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Technical note

Dynamic formulation of redundant and nonredundant parallel manipulators for dynamic parameter identification

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ABSTRACT

Utilizing the virtual work principle, this paper presents a method for the dynamic formulation of redundant and non-redundant parallel manipulators for dynamic parameter identification. In modeling, the selection of pivotal point and the computation of inertia force and moment about the pivotal point are more crucial. The selection principle of pivotal point and force transmission on a rigid body are studied. In order to validate the method, the linear form of dynamic models of a 3-DOF parallel manipulator with actuation redundancy and its corresponding non-redundant parallel manipulator are derived.

1. Introduction

It is well known that identification of dynamic parameters of a manipulator forms an essential criterion for effective control of the manipulator [1,2]. In dynamic parameter identification, it is more crucial to find the parameter linear form of the dynamic equation [3,4]. Due to the complex and coupled dynamics of parallel manipulators, the task still remains challenging. Although dynamic modeling of parallel manipulators have been studied extensively by using Langrange equation, Newton–Euler method and virtual work principle, it is difficult to rewrite the dynamic model into the linear form. Most applications for dynamic parameter identification use very simplified models, where only the dynamics of the moving platform is considered commonly [5], such that it is not practicable for all systems.

There are a few papers on the dynamics in linear form of parallel manipulators. For example, Cheng et al. [6] used the tree structure method to derive the linear dynamic model of a redundant parallel manipulator. However, the moving platform of the parallel manipulator in their study is simplified as a point such that the method is not effective for all parallel manipulators. It would be cumbersome for spatial parallel robots. Grotjahn et al. [7] used Newton–Euler equation in combination with Jourdain's principle to derive the linear dynamic model of a Stewart–Gough platform. Their study focused on the non-redundant parallel robot. Mata [8] employed Gibbs–Appell equation to derive the dynamic model of an industrial robot for experimental identification of inertial parameters. This method would be complex for parallel manipulators. There is no a general method that can be used to derive the linear dynamic model of both redundant and non-redundant parallel manipulator.

In this paper, a new method for finding the dynamic formulation in an explicit linear form is proposed. The method is suitable for the redundant and non-redundant parallel manipulator. In modeling, the selection of pivotal point and the computation of inertia force and moment about the pivotal point are more crucial. By utilizing the method, the dynamic equations in linear form of a 3-DOF redundantly actuated parallel manipulator [9,10] and its corresponding non-redundant parallel manipulator are derived, respectively.

2. Formulation of inverse dynamics

The computational procedure of inverse dynamics of redundant and nonredundant parallel manipulators for the application of dynamic parameter identification can be proposed as

(1) Based on the kinematic analysis, calculating the position, velocity and acceleration of each link of the parallel manipulator.
(2) Compute the inertia force and torque of each link about its mass center by utilizing the Newton–Euler equation.
(3) Select a pivotal point on each link that possessed the simplest form such that the partial velocity matrix and partial angular velocity matrix do not include the base dynamic parameters.
(4) The inertia force and torque about the mass center is equivalent to the pivotal point.

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(5) Choose a set of generalized coordinates to describe the manipulator system and then determine the Jacobian matrix that maps the generalized speeds to actuated joint speeds, the partial angular velocity matrix of each link, and the partial velocity matrix of the pivotal point.
(6) Apply the virtual work principle to derive the dynamic equations of motion.
(7) By extracting the base dynamic parameters, the dynamic equation is transferred to linear format with respect to the base dynamic parameters.

2.1. Partial velocity and partial angular velocity matrices

For a parallel manipulator with n DOF, n generalized coordinates are needed to specify the system completely. Then, the Jacobian matrix that maps the joint velocity to the generalized end-effector velocity can be determined.

Based on the Jacobian matrix, the partial velocity matrix [11] can be expressed as

$$\boldsymbol{H}_i = \begin{bmatrix} \frac{\partial \dot{q}_1}{\partial \boldsymbol{v}} & \frac{\partial \dot{q}_2}{\partial \boldsymbol{v}} & \cdots & \frac{\partial \dot{q}_n}{\partial \boldsymbol{v}} \end{bmatrix}^{\mathrm{T}}, \tag{1}$$

where $\dot{q}_n$ is the velocity of the nth active joint, and $\boldsymbol{v}$ is the end-effector velocity.

Accordingly, the partial angular velocity matrix can be expressed as

$$\boldsymbol{G}_i = \begin{bmatrix} \frac{\partial \omega_1}{\partial \boldsymbol{v}} & \frac{\partial \omega_2}{\partial \boldsymbol{v}} & \cdots & \frac{\partial \omega_n}{\partial \boldsymbol{v}} \end{bmatrix}^{\mathrm{T}}, \tag{2}$$

where ω_n is the angular velocity of the nth link.

