

Therefore, we have to find an appropriate control input so that the resultant trajectories of $q$, $\dot{q}$, $\ddot{q}$ can satisfy (41). However, it is also clear that if (26) does not hold, any trajectories of $q$, $\dot{q}$, $\ddot{q}$ will not satisfy (41). Hence, identifiability of the model parameters depends not only on the test trajectories but also on the structure of the identification model. Theorem 3 will emphasize that the special structure of our identification model facilitates estimation of the model parameters.

*Theorem 3:*

i) For each $i = 1, \cdots, n$, there exist at least ($p_i + 2$) data points of $(q, \dot{q}, \ddot{q})$ in $R^{3n}$ which satisfy (40) for the differential form.

ii) If the test trajectories of $q$, $\dot{q}$, $\ddot{q}$ contain data points satisfying (40) for the differential form, the same test trajectories can provide data points satisfying (40) for the integral form.

The first part of Theorem 3 implies that if the manipulator can be excited so that the trajectories of $q$, $\dot{q}$, $\ddot{q}$ travel through all points in an open subset of $R^{3n}$ including the origin, then the trajectories will contain data points satisfying (40). A trial and error method of generating the desired data points is as follows. First, we excite the manipulator fully by applying an appropriate control input. We then collect data points from the resulting trajectories with a certain sampling period. We decrease the sampling period until (40) is satisfied. If decreasing the sampling period does not lead to (40), we should try another appropriate control input. The example in Section III demonstrates that the above trial and error method works well in practice. A more efficient method of generating the desired data points may be obtained by incorporating the control input update equation in [1] into our sequential estimation algorithm in (38).

The second part of Theorem 3 states that if we get a set of the test trajectories that can provide the desired data points for the differential form, we can use the same trajectories in choosing the desired data points for the integral form. For the proof of Theorem 3, we need the following Lemma 1.

*Lemma 1:* Let $f_i$, $i = 1, \cdots, r$ be the scalar functions from $R^m$ into $R$. Let $F(x) \triangleq [f_1(x) \cdots f_r(x)]$ for $x \in R^m$. Then

$$f_i, \quad i = 1, \cdots, r \text{ are linearly independent over } R^m \tag{42}$$

if and only if for each $k = 1, \cdots, r$, there exist $a_j \in R^m$, $j = 1, \cdots, k$ such that

$$\operatorname{rank} A_k = k \tag{43}$$

where

$$A_k \triangleq \begin{bmatrix} F(a_1) \\ \vdots \\ F(a_k) \end{bmatrix}. \tag{44}$$

*Proof:* First, we prove by induction the necessity part of this lemma. By (42), there exists a point $a_1 \in R$ such that $F(a_1) \neq 0$. Hence, (43) holds for $k = 1$. We show by contradiction that if (43) holds for $k = l$, then it holds for $k = l + 1$. Suppose that (43) holds for $k = l$ but not for $k = l + 1$. In other words,

$$\text{rank}\begin{bmatrix} A_l \\ F(x) \end{bmatrix} = l, \qquad x \in R^m. \tag{45}$$

For simpler arguments, we assume without loss of generality that the first $l$ columns of $A_l$ are linearly independent. This with (45) implies that there exist constants $\lambda_j(x_1, \cdots, x_l), j = 1, \cdots, (l + 1)$ such that $\lambda_{l+1} \neq 0$ and

$$\sum_{j=1}^{l+1} \lambda_j f_j(x) = 0, \qquad x \in R^m. \tag{46}$$

Hence, (46) implies that $f_j$, $j = 1, \cdots, (l + 1)$ are linearly independent over $R^m$, which is contradictory to the assumption in (42). Next, we prove by contradiction the sufficiency part of this lemma. Suppose that $f_i$, $i = 1, \cdots, r$ are linearly dependent over $R^m$. Then, there exists a nonzero constant vector $\alpha \in R^r$ such that

$$F(x)\alpha = 0, \qquad x \in R^m. \tag{47}$$

This is contradictory to the assumption in (43).

Now, we are ready to prove Theorem 3.

