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Robust Calibration and Control of Robotic Manipulators

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Abstract

In this paper we present a robust approach for the estimation of the inertial parameters of a robotic manipulator. This approach takes into account the uncertainty in the regression model obtained via the Newton-Euler equations of the robot, and computes an ellipsoid of confidence for the unknown parameters using Semidefinite Programming (SDP). The so obtained information is then exploited in a robust control scheme for the manipulator. Results relative to a two-link planar manipulator are included in the paper.

Notation

Let $\mathcal{S}^n$ denote the set of real, symmetric matrices. For a square matrix X, $X \succ 0$ (resp. $X \succeq 0$) means X is symmetric, and positive-definite (resp. semidefinite). For given $\hat{x} \in \mathbb{R}^n$, and $P \in \mathcal{S}^n$, with $P \succ 0$, the notation $\mathcal{E}(P, \hat{x}) = \{x : (x - \hat{x})^T P^{-1}(x - \hat{x}) \leq 1\}$, denotes the ellipsoid of center $\hat{x}$ and "shape matrix" P. An alternative representation of the same ellipsoid is $\mathcal{E} = \{x : x = \hat{x} + Ez, \|z\| \leq 1\}$, with $E^T E = P$. Bold characters are used to indicate terms (vectors, matrices, etc.) that are affected by uncertainties.

1 Introduction

Robot calibration is a widely used procedure aimed at estimating the fundamental set of a manipulator dynamic parameters, from noisy experimental data. For this purpose, manipulator models which are linear in the unknown parameters are commonly used in the literature [13, 16]. These models are suitable for the application of linear regression techniques for the estimation of the parameter vector. However, these methods usually assume that the *regression matrix* is exactly known, a hardly verified assumption in robot calibration. In fact, the regression matrix is in this case built from noisy measurements, thus introducing *bias* in the parameter estimate [1, 18]. The effects of bias may be reduced to some extent by a suitable design of the calibration experiment, i.e. by a suitable choice of the reference trajectory [1, 14], but the fundamental problem remains.

In this paper we apply a *robust estimation* procedure [6, 11] to compute a *central* estimate, together with an *ellipsoid of confidence* for the manipulator parameters. The method takes into account both uncertainty on the measurements and uncertainty in the regression matrix. The additional information provided by the robust estimation procedure is then exploited to design a robust control scheme for the manipulator. The solution proposed by Spong in [17], considering a unique bound to measure the uncertainties affecting all the robot parameters (or different bounds for each of them), is extended in this paper to use the available information about the confidence ellipsoid for the parameter estimate, directly in the expression of the robust control law.

The paper is organized as follows. In Section 2, the robot regression model, suitable for the estimation of the inertial parameters, is obtained via the Newton-Euler formulation of the manipulator motion equations. Some recent results [6], related to the computation of a confidence ellipsoid for the solution set of uncertain linear equations, are briefly recalled in Section 3. A robust control algorithm, which extends the control scheme developed by Spong in [17], is proposed in Section 4, using the obtained information about the confidence ellipsoid for the parameters estimates. Finally, Section 5 is devoted to the application of the proposed robust calibration and control scheme to the case of a planar two-dof manipulator.

2 Robot calibration model

Let the kinematics of a n-dof manipulator be described according to Denavit-Hartenberg rules; let $q \in \mathbb{R}^n$ be the joint position vector, where each coordinate q_i represents the angular position of joint i, and $\tau \in \mathbb{R}^n$ the applied torques vector.

The motion equations of the manipulator can be determined following the recursive *Newton-Euler formulation*, by using the *forward* transformation law of the robot kinematics, from the base to the end–effector, and the *backward* computation of the joint torques from the end–effector to the base (see details in [4]). The so-obtained robot motion equations can be arranged in the following form:

$$\tau = D(q, \dot{q}, \ddot{q})\theta, \tag{1}$$

which is linear with respect to $\theta := [\theta_p^T \ \theta_f^T]^T \in \mathbb{R}^{n_p}$, where $\theta_p \in \mathbb{R}^p$ is the base parameter vector, i.e. the vector containing the p identifiable dynamic parameters of the robot (see [13] for details); $\theta_f \in \mathbb{R}^f$ is the friction parameter vector, as friction can be generally described as $\tau_f = F(q, \dot{q})\theta_f$, where $\tau_f \in \mathbb{R}^n$ is the friction torque vector, while the expression of $F(q, \dot{q})$ and the choice of the parameter vector θ_f depend on the considered friction model [2]. The matrix $D(q,\dot{q},\ddot{q}) \in \mathbb{R}^{n\times n_p}$ is a nonlinear function of joint position, velocity and acceleration vectors.