2.2. Force transmission on a rigid body

In general, the inertia force and torque about the mass center is easy to be derived by using the Newton–Euler equation. It is complex to obtain them about a random point. In this section, the rule of force transmission on a rigid body is derived.

Let a coordinate system $\mathcal{C}$ be attached to the mass center C of a body. In general, it is easy to compute the inertia force at the mass center of a rigid body. If the position of the mass center is not known, we can choose an arbitrary coordinate system $\mathcal{A}$ attached to point A of the body and express the inertia force relatively to this coordinate system. A wrench $\boldsymbol{F}_C$ applied to the mass center C and described with respect to coordinate system $\mathcal{C}$ on a rigid body gives the following contribution at point A with respect to coordinate system $\mathcal{A}$

$${}^{\mathcal{A}}\boldsymbol{F}_A = \boldsymbol{J}_r^{\mathrm{T}\,\mathcal{C}}\boldsymbol{F}_C, \tag{3}$$

where $\boldsymbol{J}_r^{\mathrm{T}}$ is a wrench transmission matrix defined as

$$\boldsymbol{J}_r^{\mathrm{T}} = \begin{bmatrix} {}^{\mathcal{A}}_{\mathcal{C}}\boldsymbol{R} & 0 \\ {}^{\mathcal{A}}\hat{\boldsymbol{r}} & {}^{\mathcal{A}}_{\mathcal{C}}\boldsymbol{R} \end{bmatrix} \tag{4}$$

in which

$$\hat{\boldsymbol{r}} = \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix} \tag{5}$$

is a skew symmetric matrix corresponding to the cross-product with the vector $\boldsymbol{r}$ from point A to C, ${}^{\mathcal{A}}_{\mathcal{C}}\boldsymbol{R}$ is the rotation matrix between coordinate systems $\mathcal{A}$ and $\mathcal{C}$.

The inertia force ${}^{\mathcal{C}}\boldsymbol{F}^{C}$ acting at the mass center of the body is calculated using the following equation:

$${}^{\mathcal{C}}\boldsymbol{F}_C = \begin{bmatrix} m\,{}^{\mathcal{C}}\boldsymbol{a}_C \\ {}^{\mathcal{C}}\boldsymbol{I}_C\,{}^{\mathcal{C}}\dot{\omega}_C + {}^{\mathcal{C}}\omega_C \times {}^{\mathcal{C}}\boldsymbol{I}_C\,{}^{\mathcal{C}}\omega_C \end{bmatrix}, \tag{6}$$

where m is the mass of the rigid body, ${}^{\mathcal{C}}\boldsymbol{a}_C$ is the acceleration of the body with respect to coordinate system $\mathcal{C}$, ${}^{\mathcal{C}}\omega_C$ is the angular velocity of the body, and ${}^{\mathcal{C}}\boldsymbol{I}_C$ is the moment of inertia of the body about the mass center.

Given the acceleration $\boldsymbol{a}_A$ of point A on the body, the acceleration $\boldsymbol{a}_C$ of the mass center can be expressed as

$${}^{\mathcal{A}}\boldsymbol{a}_C = {}^{\mathcal{A}}\boldsymbol{a}_A + {}^{\mathcal{A}}\omega_A \times ({}^{\mathcal{A}}\omega_A \times {}^{\mathcal{A}}\boldsymbol{r}) + {}^{\mathcal{A}}\dot{\omega}_A \times {}^{\mathcal{A}}\boldsymbol{r} \tag{7}$$

and

$${}^{\mathcal{C}}\boldsymbol{a}_C = {}_{\mathcal{A}}^{\mathcal{C}}\boldsymbol{R}\,{}^{\mathcal{A}}\boldsymbol{a}_C \tag{8}$$

Thus, the following equation can be obtained:

$${}^{\mathcal{A}}\boldsymbol{F}_A = \begin{bmatrix} m({}^{\mathcal{A}}\boldsymbol{a}_A + {}^{\mathcal{A}}\omega_A\,{}^{\mathcal{A}}\omega_A\,{}^{\mathcal{A}}\boldsymbol{r} + {}^{\mathcal{A}}\dot{\omega}_A\,{}^{\mathcal{A}}\boldsymbol{r}) \\ m\,{}^{\mathcal{A}}\boldsymbol{r}\,{}^{\mathcal{A}}\boldsymbol{a}_A + {}^{\mathcal{A}}\boldsymbol{I}_A\,{}^{\mathcal{A}}\dot{\omega}_A + {}^{\mathcal{A}}\omega_A\,{}^{\mathcal{A}}\boldsymbol{I}_A\,{}^{\mathcal{A}}\omega_A \end{bmatrix}, \tag{9}$$

where ${}^{\mathcal{A}}\boldsymbol{I}_A$ is the moment of inertia of the body about point A.

