

## TERMINAL-LINK PARAMETER ESTIMATION AND TRAJECTORY CONTROL OF ROBOTIC MANIPULATORS

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**Abstract.** This paper presents a manipulator terminal-link parameter estimation method and trajectory control technique based on the terminal-link parameter estimates. Although dynamic equations for serial-link manipulators are nonlinear, linear input-output equations concerning the unknown terminal-link parameters are developed. Unknown terminal-link parameters are estimated by an instrumental variable method based on these input-output equations. The theorem and simulation results demonstrate that the proposed instrumental variable method asymptotically yields consistent estimates. Moreover, the effectiveness of the computed torque technique based on the parameter estimates is verified by the experimental results obtained for a rotary manipulator with six degrees of freedom.

**Keywords.** Robots; manipulator; parameter estimation; trajectory control; torque control; identification.

### INTRODUCTION

Recently, a number of methods for the dynamic control of robotic manipulators (Luh, Walker, and Paul, 1980a, 1980b) have been proposed in order to ensure the trajectory control of manipulators. These include the "computed torque" technique and resolved acceleration control. All of these control schemes require an exact dynamic model of the manipulator. Naturally, when such dynamic models are inaccurate, these control schemes cannot generate effective manipulator terminal-link motion control in terms of high speed and precision.

The dynamic model for a serial-link manipulator is defined using geometric link parameters which define the homogeneous transformations between successive links, as well as the kinematic link parameters, mass, center of mass, inertia tensor matrix and friction torque, for each link. The geometric link parameter estimation method is given in Hayati (1983) as an example.

In general, the kinematic link parameters need to be measured or calculated. However, this is very difficult and necessitates considerable effort to obtain their exact values. One of the methods for solving this problem is presented by Kawasaki and Nishimura. However, estimating the parameters of the manipulator dynamic equations in real time is very difficult since the number of unknown parameters (66 for the manipulator having 6 degrees of freedom) is considerable and since the number of IVM computations may be excessive.

The manipulator may be operating under varying circumstances, for example, the payload of the objects or the end-effector may be changed when a new manipulation task is begun. In such cases, the kinematic parameters of the terminal link become unknown while other such parameters of the manipulator do not vary. Hence, if a terminal-link parameter variation estimation could be developed which is able to yield the exact estimates with less computational effort, it would find application to real-time trajectory control. This paper therefore presents a terminal-link manipulator. It also demonstrates the effectiveness of this estimation scheme through the com-

putational simulation and experimental results gained from using a manipulator having 6 degrees of freedom.

### COEFFICIENT PARAMETER ESTIMATION OF DYNAMIC EQUATION

#### Dynamic Equation

The dynamic equation for the serial-link manipulator with  $n$  joints is,

$$M(q, x)\ddot{q} + C(q, \dot{q}, x) + G(q, x) + F(\dot{q}, x) = \tau. \quad (1)$$

In Eq.(1),  $q = (q_1, q_2, \dots, q_n)^T$  is the vector representing the displacement of  $n$  joint variables,  $\dot{q}$  is the vector of joint velocities,  $\ddot{q}$  is the vector of joint accelerations,  $\tau = (\tau_1, \tau_2, \dots, \tau_n)^T$  is the vector of joint torques, and  $x$  is the unknown parameter vector which consists of the mass, center of mass, inertia tensor matrix, viscous damping friction coefficient and static friction torque. The first term  $M(q, x)\ddot{q}$ , in which  $M(q, x)$  is the  $n \times n$  generalized inertial matrix, indicates the inertial coupling. The vectors,  $C(q, \dot{q}, x)$ ,  $G(q, x)$  and  $F(\dot{q}, x)$  respectively contain the centrifugal and Coriolis, gravitational, and frictional torques at the joints. Since Eq.(1) is highly nonlinear, it is not well suited for obtaining the values of  $x$  from the motion data,  $q$ ,  $\dot{q}$ ,  $\ddot{q}$  and  $\tau$ .