*Proof of Theorem 3:* Part i) follows immediately from (26) and Lemma 1. Now, we prove part ii). By Lemma 1, the given hypothesis implies (41). On the other hand, if our claim in part ii) is false, there exists a constant vector $\alpha \in R^{p_i+2}$ such that

$$U_{ii}(t)\alpha = 0, \qquad t \in [0, \infty) \tag{48}$$

where $U_{ii}$ represents the integral form in (36). Differentiating (48), we obtain

$$U_{ii}(q_1(t), \cdots, q_i(t), \dot{q}_1(t), \cdots, \dot{q}_i(t), \ddot{q}_1(t), \cdots, \ddot{q}_i(t))\alpha = 0, \qquad t \in [0, \infty) \tag{49}$$

where $U_{ii}$ represents the differential form in (25). Clearly, (41) and (49) are contradictory. Hence, our claim is true.

Finally, we discuss the previous results closely related to ours. It is shown in [3] that the identification model for the inertial parameters can be formulated in an upper block triangular form. This result can be obtained directly from (24). In this case, however, Theorem 3 does not necessarily hold. In other words, there are unidentifiable inertial parameters. For this reason, it is suggested in [3], [13] that the inertial parameters should be estimated in linearly combined forms. Even for this purpose, our sequential estimation algorithm is still useful since the $\bar{X}_i$ in (24) are reduced simply to the linear function of inertial parameters $V_i, \cdots, V_n$ when the geometric parameters are known. Our regrouping procedure summarized in Theorem 1 was motivated by the simplification method proposed in [13], which corresponds, here, to achieving (26) by directly eliminating linearly dependent columns from $U$. We have shown in the proof of Theorem 2 that our regrouping procedure automatically guarantees (26). However, the identification model in [13] is for the inertial parameters and does not necessarily have an upper block triangular form. The sequential estimation algorithm described in (38), Theorem 1, and Theorem 3 were not obtained in [13].

## III. An Example

The 4-DOF SCARA robot shown in Fig. 2 has four links. The first, second, and fourth links are rotational while the third is translational. The first, third, and fourth links are symmetric about the $x$ axis of the joint coordinate frame while the second link is not

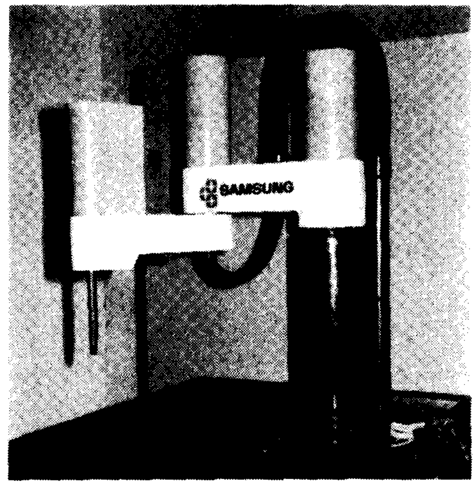



Fig. 2. A 4-DOF SCARA type robot.

symmetric about the $x$ axis of the joint coordinate frame. It has two geometric parameters and 11 inertial parameters. The geometric parameters are $a_1$ and $a_2$. The pseudo inertia matrix $J_i$ of the $i$th link is given by

$$J_i \triangleq \begin{bmatrix} \frac{I_{izz}}{2} & 0 & 0 & m_i\bar{x}_i \\ 0 & \frac{I_{izz}}{2} & 0 & m_i\bar{y}_i \\ 0 & 0 & \frac{-I_{izz}}{2} & 0 \\ m_i\bar{x}_i & m_i\bar{y}_i & 0 & m_i \end{bmatrix}, \qquad i = 1, \cdots, 4 \tag{50}$$

where $\bar{x}_3 = \bar{x}_4 = \bar{y}_1 = \bar{y}_3 = \bar{y}_4 = 0$. In this robot, $g_x = g_y = 0$ and $g_z = g = 9.8$ (m/s$^2$).

Let $S_i \triangleq \sin q_i$, and $C_i \triangleq \cos q_i$, $i = 1, \cdots, 4$. Then, $L_i$, $i = 1, \cdots, 4$ in (3) are given as follows:

$$L_1(q, \dot{q}) = \frac{1}{2}\dot{q}_1^2(I_{1zz} + m_1a_1^2 + 2m_1\bar{x}_1a_1) \tag{51}$$

$$\begin{aligned} L_2(q, \dot{q}) = &\frac{1}{2}\dot{q}_1^2(I_{2zz} + m_2a_1^2 + m_2a_2^2 + 2m_2\bar{x}_2a_2) \\ &+ \dot{q}_1^2C_2(m_2a_1a_2 + m_2\bar{x}_2a_1) - \dot{q}_1^2S_2m_2\bar{y}_2a_1 + \dot{q}_1\dot{q}_2 \\ &\cdot (I_{2zz} + m_2a_2^2 + 2m_2\bar{x}_2a_2) + \dot{q}_1\dot{q}_2C_2(m_2a_1a_2 + m_2\bar{x}_2a_1) \\ &- \dot{q}_1\dot{q}_2S_2m_2\bar{y}_2a_1 + \frac{1}{2}\dot{q}_2^2(I_{2zz} + m_2a_2^2 + 2m_2\bar{x}_2a_2) \end{aligned} \tag{52}$$