The wrench ${}^{\mathcal{A}}\boldsymbol{F}_A$ can finally be calculated in the base coordinate system O–XY by

$${}^{O}\boldsymbol{F}_A = \begin{bmatrix} {}^{O}_{\mathcal{A}}\boldsymbol{R} & 0 \\ 0 & {}^{O}_{\mathcal{A}}\boldsymbol{R} \end{bmatrix} \cdot {}^{\mathcal{A}}\boldsymbol{F}_A, \tag{10}$$

where ${}^{O}_{\mathcal{A}}\boldsymbol{R}$ is the rotation matrix between coordinate systems O–XY and $\mathcal{A}$.

2.3. Linear dynamic model based on virtual work principle

It is assumed that the mechanism undergoes a virtual motion $\dot{\boldsymbol{q}}^*$. The following equations can be obtained

$$\dot{\theta}^* = \boldsymbol{J}\dot{\boldsymbol{q}}^*, \tag{11}$$

$$\omega_i^* = \boldsymbol{G}_i\dot{\boldsymbol{q}}^*, \tag{12}$$

$$\boldsymbol{v}_i^* = \boldsymbol{H}_i\dot{\boldsymbol{q}}^*, \tag{13}$$

where $\dot{\theta}^*$ is the virtual velocity of active joint, ω_i^* is the virtual angular velocity of link, and v_i^* is the virtual velocity.

Based on the virtual work principle, the sum of the virtual work done by all forces and torques should be zero during the virtual time interval δt.

$$\dot{\theta}^*\tau\delta t + \left(\sum_{i=1}^{n}\boldsymbol{v}_i\boldsymbol{R}_i + \sum_{i=1}^{n}\omega_i\boldsymbol{T}_i\right)\delta t = \boldsymbol{0}, \tag{14}$$

where $\boldsymbol{R}_i$ and $\boldsymbol{T}_i$ are the resultant force and torque.

Eq. (14) can be rewritten as

$$\boldsymbol{J}^{\mathrm{T}}\boldsymbol{\tau} + \sum_{i=1}^{n}\boldsymbol{G}_i\boldsymbol{T}_i + \sum_{i=1}^{n}\boldsymbol{H}_i\boldsymbol{R}_i = \boldsymbol{0}, \tag{15}$$

where $\boldsymbol{J}$ is the Jacobian matrix.

By extracting the base dynamic parameters from $\boldsymbol{R}_i$ and $\boldsymbol{T}_i$, Eq. (15) can be rewritten in linear form as

$$\boldsymbol{J}^{\mathrm{T}}\boldsymbol{\tau} - \boldsymbol{\Omega}p = 0, \tag{16}$$

where $\boldsymbol{\Omega}$ is the observation matrix containing kinematics information, and $\boldsymbol{p}$ is the base dynamic parameters.

In the following sections, the proposed method is used to a 3-DOF redundant parallel manipulator and its corresponding non-redundant parallel manipulator.

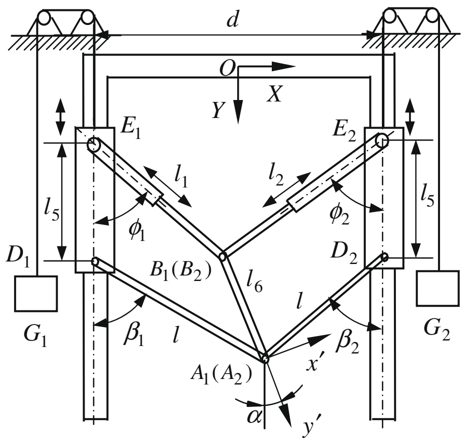

Fig. 1. The kinematic model.

3. Kinematic analysis of the 3-DOF parallel manipulator

3.1. Structure description

The kinematic model of the 3-DOF parallel manipulator is shown in Fig. 1. Sliders E_1D_1 and E_2D_2 drive links A_1D_1 and A_2D_2 when they slide along the vertical guide ways. Links E_1B_1 and E_2B_2, to be driven by two actuators, are extendible struts. G_1 and G_2 are counterweights. The whole construction enables movement of the moving platform in a plane and its rotation about the axis normal to the motion plane of the manipulator. Link E_1B_1 is a redundant link. In this paper, the corresponding non-redundant parallel manipulator is the one shown in Fig. 1 with link E_1B_1 removed.

