Kaunas University of Technology
Faculty of Informatics

Numerical Methods and Algorithms

Engeneering Project 4

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Ordinary Differential Equations

Task.

1. Construct an ordinary differential equation (ODE) for the given problem. You should use the theory in Section 1 Newton's Laws of Motion. Explain the process.
2. Solve the constructed ODE using Euler's method.
3. Demonstrate that the solution is accurate, that is, provide solutions with timesteps 1, 0.5, 0.25, … in the same figure.
4. Find the largest possible timestep for the solution to maintain stability and provide the figure to demonstrate instability.
5. Solve the ODE with the standard library functions, for example, Python scipy.integrate solve_ivp.

1. ODE: Mathematical expression of Ordinary Differential Equation contains the function and its derivatives; • Solution of ODE is a function which has one or several integration constants; • When integration constants are defined by known initial values, it is initial value problem.

   Many differential equations describe real physical, biological, economical and social systems. An important feature of such ODE is that they can describe the movement or status changes over time;

$$\frac{dy}{dx} = f(x, y), \qquad y(x_0) = y_0$$

For the parachutist and equipment with masses m1 and m2 respectively,

the forces acting on the system during the fall will be gravity and air resistance. The air resistance force is proportional to the square of the velocity, v^2, with a different proportionality constant before and after the parachute is deployed (k1 and k2 respectively).

The net force F acting on the system at any time t can be described by Newton's second law: F= m · a ;where m=m1+m2m=m1+m2 is the total mass and a is the acceleration.

Before the parachute is deployed, the air resistance is k1 · v^2, and after deployment, it is

k2 · v^2. The gravitational force is m·gm·g. The ODE for the system can be written as:
$m \cdot \frac{dv}{dt} = m \cdot g - k \cdot v^2$;where k is either k1 or k2 depending on whether the parachute is deployed or not.

Equation during free fall:

$$\frac{dv}{dt} = \frac{(m_1 + m_2)g - k_1 v^2}{m_1 + m_2}$$

This is the ordinary differential equation (ODE) that describes the velocity v of the parachutist during free fall, where:

- dv/dt is the acceleration,
- m1 and m2 are the masses of the parachutist and the equipment, respectively,
- g is the acceleration due to gravity,
- k1 is the drag coefficient during free fall,
- v is the velocity.

The first and the second ODE are similar, except for the air resistance coefficient changes:

$$\frac{dv}{dt} = \frac{(m_1 + m_2)g - k_2 v^2}{m_1 + m_2}$$

This equation represents the acceleration dv/dt of the parachutist after the parachute is extended, with the following parameters:

- m1 and m2 are the masses of the parachutist and equipment, respectively.
- g is the acceleration due to gravity.
- k2 is the drag coefficient after the parachute is extended.
- v is the velocity.

$$h(t) = h_0 - \int_0^t v(\tau)\, d\tau$$

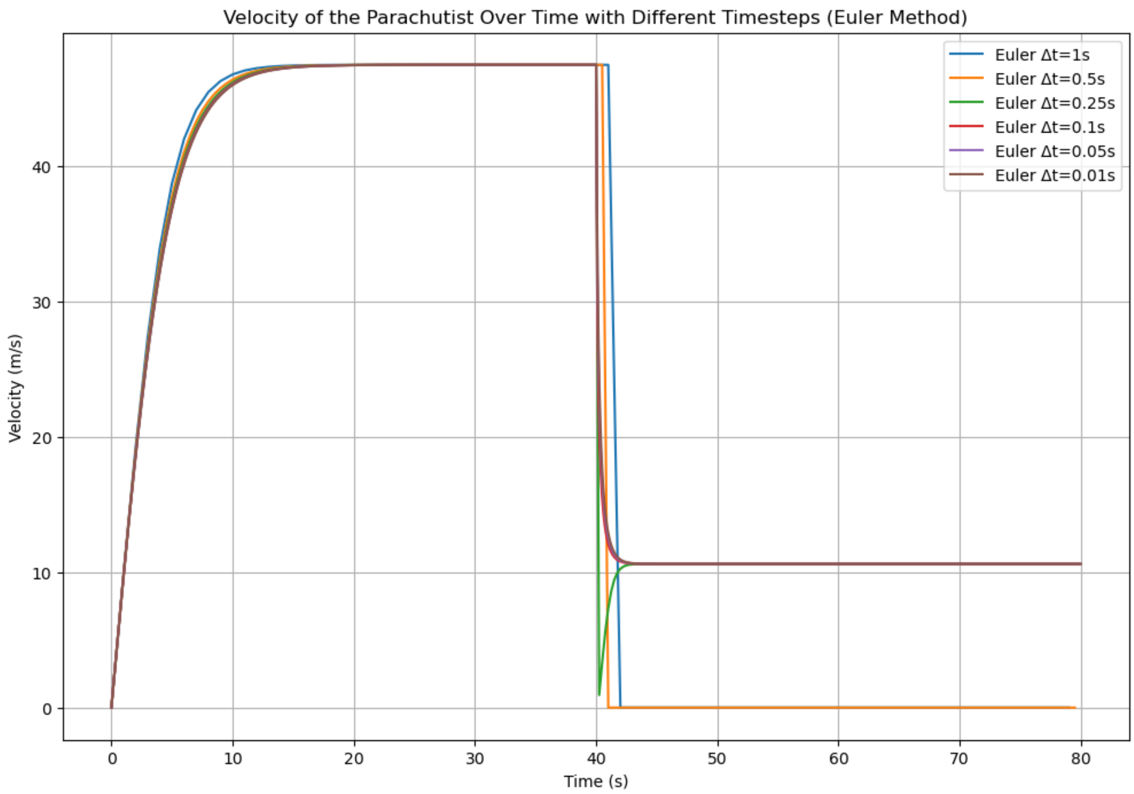
This equation determines the height h(t) of the parachutist at any time t by integrating the velocity v(t) over time, where:

- h0 is the initial height from which the parachutist starts the fall.
- v(t) is the velocity as a function of time t.
- The integral subtracts the accumulated distance fallen from the initial height h0 to give the current height at time t.

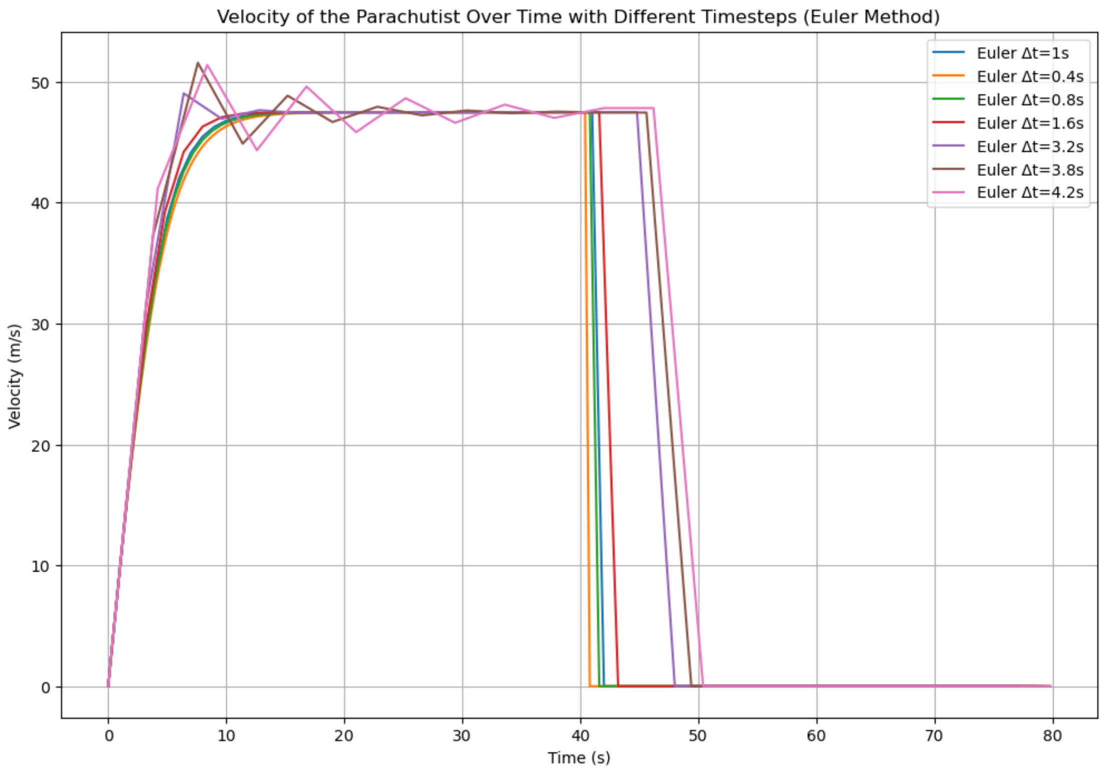
2. 3. Euler's method is a numerical technique to solve ODEs. It uses the current velocity and acceleration to estimate the velocity at the next time step. The general form of Euler's method for our problem is:

```python
g = 9.8  # Acceleration due to gravity (m/s^2)
m1 = 100  # Mass of parachutist (kg)
m2 = 15  # Mass of equipment (kg)
k1 = 0.5  # Air resistance coefficient during free fall (kg/m)
k2 = 10  # Air resistance coefficient after parachute is extended (kg/m)
h0 = 3000  # Initial altitude (m)
tg = 40  # Time when parachute is extended (s)

# Total mass
m = m1 + m2

# The ODE to solve is dv/dt = g - k/m * v^2, with k changing at t=tg
def ode(t, v):
    if t < tg:
        k = k1
    else:
        k = k2
    return g - (k/m) * v**2
```