Relation (1) can be used for the estimation of θ by means of a Least-Squares algorithm [15], by collecting the values of τ, q, $\dot{q}$, $\ddot{q}$ (by measurements and/or reconstructed via software) at n_m time instants (with $n_m n > n_p$) [1], [4], [14], [18]. The torque data and the regression matrix D are affected by uncertainties, due to measurement noise and to computational errors in the reconstruction of the not directly available signals. One of the most common way to deal with such uncertainties consists in the insertion of a zero mean noise vector in equation (1), which is uncorrelated from θ and collects all the uncertainties effects, while the data matrix D is assumed to be deterministic [4].

In this paper, the estimation of the inertial parameter vector θ is performed by taking directly into account uncertanties in the regression matrix and in the torque data, on the basis of some recent results [6] related to the computation of a confidence ellipsoid for the solution set of uncertain linear equations, briefly recalled in the next section.

3 Confidence ellipsoid for uncertain linear equations

The results developed in [6] for the computation of a confidence ellipsoid for the solution set of uncertain linear equations, are briefly recalled hereafter, to show how they can be applied to solve equation (1) for the estimation of a robot inertial parameters.

In [6], the authors consider the problem of computing an ellipsoidal outer approximation for the set of solutions, if any, to the "uncertain linear equation"

$$\mathbf{A}x = \mathbf{b}, \;\; [\; \mathbf{A} \;\; \mathbf{b} \;] \in \mathcal{U}, \tag{2}$$

where $\mathcal{U}$ is a given subset of $\mathbb{R}^{n\times(m+1)}$, described in Linear Fractional form (LFR)

$$\mathcal{U} := \left\{ [A \quad b] + L\Delta\,(I - H\Delta)^{-1}\,[R_A \quad R_b],\; \Delta \in \mathbf{\Delta} \right\}, \tag{3}$$

where $A \in \mathbb{R}^{n\times m}$, $b \in \mathbb{R}^n$, $L \in \mathbb{R}^{n\times n_p}$, $R_A \in \mathbb{R}^{n_q\times m}$, $R_b \in \mathbb{R}^{n_q}$, and $H \in \mathbb{R}^{n_q\times n_p}$ are given, and $\mathbf{\Delta}$ is a subset of $\mathbb{R}^{n_p\times n_q}$. The use and motivation of this LFR description of the uncertainty is detailed also in [7, 8, 11]. The solution ellipsoid $\mathcal{E} = \{x :\; x = \hat{x} + Ez,\; \|z\| \le 1\}$, of size minimal in a certain geometrical sense (the sum of squared semi-axes lenghts is minimized), is computed via Semidefinite Programming (SDP), a particular class of convex optimization problems for which efficient solution algorithms are available [9, 19].

3.1 Confidence ellipsoid for LS solutions

For the estimation problem in this paper, we need to compute a confidence ellipsoid for the set $\mathcal{X}$ of Least-Squares solutions to the equation (2),

$$\mathcal{X} = \{x : \|\mathbf{A}x - \mathbf{b}\| \text{ is minimal, and } [\; \mathbf{A} \;\; \mathbf{b} \;] \in \mathcal{U}\}.$$