3.2. Inverse kinematics

The kinematic model is shown in Fig. 1. Let the position vector of joint point A_1 be $\boldsymbol{r}_{O'} = [x \quad y]^{\mathrm{T}}$ in the base coordinate system, and the position vector of point B_i ($i = 1,2$) can be expressed as

$$\boldsymbol{r}_{\mathrm{B}i} = [x_{\mathrm{B}i} \quad y_{\mathrm{B}i}]^{\mathrm{T}} = \boldsymbol{r}_{O'} + \boldsymbol{R}\boldsymbol{r}'_{\mathrm{B}i}, \tag{17}$$

where $\boldsymbol{R}$ is the rotation matrix from coordinate system $O'–x'y'$ to $O–XY$, $\boldsymbol{r}'_{\mathrm{B}i}$ is the position vector of point $B_1(B_2)$ in $O'–x'y'$.

Based on Fig. 1, the following equations can be obtained:

$$\sin\phi_i = \frac{x_{\mathrm{B}i} - x_{\mathrm{E}i}}{l_i}, \quad \cos\phi_i = \frac{y_{\mathrm{B}i} - q_i}{l_i}, \quad i = 1,2, \tag{18}$$

$$\sin\beta_i = \frac{x - x_{\mathrm{D}i}}{l}, \quad \cos\beta_i = \frac{y - (q_i + l_5)}{l}, \quad i = 1,2, \tag{19}$$

where ϕ_i is the angle between link E_iB_i and the vertical axis parallel to the Y-axis, β_i is the angle between link A_iD_i and the vertical axis parallel to the Y-axis, d is the distance between two columns, q_i and $y_{\mathrm{B}i}$ are the Y coordinates of points E_i and B_i, l_i is the length of the ith extendible link, l is the length of the constant length link, and $x_{\mathrm{E}i}$, $x_{\mathrm{D}i}$ and $x_{\mathrm{B}i}$ are the X coordinates of points E_i, D_i and B_i in $O–XY$.

Based on Eqs. (18) and (19), the inverse kinematic solutions of the redundant manipulator can be written as

$$q_1 = y - l_5 \pm \sqrt{l^2 - (x + d/2)^2}, \tag{20a}$$

$$q_2 = y - l_5 \pm \sqrt{l^2 - (x + d/2)^2}, \tag{20b}$$

$$l_1 = \sqrt{(x - l_6\sin\alpha + d/2)^2 + (y - l_6\cos\alpha - q_1)^2}, \tag{20c}$$

$$l_2 = \sqrt{(x - l_6\sin\alpha + d/2)^2 + (y - l_6\cos\alpha - q_2)^2}, \tag{20d}$$

where l_5 is the height of the slider, α and l_6 are the rotation angle and length of the moving platform, respectively. For the configuration shown in Fig. 1, the "±" of Eq. (20) should be '−'.

For the corresponding non-redundant parallel manipulator, the inverse kinematic solution is shown as Eqs. (20a), (20b) and (20d).

3.3. Jacobian matrix

Taking the time derivative of Eqs. (18) and (19) leads to

$$\dot{q}_i = \dot{y} + l\sin\beta_i\dot{\beta}_i = \boldsymbol{J}_i[\dot{x} \quad \dot{y} \quad \dot{\alpha}]^{\mathrm{T}}, \tag{21}$$

$$\dot{l}_i = \dot{x}_{\mathrm{B}i}\sin\phi_i + (\dot{y}_{\mathrm{B}i} - \dot{q}_i)\cos\phi_i = \boldsymbol{J}_{i+2}[\dot{x} \quad \dot{y} \quad \dot{\alpha}]^{\mathrm{T}}, \tag{22}$$

$$\dot{\beta}_i = \frac{\dot{x}}{l\cos\beta_i}, \tag{23}$$

where $\boldsymbol{J}_i = [\tan\beta_i \quad 1 \quad 0]$, $\boldsymbol{E} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}^{\mathrm{T}}$,

$$\boldsymbol{J}_{i+2} = [\sin\phi_i \quad \cos\phi_i]\left(\begin{bmatrix} \boldsymbol{e}_1^{\mathrm{T}} \\ \boldsymbol{e}_2^{\mathrm{T}} \end{bmatrix} + \boldsymbol{E}\boldsymbol{R}\boldsymbol{r}'_{\mathrm{B}i}\boldsymbol{e}_3^{\mathrm{T}}\right) - \boldsymbol{J}_i\cos\phi_i,$$

$$\boldsymbol{e}_1 = [1 \quad 0 \quad 0]^{\mathrm{T}}, \quad \boldsymbol{e}_2 = [0 \quad 1 \quad 0]^{\mathrm{T}}, \quad \boldsymbol{e}_3 = [0 \quad 0 \quad 1]^{\mathrm{T}}.$$