$$\begin{aligned} L_3(q, \dot{q}) = &\frac{1}{2}\dot{q}_1^2(I_{3zz} + m_3a_1^2 + m_3a_2^2) + \dot{q}_1^2C_2m_3a_1a_2 + \frac{1}{2}\dot{q}_2^2 \\ &\cdot (I_{3zz} + m_3a_2^2) + \frac{1}{2}\dot{q}_3^2m_3 + q_3gm_3 + \dot{q}_1\dot{q}_2(I_{3zz} + m_3a_2^2) \\ &+ \dot{q}_1\dot{q}_2C_2m_3a_1a_2 \end{aligned} \tag{53}$$

$$\begin{aligned} L_4(q, \dot{q}) = &\frac{1}{2}\dot{q}_1^2(I_{4zz} + m_4a_1^2 + m_4a_2^2) + \dot{q}_1^2C_2m_4a_1a_2 + \frac{1}{2}\dot{q}_2^2 \\ &\cdot (I_{4zz} + m_4a_2^2) + \frac{1}{2}\dot{q}_3^2m_4 + q_3gm_4 + \dot{q}_1\dot{q}_2(I_{4zz} + m_4a_2^2) \\ &+ \dot{q}_1\dot{q}_2C_2m_4a_1a_2 + \frac{1}{2}\dot{q}_4^2I_{4zz} + \dot{q}_1\dot{q}_4I_{4zz} + \dot{q}_2\dot{q}_4I_{4zz}. \end{aligned} \tag{54}$$

By the regrouping procedure suggested in Section II, we obtain

$$p_1=p_3=p_4=1, \quad p_2=3, \qquad p=\sum_{i=1}^{4}(p_i+2)=14 \tag{55}$$

and

$$\hat{L}_1=\hat{F}_{11}\hat{X}_{11} \qquad \hat{L}_2=\hat{F}_{21}\hat{X}_{21}+\hat{F}_{22}\hat{X}_{22}+\hat{F}_{23}\hat{X}_{23}$$
$$\hat{L}_3=\hat{F}_{31}\hat{X}_{31} \qquad \hat{L}_4=\hat{F}_{41}\hat{X}_{41} \tag{56}$$

where

$$\hat{F}_{11}=\frac{1}{2}\dot{q}_1^2,\ \hat{F}_{21}=\frac{1}{2}\dot{q}_2^2+\dot{q}_1\dot{q}_2,\ \hat{F}_{22}=\dot{q}_1^2C_2+\dot{q}_1\dot{q}_2C_2 \tag{57}$$

$$\hat{F}_{23}=-(\dot{q}_1^2S_2+\dot{q}_1\dot{q}_2S_2),\ \hat{F}_{31}=\frac{1}{2}\dot{q}_3^2+q_3g,\ \hat{F}_{41}=\frac{1}{2}\dot{q}_4^2+\dot{q}_1\dot{q}_4+\dot{q}_2\dot{q}_4$$

$$\hat{X}_{11}=I_{1zz}+m_1a_1^2+2m_1\bar{x}_1a_1+I_{2zz}+m_2a_1^2+m_2a_2^2+2m_2\bar{x}_2a_2+I_{3zz}$$
$$+m_3a_1^2+m_3a_2^2+I_{4zz}+m_4a_1^2+m_4a_2^2$$
$$\hat{X}_{21}=I_{2zz}+m_2a_2^2+2m_2\bar{x}_2a_2+I_{3zz}+m_3a_2^2+I_{4zz}+m_4a_2^2 \tag{58}$$
$$\hat{X}_{22}=m_2a_1a_2+m_2\bar{x}_2a_1+m_3a_1a_2+m_4a_1a_2,\ \hat{X}_{23}=m_2\bar{y}_2a_1$$
$$\hat{X}_{31}=m_3+m_4,\ \hat{X}_{41}=I_{4zz}.$$