The relationships between the kinematic link parameters and joint torques can be found in the recursive Newton-Euler formulation. For the purpose of our application, we provide an outline of the algorithm for a serial-link rotary manipulator. To formulate the dynamic equation for an  $n$  degree-of-freedom manipulator, the Denavit-Hartenberg convention defines the  $n+1$  coordinate systems (the  $i$ -th coordinate system being attached to the  $i$ -th link, beginning with link 0, the base, and ending with link  $n$ , the end-effector).

The Newton-Euler equations of motion for manipulators having all rotary joints are listed below.

Forward equation: for  $i = 1, 2, \dots, n$ ,

$$\omega_i = A_i^{i-1}(\omega_{i-1} + Z_0 \dot{q}_i) \quad (2)$$

$$\dot{\omega}_i = A_i^{i-1}(\dot{\omega}_{i-1} + Z_0 \ddot{q}_i + \omega_i \times Z_0 \dot{q}_i) \quad (3)$$

$$a_i = \sigma(r_i) + A_i^{i-1} a_{i-1} \quad (4)$$

$$\bar{a}_i = \sigma(s_i) + a_i \quad (5)$$

Backward equation: for  $i = n, n-1, \dots, 1$ ,

$$f_i = m_i a_i + A_i^{i+1} f_{i+1} \quad (6)$$

$$n_i = I_i \dot{\omega}_i + \omega_i \times (I_i \omega_i) + m_i (r_i + s_i) \times \bar{a}_i + r_i \times A_i^{i+1} f_{i+1} + A_i^{i+1} n_{i+1} \quad (7)$$

$$\tau_i = (A_i^{i-1} Z_0)^T n_i + F_i \text{sign}(\dot{q}_i) + d_i \dot{q}_i \quad (8)$$

where

$A_{i-1}^i$ : The rotation matrix that maps position vectors from the  $i$ -th coordinate system to the  $(i-1)$ th coordinate system

$\omega_i, \dot{\omega}_i$ : The angular velocity and accelerations of link  $i$

$a_i$ : The linear acceleration of link  $i$

$\bar{a}_i$ : The linear acceleration of the center of mass of link  $i$

$r_i$ : The origin of the  $i$ -th coordinate system with respect to the  $(i-1)$ th coordinate system

$s_i$ : The center of mass of link  $i$

$I_i$ : The inertia tensor matrix about the center of mass of link  $i$

$f_i, n_i$ : The force and moment exerted on link  $i$  by link  $(i-1)$  with respect to the  $i$ -th coordinate system

$F_i$ : The static friction torque

$d_i$ : The viscous damping coefficient

$\sigma_i(a)$ : The functional vector defined as  $\dot{\omega}_i \times a + \omega_i \times (\omega_i \times a)$

$Z_0$ : The vector defined as  $(0, 0, 1)^T$ .

#### Linear Input-Output Equation for Coefficient Parameters

When geometric link parameters and motion data are given, the coefficient parameters for the manipulator dynamic equation, which are composed from the kinematic link parameters, can be estimated by the instrument variable method (IVM).

This estimation scheme is based on the linear input-output equations for the unknown coefficient parameters of the manipulator dynamic equation. Four assumptions are made when estimating the coefficient parameters:

- 1) The geometric link parameters except joint variables are known exactly.
- 2) The actuator parameters are known exactly.
- 3) The joint variable and joint velocity measurement noises are negligible.
- 4) The joint acceleration measurement noises are white noises having a zero mean.

