Exciting Trajectories for the Identification of Base Inertial Parameters of Robots

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Abstract

A common way to identify the inertial parameters of robots is to use a linear model in relation to the parameters and standard least-squares (LS) techniques. This article presents a method to generate exciting identification trajectories in order to minimize the effect of noise and error modeling on the LS solution. Using nonlinear optimization techniques, the condition number of a matrix **W** *obtained from the energy model is minimized, and the scaling of its terms is carried out. An example of a three-degree-of-freedom robot is presented.*

1. Introduction

The numerical values of the inertial parameters must be known in order to improve the performance of robots by the use of advanced control methods such as computed torque control. Identification methods are attractive, because parameters may be difficult to measure directly. Least-squares (LS) techniques, on dynamic or energetic models, are generally used to estimate the values of the inertial parameters.

There are two problems related to using LS techniques.

1. First, there is a structural problem related to the loss of identifiability of some parameters for any trajectory used during the process of identification. This is called *model structure identifiability* by Ljung (1987). Both energetic and dynamic models are linear in relation to standard inertial parameters such as the mass, components of first moment, and inertia tensor of each link. Some of these parameters can be eliminated or regrouped to provide the minimum set of inertial parameters. These are also called the *base parameters* and are the identifiable parameters of the model.

   Fortunately, this problem has now been solved by symbolic or numerical methods for both open- and closed-loop structure robots (Gautier and Khalil 1988a; 1989; 1990; Gautier 1990a,b; Mayeda et al. 1988; 1989; 1990; Sheu and Walker 1989).
2. The second problem concerns the trajectories used to identify the base parameters of the model. We must use suitable trajectories; otherwise some parameters become unidentifiable or very sensitive to noisy data. Several authors such as Mayeda et al. (1984), Olsen and Beckey (1985), Atkeson et al. (1986), and Khosla (1986) had the idea of using special test motions based on moving only one or two axes at a time to improve general test motions results. Each test allows the identification of a small number of parameters at a time. There is no study about the difference to the global LS solution, and there is no analytical study about the optimality of the trajectories.

   The first study concerning the problem of exciting trajectories has been performed by Armstrong (1987; 1988; 1989), who developed a method to minimize a condition number of an excitation matrix computed from the dynamic model. The degrees of freedom are the points of a sequence of joint acceleration $\ddot{\mathbf{q}}$. Then joint velocity $\dot{\mathbf{q}}$ and joint position $\mathbf{q}$ are obtained by numerical integration. This is a nonlinear path optimization problem, which can be solved by the Lagrangian technique. It requires a large number of variables, and joint constraints are difficult to satisfy. Finding an optimal trajectory of 300 points for a three-degree-of-freedom robot (900 optimization variables), using a gradient method, requires 40 hours of VAX time.

In this article we present a new method to generate exciting trajectories for the identification of inertial parameters using the minimum set of parameters and the energy model, because it is a function of the joint positions and velocities and doesn't require calculating or measuring the accelerations (Gautier and Khalil, 1988b; Sheu and Walker 1989). By sampling the linear minimum model on a trajectory $(\mathbf{q}, \dot{\mathbf{q}})$, base parameters $\mathbf{X}$ can be estimated as the LS solution of a full-rank overdetermined

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linear system (Gautier and Khalil 1988b), $\mathbf{Y} = \mathbf{WX} + \rho$. The sensitivity of the estimated parameters $\hat{\mathbf{X}}$ to errors on $\mathbf{Y}$ or $\mathbf{W}$ can be measured by the condition number of $\mathbf{W}$ provided that $\mathbf{W}$ is well equilibrated (Dongarra et al. 1979).

Nonlinear optimization techniques are used to find a trajectory that minimizes the condition number of $\mathbf{W}(\mathbf{q}, \dot{\mathbf{q}})$ and scales its rows and columns. At first the algorithm finds a sequence of optimum points $(\mathbf{q}, \dot{\mathbf{q}})$. Then a continuous trajectory is defined on these optimum points using fifth-order polynomial functions. The joint positions, velocities, and acceleration constraints are taken into account.

We have extensively used commercially available software packages in developing our idea about exciting trajectory. Subroutines from Harwell (1988) have been used for the solution of the optimization problem. Ctrl-C (1989) software and routines from IMSL (1990) and Harwell (1988) have been used for the solution of linear algebra and other problems. The corresponding algorithms are not detailed in this article but can be found in the given references.

2. The Identification Model

Details about this section can be found in Gautier and Khalil (1988a,b; 1989) and Gautier (1990a,b). The system to be considered is an open-loop structure mechanism. The description of the system will be carried out by the use of the modified Denavit and Hartenberg notation (Khalil and Kleinfinger 1986; Dombre and Khalil 1988).

The system is composed of n joints and $n + 1$ links; link 0 is the base, while link n is the end effector. A coordinate frame j is assigned, fixed with respect to link j. The 10 standard inertial parameters of link j are composed of:

- $\mathbf{XX_j}, \mathbf{XY_j}, \mathbf{XZ_j}, \mathbf{YY_j}, \mathbf{YZ_j}, \mathbf{ZZ_j}$, representing the inertia matrix elements of link j about the origin of frame j, referred to frame j;
- $\mathbf{MX_j}, \mathbf{MY_j}, \mathbf{MZ_j}$, the first moments of link j about the origin of frame j, referred to frame j; and
- $\mathbf{M_j}$, the mass of link j.

The $10n$ standard parameters of the robot can be reduced to b base parameters by eliminating some of them and grouping some others. We define $\mathbf{X}$ as the $b \times 1$ vector containing the base (identifiable) parameters.

Different dynamic models linear in the inertial parameters can be obtained from Newton-Euler or Lagrangian formulation and from the energy theorem. In this article the energy model is used, because it is easy to calculate and doesn't depend on the joint accelerations.

From the energy theorem comes:

$$y = \Delta\mathbf{H} \tag{1}$$

where:

$$y = \int_{t_a}^{t_b} \mathbf{T}^T \dot{\mathbf{q}}\, dt \tag{2}$$

$$\Delta\mathbf{H} = \mathbf{H}(t_b) - \mathbf{H}(t_a) \tag{3}$$

where $\mathbf{T}$ is the $n \times 1$ vector of joint torques or forces not derived from a potential; $\mathbf{q}, \dot{\mathbf{q}}$ are the $n \times 1$ vectors of joint positions and velocities, respectively; H is the total energy, (the Hamiltonian), of the robot; H=E+U; $\mathrm{E}(\mathbf{q}, \dot{\mathbf{q}})$ is the kinetic energy; and $\mathrm{U}(\mathbf{q})$ is the potential energy.

From the linearity of the energy with respect to the inertial parameters comes

$$\mathrm{H} = \mathbf{h}\ \mathbf{X} = \sum_{i=1}^{b} h_i \mathrm{X_i} \tag{4}$$

$$\Delta\mathrm{H} = [\mathbf{h}(t_b) - \mathbf{h}(t_a)]\mathbf{X} = \Delta\mathbf{h}\ \mathbf{X} \tag{5}$$

where $\mathbf{h}$ is a $1 \times \mathrm{b}$ row matrix composed of the partial derivatives h_i:

$$h_i = \frac{\partial \mathrm{H}}{\partial \mathrm{X_i}}. \tag{6}$$

The identification model is the energy difference equation:

$$y = \int_{t_a}^{t_b} \mathbf{T}^{\mathrm{T}} \dot{\mathbf{q}} dt = \Delta\mathbf{h}\ (\mathbf{q}, \dot{\mathbf{q}})\ \mathbf{X} \tag{7}$$

Let us define $(\mathbf{q}, \dot{\mathbf{q}})_{\mathrm{ab(i)}}$ as a sample of $(\mathbf{q}, \dot{\mathbf{q}})$ at times $(t_{\mathrm{a}}, t_{\mathrm{b}})_{\mathrm{(i)}}$.