Hence, the minimal number of the model parameters to be identified is $p = 14$. Note that $L$ has only six terms after regrouping, while it has 25 terms before regrouping. From (57) and (58) we can obtain the identification model

$$\begin{bmatrix} U_{11} & U_{12} & 0 & U_{14} \\ 0 & U_{22} & 0 & U_{24} \\ 0 & 0 & U_{33} & 0 \\ 0 & 0 & 0 & U_{44} \end{bmatrix}\begin{bmatrix} \tilde{X}_1 \\ \tilde{X}_2 \\ \tilde{X}_3 \\ \tilde{X}_4 \end{bmatrix}=\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix} \tag{59}$$

where, in the differential form,

$$U_{11} \triangleq [\ddot{q}_1 \vdots \dot{q}_1 \vdots \operatorname{sgn}(\dot{q}_1)]$$
$$U_{14} \triangleq U_{24} \triangleq [\ddot{q}_4 \quad 0 \quad 0]$$
$$U_{33} \triangleq [(\ddot{q}_3-g) \vdots \dot{q}_3 \vdots \operatorname{sgn}(\dot{q}_3)]$$
$$U_{44} \triangleq [(\ddot{q}_1+\ddot{q}_2+\ddot{q}_4) \vdots \dot{q}_4 \vdots \operatorname{sgn}(\dot{q}_4)]$$
$$U_{12} \triangleq [\ddot{q}_2 \vdots \{-2\dot{q}_1\dot{q}_2S_2-\dot{q}_2^2S_2+(2\ddot{q}_1+\ddot{q}_2)C_2\} \vdots$$
$$\{-2\dot{q}_1\dot{q}_2C_2-\dot{q}_2^2C_2-(2\ddot{q}_1+\ddot{q}_2)S_2\} \vdots 0 \quad 0 \quad ]$$
$$U_{22} \triangleq [(\ddot{q}_1+\ddot{q}_2) \vdots (\ddot{q}_1C_2+\dot{q}_1^2S_2) \vdots (-\ddot{q}_1S_2+\dot{q}_1^2C_2) \vdots \dot{q}_2 \vdots \operatorname{sgn}(\dot{q}_2)] \tag{60}$$

and, in the integral form,

$$U_{11} \triangleq \left[\dot{q}_1|_{t_0}^{t} \vdots q_1|_{t_0}^{t} \vdots \int_{t_0}^{t}\operatorname{sgn}(\dot{q}_1)\,dt\right]$$
$$U_{14} \triangleq U_{24} \triangleq [\dot{q}_4|_{t_0}^{t} \vdots 0 \quad 0]$$
$$U_{33} \triangleq \left[\{\dot{q}_3|_{t_0}^{t}-g(t-t_0)\} \vdots q_3|_{t_0}^{t} \vdots \int_{t_0}^{t}\operatorname{sgn}(\dot{q}_3)\,dt\right]$$
$$U_{44} \triangleq \left[(\dot{q}_1+\dot{q}_2+\dot{q}_4)|_{t_0}^{t} \vdots q_4|_{t_0}^{t} \vdots \int_{t_0}^{t}\operatorname{sgn}(\dot{q}_4)\,dt\right]$$
$$U_{12} \triangleq [\dot{q}_2|_{t_0}^{t} \vdots \{(2\dot{q}_1+\dot{q}_2)C_2\}|_{t_0}^{t} \vdots \{-(2\dot{q}_1+\dot{q}_2)S_2\}|_{t_0}^{t} \vdots 0 \quad 0]$$

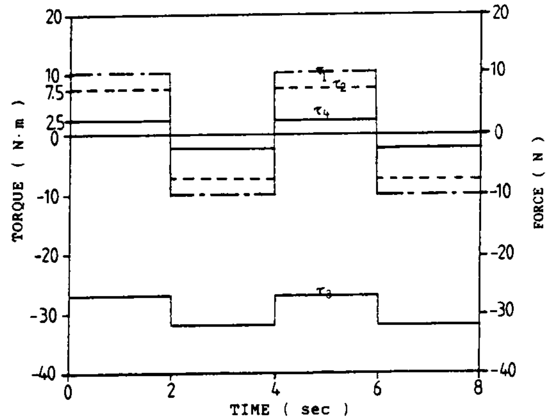


Fig. 3. Trajectories of $\tau$.