Thus, the Jacobian matrix of the redundantly actuated parallel manipulator studied here can be expressed as

$$\boldsymbol{J} = [\boldsymbol{J}_1^{\mathrm{T}} \quad \boldsymbol{J}_2^{\mathrm{T}} \quad \boldsymbol{J}_3^{\mathrm{T}} \quad \boldsymbol{J}_4^{\mathrm{T}}]^{\mathrm{T}}. \tag{24}$$

For the corresponding non-redundant parallel manipulator, the Jacobian matrix can be expressed as

$$\boldsymbol{J}_{\mathrm{N}} = [\boldsymbol{J}_1^{\mathrm{T}} \quad \boldsymbol{J}_2^{\mathrm{T}} \quad \boldsymbol{J}_4^{\mathrm{T}}]^{\mathrm{T}}. \tag{25}$$

3.4. Partial velocity matrix and partial angular velocity matrix of the 3-DOF manipulator

The base dynamic parameters of the parallel manipulator include mass, the first moment and moment of inertia. To avoid the appearance of the base dynamic parameters in the partial velocity and partial angular velocity matrices used in dynamic modeling, point E_i is regarded as the pivotal point of the slider and the upper part of the extendible link, points B_i, D_i and A_1 are the pivotal points of the lower part of the extendible link, the constant length link and the moving platform in modeling, respectively. The partial velocity matrix of each pivotal point and partial angular velocity matrix of each moving part are computed, respectively.

Since the slider has only the translational capability, the partial angular velocity matrix is

$$\boldsymbol{G}_{i1} = 0. \tag{26}$$

According to Eqs. (1) and (21), the partial velocity matrix of point E_i can be expressed as

$$\boldsymbol{H}_{i1} = [0 \quad 1]^{\mathrm{T}}\boldsymbol{J}_i. \tag{27}$$

Based on Eqs. (2) and (23), the partial angular velocity matrix of the constant length link and partial velocity matrix of point D_i can be written as

$$\boldsymbol{G}_{i2} = \left[\tfrac{1}{l\cdot\cos\beta_i} \quad 0\right][\boldsymbol{e}_1 \quad \boldsymbol{e}_2]^{\mathrm{T}}, \tag{28}$$

$$\boldsymbol{H}_{i2} = \boldsymbol{H}_{i1}. \tag{29}$$

Let $\boldsymbol{H}_{i3}$, $\boldsymbol{H}_{i4}$, $\boldsymbol{H}_{i5}$ and $\boldsymbol{H}_{\mathrm{N}}$ denote the partial velocity matrix of point E_i, B_i, mass center of the counterweight and point O_{N}, and $\boldsymbol{G}_{i3}$, $\boldsymbol{G}_{i4}$, $\boldsymbol{G}_{i5}$, and $\boldsymbol{G}_{\mathrm{N}}$ denote the partial angular velocity matrix of the upper part and lower part of chain E_i B_i, counterweight and moving platform, they can be achieved in the same way.

4. Dynamic modeling

4.1. Inertia forces and torques of moving parts

Here, we denote that m_{i1}, m_{i2}, m_{i3}, m_{i4} and m_{i5} are the masses of the slider, constant length link, the upper part of the extendible link, the lower part of the extendible and the counterweight, respectively, m_N is the mass of the moving platform, and $\boldsymbol{g}$ is the gravitational acceleration vector. Based on Eq. (10), the inertia force and torque of each moving part about the pivotal point can be determined.

The inertia force and torque of the slider about point E_i can be expressed as

$$\boldsymbol{F}_{i1} = -m_{i1}(\boldsymbol{a}_{Ei} - \boldsymbol{g}), \quad M_{i1} = 0, \tag{30}$$

where $\boldsymbol{a}_{Ei}$ is the acceleration of point E_i.