$$\begin{aligned}(\mathbf{q}, \dot{\mathbf{q}})_{\mathrm{a(i)}} &= [\mathbf{q}(t_{a(\mathrm{i})}), \dot{\mathbf{q}}(t_{a(\mathrm{i})}], (\mathbf{q}, \dot{\mathbf{q}})_{\mathbf{ab(i)}} \\ &= [(\mathbf{q}, \dot{\mathbf{q}})_{\mathbf{a(i)}}, (\mathbf{q}, \dot{\mathbf{q}})_{\mathbf{b(i)}}]\end{aligned}$$

From a sequence of r samples $(\mathbf{q}, \dot{\mathbf{q}})_{\mathbf{ab(i)}}, i = 1, \ldots, r$, we can obtain r samples of the energy difference equation.

$$\Delta\mathrm{H(i)} = \Delta\mathbf{h(i)}\mathbf{X} \tag{8}$$

We define the $r \times b$ matrix $\mathbf{W}, r \geq \mathrm{b}$ and the $r \times 1$ vector $\mathbf{Y}$:

$$\mathbf{Y} = \begin{bmatrix} \mathrm{y(1)} \\ \ldots \\ \mathrm{y(r)} \end{bmatrix}, \qquad \mathbf{W} = \begin{bmatrix} \Delta\mathbf{h(1)} \\ \ldots \\ \Delta\mathbf{h(r)} \end{bmatrix} \tag{9}$$

$$y(\mathrm{i}) = \int_{t_{a(\mathrm{i})}}^{t_{b(\mathrm{i})}} \mathbf{T}^{\mathrm{T}} \dot{\mathbf{q}} \mathrm{dt},$$

$$\Delta\mathbf{h(i)} = \mathbf{h}((\mathbf{q}, \dot{\mathbf{q}})_{\mathbf{b(i)}}) - \mathbf{h}((\mathbf{q}, \dot{\mathbf{q}})_{\mathbf{a(i)}})$$

We obtain an overdetermined linear system:

$$\mathbf{Y}(\mathrm{T}, \dot{\mathbf{q}}) = \mathbf{W}(\mathbf{q}, \dot{\mathbf{q}})\mathbf{X} + \rho, \tag{10}$$

where ρ is the vector of errors.

To simplify the writing, we suppose that there is no friction. Thus $\mathbf{T}$ will be equal to the vector of actuator torques Γ. If the friction cannot be neglected, its coefficients can be taken into account in the identification process, as presented in Gautier and Khalil (1988b). A good choice of $(\mathbf{q}, \dot{\mathbf{q}})_{ab(i)}$ and the use of the base parameters ensures that $\mathbf{W}$ is a full-rank matrix. The base parameters can be identified using the LS solution $\hat{\mathbf{X}}$ of the linear system (10).

$$\hat{\mathbf{X}} = \underset{\mathbf{X}}{\text{Arg. min}} \|\rho\|^2$$

The sequential LS solution using the efficient numerical algorithm developed by Bierman (1977) is well suited for on-line identification (Gautier 1986). The most efficient method to calculate the batch LS solution by direct method uses orthogonal decomposition of $\mathbf{W}$ by Householder transformation QR and by singular value decompositions (SVD) (Dongarra et al. 1979). Moreover, these factorizations provide efficient tools for practical analysis of LS problems (Lawson and Hanson 1974; Golub and Van Loan 1983).

3. Perturbation Bounds for the LS Solution

In practical application, $\mathbf{Y}$ and $\mathbf{W}$ are perturbated by noise measurements on $\Gamma, \mathbf{q}, \dot{\mathbf{q}}$ and by error modeling. In this section we study the effect of these perturbations on the LS solution of (10).

Let $\hat{\mathbf{X}} + \delta\hat{\mathbf{X}}$ be the LS solution of the perturbated system:

$$\mathbf{Y} + \delta\mathbf{Y} = (\mathbf{W} + \delta\mathbf{W})\mathbf{X} + \rho. \tag{11}$$

In the case $r = b$, simple bounds for $\delta\hat{\mathbf{X}}$ are given by Ciarlet (1982)[1]:

$$\frac{\|\delta\hat{\mathbf{X}}\|}{\|\hat{\mathbf{X}}\|} \leq \text{Cond}(\mathbf{W}) \frac{\|\delta\mathbf{Y}\|}{\|\mathbf{Y}\|} \quad \text{with } \delta\mathbf{W} = \mathbf{0} \tag{12}$$

$$\frac{\|\delta\hat{\mathbf{X}}\|}{\|\hat{\mathbf{X}} + \delta\hat{\mathbf{X}}\|} \leq \text{Cond}(\mathbf{W}) \frac{\|\delta\mathbf{W}\|}{\|\mathbf{W}\|} \quad \text{with } \delta\mathbf{Y} = \mathbf{0} \tag{13}$$

$\|.\|$ is a p vector norm or its subordinate p matrix norm. $\text{Cond}(\mathbf{W})$ is the condition number of $\mathbf{W}$ with respect to the p norm.

Relations (12) and (13) show that the condition number is a quantity that measures the sensitivity of the solution $\hat{\mathbf{X}}$ to errors in $\mathbf{W}$ or $\mathbf{Y}$. Thus it is very important that $\text{Cond}(\mathbf{W})$ be as small as possible before computing $\hat{\mathbf{X}}$. In the appendix we give an example with $r = c = 2$ and $\text{Cond}(\mathbf{W}) = 2249$. This illustrates that a relative norm error about 7.10^{-4} on $\mathbf{Y}$ will lead to a relative norm error about 1.6 on $\mathbf{X}$.

$\text{Cond}(\mathbf{W})$ in the p norm is given by:

$$\text{Cond}_p(\mathbf{W}) = \|\mathbf{W}\|_p \|\mathbf{W}^+\|_p, \tag{14}$$

where $\mathbf{W}^+$ is the pseudoinverse of $\mathbf{W}$. It can be seen that:

$$\text{Cond}(\mathbf{W}) \geq 1. \tag{15}$$

The usual p norms are the 1-norm, the 2-norm and the ∞-norm, but any two condition numbers in two different norms are equivalent. Thus, if a matrix is well conditioned in a norm, it is well conditioned in all the other norms, depending on some constants (Golub and Van Loan 1983).

The 2-norm condition is easy to calculate using the SVD of $\mathbf{W}$ (Dongarra et al. 1979; Lawson and Hanson 1974; Golub and Van Loan 1983; Klema and Laub 1980):

$$\text{Cond}_2(\mathbf{W}) = \frac{\sigma_1}{\sigma_b}, \tag{16}$$

where σ_1 is the largest singular value, and σ_b is the smallest singular value.

The SVD of $\mathbf{W}$ is given by:

$$\mathbf{U}^T\mathbf{W} = \mathbf{S}\mathbf{V}^T \tag{17}$$

with

$$\mathbf{S} = \begin{bmatrix} \mathbf{\Sigma} \\ \mathbf{0} \end{bmatrix},$$

where $\mathbf{U}$ is an r$\times$ r orthogonal matrix, $\mathbf{V}$ is a b$\times$ b orthogonal matrix, and $\mathbf{\Sigma}$ is a $b \times b$ diagonal matrix whose elements are the singular values of $\mathbf{W}$ in non-increasing order ($\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_b > 0$).

In the 2-norm, the minimum condition is obtained for an orthogonal matrix:

$$\text{Cond}(\mathbf{W}) = 1 \Leftrightarrow \mathbf{W}^T\mathbf{W} = \mathbf{I},$$

where $\mathbf{I}$ is the identity matrix.