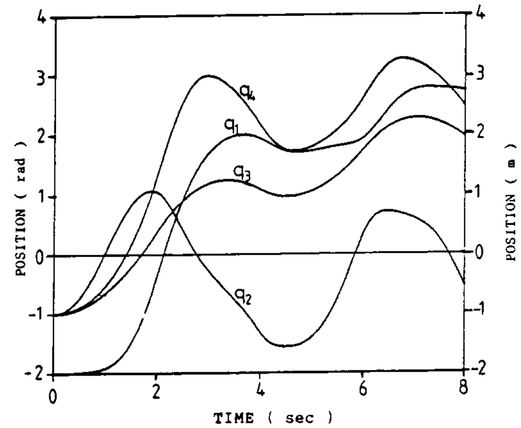
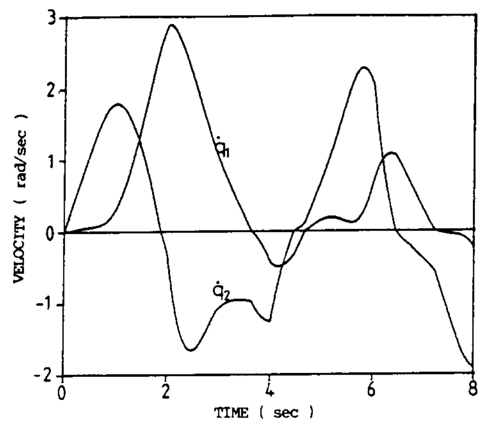
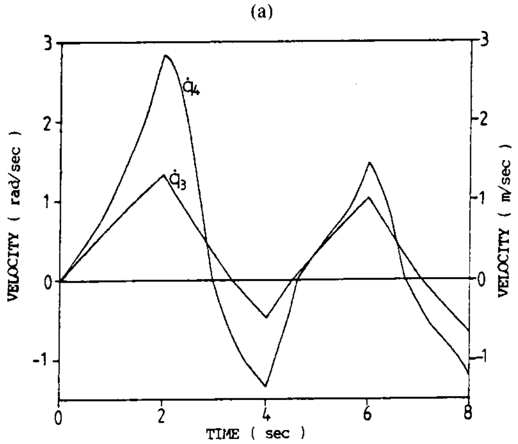


Fig. 4. Trajectories of $q$.



(a)



(b)

Fig. 5. Trajectories of $\dot{q}$.