This problem is reduced to the previous problem, setting $\tilde{\mathbf{A}} = \mathbf{A}^T\mathbf{A}$ and $\tilde{\mathbf{b}} = \mathbf{A}^T\mathbf{b}$. The LFR's of the new elements $\tilde{\mathbf{A}}, \tilde{\mathbf{b}}$ may be constructed from the original LFR's of $\mathbf{A}, \mathbf{b}$ using the LFR product rule: let

$$\mathbf{M}_i(\Delta_i) = M_i + L_i\Delta_i(I - H_i\Delta_i)^{-1}R_i,\; i = 1, 2,$$

then the product $\mathbf{M} = \mathbf{M}_1\mathbf{M}_2$ may be expressed in LFR form as

$$\mathbf{M}(\widetilde{\Delta}) = M + L\widetilde{\Delta}(I - H\widetilde{\Delta})^{-1}R, \tag{4}$$

where

$$M = M_1M_2,\; L = [\; L_1 \quad M_1L_2 \;],\; R = \begin{bmatrix} R_1M_2 \\ R_2 \end{bmatrix},$$

$$H = \begin{bmatrix} H_1 & R_1L_2 \\ 0 & H_2 \end{bmatrix},\; \widetilde{\Delta} = \mathbf{diag}\,(\Delta_1,\; \Delta_2).$$

In the experimental estimation problem we will make the further assumption that $\mathbf{A}$ is full-rank for any value of the uncertainty. This condition may be checked via a sufficient LMI condition [3, 10].

Given the above structure for the uncertainty, the results of [6] may be directly applied to determine a central estimate θ_0 and its ellipsoid of confidence $\mathcal{E}$, as reported in Section 5. In the next section we propose a robust control scheme which incorporates the information provided by the robust estimate.

4 Robust robot control

Let the motion equations (1) of the robot be rewritten in the following well-known form:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = D(q,\dot{q},\ddot{q})\theta = \tau, \tag{5}$$

in which $M(q)$ is the inertia matrix, $C(q,\dot{q})\dot{q}$ are the centrifugal and Coriolis torques, and $g(q)$ is the gravitational term (for the considered *planar* manipulator $g(q) = 0$).

According to the control scheme developed by Spong in [17], the following "nominal" control vector τ_0 is defined as

$$\tau_0 = M_0(q)a + C_0(q,\dot{q})v + g_0(q) - Kr = D(q,\dot{q},v,a)\theta_0 - Kr, \tag{6}$$

with $M_0(\cdot)$, $C_0(\cdot,\cdot)$, and $g_0(\cdot)$ computed plugging the central estimate θ_0 into their expressions, and

$$v = \dot{q}_d - \Lambda e,\;\; a = \dot{v},\;\; r = \dot{e} + \Lambda e,\;\; e = q - q_d, \tag{7}$$

being q_d a given twice continuously differentiable reference trajectory, and being $K \succ 0$, $\Lambda \succ 0$ diagonal matrices.

The complete control τ is then defined, on the basis of the nominal one, as:

$$\tau = \tau_0 + D(q,\dot{q},v,a)u, \tag{8}$$

where u is a new control input, which will be defined in order to guarantee the uniformly ultimately boundedness of the closed-loop system, by using the available information about the confidence ellipsoid of the estimate θ_0, employed in the nominal control input.

In [17], Spong designed u under the assumption of the existence of $\theta_0 \in \mathbb{R}^{n_p}$ and $\rho \in \mathbb{R}_+$, *both known*, such that:

$$\|\theta - \theta_0\| \leq \rho, \tag{9}$$

where θ_0 is the estimate of θ used to define τ_0. In order to avoid an overly conservative design, due to the use of a unique bound ρ to measure all the parameter uncertainties, Spong suggested a modification in the control law, which employed a different measure ρ_i of uncertainty for each parameter θ_i, under the assumption that every ρ_i is known.

In the subsequent developments, we will design a control law for u, which extends Spong's results, by using the information about the estimate confidence ellipsoid, obtained by the previously developed estimation procedure.

According to the results of this procedure, the confidence ellipsoid $\mathcal{E}(P, \theta_0)$ previously determined, corresponds to parameter vectors θ such that:

$$\theta = \theta_0 + Ez, \quad \|z\| \leq 1, \tag{10}$$

with $P = E^T E \succ 0$.