The inertia force and torque of the constant length link about point D_i can be expressed as

$$\boldsymbol{F}_{i2} = -m_{i2}\left(\boldsymbol{a}_{Di} + s_{i2}\ddot{\beta}_i\boldsymbol{E}\begin{bmatrix}\sin\beta_i\\ \cos\beta_i\end{bmatrix} - s_{i2}\dot{\beta}_i^2\begin{bmatrix}\sin\beta_i\\ \cos\beta_i\end{bmatrix} - \boldsymbol{g}\right), \tag{31}$$

$$M_{i2} = -\ddot{\beta}_i I_{i2} + m_{i2}s_{i2}[\sin\beta_i \quad \cos\beta_i]\boldsymbol{E}(\boldsymbol{a}_{Di} - \boldsymbol{g}), \tag{32}$$

where $\boldsymbol{a}_{Di}$ is the acceleration of point D_i, s_{i2} is the distance between the mass center of the constant length link and point D_i, and I_{i2} is the moment of inertia of the link about point D_i.

Here, we suppose that $\boldsymbol{F}_{i3}$, M_{i3}, $\boldsymbol{F}_{i4}$, M_{i4}, $\boldsymbol{F}_{i5}$, M_{i5}, $\boldsymbol{F}_N$, M_N denote the inertia force and moment of the upper part of chain E_iB_i about point E_i, inertia force and moment of the lower part of chain E_iB_i about point B_i, inertia force and moment of the counterweight about its mass center, inertia force and moment of the moving platform about point O_N, respectively. Accordingly, $\boldsymbol{F}_{i3}$, M_{i3}, $\boldsymbol{F}_{i4}$, M_{i4}, $\boldsymbol{F}_{i5}$, M_{i5}, $\boldsymbol{F}_N$ and M_N can be determined.

For the corresponding non-redundant parallel manipulator, $\boldsymbol{F}_{13}$, M_{13}, $\boldsymbol{F}_{14}$ and M_{14} cannot occur.

4.2. Dynamic model of redundant parallel manipulator

Eqs. (31) and (32) can be rearranged in matrix form

$$\begin{bmatrix}\boldsymbol{F}_{i2}\\ M_{i2}\end{bmatrix} = -\begin{bmatrix}\boldsymbol{a}_{Di} - \boldsymbol{g} & \boldsymbol{A}_{i2} & 0\\ 0 & B_{i2} & \ddot{\beta}_i\end{bmatrix}\begin{bmatrix}m_{i2}\\ m_{i2}s_{i2}\\ I_{i2}\end{bmatrix}, \tag{33}$$

where

$$\boldsymbol{A}_{i2} = \ddot{\beta}_i\boldsymbol{E}[\sin\beta_i \quad \cos\beta_i]^{\mathrm{T}} - \dot{\beta}_i^2[\sin\beta_i \quad \cos\beta_i]^{\mathrm{T}}$$
$$B_{i2} = -[\sin\beta_i \quad \cos\beta_i]\boldsymbol{E}(\boldsymbol{a}_{Di} - \boldsymbol{g}).$$

Furthermore, we can obtain

$$[\boldsymbol{H}_{i2}^{\mathrm{T}} \quad \boldsymbol{G}_{i2}^{\mathrm{T}}]\begin{bmatrix}\boldsymbol{F}_{i2}\\ M_{i2}\end{bmatrix} = -\boldsymbol{\Omega}_{i2}\boldsymbol{p}_{i2}, \tag{34}$$

where $\boldsymbol{\Omega}_{i2} = [\boldsymbol{H}_{i2}^{\mathrm{T}} \quad \boldsymbol{G}_{i2}^{\mathrm{T}}]\begin{bmatrix}\boldsymbol{a}_{Di} - \boldsymbol{g} & \boldsymbol{A}_{i2} & 0\\ 0 & B_{i2} & \ddot{\beta}_i\end{bmatrix}$ and $\boldsymbol{p}_{i2} = [m_{i2} \quad m_{i2}s_{i2} \quad I_{i2}]^{\mathrm{T}}$.

In the same way, the dynamic parameters of other moving parts can be extracted from the corresponding inertia forces and torques.

Based on Eq. (15), the dynamic formulation of the redundantly actuated parallel manipulator can be expressed as

$$\boldsymbol{J}^{\mathrm{T}}\boldsymbol{\tau} + \sum_{i=1}^{2}\sum_{j=1}^{5}[\boldsymbol{H}_{ij}^{\mathrm{T}} \quad \boldsymbol{G}_{ij}^{\mathrm{T}}]\begin{bmatrix}\boldsymbol{F}_{ij}\\ M_{ij}\end{bmatrix} + [\boldsymbol{H}_N^{\mathrm{T}} \quad \boldsymbol{G}_N^{\mathrm{T}}]\begin{bmatrix}\boldsymbol{F}_N\\ M_N\end{bmatrix} = \boldsymbol{0}, \tag{35}$$

where $\boldsymbol{\tau} = [F_1 \quad F_2 \quad F_3 \quad F_4]^{\mathrm{T}}, F_1, F_2, F_3$ and F_4 are the driving forces that act on sliders E_1D_1 and E_2D_2, extendible links E_1B_1 and E_2B_2, respectively.