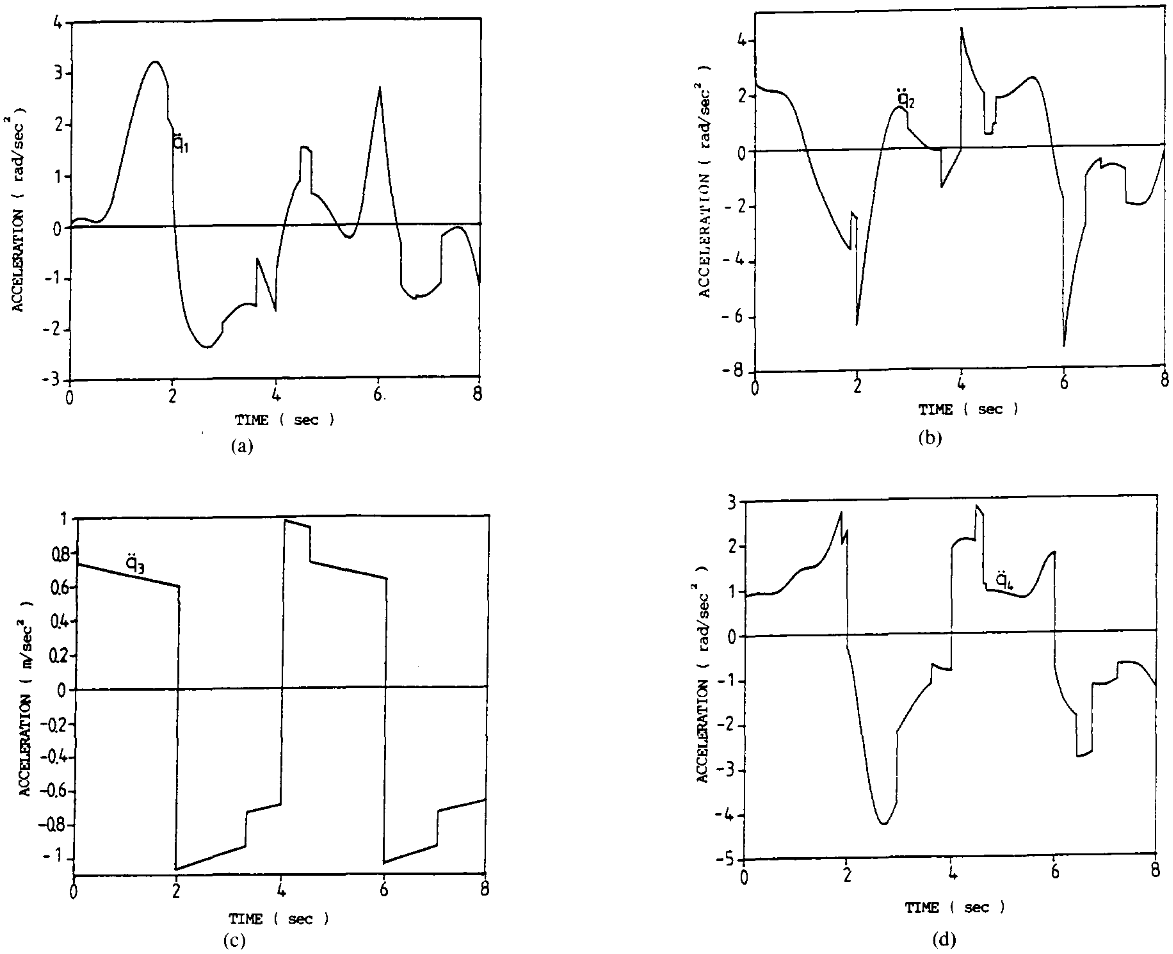



Fig. 6. Trajectories of $\ddot{q}$.

TABLE I
SIMULATION RESULTS OF THE SEQUENTIAL ESTIMATION ALGORITHM (38)

| $\tilde{X}$ | True Values | No Measurement Errors: Differential Form | No Measurement Errors: Integral Form | With Measurement Errors: Differential Form | With Measurement Errors: Integral Form |
|---|---|---|---|---|---|
| $\hat{X}_{11}$ | 5.0000 (kg·m$^2$) | 5.0000 | 5.0000 | 4.9090 | 5.2405 |
| $b_1$ | 1.0000 (kg·m$^2$/s) | 1.0000 | 1.0000 | 1.2839 | 1.2672 |
| $c_1$ | 1.0000 (N·m) | 1.0000 | 1.0000 | 0.6918 | 0.8814 |
| $\hat{X}_{21}$ | 2.5000 (kg·m$^2$) | 2.5000 | 2.5000 | 2.4533 | 2.5528 |
| $\hat{X}_{22}$ | 1.0000 (kg·m$^2$) | 1.0000 | 1.0000 | 0.9385 | 1.0226 |
| $\hat{X}_{23}$ | 0.3000 (kg·m$^2$) | 0.3000 | 0.3000 | 0.2502 | 0.3165 |
| $b_2$ | 0.7000 (kg·m$^2$/s) | 0.7000 | 0.7000 | 0.6345 | 0.8914 |
| $c_2$ | 0.7000 (N·m) | 0.7000 | 0.7000 | 0.8325 | 0.4961 |
| $\hat{X}_{31}$ | 3.0000 (kg) | 3.0000 | 3.0000 | 3.1165 | 3.0825 |
| $b_3$ | 0.3000 (kg/s) | 0.3000 | 0.3000 | 0.3793 | 0.3793 |
| $c_3$ | 0.3000 (N) | 0.3000 | 0.3000 | 0.2163 | 0.2520 |
| $\hat{X}_{41}$ | 0.5000 (kg·m$^2$) | 0.5000 | 0.5000 | 0.4979 | 0.4830 |
| $b_4$ | 0.3000 (kg·m$^2$/s) | 0.3000 | 0.3000 | 0.3256 | 0.3645 |
| $c_4$ | 0.3000 (N·m) | 0.3000 | 0.3000 | 0.2660 | 0.3437 |

$$U_{22} \triangleq \left[ (\dot{q}_1+\dot{q}_2)\big|_{t_0}^{t} \,\vdots\, \dot{q}_1 C_2\big|_{t_0}^{t} \,\vdots\, (-\dot{q}_1 S_2)\big|_{t_0}^{t} \,\vdots\, q_2\big|_{t_0}^{t} \,\vdots\, \int_{t_0}^{t} \operatorname{sgn}(\dot{q}_2)\, dt \right] + \left[ 0 \,\vdots\, \int_{t_0}^{t} (\dot{q}_1\dot{q}_2+\dot{q}_1^2) S_2\, dt \,\vdots\, \int_{t_0}^{t} (\dot{q}_1\dot{q}_2+\dot{q}_1^2) C_2\, dt \,\vdots\, 0 \quad 0 \right]. \tag{61}$$