It can be easily seen that equations (5), (6), (8) lead to the following equation:

$$\dot{r} = -M(q)^{-1}(K + C(q,\dot{q}))r + M(q)^{-1}D(q,\dot{q},v,a)(\tilde{\theta} + u), \tag{11}$$

with $\tilde{\theta} := \theta - \theta_0 = Ez$, $\|z\| \leq 1$, which can be rewritten as:

$$\dot{r} = f(q,\dot{q},r) + \mathcal{N}u + \mathcal{N}Ez, \tag{12}$$

where:

$$f(q,\dot{q},r) \triangleq -M(q)^{-1}\left(K + C(q,\ \dot{q})\right)r, \tag{13}$$

$$\mathcal{N} \triangleq M(q)^{-1}D(q,\ \dot{q},\ v,\ a). \tag{14}$$

The following theorem then holds:
Theorem 1. Let the command input u be defined as:

$$u(r,t) = \begin{cases} -\dfrac{E(D(q,\dot{q},v,a)E)^T r}{\|(D(q,\dot{q},v,a)E)^T r\|}, & \text{if } \|(D(q,\dot{q},v,a)E)^T r\| > \varepsilon, \\ -\dfrac{E(D(q,\dot{q},v,a)E)^T r}{\varepsilon}, & \text{if } \|(D(q,\dot{q},v,a)E)^T r\| \leq \varepsilon, \end{cases} \tag{15}$$

with $\varepsilon > 0$. Then the control law (8) is continuous, and the closed-loop system (11), (15) is unformly ultimately bounded as defined in [5].

The proof of Theorem 1 follows directly from [5] and [17]. In particular, the control law developed by Spong in [17] can be determined as a particular case of (15) assuming $P = \rho^2 I$, i.e. when a confidence sphere with radius ρ is considered for the estimate vector θ_0.

5 Robust calibration and control of a planar 2-dof manipulator

In this section we will compute from experimental data an ellipsoid of confidence for the inertial parameters of a SCARA two-link planar manipulator produced by IMI (*Integrated Motions Inc.*, Berkeley, USA). The considered manipulator has two revolute joints equipped with NSK brushless motors, incorporating resolvers for position measurements. The control unit of the robot is constituted by a Pentium-90 PC with a TMS320C30 DSP board, and the control algorithms are implemented in C. The joint coordinates are defined according to Denavit-Hartenberg notation, and collected in vector $q := [q_1\ q_2]^T$, where q_i represents the angular position of joint i; the command torque vector is defined as $\tau := [\tau_1\ \tau_2]^T$, where τ_i is the torque applied to joint i; the lengths of the links are respectively given by: $l_1 = 0.359$ m and $l_2 = 0.24$ m. A schematic model of the robot is shown in Figure 1.

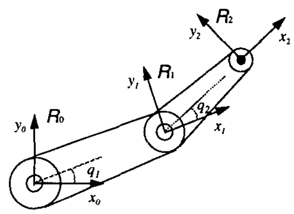

Figure 1: Schematical model of the IMI manipulator.

The control of the two-link robot is performed by an independent joint feedback control law, which is implemented by inserting a high-gain (not model-based) observer for the estimation of the joint velocities, as only position measurements are available. The joint accelerations are computed via software from the velocities, after the insertion of a discrete time filter; the torques are reconstructed by their designed expression, and filtered on-line, using the same discrete time filter employed for the velocities. The interested reader can find more details about this matter in [4]. The motion equations of the manipulator can be determined by introducing a base inertial parameter set, constituted by four parameters: the inertia moments of the links Γ_{1z}, Γ_{2z}, with respect to the z_0-axis (i.e. the axis perpendicular to the motion plane), and the first order moments $m_2 s_{2x}$, $m_2 s_{2y}$ of the second link (m_2 is the mass of the link, while s_{2x} and s_{2y} are the coordinates of its center of mass).

By modelling only Coulomb and viscous friction, under the assumption that other components (e.g. hysteresis effects and/or stiction) can be considered as negligible, the friction torque acting on each joint can be described as:

$$\tau_{f,i}(\dot{q}_i) = \sigma_{s,i}\mathrm{sgn}(\dot{q}_i) + \sigma_{v,i}\dot{q}_i, \tag{16}$$

where the parameters $\sigma_{s,i}$ and $\sigma_{v,i}$ represent Coulomb dynamic friction and the viscous friction coefficient, respectively. The complete parameter vector $\theta \in \mathbb{R}^8$ is then given by:

$$\theta = \left[\Gamma_{1z} + m_2 l_1^2\ \ m_2 s_{2x}\ \ m_2 s_{2y}\ \ \Gamma_{2z}\ \ \sigma_{s,1},\ \ \sigma_{v,1},\ \ \sigma_{s,2},\ \ \sigma_{v,2}\right]^T.$$