The unknown parameters are  $m_i, s_i, I_i, F_i$  and  $d_i$ . Let us define the following parameter vectors and matrix to describe the linear input-output equations for the coefficient parameters of the manipulator dynamic equation.

$$\alpha_i = m_i s_i + \sum_{j=1}^n m_j r_j, \quad (9)$$

$$\begin{aligned} \bar{I}_i &= I_i + m_i ((r_i + s_i)^T (r_i + s_i) \delta_3 \\ &\quad - (r_i + s_i)(r_i + s_i)^T) \\ &\quad + \sum_{j=i+1}^n m_j (r_i^T r_j \delta_3 - r_i r_j^T), \end{aligned} \quad (10)$$

$$\beta_i = (\bar{I}_{11}^i, \bar{I}_{22}^i, \bar{I}_{33}^i, \bar{I}_{12}^i, \bar{I}_{13}^i, \bar{I}_{23}^i), \quad (11)$$

$$x^i = (\alpha_i^T, \beta_i^T, F_i, d_i)^T, \quad (12)$$

and

$$x = ((x^1)^T, (x^2)^T, \dots, (x^n)^T)^T. \quad (13)$$

$\delta_i$  is an  $i \times i$  unit vector,  $\bar{I}_{jk}^i$  is the  $j$  column and  $k$  row element of  $\bar{I}_i$ . Parameter  $x$  is constant where all of the joints are completely rotational. Moreover, let us define the vectors and matrixes as

$$B_i = [\sigma_i(e_1), \sigma_i(e_2), \sigma_i(e_3)], \quad (14)$$

$$C_i = [e_1 \times A_i^{i-1} a_{i-1}, e_2 \times A_i^{i-1} a_{i-1}, e_3 \times A_i^{i-1} a_{i-1}], \quad (15)$$

$$D_i = \begin{bmatrix} \dot{\omega}_1^i & -\omega_2^i \omega_3^i & \omega_2^i \omega_3^i & \dot{\omega}_2^i - \omega_1^i \omega_3^i \\ \omega_1^i \omega_3^i & \dot{\omega}_2^i & -\omega_1^i \omega_3^i & \dot{\omega}_1^i + \omega_2^i \omega_3^i \\ -\omega_1^i \omega_2^i & \omega_1^i \omega_2^i & \dot{\omega}_3^i & (\omega_1^i)^2 - (\omega_2^i)^2 \\ \dot{\omega}_3^i + \omega_1^i \omega_2^i & (\omega_2^i)^2 - (\omega_3^i)^2 & \dot{\omega}_3^i - \omega_1^i \omega_2^i & \dot{\omega}_1^i - \omega_2^i \omega_3^i \\ \dot{\omega}_1^i - \omega_2^i \omega_3^i & \dot{\omega}_2^i + \omega_1^i \omega_3^i & \dot{\omega}_3^i - \omega_1^i \omega_2^i & \dot{\omega}_1^i + \omega_2^i \omega_3^i \end{bmatrix}, \quad (16)$$

$$R_{jj}^i = \begin{cases} A_j^{i-1} Z_0 & \text{if } j=i \\ 0 & \text{if } j < i \end{cases} \quad (17)$$

$$K_{jj}^i = \begin{cases} (A_j^{i-1} Z_0) \times \sum_{k=i}^{j-1} A_j^{k-1} r_k & \text{if } j > i \\ 0 & \text{if } j \leq i \end{cases}, \quad (18)$$

$$w_{ij} = \left( \begin{matrix} \dot{q}_j^i \times \ddot{q}_j^i \\ \ddot{q}_j^i \end{matrix} \right)^T \begin{bmatrix} B_i & 0 \\ C_i & D_i \end{bmatrix}, \quad \ddot{q}_j^i \text{sign}(\dot{q}_j), \quad \ddot{q}_j^i \dot{q}_j, \quad (19)$$

and

$$W = \begin{bmatrix} w_{11} & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ w_{ln} & \dots & \dots & w_{nn} \end{bmatrix}^T, \quad (20)$$



where  $e_i$  is a unit vector in which the  $i$ -th element is a unit,  $\hat{x}_i^j$  and  $\hat{z}_i^j$  are elements of  $\hat{x}_i$  and  $\hat{z}_i$  respectively, and  $\delta_{ij}$  is a Kronecker delta.