It may be noted that the bounds given by relations (12) and (13) should be meaningless unless the errors $\delta\mathbf{W}_{ij}$ on the elements $\mathbf{W}_{ij}$ of the matrix $\mathbf{W}$ are all about the same size; that is to say, $\mathbf{W}$ must be scaled before computing $\hat{\mathbf{X}}$ (Dongarra et al. 1979). (See example 2 in the Appendix.)

4. The Optimization Problem

The problem is to find a trajectory $(\mathbf{q}, \dot{\mathbf{q}})$ that gives a well-conditioned and well-equilibrated $\mathbf{W}(\mathbf{q}, \dot{\mathbf{q}})$ matrix.

The trajectory is defined by a sequence of points $(\mathbf{q}, \dot{\mathbf{q}})_{ab(i)}, i = 1, \ldots, r$ and $r \geq b$, corresponding to samples of $(\mathbf{q}, \dot{\mathbf{q}})$ at times $t_{a(i)}$ and $t_{b(i)}$.

1. In the case $r > b$, the general relations function, also of $\text{Cond}(\mathbf{W})$, is given by Lawson and Hanson (1974).

4.1. Cost Function

The cost function f can be defined as:

$$f(\mathbf{q}, \dot{\mathbf{q}}) = \lambda_1 \text{ Cond}(\mathbf{W}(\mathbf{q}, \dot{\mathbf{q}})) + \lambda_2 \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}}) \qquad (18)$$

where λ_1 and λ_2 are two weighting scalars, and **S** is the measure of the equilibrium.

When **W** is computed highly accurately, the absolute error in an element is proportional to its size, and the strategy of equilibrium reduces to scale **W** so that all its elements are of the same order of magnitude (Dongarra et al. 1979). Thus **S** can be defined as:

$$S = \frac{|\mathbf{W}_{ij}|_{\max}}{|\mathbf{W}_{ij}|_{\min}}, \qquad (19)$$

where $|\mathbf{W}_{ij}|_{\max}$ and $|\mathbf{W}_{ij}|_{\min}$ are the maximum and minimum absolute values of the elements $\mathbf{W}_{ij}$ of **W**, and $|\mathbf{W}_{ij}|_{\min} \neq 0$. The criterion f is a nonlinear function of $(\mathbf{q}, \dot{\mathbf{q}})_{ab(i)}, i = 1, \ldots, r$.

Taking into account the joint positions and velocities constraints, the problem is a nonlinear optimization problem with constraints. It can be formulated as shown in the text that follows.

Find $(\mathbf{q}, \dot{\mathbf{q}})_{ab(i)}, i = 1, \ldots, r$ that minimizes $f(\mathbf{q}, \dot{\mathbf{q}})$ under the constraints:

$$q_{lj} \leq q_j \leq q_{Fj} \quad \text{and} \quad |\dot{q}_j| \leq \dot{q}_{Mj}, j = 1, \ldots, n, \qquad (20)$$

where $\mathbf{q_l}, \mathbf{q_F}$ define the joint limits vectors, and $\dot{q}_{Mj}$ is the maximum absolute value of the velocity of joint j. Three measures of the condition number are tested:

1. The 2-norm condition number using SVD decomposition of **W**, (eq. (16)) calculated by a program from the IMSL (1990) software package,
2. An estimate of the 2-norm condition from the QR decomposition of **W** with column pivoting, (Dongarra et al. 1979; Lawson and Hanson 1974; IMSL 1990),
3. A condition of the matrix $\mathbf{A} = \mathbf{W}^T\mathbf{W}$ in the Frobenius norm, given by:

$$\text{Cond}_F(\mathbf{A}) = \|\mathbf{A}\|_F \|\mathbf{A}^{-1}\|_F \qquad (21)$$

$$\|\mathbf{A}\|_F = \left(\sum_{i=1}^{b} \sum_{j=1}^{b} \mathbf{A}_{ij}^2 \right)^{1/2} \qquad (22)$$

Although similar results were obtained in the three cases, the last measure gives the best convergence rate.

4.2. Constraints

In order to transform the problem into another simpler one without constraints, the following change of variables can be used (Minoux 1983):

$$q_{lj} \leq q_j \leq q_{Fj} \Leftrightarrow q_j = q_{lJ} + (q_{Fj} - q_{lj}) \sin^2(x_k), \quad j = 1, \ldots, n$$

$$|\dot{q}_j| \leq \dot{q}_{Mj} \Leftrightarrow \dot{q}_j = \dot{q}_{Mj} \sin(x_k), \quad j = 1, \ldots, n$$

The n_d new variables x_k are not constrained anymore.

4.3. Optimization method

Symbolic expressions of the derivatives of the criterion cannot be obtained for this problem. The problem is also characterized by a large number of degrees of freedom n_d. To reduce the number of points, every sample $(\mathbf{q}, \dot{\mathbf{q}})_{(i)}$ is used twice, such that:

$$(\mathbf{q}, \dot{\mathbf{q}})_{a(i+1)} = (\mathbf{q}, \dot{\mathbf{q}})_{b(i)}$$

To obtain r rows of **W** requires:

$$n_d = (r + 1)2n, \qquad r \geq b \qquad (23a)$$

In many industrial robots in which the first axis is rotational and vertical, **W** is independent of q_1, thus equation (23a) reduces to:

$$n_d = (r + 1)(2n - 1), \qquad r \geq b \qquad (23b)$$

For the 3-DOF robot given in the example in section 6, there are 15 base parameters. Thus, using (23b), we find $n_d \geq 80$.

Three optimizing methods have been tested:

1. A heuristic method (Nedler and Mead 1965).
2. A quasi-Newton method in which derivatives are estimated by difference and implemented in program VA10AD of Harwell (1988) software package.
3. A method developed by Powell (1964) and Minoux (1983) and implemented in program VA04AD of the Harwell (1988) software package. It is a gradient conjugate type method that approaches the conjugate directions by an iterative procedure.

The last method has given the best results.

The n_d of $x_k, k = 1, \ldots, n_d$, are initialized by normal random sequences and converge to n_d optimum values, which correspond to $(r + 1)$ optimum pairs $(q_j, \dot{q}_j)$ for joints $j, j = 1, \ldots, n$, within joint positions and velocities limits.

5. Interpolation

As a result of the optimization procedure, we can obtain a set of optimum points $(\mathbf{q}, \dot{\mathbf{q}})_{(i)}$. A continuous and smooth trajectory is calculated by interpolating a line between these points, assuming zero initial and final accelerations

and using a fifth-order polynomial (Gautier 1990a). The following notations are used:

- t is the current time.
- $[t_{a(i)}, t_{b(i)}]$ defines an interval (i).
- u(i) is the relative time on an interval (i):

$$u(\mathrm{i}) = t - t_{a(\mathrm{i})}, u(\mathrm{i}) \in [0, u_{f(\mathrm{i})}],$$

with $u_{f(\mathrm{i})} = t_{b(\mathrm{i})} - t_{a(\mathrm{i})}$.

- $u_{f(\mathrm{i})}$ is the time necessary to move from $(\mathbf{q}, \dot{\mathbf{q}})_{\mathrm{a(i)}}$ to $(\mathbf{q}, \dot{\mathbf{q}})_{\mathrm{b(i)}}$ and is calculated in the following section.