Following the Newton-Euler approach, the robot dynamic model is then determined as in (1), with $D_{1,1} = \ddot{q}_1$, $D_{1,2} = l_1(c_2(2\ddot{q}_1 + \ddot{q}_2) - s_2\dot{q}_2(2\dot{q}_1 + \dot{q}_2))$, $D_{1,3} =$

$l_1(-s_2(2\ddot{q}_1 + \ddot{q}_2) - c_2\dot{q}_2(2\dot{q}_1 + \dot{q}_2))$, $D_{1,4} = \ddot{q}_1 + \ddot{q}_2$, $D_{1,5} = \mathrm{sgn}(\dot{q}_1)$, $D_{1,6} = \dot{q}_1$, $D_{1,7} = D_{1,8} = 0$, $D_{2,1} = 0$, $D_{2,2} = l_1(c_2\ddot{q}_1 + s_2\dot{q}_1^2)$, $D_{2,3} = l_1(-s_2\ddot{q}_1 + c_2\dot{q}_1^2)$, $D_{2,4} = \ddot{q}_1 + \ddot{q}_2$, $D_{2,5} = D_{2,6} = 0$, $D_{2,7} = \mathrm{sgn}(\dot{q}_2)$, $D_{2,8} = \dot{q}_2$ (with $s_i := \sin q_i$ and $c_i := \cos q_i$). The joint reference trajectories shown in Figure 2, which were determined in order to properly "excite" the robot dynamics (see [4] for details), have been used to collect data for the subsequent developments. As previously explained, the torque data and the regression matrix D are affected by uncertainty due to noisy measurements (or computational errors) of q, $\dot{q}$, $\ddot{q}$; in order to *highlight such uncertainties*, the regression model of the manipulator can be rewritten as $\tau(\delta) = D(\delta)\theta$. In particular, the "true value" of the acquired data is assumed to lie in an interval: $\tau_1 = \tau_{10} + \delta_{\tau_1}^{\max}\delta_1$, $\tau_2 = \tau_{20} + \delta_{\tau_2}^{\max}\delta_2$, $q_1 = q_{10} + \delta_1^{\max}\delta_3$, $q_2 = q_{20} + \delta_2^{\max}\delta_4$, $\dot{q}_1 = \dot{q}_{10} + \dot{\delta}_1^{\max}\delta_5$, $\dot{q}_2 = \dot{q}_{20} + \dot{\delta}_2^{\max}\delta_6$, $\ddot{q}_1 = \ddot{q}_{10} + \ddot{\delta}_1^{\max}\delta_7$, $\ddot{q}_2 = \ddot{q}_{20} + \ddot{\delta}_2^{\max}\delta_8$, where $\tau_0 = [\tau_{10},\ \tau_{20}]^T$, $q_0 = [q_{10},\ q_{20}]^T$, $\dot{q}_0 = [\dot{q}_{10},\ \dot{q}_{20}]^T$, $\ddot{q}_0 = [\ddot{q}_{10},\ \ddot{q}_{20}]^T$ are the measured values, $\delta_{\tau_i}^{\max}$, $\delta_i^{\max}$, $\dot{\delta}_i^{\max}$, $\ddot{\delta}_i^{\max}$ represent the confidence bounds on the measurements, and $|\delta_i| \leq 1$. These bounds have been experimentally estimated (see [4] for further details on the experimental setup) as $\delta_{\tau_1}^{\max} = 4.5744$ Nm, $\delta_{\tau_2}^{\max} = 0.5459$ Nm, $\delta_1^{\max} = 0.0529$ rad, $\delta_2^{\max} = 0.0566$ rad, $\dot{\delta}_1^{\max} = 0.0825$ rad/s, $\dot{\delta}_2^{\max} = 0.0849$ rad/s, $\ddot{\delta}_1^{\max} = 0.5892$ rad/s^2, $\ddot{\delta}_2^{\max} = 1.6478$ rad/s^2.