The test trajectories of $\tau$, $q$, $\dot{q}$, and $\ddot{q}$ used for the identification of the model parameters are shown in Figs. 3–6. The true values of the model parameters are assumed to be given as in Table I. When data points are chosen from the test trajectories with the sampling period 10 ms, the condition (40) for the differential form was satisfied. In the integral form, when the data points $t_j$ are chosen so that $t_0 = 0$ and $t_j - t_{j-1} = 0.5$ s, (40) was satisfied. Simulation results in Table I show that the sequential algorithm (38) works well. To investigate the influence of measurement errors on estimation accuracy, the constant bias errors, 3.0°, 3.0°/s, 3.0°/s$^2$, 0.15 N·m are added in the data of $q_i$, $\dot{q}_i$, $\ddot{q}_i$, $\tau_i$, $i = 1, 2, 4$, respectively, while the constant bias errors, 0.10 m, 0.10 m/s, 0.10 m/s$^2$, 0.1 N are added in the data of $q_3$, $\dot{q}_3$, $\ddot{q}_3$, $\tau_3$, respectively. Also, 3.0-percent constant scale factor errors are assumed in the data of $q$, $\dot{q}$, $\ddot{q}$, and $\tau$. The simulation results in Table I reveal some degeneration of estimation accuracy in the face of such measurement errors. In particular, the bias errors degenerated the estimation accuracy much more severely than the scale factor errors.

## IV. Conclusion

We have presented an efficient estimation method for the model parameters of robotic manipulators. Computational efficiency is gained by regrouping the Lagrangian equation and by formulating the identification models in an upper block triangular form with the properties (26), (27). Simulation study shows that measurement and computation errors can degenerate estimation accuracy. Since the least squares methods are, in general, sensitive to the bias errors, the bias errors that occur during the measurements should be kept small through offset compensation or other methods.

## References


[1] B. Armstrong, "On finding exciting trajectories for identification experiments involving systems with nonlinear dynamics," in *Proc. IEEE Conf. on Robotics and Automation*, pp. 1131–1139, 1987.

[2] B. Armstrong, O. Khatib, and J. Burdick, "The explicit dynamic model and inertial parameters of the PUMA 560 Arm," in *Proc. IEEE Conf. on Robotics and Automation*, pp. 510–518, 1986.

[3] C. G. Atkeson, C. H. An, and J. M. Hollerbach, "Estimation of inertial parameters of manipulator loads and links," *Int. J. Robotics Res.*, vol. 5, pp. 101–119, 1986.

[4] J. W. Burdick, "An algorithm for generation of efficient manipulator dynamic equations," in *Proc. IEEE Conf. on Robotics and Automation*, pp. 212–218, 1986.

[5] G. Cesareo, F. Nicolo, and S. Nicosia, "DYMIR: A code for generating dynamic model of robots," in *Proc. IEEE Conf. on Robotics and Automation*, pp. 1159–1209, 1984.

[6] J. J. Craig, P. Hsu, and S. S. Sastry, "Adaptive control of mechanical manipulators," in *Proc. IEEE Conf. on Robotics and Automation*, pp. 190–195, 1986.

[7] E. G. Gilbert and I. J. Ha, "An approach to nonlinear feedback control with applications to robotics," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-14, pp. 879–884, 1984.

[8] G. C. Goodwin and R. L. Payne, *Dynamic System Identification: Experiment Design and Data Analysis.* New York, NY: Academic Press, 1977.

[9] D. T. Greenwood, *Principles of Dynamics.* Englewood Cliffs, NJ: Prentice-Hall, 1980.

[10] I. J. Ha and E. G. Gilbert, "Robust tracking in nonlinear systems," *IEEE Trans. Automat. Contr.*, vol. AC-32, pp. 763–771, 1987.

[11] K. Hironoki and N. Seinosuke, "Parallel processing of robot-arm control computation on a multi-microprocessor system," *IEEE J. Robotics Automat.*, vol. RA-1, pp. 104–113, 1985.

[12] J. M. Hollerbach, "A recursive lagrangian formulation of manipulator dynamics and comparative study of dynamics formulation complexity," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-10, pp. 730–736, 1980.

[13] W. Khalil, M. Gautier, and J. F. Kleinfinger, "Automatic generation of identification models of robots," *Int. J. Robotics Automat.*, vol. 1, no. 1, pp. 2–6, 1986.