The relationships between unknown parameters and joint torques can then be described using the above defined functions as

$$\tau(t) = W(t) x \quad (21)$$

Since the elements of  $W(t)$  do not include the kinematic parameters, Eq.(21) implies the linear input-output equation concerning the unknown coefficient parameters of the dynamic equations for the manipulator.

#### Instrumental Variable Method

Let us use a tilde to distinguish the measurement values, including the calculated values which use measurement values, from the true values. The measured joint acceleration at the sampling time,  $t = j\Delta t$ , in which  $\Delta t$  is the sampling period, is then given as

$$\ddot{q}(j) = \ddot{q}(j) + \varepsilon(j) \quad (22)$$

where  $\varepsilon(j)$  is the white noise process having a zero mean. The joint torques can be calculated based on the actuator equations using measurement motion data,  $q(j)$ ,  $\dot{q}(j)$ ,  $\ddot{q}(j)$ , and the actuator input. Since the output torque of the actuator similar to that of a DC servomotor is proportional to joint acceleration, the measured joint torques can be expressed by

$$\tilde{\tau}(j) = \tau(j) + \xi(j) \quad (23)$$

where  $\xi(j)$  is the measurement noise process having a zero mean and is furthermore not correlative to  $\xi(i)$ , where  $(i \neq j)$ .

The values of  $W(j)$  can be calculated from Eqs.(2) ~ (5) and Eqs.(14) ~ (19). From Eq.(1),  $\tilde{W}(j)x$  is given as

$$\tilde{W}(j)x = W(j)x + M(q(j), x)\varepsilon(j) \quad (24)$$

Upon substitution of Eqs.(23) and (24), Eq.(21) becomes

$$\tilde{\tau}(j) = \tilde{W}(j)x + \varepsilon(j) \quad (25)$$

where

$$\varepsilon(j) = \xi(j) - M(q(j), x)\varepsilon(j) \quad (26)$$

Obviously,  $\varepsilon(j)$  is the zero mean and is not correlative to  $\varepsilon(i)$ , where  $(i \neq j)$ . However, it is correlative to  $\tilde{W}(j)$ . Therefore, the least square estimation method (LSM), which is very simple, does not yield the true values of the unknown parameters.

Let us define the following vectors and matrix to described the IVM:

$$y = (\tilde{\tau}^T(1), \tilde{\tau}^T(2), \dots, \tilde{\tau}^T(N))^T \quad (27)$$

$$A = (\tilde{W}^T(1), \tilde{W}^T(2), \dots, \tilde{W}^T(N))^T \quad (28)$$

$$\psi = (\varepsilon^T(1), \varepsilon^T(2), \dots, \varepsilon^T(N))^T \quad (29)$$

The input-output equations from sampling number 1 to  $N$  are

$$y = Ax + \psi \quad (30)$$

The optimum parameter estimates are determined by minimizing the criterion,

$$PI = E\{\hat{x}^T A A^T \hat{x}\} \quad (31)$$

where  $E\{\cdot\}$  is the expected value. The optimum parameter estimate is then

$$\hat{x} = (A^T A A^T A)^{-1} A^T A A^T y \quad (32)$$

where  $\hat{A}$  is the instrumental variable matrix defined as

$$\hat{A} = \text{quasi diag } (\hat{W}(1), \hat{W}(2), \dots, \hat{W}(N)) \quad (33)$$

with

$$\hat{W}(j) = W(q(j), \dot{q}(j), \ddot{q}(j)) \quad (34)$$

In Eq.(34),  $\ddot{q}(j)$  is the instrumental variable and is obtained from the instrumental model on the assumption that the dynamic equation of each link motion is linear and time-invariant. The dynamic equation is described as