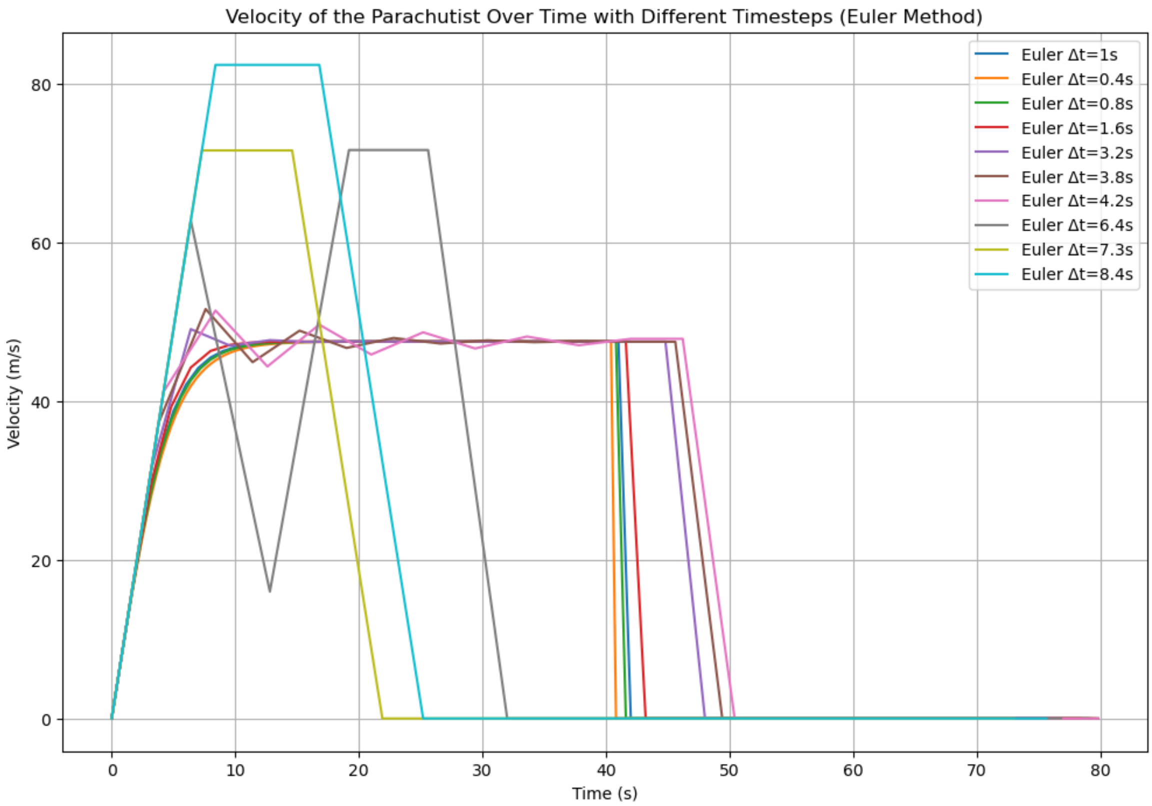
The representation of the Velocity of the Parachutist Over Time with Different Timesteps (Euler Method).
To demonstrate the accuracy of the solution, we solved the ODE using Euler's method with different timesteps, like Δt=1,0.5,0.25,… and we did plot the velocity as a function of time for each timestep on the same graph (figure) as it is required.



4. The largest timestep that maintains the stability of the solution needs to be found empirically by gradually increasing the timestep until the solution becomes unstable. We would then plot the solutions to show the instability.



Here's an enhanced instable version:



5. We can solve the ODE using Python's scipy.integrate.solve_ivp function, which is a more accurate method than Euler's method because it uses adaptive timesteps and more sophisticated integration techniques.
We did compare the Euler's method solution with different timesteps and use solve_ivp to find the solution.
The plot above shows the velocity of the parachutist over time as calculated by the solve_ivp function from the Python scipy.integrate library. This function uses an adaptive method to solve the ordinary differential equation (ODE) that models the parachutist's motion. The velocity increases rapidly until the parachute is deployed at 40 seconds, after which the velocity levels off due to the increased air resistance from the deployed parachute.