Eq. (35) can be rewritten as

$$\boldsymbol{J}^{\mathrm{T}}\boldsymbol{\tau} - \boldsymbol{\Omega}\boldsymbol{p} = 0, \tag{36}$$

where $\boldsymbol{p} = [m_{11} \quad m_{21} \quad \boldsymbol{p}_{12}^{\mathrm{T}} \quad \boldsymbol{p}_{22}^{\mathrm{T}} \quad \cdots \quad \boldsymbol{p}_{15}^{\mathrm{T}} \quad \boldsymbol{p}_{25}^{\mathrm{T}} \quad \boldsymbol{p}_N^{\mathrm{T}}]^{\mathrm{T}}$ is the dynamic parameters of the redundantly actuated parallel manipulator, $\boldsymbol{p}_N$ is the dynamic parameter of the moving platform, $\boldsymbol{p}_{i1}$, $\boldsymbol{p}_{i2}$, $\boldsymbol{p}_{i3}$, $\boldsymbol{p}_{i4}$ and $\boldsymbol{p}_{i5}$ are the base dynamic parameters of the slider, constant length link, the upper part of the extendible link, the lower part of the extendible and the counterweight, respectively.

Eq. (36) is the dynamic formulation in the linear form of the dynamic parameters and can be applied to the dynamic parameter identification.

4.3. Dynamic model of non-redundant parallel manipulator

The dynamic formulation of the non-redundant parallel manipulator can be expressed as

$$\boldsymbol{J}_{\mathrm{N}}^{\mathrm{T}}\boldsymbol{\tau}_{\mathrm{N}} + \sum_{i=1}^{2}\sum_{j=1}^{5}[\boldsymbol{H}_{ij}^{\mathrm{T}} \quad \boldsymbol{G}_{ij}^{\mathrm{T}}]\begin{bmatrix}\boldsymbol{F}_{ij}\\ M_{ij}\end{bmatrix} - \sum_{k=3}^{4}[\boldsymbol{H}_{1k}^{\mathrm{T}} \quad \boldsymbol{G}_{1k}^{\mathrm{T}}]\begin{bmatrix}\boldsymbol{F}_{1k}\\ M_{1k}\end{bmatrix} + [\boldsymbol{H}_N^{\mathrm{T}} \quad \boldsymbol{G}_N^{\mathrm{T}}]\begin{bmatrix}\boldsymbol{F}_N\\ M_N\end{bmatrix} = \boldsymbol{0}, \tag{37}$$

where $\boldsymbol{\tau}_{\mathrm{N}} = [F_1 \quad F_2 \quad F_4]^{\mathrm{T}}$.

In the same way, Eq. (37) can be rewritten in the linear form of the base dynamic parameters

$$\boldsymbol{J}_{\mathrm{N}}^{\mathrm{T}}\boldsymbol{\tau}_{\mathrm{N}} - \boldsymbol{\Omega}_{\mathrm{N}}\boldsymbol{p}_{\mathrm{NN}} = 0 \tag{38}$$

where $\boldsymbol{\Omega}_{\mathrm{N}}$ is the observation matrix of the non-redundant manipulator, $\boldsymbol{p}_{\mathrm{NN}} = [m_{11} \quad m_{21} \quad \boldsymbol{p}_{12}^{\mathrm{T}} \quad \boldsymbol{p}_{22}^{\mathrm{T}} \quad \boldsymbol{p}_{23}^{\mathrm{T}} \quad \boldsymbol{p}_{24}^{\mathrm{T}} \quad \boldsymbol{p}_{15}^{\mathrm{T}} \quad \boldsymbol{p}_{25}^{\mathrm{T}} \quad \boldsymbol{p}_N^{\mathrm{T}}]^{\mathrm{T}}$ is the base dynamic parameters of the non-redundant parallel manipulator.

5. Driving force optimization for redundant manipulator

Since the 3-DOF redundant manipulator has one redundant actuator and $\boldsymbol{J}^{\mathrm{T}}$ is non-square, the driving force $\boldsymbol{\tau}$ in Eq. (36) has infinite solutions. To obtain a unique solution, optimization technique has to be applied. For different demands, it has different optimizing objective. In this paper, the optimizing objective is minimizing the 2 norm of $\boldsymbol{\tau}$.