$$\ddot{q}_i(j) + a_{i1}^0 \dot{q}_i(j-1) = b_{i1}^0 u_i(j) + b_{i1}^1 \dot{q}_i(j-1) \quad (35)$$

where  $u_i(j)$  is the actuator input of joint  $i$ , and  $a_{i1}^0, b_{i1}^0$  and  $b_{i1}^1$  are constant coefficients of the instrumental model. If  $u_i(j)$  does not contain  $\ddot{q}_i(j)$  and the coefficients of the instrumental model are determined from the previous rough information for the manipulator,  $\ddot{q}_i(j)$  is independent of  $\varepsilon(j)$  and is strongly correlative to  $\ddot{q}_i(j)$ . These conditions indicate that

$$(i) \lim_{N \rightarrow \infty} \frac{1}{N} (A^T A A^T A)^{-1} \text{ is non-singular,} \quad (36)$$

and that

$$(ii) \lim_{N \rightarrow \infty} \frac{1}{N} A^T A A^T \psi = 0 \quad (37)$$

Therefore,  $\hat{x}$  becomes an asymptotically consistent estimate as  $N \rightarrow \infty$ .

During actual calculation,  $\hat{x}$  can be obtained from the recursive parameter estimate algorithm. Notice that it is impossible to estimate some the unknown coefficient parameters, which do not influence joint torque. However, these parameters are not necessary for dynamic motion control of the manipulator or for the simulation of its dynamic motion.

#### Terminal-Link Parameter Estimation

When a manipulator grips an unknown object or the end-effector of the manipulator is exchanged, the kinematic parameters of the terminal link, which becomes a single body with the unknown object and the end-effector, change while the other link parameters do not. In such cases, an estimation method which can estimate the terminal-link parameter variations effectively and precisely is necessary for the control and motion simulation of the manipulator. This is because estimating terminal-link parameters with the IVM is not effective.

Let us denote the variation of a function,  $y$ , by  $\Delta y$ . The variations of joint torques are then described as

$$\Delta \tau(j) = W(j) \Delta x \quad (38)$$

It is possible to set the origin of the  $n$ -th coordinate system at the origin of the  $(n-1)$ th coordinate system without a loss of generality. This means that  $r_n = 0$ . From this condition,  $\Delta x_1 = 0$ ,  $\Delta x_2 = 0$  and finally  $\Delta x_i = 0$  for  $i \leq n-1$ . Hence, Eq.(38) becomes

$$\Delta \tau(j) = R(j) \Delta x^n, \quad (39)$$

where

$$R(j) = (w_{1n}, w_{2n}, \dots, w_{nn})^T. \quad (40)$$

The left-hand term of Eq.(39) may be computed as the difference between the output torques calculated from the actuator equations and the joint torque calculated from the recursive Newton-Euler equation using the nominal coefficient parameters. Even if the nominal coefficient parameters,  $x_{nomi}$ , are estimated by the IVM of Section 2, they do not equal  $x$  because of the finiteness of the sampling number. From Eqs.(23) ~ (26), the measurement variation torque becomes

$$\Delta \tau(j) = \Delta \tau(j) + \psi(j) + \tilde{W}(j) \delta x, \quad (41)$$

where

$$\delta x = x - x_{nomi} \quad (42)$$

The right-hand term of Eq.(39) is calculated as

$$\tilde{R}(j) \Delta x^n = R(j) \Delta x^n + M(q(j), x^*) \epsilon(j), \quad (43)$$

where

$$x^* = (0, \dots, 0, (x^n)^T)^T. \quad (44)$$

By substituting Eqs.(41) and (43), Eq.(39) is given as

$$\Delta \tau(j) = \tilde{R}(j) x^n + \psi(j) - \tilde{M}(q(j), x^*) \epsilon(j) + \tilde{W}(j) \delta x \quad (45)$$

The second, third and fourth terms of the right-hand side of Eq.(45) are equation errors. Since the fourth term of Eq.(45) is not a zero mean, the true parameter values can not be obtained using this equation via the IVM.