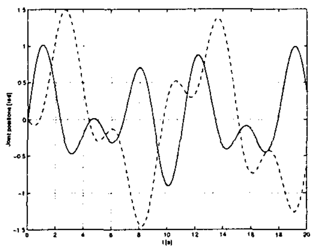

Figure 2: *Joint reference trajectories. Solid line: first joint; dashed line: second joint.*

To reduce the size of the problem, it is reasonable to neglect the uncertainties on positions and velocities in comparison with those on torques and accelerations; the total number of normalized uncertain parameters is therefore $N = 4n_m$,

$$\delta^T = [\delta_1^{(1)}, \ldots, \delta_1^{(4)}, \delta_2^{(1)}, \ldots, \delta_2^{(4)}, \cdots, \delta_{n_m}^{(1)}, \ldots, \delta_{n_m}^{(4)}] \in \mathbf{R}^{4n_m}, \tag{17}$$

being $\delta_i^{(j)}$ the j-th uncertain parameter at the i-th measurement, and being n_m the number of samples acquired over the manipulator reference trajectory. We then compute a linearized model of the uncertainty in the form

$$D(\delta) = D_0 + \sum_{k=1}^{n_m}\sum_{j=1}^{4} \delta_k^{(j)} D_k^{(j)}; \quad \tau(\delta) = \tau_0 + \sum_{k=1}^{n_m}\sum_{j=1}^{4} \delta_k^{(j)} \tau_k^{(j)},$$

where $D_0 = D(0)$, and $D_k^{(j)} = \frac{\partial D(\delta)}{\partial \delta_k^{(j)}}$, $\tau_k^{(j)} = \frac{\partial \tau(\delta)}{\partial \delta_k^{(j)}}$. The above uncertainty model can then be set in LFR form as

$$\mathbf{M}(\delta) = [\ D_0 \quad \tau_0\ ] + L\Delta[\ R_D \quad R_\tau\ ], \tag{18}$$

with $\Delta \in \mathbf{R}^{p,q}$,

$$\Delta = \mathbf{diag}\,(\delta_1^{(1)} I_{r_1^{(1)}}, \ldots, \delta_1^{(4)} I_{r_1^{(4)}}, \ldots, \delta_{n_m}^{(1)} I_{r_{n_m}^{(1)}}, \ldots, \delta_{n_m}^{(4)} I_{r_{n_m}^{(4)}}),$$

$r_k^{(j)} = \mathbf{rank}(D_k^{(j)})$, $D_0 \in \mathbf{R}^{2n_m,m}$, $\tau_0 \in \mathbf{R}^{2n_m}$, $M_0 = [\ D_0 \quad \tau_0\ ]$, $L \in \mathbf{R}^{2n_m,p}$, $R_D \in \mathbf{R}^{q,m}$, $R_\tau \in \mathbf{R}^{q,1}$ and $R = [\ R_D \quad R_\tau\ ]$. Using the product rule for LFR, we determine the LFR of $\widehat{\mathbf{M}}(\delta) = \mathbf{D}(\delta)^T\mathbf{D}(\delta)$ which is needed for the application of the results in [6]:

$$\widehat{\mathbf{M}}(\delta) = \widehat{M}_0 + \widehat{L}\widehat{\Delta}(I - \widehat{H}\widehat{\Delta})\widehat{R},$$

with $\widehat{\Delta} = \begin{bmatrix} \Delta & 0 \\ 0 & \Delta \end{bmatrix}$, $\widehat{M}_0 = D_0^T [\ D_0 \quad \tau_0\ ]$, $\widehat{L} = [\ R^T \quad D_0^T L\ ]$, $\widehat{R} = [\ (L^T M_0)^T \quad R^T\ ]^T$, $\widehat{H} = \begin{bmatrix} 0 & L^T L \\ 0 & 0 \end{bmatrix}$.

5.1 Numerical results

Using the model developed in the previous section, we solved the robust estimation problem via the algorithm detailed in [6]. The central estimate and the ellipsoid of confidence are computed as the solution of a Semidefinite Programming problem; the implementation is based on the Matlab package `lmitool` and has been developed by the authors of [6].

