Start out with simple advection:

$$\frac{\partial a}{\partial t} + u\frac{\partial a}{\partial x} + v\frac{\partial a}{\partial y} = 0$$

$$\hookrightarrow \quad \frac{\partial a}{\partial t} = -\left(\frac{\partial F(a)}{\partial x} + \frac{\partial F(a)}{\partial y}\right) = -\vec{\nabla}\cdot\vec{F}(a)$$

After averaging:

$$\frac{1}{\Delta x \Delta y}\iint_{x_{i-1/2}}^{x_{i+1/2}} \frac{\partial a}{\partial t}\,dx\,dy = \frac{-1}{\Delta x \Delta y}\iint_{x_{i-1/2}}^{x_{i+1/2}} \frac{\partial F(a)}{\partial x} + \frac{\partial F(a)}{\partial y}\,dx\,dy$$

Apply Divergence Theorem

$$\frac{\partial}{\partial t}\langle a\rangle_i = \frac{-1}{\Delta x}\left(F_{i+1/2} - F_{i-1/2}\right)$$

$$\hookrightarrow \quad \frac{a_i^{n+1} - a_i^n}{\Delta t} = -\frac{1}{\Delta x}\left[F(a_{i+1/2}) - F(a_{i-1/2})\right]$$

predictor - corrector:

Each interface, $i \pm 1/2$, has both L and R states.

Left: $$a_{i+1/2,i,L}^{n+1/2} = a_i^n + \frac{\Delta x}{2}\frac{\partial a}{\partial x}\bigg|_i + \frac{\Delta t}{2}\frac{\partial a}{\partial t}\bigg|_i + \mathcal{O}(\Delta x^2) + \mathcal{O}(\Delta y^2)$$

$$= a_i^n + \frac{\Delta x}{2}\left(1 - \frac{\Delta t}{\Delta x}u\right)\frac{\partial a}{\partial x}\bigg|_i$$