To solve this problem, not only the terminal-link parameter variation but also some parts of the nominal coefficient parameter errors must be estimated. That is, the unknown parameters,  $\Delta x^i$  (for  $k \leq i \leq n$ ), are estimated to obtain asymptotically consistent estimates of the terminal-link parameter variations from

$$\Delta \tau_i(j) = \sum_{k=i}^n w_{ik}^T(j) \Delta x^k \quad (46)$$

(for  $i=k, k+1, \dots, n$ ).

Let us define the following vectors and matrixes for the estimation:

$$P = \begin{bmatrix} p_{11} & p_{12} & \dots & p_{1n} \\ 0 & p_{22} & \dots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & p_{nn} \end{bmatrix}, \quad n-k+1, \quad (47)$$

$$x = (\Delta x^k)^T, (\Delta x^{k+1})^T, \dots, (\Delta x^n)^T)^T, \quad (48)$$

$$z = (\Delta \tau_k(1), \dots, \Delta \tau_k(N), \dots, \Delta \tau_n(N))^T, \quad (49)$$

$$\tilde{A} = (\tilde{W}^T(1)P^T, \tilde{W}^T(2)P^T, \dots, \tilde{W}^T(N)P^T)^T, \quad (50)$$

and

$$\tilde{v} = (\psi^T(1)P^T, \psi^T(2)P^T, \dots, \psi^T(N)P^T)^T, \quad (51)$$

where  $P$  is the weighting matrix and  $\psi$  is the measurement noise having a zero mean. The relationship between the unknown parameters

and the joint torque variations is expressed by

$$y = \tilde{A} x + \tilde{v}. \quad (52)$$

The optimum parameter estimates are determined by minimizing the criterion.

$$PI = E\{\tilde{v}^T \tilde{A} \tilde{A}^T \tilde{v}\}. \quad (53)$$

where

$$\tilde{A} = \text{quasi diag} (P\tilde{W}(1), P\tilde{W}(2), \dots, P\tilde{W}(N)). \quad (54)$$

As the conditions which are similar to those of Eqs.(36) and (37) are satisfied, the optimum parameter estimates equal the true values asymptotically as the sampling number increases. When  $k=1$ , this estimation coincides with the coefficient parameter estimated, as described in Section 2. When  $k=n$ , some parts of the terminal-link parameter variations, which influence the terminal joint, may be estimated. If the manipulator has six degrees of freedom, the number of computations of the parameter estimation method will be at least  $k=5$ , on the condition that all of the terminal-link parameter variation elements can be estimated.

#### COMPUTATION SIMULATION

Terminal-link parameter estimations were simulated numerically using a manipulator having six degrees of freedom as outlined in Fig. 1. The geometric link parameters of the manipulator are given in Table 1, while the kinematic link parameters of the manipulator are presented in Table 2. The simulation conditions are that

- o The true terminal parameter variations are  $\Delta a_6 = 60$ ,  $\Delta S_6 = (0.03, 0.03, 0)^T$ ,  $\Delta F_6 = 150$ ,  $\Delta d_6 = 0$  and  $\Delta \phi_6 = \text{diag}(0.3, 0.2, 0.3)$ .
- o The nominal coefficient parameters have random 10 % errors.
- o The standard deviation of joint acceleration measurement noise is  $\sigma = 10 \text{ rad/S}^2$ .
- o The unknown parameters are  $\Delta x^5$  and  $\Delta x^6$ , that is,  $k=5$ .
- o  $P = \begin{bmatrix} 0.01 & 0 \\ 0 & 1 \end{bmatrix}$ .
- o The motion trajectories are adequately preplanned.