For any interval (i) and for each joint j we define (subscripts j and i are omitted):

$$q(u) = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4 + a_5 u^5 \quad (24)$$

$$\dot{q}(u) = a_1 + 2a_2 u + 3a_3 u^2 + 4a_4 u^3 + 5a_5 u^4 \quad (25)$$

$$\ddot{q}(u) = 2a_2 + 6a_3 u + 12a_4 u^2 + 20a_5 u^3 \quad (26)$$

5.1. *Calculation of the Coefficients* a_i

The coefficients a_i are determined to satisfy the following initial and final conditions:

$$q(0) = q_a, \quad q(u_f) = q_b, \quad \dot{q}(0) = \dot{q}_a,$$

$$\dot{q}(u_f) = \dot{q}_b, \quad \ddot{q}(0) = \ddot{q}(u_f) = 0 \quad (27)$$

From (24, ..., 27) we obtain:

$$\begin{aligned} a_0 &= q_a \\ a_1 &= \dot{q}_a \\ a_2 &= 0 \\ a_3 &= 10\mathbf{A}u_f^{-3} - (6\dot{q}_a + 4\dot{q}_b)u_f^{-2} \\ a_4 &= -15\mathbf{A}u_f^{-4} + (8\dot{q}_a + 7\dot{q}_b)u_f^{-3} \\ a_5 &= 6\mathbf{A}u_f^{-5} - 3(\dot{q}_a + \dot{q}_b)u_f^{-4} \end{aligned} \quad (28)$$

where $\mathbf{A} = q_b - q_a$.

5.2. *Calculation of* u_f

To minimize the identification time, u_f is calculated so that maximum velocity or maximum acceleration is reached on one joint, and velocity and acceleration constraints (eqs. (20) and (29)) are also satisfied. The acceleration constraint of joint j is given as:

$$|\ddot{q}_j| \leq \ddot{q}_{Mj} \quad (29)$$

This is a univariate nonlinear optimization problem that is solved using a quadratic interpolation method implemented in program VD06AD of the Harwell (1988) library.

The optimization is carried out in three steps.

Calculating u_f to Satisfy the Acceleration Constraints

For each joint j, we look for the value u_{fj} that minimizes the cost function:

$$(|\ddot{q}_j|_{max} - \ddot{q}_{Mj})^2$$

where $|\ddot{q}_j|_{max}$ is the maximum absolute value of $\ddot{q}_j$ during the interval (i). For a given value u_f, this is calculated by putting, in eq. (26), the roots of the second-order equation defined by $\ddot{q}_j(t) = 0$. Efficient program (Harwell 1988), is used to find all the real roots of a polynomial with real coefficients that fall within a given interval.

Taking $u_f = u_{fa}$ such that:

$$u_{fa} = \max_j u_{fj}$$

and because $|\ddot{q}_j|_{max}$ is a decreasing function of u_f, we have:

$$|\ddot{q}_a|_{max} = \ddot{q}_{M_a}$$
$$|\ddot{q}_j|_{max} \leq \ddot{q}_{M_j}, \quad j \neq a$$

Calculating u_f to Satisfy the Velocity Constraints

Given u_{f_a}, we calculate the value of $|\dot{q}_j|_{max}$, where $|\dot{q}_j|_{max}$ is the maximum absolute value of $\dot{q}_j$ during the interval (i). This is calcuted by putting, in equation (25), the nonzero roots of the following equation defined by $\ddot{q}_j(t) = 0$:

$$3a_3 + 6a_4 + 10a_5 u^2 = 0$$

The solution is carried out as previously using Harwell PA02BD (1988).

If $|\dot{q}_j|_{max} \leq \dot{q}_{M_j}, j = 1, \ldots, n$, then we take $u_f = u_{f_a}$. If not, we find joint v such that:

$$|\dot{q}_v|_{max} - \dot{q}_{M_v} = \max_j(|\dot{q}_j|_{max} - \dot{q}_{M_j})$$

Then we look for the value u_{f_v} that minimizes the cost function:

$$(|\dot{q}_v|_{max} - \dot{q}_{M_v})^2$$

Taking $u_f = u_{f_v}$, and because $|\dot{q}_j|_{max}$ is a decreasing function of u_f, we have:

$$|\dot{q}_v|_{max} = \dot{q}_{M_v},$$
$$|\dot{q}_j|_{max} \leq \dot{q}_{M_j}, \quad |\ddot{q}_j|_{max} \leq \ddot{q}_{M_j}, \qquad j \neq v$$

Verifying the Position Constraints

After this, the calculated position trajectory has to be checked to see that it doesn't violate the position constraints. If it does, a new sequence of $(\mathbf{q}, \dot{\mathbf{q}})_{ab(i)}$ must be determined using position and velocity constraints smaller than $\mathbf{q}_I$, $\mathbf{q}_F$, and $\dot{\mathbf{q}}_M$, respectively. This is because extrema of $\mathbf{q}$ are increasing functions of u_f, whereas extrema of $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ are decreasing functions of u_f (Gautier 1990a).

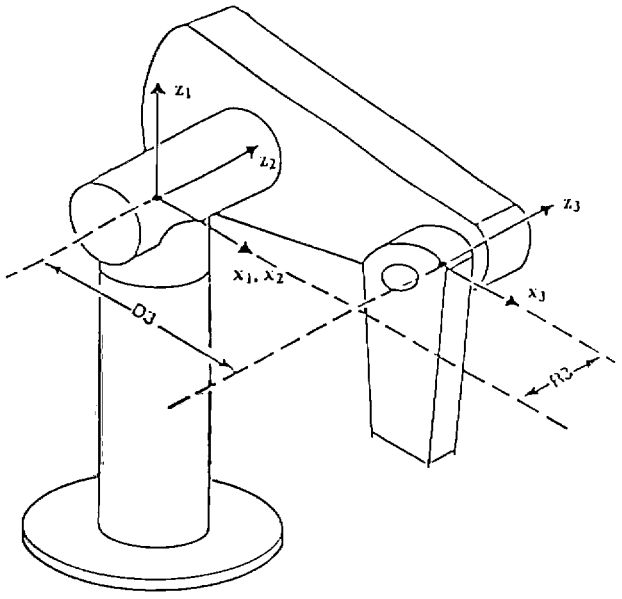

Fig. 1. Three-degree-of-freedom robot.

Table 1. The Geometric Parameters of the 3-DOF Robot

j	σ_i	α_j	d_j	θ_j	r_j
1	0	0	0	θ_1	0
2	0	−90	0	θ_2	0
3	0	0	D3 = 0.5 m	θ_3	R3 = 0.2 m

6. Application

The method is simulated on a 3-DOF robot similar to the first three DOFs of the PUMA 560 robot (Fig. 1). The geometric parameters of the robot are given in Table 1.

The symbolic expressions of the functions h_i are automatically computed using the SYMORO software package (Khalil 1990). They are also easy to calculate numerically (Gautier 1990a).

6.1. Influence of the Number of Equations

Cond(**W**) has been calculated using random sequences $(\mathbf{q}, \dot{\mathbf{q}})_{ab(i)}$ for different values of r. Minimum, maximum, and mean values over 10 samples are reported in Table 2 and Figure 2. It can be seen that Cond(**W**) decreases with r, but doesn't vary any more for $r \geq 50$.

Table 2 gives optimization results with respect to r. For $r \geq 50$, the optimization fails because of the great number of variables n_d. For $r = 30$, the condition number reduces to 11 as a result of optimization; thus $r = 30$ can be considered as a good compromise between the difficulty of the optimization and the minimum value of Cond(**W**).

Table 2. Effect of the Number of Equations r

	Cond(W)			
	Random Sequences			Optimum
r	Minimum	Mean	Maximum	Sequence
15	5.6650D+03	2.7994D+04	9.4585D+04	53
20	9.3734D+02	1.9450D+03	2.9600D+03	30
30	4.0261D+02	9.5398D+02	1.6989D+03	11
50	4.7723D+02	5.8609D+02	7.5051D+02	
100	4.0985D+02	4.9641D+02	6.9085D+02	
200	3.8291D+02	4.3867D+02	5.1616D+02	
300	3.6296D+02	4.1815D+02	4.8995D+02	
500	3.8040D+02	4.2205D+02	4.6090D+02	

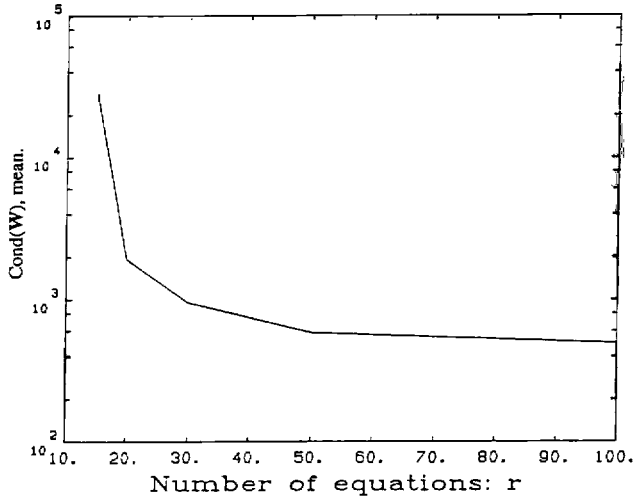

Fig. 2. Effect of the number of equations r.