The solution of Eq. (36) to minimize the 2 norm of $\boldsymbol{\tau}$ [12] can be determined by

$$\boldsymbol{\tau}_{\min} = \boldsymbol{J}(\boldsymbol{J}^{\mathrm{T}}\boldsymbol{J})^{-1}\boldsymbol{\Omega}\boldsymbol{p}. \tag{39}$$

As an example to compute the driving force, the geometric and inertia parameters of the parallel manipulator are given in Tables 1 and 2, respectively.

Let the moving platform move from the point with the coordinate $[0.3\,\mathrm{m} \quad 3.2\,\mathrm{m} \quad \pi/3]^{\mathrm{T}}$ to another point with the coordinate $[0.9\,\mathrm{m} \quad 2.3\,\mathrm{m} \quad 0]^{\mathrm{T}}$ through accelerating, constant velocity and decelerating phases. According to Eq. (39), the driving forces of the redundantly actuated parallel manipulator are determined and shown in Fig. 2. Moreover, the same motion is simulated on its corresponding non-redundant parallel manipulator with redundant link E_1B_1 removed and the driving force is given in Fig. 3.

Table 1
Geometric parameters.

Parameter	Value
d (mm)	1200
l (mm)	1200
l_5 (mm)	250
l_6 (mm)	250

Table 2
Inertia parameters.

Parameter	Value
m_N (kg)	150
m_{11}, m_{21} (kg)	120
m_{12}, m_{22} (kg)	220
m_{13}, m_{23} (kg)	60
m_{14}, m_{24} (kg)	20
m_{15}, m_{25} (kg)	495

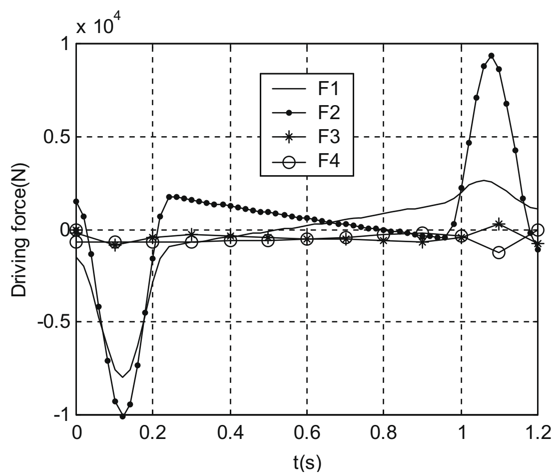

Fig. 2. Driving forces of the redundant manipulator.

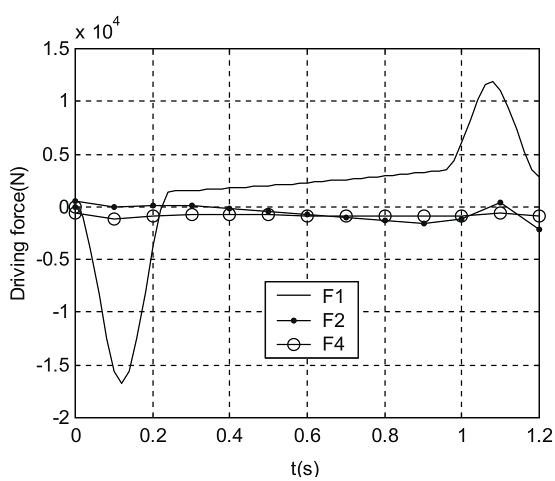

Fig. 3. Driving forces of the non-redundant manipulator.

From Fig. 2, it can be seen that the difference between F_1 acting on slider E_1D_1 and F_2 acting on slider E_2D_2 is very small. From Fig. 3, one may see that F_1 of the corresponding non-redundant parallel manipulator is much bigger than F_2. Thus, a big striking on the non-redundant parallel manipulator is produced and the performance of the non-redundant parallel manipulator is worse. Thus, the dynamic performance of the redundant parallel manipulator is better than that of its corresponding non-redundant parallel manipulator.

6. Conclusions

This paper has investigated the dynamic formulation of parallel manipulators for dynamic parameter identification. A method for finding the linear form of dynamic equation is proposed for both redundant and non-redundant parallel manipulator. In modeling, it is important for selecting proper pivotal points to avoid the existence of the base dynamic parameters in the partial velocity matrix of the pivotal point and partial angular velocity matrix of each link. As examples to validate the method, the linear form of dynamic models of a 3-DOF redundantly actuated parallel manipulator and its corresponding non-redundant manipulator are derived.

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