[14] P. K. Khosla and T. Kanade, "Experimental evaluation of the feedforward compensation and computed-torque control schemes," in *Proc. Automatic Contr. Conf.*, pp. 790–798, 1986.

[15] J. Y. S. Luh, M. W. Walker, and R. P. Paul, "Resolved acceleration control of mechanical manipulators," *IEEE Trans. Automat. Contr.*, vol. AC-25, pp. 468–474, 1980.

[16] J. Y. S. Luh, M. W. Walker, and R. P. C. Paul, "On-line computational scheme for mechanical manipulators," *ASME J. Dynamic Systems Meas. Contr.*, vol. 102, pp. 69–76, 1980.

[17] H. Mayeda, K. Osuka, and A. Kangawa, "A new identification method for serial manipulator arms," in *Proc. IFAC 9th World Congress*, vol. 6, pp. 74–79, 1984.

[18] A. Mukerjee and D. H. Ballard, "Self-calibration in robot manipulators," in *Proc. IEEE Conf. on Robotics and Automation*, pp. 1050–1057, 1985.

[19] J. J. Murray and C. P. Neuman, "Arm: An algebraic robot dynamic modeling program," in *Proc. Int. Conf. on Robotics*, pp. 115–120, 1984.

[20] C. P. Neuman and P. K. Khosia, "Identification of robot dynamics: An application of recursive estimation," in *Proc. 4th Yale Workshop on Applications of Adaptive Systems Theory*, pp. 42–49, 1985.

[21] H. B. Olsen and G. A. Bekey, "Identification of robot dynamics," in *Proc. IEEE Conf. on Robotics and Automation*, pp. 1004–1010, 1986.

[22] R. P. Paul, *Robot Manipulators.* Cambridge, MA: MIT Press, 1981.

[23] E. P. Ryan, G. Leitmann, and M. Corless, "Practical stabilizability of uncertain dynamical systems: Application to robotic tracking," *J. Optimiz. Theory Appl.*, vol. 47, pp. 235–252, 1985.

[24] C. Samson, "Robust nonlinear control of robotic manipulators," in *Proc. IEEE Conf. on Decision Control*, 1983.

[25] J.-J. Slotine, "The robust control of robot manipulators," *Int. J. Robotics Res.*, vol. 4, no. 2, pp. 49–64, 1985.

[26] M. W. Spong, J. S. Thorp, and J. M. Kleinwaks, "The control of robot manipulators with bounded input, Part II: Robustness and disturbance rejection," in *Proc. IEEE Conf. on Decision Control*, 1984.

[27] M. Takegaki and S. Arimoto, "A new feedback method for dynamic control of manipulators," *ASME J. Dynamic Systems Meas. Contr.*, vol. 102, pp. 119–125, 1981.

[28] D. E. Whitney, C. A. Lozinski, and J. M. Rourke, "Industrial robot calibration method and results," *ASME J. Dynamic Systems Meas. Contr.*, vol. 108, pp. 1–8, 1986.


# Comments on "Stabilization of Uncertain Systems Subject to Hard Bounds on Control with Application to a Robot Manipulator"

M. ARSHAD AND Z. BAHRI

In the above paper,[1] Example 1, the following modifications seem necessary in order to make the derivations of this problem consistent with its earlier analysis.

According to (4) in paper,[1] $M_1$ and $S$ should read as follows:

$$\begin{bmatrix} Z \\ Y \end{bmatrix} = \begin{bmatrix} M_1 \\ S \end{bmatrix} X.$$

Where in this example $M_1$ and $S$ are chosen

$$M_1 = [-2 \quad 1] \tag{1}$$

$$S = [1 \quad 1] \tag{2}$$

with $P = 3/2$; $d_1 = 0.954$; and $\| y_0 \| = 7.826$.

Based on the above (1) and (2), the (29) of paper[1] for the attracting region

$$\frac{1}{2} d_1 y^T y + [1 - d_1] z^T P z < \frac{1}{2} d_1 \| y_0 \|^2 \tag{3a}$$

Manuscript received August 15, 1988; revised October 5, 1988.

The authors were with the Department of Electrical Engineering and Computer Science (M/C 154), The University of Illinois at Chicago, Chicago, IL 60680-4348.

IEEE Log Number 8826218.

[1] M. Hached, S. M. Madani-Esfahani, and S. H. Żak, *IEEE J. Robotics Automation.* vol. 4, no. 3, pp. 310–323, June 1988.