6.2. Exciting Trajectories

To obtain a matrix **W** (30 × 15), the optimization algorithm provides 155 values of x_k corresponding to 31 points $\dot{q}_1$ and to 31 pairs $(q_j, \dot{q}_j)$ for joint 2 and 3. The initial values of the random points give a condition number Cond(**W**) ≈ 10^3 and a scaling **S** ≈ 10^5 and converge to optimum points that give Cond(**W**) ≈ 10 and **S** ≈ 175.

The optimization required about 30 minutes of VAX 8700 CPU time (6 Mips, 1.2 MFlops, test LINPACK 64 bits). The position q_1 has no effect on **W**; therefore, it was taken using a uniform random sequence of 31 points between q_{I1} and q_{F1}.

Starting point $(\mathbf{q}, \dot{\mathbf{q}})_{a(0)}$ and end point $(\mathbf{q}, \dot{\mathbf{q}})_{b(r+1)}$ corresponding to $\mathbf{q} = \dot{\mathbf{q}} = \mathbf{0}$ were added.

Trajectories are given in Figures 3, 4, and 5.

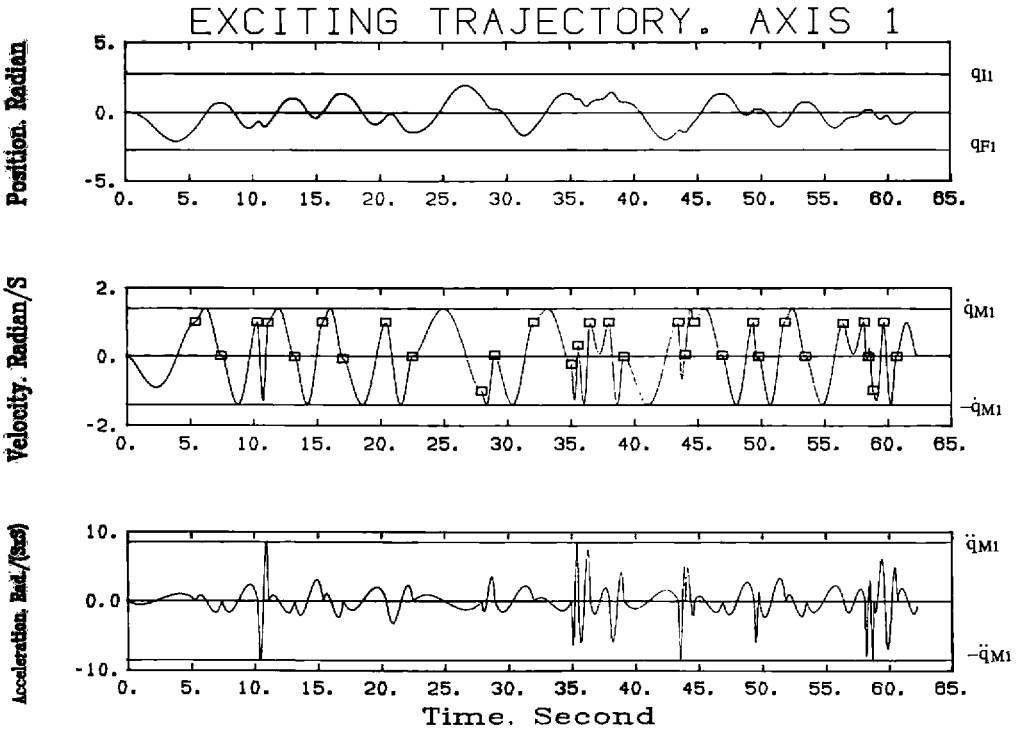

Fig. 3. Exciting trajectory, axis 1.

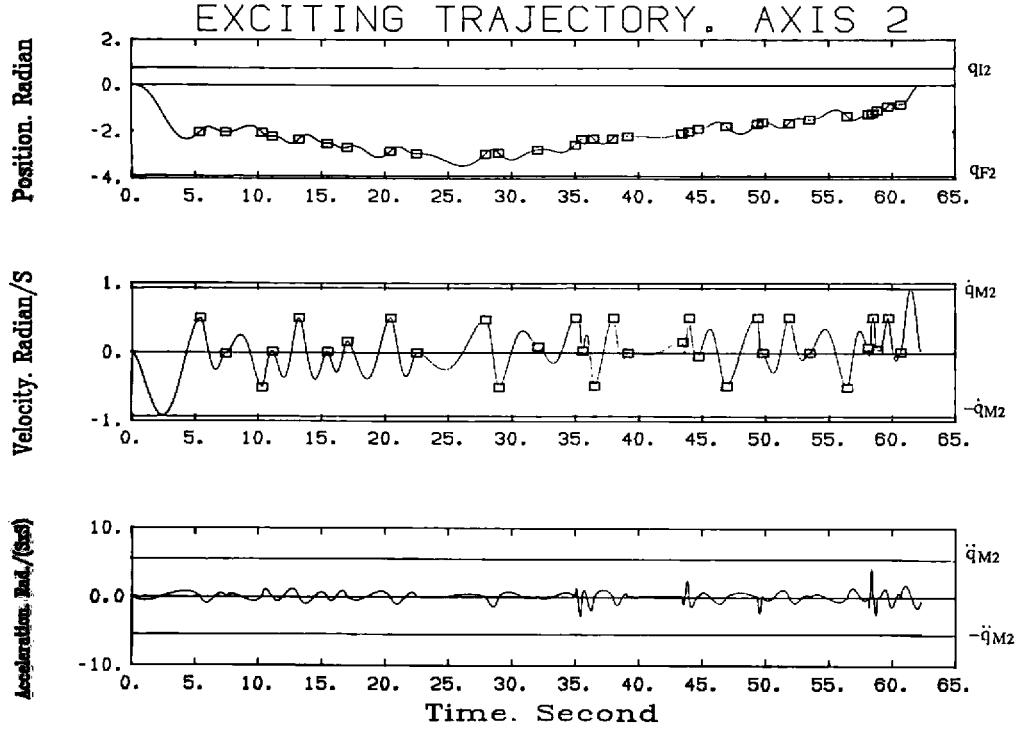

Fig. 4. Exciting trajectory, axis 2.