We considered $n_m = 5$ samples along the trajectory, and obtained the following results for the confidence ellipsoid

$$P = \begin{bmatrix} 1.62 & 2.84 & 1.56 & 0.1 & -1.13 & 4.03 & -0.13 & 0.06 \\ 2.84 & 9.06 & 5.31 & 0.19 & -1.38 & 2.75 & -0.38 & -0.08 \\ 1.56 & 5.31 & 3.95 & 0.05 & -0.57 & 0.39 & -0.11 & -0.49 \\ 0.1 & 0.19 & 0.05 & 0.03 & -0.06 & 0.22 & -0.02 & 0.04 \\ -1.13 & -1.38 & -0.57 & -0.06 & 3.66 & -9.49 & 0.07 & -0.08 \\ 4.03 & 2.75 & 0.39 & 0.22 & -9.49 & 32.2 & -0.22 & 0.56 \\ -0.13 & -0.38 & -0.11 & -0.02 & 0.07 & -0.22 & 0.26 & -0.32 \\ 0.06 & -0.08 & -0.49 & 0.04 & -0.08 & 0.56 & -0.32 & 0.74 \end{bmatrix},$$

$\theta_0 = [4.69,\ -0.4,\ 0.65,\ -0.09,\ 1.4,\ 12.12,\ 1.9,\ -0.24]^T$.

Figure 3 compares the measured torques (dots) with the reconstructed ones, obtained plugging the central estimate θ_0 into the manipulator model. The estimate which is satisfactory in terms of torque reconstruction, could be improved by taking into account a larger number of samples. The information provided by the robust estimate (central estimate and ellipsoid of confidence on the parameters) will be exploited for robust control of the manipulator in the following section.

5.2 Simulations of the robust control scheme

In the preliminary simulation tests, whose results are reported in this section, the considered reference trajectory for both joints is simply given by a step function, smoothed by a second order filter (see Figure 4). In the first test, whose results are reported in Figure 5, only the *nominal command input* (6), is applied to the robot, assuming θ_0 given by the central estimate previously obtained from the experimental data. In the second test (Figure 6), the robust control scheme (15) is applied,

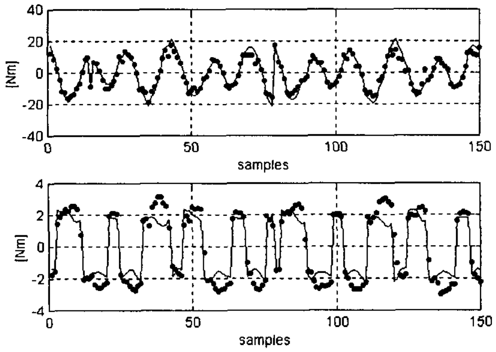

Figure 3: Measured and reconstructed (solid line) torque using the central estimate, with *only* 5 samples.

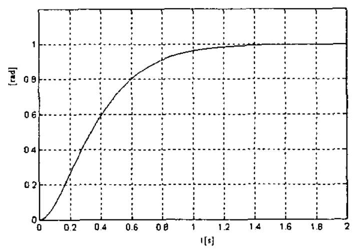

Figure 4: Joint reference.

with $E = P^{1/2}$ corresponding to the computed confidence ellipsoid $\mathcal{E}(P, \theta_0)$, and $\varepsilon = 1$ (the choice of ε has been made as trade-off, to reduce the ultimate tracking error and to avoid the command chattering).

Comparing the results of the two tests, it can be seen that the insertion of the robust control input has significantly reduced the joint tracking errors during the transient time; the price paid for such reduction is given by a longer settling time of the system response.

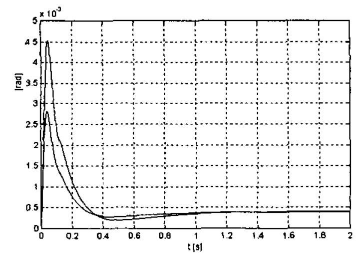

Figure 5: Simulation results. Joint tracking errors with only the nominal command input.

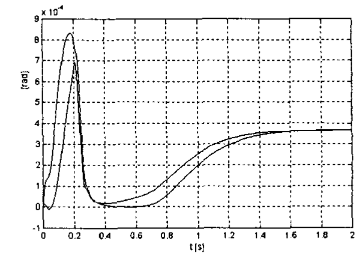

Figure 6: Simulation results. Joint tracking errors with the robust control scheme.

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