Table 1 Geometric Mechanical Parameters

JOINT i	$a_i$ (m)	$d_i$ (m)	$\phi_i$ (deg.)
1	-0.011	0	90°
2	-0.08	0	0°
3	-0.0886	0	0°
4	0	0	90°
5	0	0.103	90°
6	0	0	0°

where,  
( $a_i$  = the distance between the coordinate systems  $i-1$  and  $i$  measured along  $x_i$ ,  $d_i$  = the distance between  $x_{i-1}$  and  $x_i$  measured along  $z_{i-1}$ , and  $\phi_i$  = the angle between the  $z_{i-1}$  and  $z_i$  axes measured about  $x_i$ .)



Table 2 Kinematic Link Parameters

JOINT $i$	$m_i$ (g)	$S_i$ (m)	$I_i$ (gm <sup>2</sup> )	$\ddot{\tau}_i$ (gm <sup>2</sup> /s <sup>2</sup> )	$d_i$ (gm/s)
1	800	(-0.04, -0.08, 0.0)	diag (10, 10, 10)	470	0
2	100	(-0.04, 0.02, 0.0)	diag (5, 5, 5)	1250	0
3	350	(-0.07, 0.035, 0.0)	diag (5, 5, 5)	1250	0
4	200	(-0.0, 0.0, 0.0)	diag (1, 1, 1)	450	0
5	50	(-0.01, -0.03, 0.0)	diag (0.5, 0.5, 0.5)	250	0
6	20	(0.01, 0.01, 0.0)	diag (0.1, 0.1, 0.1)	100	0

The simulation results using the LSM are shown in Fig. 2. The LSM obviously did not converge on the true value. The simulation results using the IVM are shown in Fig. 3. In this case, the parameter estimates converge with the true values, although the variances of the estimates were relatively large in the beginning.

EXPERIMENTAL

Several experiments were performed to study the effectiveness of the IVM. The geometric link parameters of the manipulator used in the experiment are the same as those in Table 1 for the simulation.

The experimental conditions were that the nominal coefficient parameters were experimentally obtained using the IVM in the no-payload condition, that the additional payload was 250 g, and that the motion data,  $\dot{q}$  and  $\ddot{q}$ , were respectively obtained by the first and second differentiations of  $\ddot{q}$ , which were measured from the pulse encoders. The other conditions were the same as those during the simulation.

The experimental parameter estimation results are given in Fig. 4. Parameter estimates using the LSM and IVM nearly converge to the same value. This implies that the acceleration measurement noises are very small. System responses via the computed torque technique based on the IVM parameter estimates are plotted in Fig. 5. For comparison, system responses from position and velocity feedback only are also shown. In this experiment, other joint responses which are not presented, have been driven along the trajectories similar to the desired trajectory of joint 3. The trajectory error of the computed torque technique is clearly smaller than that of the feedback control.

CONCLUSION

The terminal-link parameter estimation of a serial-link manipulator has been presented. The proposed estimation method was verified asymptotically to yield consistent estimates. The number of operations required to estimate the terminal-link parameters of a manipulator having six degrees of freedom is about 1800 multiplications and 1500 additions using the recursive IVM at  $k = 5$ . These estimation calculations will be realized in real time through the assignment of an exclusively used VLSI for the estimation

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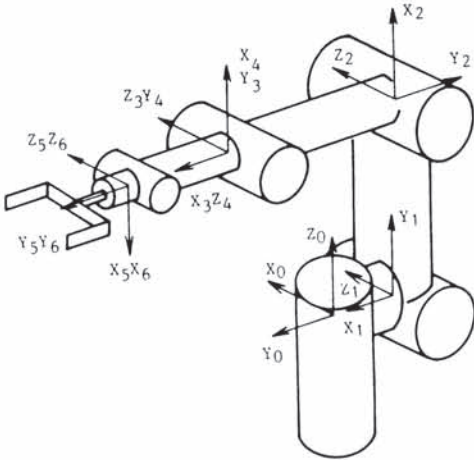


Fig. 1. Coordinate frame for manipulator having 6 degrees of freedom