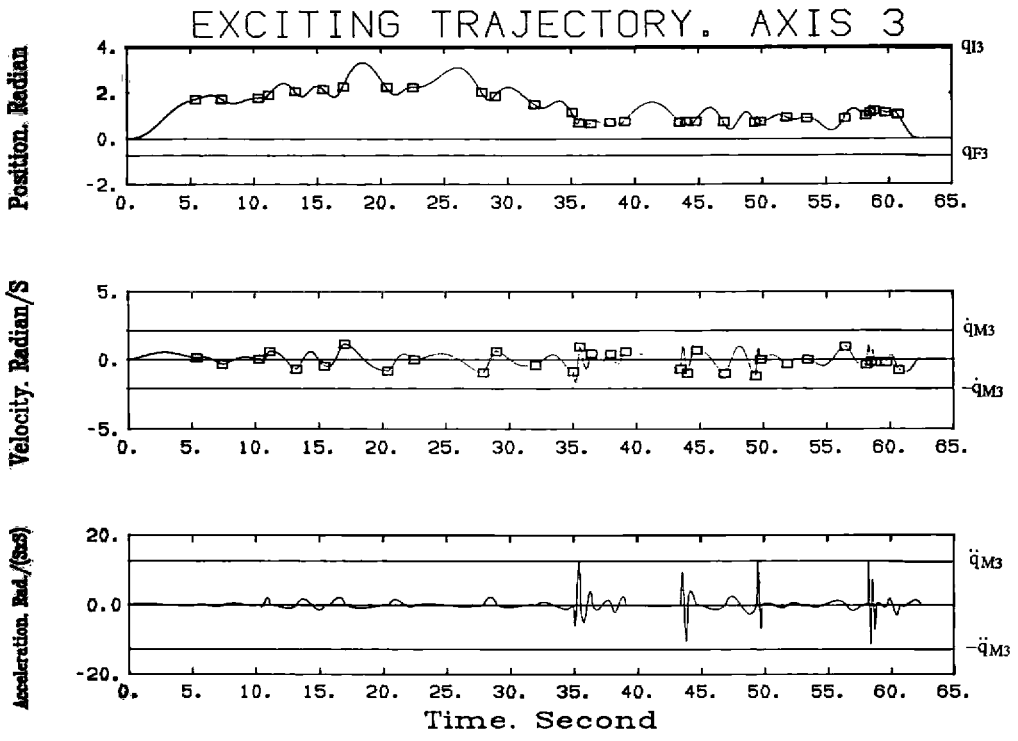

Fig. 5. Exciting trajectory, axis 3.

6.3. Identification

Tables 3 through 7 compare the results of the identification using a $(\mathbf{q}, \dot{\mathbf{q}})$ random trajectory with $\text{Cond}(\mathbf{W}) \approx 10^3$ and the optimum trajectory. $\mathbf{X}$ is the vector of the base parameters, and $\hat{\mathbf{X}}$ is the vector of estimated parameters.

The influence of noise was tested. The noise is supposed a zero mean Gaussian random sequence whose standard deviation σ_ρ was equal to $k/4$ times the maximum absolute value of $\mathbf{Y}$ to generate $\delta\mathbf{Y}$ or $k/4$ times $|\mathbf{q_F} - \mathbf{q_I}|$ and $k/4$ times $\dot{\mathbf{q}}_\mathbf{M}$ to generate $\delta\mathbf{W}$.

Tables 3, 4, and 5 give the comparison with respect to the 2-norm; Table 4 shows that the condition number is not sensitive to perturbations. Tables 6 and 7 give the comparison with respect to the components of $\hat{\mathbf{X}}$. Standard deviations $\sigma_{\mathrm{X}i}$ were computed using SVD factorization of $\mathbf{W}$ and assuming that $\delta\mathbf{W} = \mathbf{0}$ and that ρ is a zero mean noise with a covariance matrix:

$$\mathbf{C}_{\rho\rho} = \mathbf{E}(\rho\rho^{\mathrm{T}}) = \sigma_{\rho^2}\mathbf{I}.$$

Then the covariance matrix of the estimation error is given by Lawson and Hanson (1974):

$$\begin{aligned}\mathbf{C_{XX}} &= \mathrm{E}([\mathbf{X} - \hat{\mathbf{X}}][\mathbf{X} - \hat{\mathbf{X}}]^{\mathrm{T}}) \\ &= \sigma_{\rho^2}[\mathbf{W}^{\mathrm{T}}\mathbf{W}]^{-1} \\ &= \sigma_{\rho^2}\mathbf{V}\boldsymbol{\Sigma}^{-2}\mathbf{V}^{\mathrm{T}}, \qquad \sigma_{\mathrm{X}i^2} = \mathbf{C_{XX}}_{ii},\end{aligned}$$

where E is the expectation operator.

With $k = 0.01$, the confidence interval $(2\sigma_{\mathbf{X}_i})$ show that some components of $\hat{\mathbf{X}}$ (in bold type in Tables 7 and 12) are not significant when $\mathbf{W}$ is ill conditioned.

Tables 8 through 12 give similar results obtained with $r = 15$. Table 8 shows that the relation (12) is significant. Tables 11 and 12 show the results obtained with $k = 0.001$. Standard deviations in Tables 6 and 11 are the same size, whereas the relative perturbation on $\mathbf{Y}$ is 10 times larger in Table 6. This shows the importance of r in decreasing the condition number and increasing the immunity to noise. This can also be seen when comparing Tables 7 and 12, where the condition number in Table 12 is 10 times larger than in Table 7. The standard deviations in Table 12 are also much larger.

7. Conclusion

Using the linear model and the LS technique for inertial parameters identification without choosing the trajectories carefully may lead to a poor estimation of the parameters. To overcome this difficulty, we have proposed a new method for generating trajectories using the energy model. The generated trajectories give acceptable small condition numbers and a well-equilibrated design matrix. Identification results are improved when the number of equations increases but are no more than twice the number of unknown parameters. The calculated trajectory can be applied to a real robot, because it respects the joint

Table 3. ($r = 30$)

	$\delta \mathbf{W} = \mathbf{0}$					
k	$\frac{\|\delta \mathbf{Y}\|}{\|\mathbf{Y}\|}$	$\frac{\|\delta \mathbf{X}\|}{\|\mathbf{X}\|}$	$\kappa \frac{\|\delta \mathbf{Y}\| \|\mathbf{X}\|}{\|\mathbf{Y}\| \|\delta \mathbf{X}\|}$	$\frac{\|\delta \mathbf{Y}\|}{\|\mathbf{Y}\|}$	$\frac{\|\delta \mathbf{X}\|}{\|\mathbf{X}\|}$	$\kappa \frac{\|\delta \mathbf{Y}\| \|\mathbf{X}\|}{\|\mathbf{Y}\| \|\delta \mathbf{X}\|}$
0.001	$5.35\ 10^{-4}$	$4.7\ 10^{-4}$	12.7	$6.48\ 10^{-4}$	0.018	37
0.01	0.0047	0.007	7.6	0.0071	0.161	45
0.1	0.05	0.059	9.5	0.0649	1.796	39
	$\kappa = \text{Cond}(\mathbf{W}) = 11.16,\ \mathbf{S} = 175$			$\kappa = \text{Cond}(\mathbf{W}) = 1.02\ 10^3,\ \mathbf{S} = 8.9\ 10^4$		

Table 4. (r=30)

	$\delta \mathbf{Y} = \mathbf{0}$					
k	$\frac{\|\delta \mathbf{W}\|}{\|\mathbf{W}\|}$	$\frac{\|\delta \mathbf{X}\|}{\|\mathbf{X}\|}$	$\text{Cond}(\mathbf{W} + \delta \mathbf{W})$	$\frac{\|\delta \mathbf{W}\|}{\|\mathbf{W}\|}$	$\frac{\|\delta \mathbf{X}\|}{\|\mathbf{X}\|}$	$\text{Cond}(\mathbf{W} + \delta \mathbf{W})$
0.001	0.005	0.0041	11.162	$5.26\ 10^{-4}$	0.013	$1.0196\ 10^3$
0.01	0.055	0.0384	11.177	0.004	0.1853	$1.0200\ 10^3$
0.1	0.5767	0.4691	13.295	0.056	2.0129	$0.8578\ 10^3$
	$\text{Cond}(\mathbf{W}) = 11.158,\ \mathbf{S} = 175$			$\text{Cond}(\mathbf{W}) = 1.0208\ 10^3,\ \mathbf{S} = 8.9\ 10^4$		

Table 5. (r=30)

k	$\frac{\|\delta \mathbf{Y}\|}{\|\mathbf{Y}\|}$	$\frac{\|\delta \mathbf{W}\|}{\|\mathbf{W}\|}$	$\frac{\|\delta \mathbf{X}\|}{\|\mathbf{X}\|}$	$\frac{\|\delta \mathbf{Y}\|}{\|\mathbf{Y}\|}$	$\frac{\|\delta \mathbf{W}\|}{\|\mathbf{W}\|}$	$\frac{\|\delta \mathbf{X}\|}{\|\mathbf{X}\|}$
0.001	$3.68\ 10^{-4}$	0.0063	0.0042	$4.97\ 10^{-4}$	$4.64\ 10^{-4}$	0.010
0.01	0.007	0.056	0.0559	0.0055	0.0057	0.129
0.1	0.054	0.633	0.4749	0.0717	0.053	1.662
	$\text{Cond}(\mathbf{W}) = 11.158,\ \mathbf{S} = 175$			$\text{Cond}(\mathbf{W}) = 1.0208\ 10^3,\ \mathbf{S} = 8.9\ 10^4$		

position, velocity, and acceleration limits. These good simulation results are encouraging for future experimental work.

Appendix

Example 1

The following example illustrates the effect of the condition number in the 1-norm, chosen for simplicity (Forsythe et al. 1977).

Let us define:

$$\mathbf{A} = \begin{bmatrix} 4.1 & 2.8 \\ 9.7 & 6.6 \end{bmatrix}, \qquad \mathbf{Y} = \begin{bmatrix} 4.1 \\ 9.7 \end{bmatrix} \tag{A1}$$

The matrix $\mathbf{A}$ appears to be without problem:

$$\text{Det}(\mathbf{A}) = -0.1, \quad \mathbf{A}^{-1} = \begin{bmatrix} -66 & 28 \\ 97 & -41 \end{bmatrix} \tag{A2}$$

The solution of the square regular system:

$$\mathbf{Y} = \mathbf{A}\mathbf{X} \tag{A3}$$

is given by:

$$\mathbf{X} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{A4}$$

Let us suppose a small perturbation $\delta \mathbf{Y}$:

$$\delta \mathbf{Y} = \begin{bmatrix} 0.01 \\ 0 \end{bmatrix}, \quad \|\delta \mathbf{Y}\| = 0.01, \quad \frac{\|\delta \mathbf{Y}\|}{\|\mathbf{Y}\|} = 7.246\ 10^{-4} \tag{A5}$$

The solution of the perturbed system (A3) becomes:

$$\mathbf{A}(\mathbf{X} + \delta \mathbf{X}) = \mathbf{Y} + \delta \mathbf{Y} \tag{A6}$$

$$\mathbf{X} + \delta \mathbf{X} = \begin{bmatrix} 0.34 \\ 0.97 \end{bmatrix}, \quad \delta \mathbf{X} = \begin{bmatrix} -0.66 \\ 0.97 \end{bmatrix}, \quad \frac{\|\delta \mathbf{X}\|}{\|\mathbf{X}\|} = 1.63 \tag{A7}$$

Table 6. (r=30)

	Cond(W) = 11.16				
	X_i	$\hat{X}_i$	$X_i - \hat{X}_i$	$\frac{X_i - \hat{X}_i}{X_i}$	σX_i
ZZR_1	5.0186	5.0251	−0.0065	−0.0013	0.0281
XXR_2	−2.0500	−2.0436	−0.0064	0.0031	0.0290
XY_2	0.7000	0.6848	0.0152	0.0218	0.0209
XZR_2	−1.0700	−1.0767	0.0067	−0.0063	0.0148
YZ_2	0.6500	0.6473	0.0027	0.0041	0.0191
ZZR_2	6.5500	6.4994	0.0506	0.0077	0.0388
MXR_2	4.3000	4.3118	−0.0118	−0.0027	0.0066
MY_2	0.6000	0.6065	−0.0065	−0.0109	0.0069
XXR_3	0.7634	0.7906	−0.0272	−0.0356	0.0301
XY_3	0.6872	0.7038	−0.0166	−0.0242	0.0253
XZ_3	0.5500	0.5427	0.0073	0.0133	0.0135
YZ_3	−0.6000	−0.6056	0.0056	−0.0093	0.0140
ZZ_3	0.9646	0.9758	−0.0112	−0.0116	0.0134
MX_3	0.5280	0.5281	−0.0001	−0.0002	0.0075
MY_3	1.1400	1.1278	0.0122	0.0107	0.0064

$$\delta \mathbf{W} = \mathbf{0}, \quad \frac{\|\delta \mathbf{Y}\|}{\|\mathbf{Y}\|} = 4.7\ 10^{-3}$$

The condition number of **A** is equal to:

$$\mathrm{Cond}_1(\mathbf{A}) = \|\mathbf{A}\|_1 \|\mathbf{A}^{-1}\|_1 = 13.8 * 163 = 2249.4$$

The perturbation bound relation (12) of Section 3 gives:

$$1.63 \leq 2249.4 * 7.246\ 10^{-4}$$

Thus a fairly small perturbation in the measure **Y** completely changes the solution **X** of (A3). The example has been chosen to have the equality.

It is important to realize that the two solutions (A4) and (A7) are exact, without any rounding error. The only reason is that the system (A3) is ill conditioned so that it magnifies any perturbation in **Y** or **A** in the solution **X**.

Example 2

This example (Dongarra et al. 1979; Golub and Van Loan 1983) illustrates the effect of equilibrium on the meaning of the condition number.

Let us consider the problem $\mathbf{AX} = \mathbf{Y}$ with:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 10^{-6} \end{bmatrix}, \qquad \mathbf{Y} = \begin{bmatrix} 1 \\ 10^{-6} \end{bmatrix} \qquad \text{(A8)}$$

The solution of this system is:

$$\mathbf{X} = [\,1 \quad 1\,]^{\mathrm{T}}$$

and condition $\mathrm{Cond}_2(\mathbf{A}) = 10^6$. If $\delta \mathbf{Y} = [\,10^{-6} \quad 0\,]^{\mathrm{T}}$, then the solution of (A6) gives:

$$\frac{\|\delta \mathbf{Y}\|}{\|\mathbf{Y}\|} = \frac{10^{-6}}{(1 + 10^{-12})^{1/2}} \cong 10^{-6}, \qquad \frac{\|\delta \mathbf{X}\|}{\|\mathbf{X}\|} = \frac{10^{-6}}{\sqrt{2}}$$

Then the inequality (12) in the 2-norm has the form:

$$\frac{10^{-6}}{\sqrt{2}} \leq 10^6 * 10^{-6}$$

Thus the upper bound can be a gross overestimate. On the other hand, if $\delta \mathbf{Y} = [\,0 \quad 10^{-6}\,]^{\mathrm{T}}$, then

$$\frac{\|\delta \mathbf{Y}\|}{\|\mathbf{Y}\|} = \cong 10^{-6}, \qquad \frac{\|\delta \mathbf{X}\|}{\|\mathbf{X}\|} = \frac{1}{\sqrt{2}}.$$

The inequality says:

$$\frac{1}{\sqrt{2}} \leq 10^6 * 10^{-6}.$$

There are perturbations for which the bound in (12) is essentially attained. This example suggests that the condition number should be meaningless unless the matrix **A** be properly scaled.

Table 7. ($r = 30$)

	$\mathbf{X_i}$	$\hat{\mathbf{X}}_\mathbf{i}$	$X_i - \hat{X}_i$	$\frac{X_i - \hat{X}_i}{X_i}$	σX_i
			Cond(**W**) = 1.02 10^3		
ZZR_1	5.0186	5.3447	−0.3261	−0.0650	0.4688
XXR_2	−2.0550	−2.2357	0.1857	−0.0906	0.4544
XY_2	0.7000	0.5842	0.1158	0.1654	**0.4379**
XZR_2	−1.0700	−2.2074	1.1374	−1.0630	0.8755
YZ_2	0.6500	0.5710	0.0790	0.1215	**0.5830**
ZZR_2	6.5500	7.5616	−1.0116	−0.1544	1.5245
MXR_2	4.3000	4.3138	−0.0138	−0.0032	0.0159
MY_2	0.6000	0.5967	0.0033	0.0056	0.0075
XXR_3	0.7634	0.5788	0.1846	0.2418	**0.4319**
XY_3	0.6872	0.6953	−0.0081	−0.0118	0.1364
XZ_3	0.5500	0.5252	0.0248	0.0450	0.1346
YZ_3	−0.6000	−0.7103	0.1103	−0.1839	0.2298
ZZ_3	0.9646	0.9366	0.0280	0.0290	0.0600
MX_3	0.5280	0.5351	−0.0071	−0.0135	0.0087
MY_3	1.1400	1.1516	−0.0116	−0.0102	0.0167
			$\delta\mathbf{W} = 0,\ \frac{\|\delta\mathbf{Y}\|}{\|\mathbf{Y}\|} = 7\ 10^{-3}$		

Table 8. ($r = 15$)

	$\delta\mathbf{W} = 0$					
k	$\frac{\|\delta\mathbf{Y}\|}{\|\mathbf{Y}\|}$	$\frac{\|\delta\mathbf{X}\|}{\|\mathbf{X}\|}$	$\kappa\frac{\|\delta\mathbf{Y}\|}{\|\mathbf{Y}\|}\frac{\|\mathbf{X}\|}{\|\delta\mathbf{X}\|}$	$\frac{\|\delta\mathbf{Y}\|}{\|\mathbf{Y}\|}$	$\frac{\|\delta\mathbf{X}\|}{\|\mathbf{X}\|}$	$\kappa\frac{\|\delta\mathbf{Y}\|}{\|\mathbf{Y}\|}\frac{\|\mathbf{X}\|}{\|\delta\mathbf{X}\|}$
0.001	7.9 10^{-4}	0.0047	**8.9**	4.25 10^{-4}	0.39	**33**
0.01	0.008	0.05	**8.6**	0.0055	5.35	**31**
0.1	0.076	0.53	**7.6**	0.055	53.55	**31**
	κ = Cond(**W**) = 53.27, **S** = 968			κ=Cond(**W**)=3.079 10^4, **S** = 6006		

Table 9. ($r = 15$)

	$\delta\mathbf{Y} = 0$					
k	$\frac{\|\delta\mathbf{W}\|}{\|\mathbf{W}\|}$	$\frac{\|\delta\mathbf{X}\|}{\|\mathbf{X}\|}$	Cond(**W** + δ**W**)	$\frac{\|\delta\mathbf{W}\|}{\|\mathbf{W}\|}$	$\frac{\|\delta\mathbf{X}\|}{\|\mathbf{X}\|}$	Cond(**W** + δ**W**)
0.001	0.0012	0.0057	53.30	4.7 10^{-4}	0.75	3.055 10^4
0.01	0.0231	0.0923	53.55	0.037	1.57	3.166 10^4
0.1	0.1697	0.7935	58.15	0.068	104.5	11.32 10^4
	Cond(**W**) = 53.27, **S** = 968			Cond(**W**) = 3.079 10^4, **S** = 6006		

Table 10. ($r = 15$)

k	$\frac{\|\delta \mathbf{Y}\|}{\|\mathbf{Y}\|}$	$\frac{\|\delta \mathbf{W}\|}{\|\mathbf{W}\|}$	$\frac{\|\delta \mathbf{X}\|}{\|\mathbf{X}\|}$	$\frac{\|\delta \mathbf{Y}\|}{\|\mathbf{Y}\|}$	$\frac{\|\delta \mathbf{W}\|}{\|\mathbf{W}\|}$	$\frac{\|\delta \mathbf{X}\|}{\|\mathbf{X}\|}$
0.001	$5.9\ 10^{-4}$	0.002	0.016	$6.8\ 10^{-4}$	$4.15\ 10^{-4}$	0.548
0.01	0.007	0.014	0.121	0.006	0.055	3.64
0.1	0.067	0.231	0.677	0.05	0.05	86
	$\text{Cond}(\mathbf{W}) = 53.27$, $\mathbf{S} = 968$			$\text{Cond}(\mathbf{W}) = 3.079\ 10^4$, $\mathbf{S} = 6006$		

Table 11. ($r = 15$)

	$\text{Cond}(\mathbf{W}) = 53.27$				
	X_i	$\hat{X}_i$	$X_i - \hat{X}_i$	$\frac{X_i - \hat{X}_i}{X_i}$	σX_i
ZZR_1	5.0186	5.0251	−0.0111	−0.0022	0.0155
XXR_2	−2.0500	−2.0768	0.0268	−0.0131	0.0180
XY_2	0.7000	0.6996	0.0004	0.0005	0.0162
XZR_2	−1.0700	−1.0707	0.0007	−0.0006	0.0056
YZ_2	0.6500	0.6455	0.0045	0.0069	0.0105
ZZR_2	6.5500	6.5733	−0.0233	−0.0036	0.0204
MXR_2	4.3000	4.3023	−0.0023	−0.0005	0.0019
MY_2	0.6000	0.6013	−0.0013	−0.0022	0.0012
XXR_3	0.7634	0.7830	−0.0195	−0.0256	0.0159
XY_3	0.6872	0.6778	0.0094	0.0136	0.0170
XZ_3	0.5500	0.5617	−0.0117	−0.0213	0.0076
YZ_3	−0.6000	−0.5891	−0.0109	0.0181	0.0127
ZZ_3	0.9646	0.9695	−0.0049	−0.0051	0.0166
MX_3	0.5280	0.5240	0.0040	0.0075	0.0027
MY_3	1.1400	1.1405	−0.0005	−0.0005	0.0014
	$\delta \mathbf{W} = \mathbf{0}$, $\frac{\|\delta \mathbf{Y}\|}{\|\mathbf{Y}\|} = 7.9\ 10^{-4}$				

Table 12. ($r = 15$)

	Cond(W) = 3 10^4				
	X_i	$\hat{X}_i$	$X_i - \hat{X}_i$	$\frac{X_i - \hat{X}_i}{X_i}$	σX_i
ZZR_1	5.0186	5.9268	−0.9083	−0.1810	**3.1098**
XXR_2	−2.0500	−0.9635	−1.0865	0.5300	**1.7504**
XY_2	0.7000	2.2263	−1.5263	−2.1804	**2.4889**
XZR_2	−1.0700	−1.4013	0.3313	−0.3097	0.2432
YZ_2	0.6500	−1.9471	2.5971	3.9956	**3.8134**
ZZR_2	6.5500	7.3205	−0.7705	−0.1176	0.6424
MXR_2	4.3000	4.2844	0.0156	0.0036	0.0206
MY_2	0.6000	0.6120	−0.0120	−0.0201	0.0138
XXR_3	0.7634	−0.9363	1.6997	2.2264	**5.0252**
XY_3	0.6872	0.3805	0.3067	0.4462	**0.3235**
XZ_3	0.5500	0.4340	0.1160	0.2108	**0.3292**
YZ_3	−0.6000	−0.1483	−0.4517	0.7529	**1.2149**
ZZ_3	0.9646	1.0737	−0.1091	−0.1131	0.2598
MX_3	0.5280	0.5247	0.0033	0.0062	0.0073
MY_3	1.1400	1.1519	−0.0119	−0.0105	0.0375
$\delta \mathbf{W} = \mathbf{0}, \ \frac{\|\delta \mathbf{Y}\|}{\|\mathbf{Y}\|} = 4.2\ 10^{-4}$					

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