CSC343: Assignment #3, Part 2

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Question 1

Relation R with attributes $ABCDEFGH$ and functional dependencies S:

$$S = \{A \to CF,\ BCG \to D,\ CF \to AH,\ D \to B,\ H \to DEG\}$$

(a)

Which of these functional dependencies violate BCNF?

We first need to determine whether the sets of attributes on the left sides of each rule constitute superkeys. We will do so by finding the closure sets of each:

X	X_S^+
A	$\{A, B, C, D, E, F, G, H\}$
BCG	$\{B, C, D, G\}$
CF	$\{A, B, C, D, E, F, G, H\}$
D	$\{B, D\}$
H	$\{B, D, E, G, H\}$

$\therefore$ The functional dependencies that violate BCNF are: $\boxed{BCG \to D,\ D \to B, \text{ and } H \to DEG}$.

(b)

Obtain a lossless decomposition of R into a collection of relations that are in BCNF. Project the dependencies onto each relation in that final decomposition.

Decomposition of R

R violates BCNF, so proceed with the algorithm.

Pick the BCNF-violating rule $H \to DEG$.

Found earlier: $H^+ = \{B, D, E, G, H\}$.

Pick R_1 to have attributes $BDEGH$. Pick R_2 to have attributes $ACFH$.

Decomposition of $R_1(B, D, E, G, H)$:

Projecting the FDs of R onto R_1:

B	D	E	G	H	closure	FDs
✓					$B^+ = B$	nothing
	✓				$D^+ = BD$	$B \to BD$: violates BCNF; abort the projection

R_1 has a rule that violates BCNF: $D \to B$. Will need to decompose R_1 further.

Pick the BCNF-violating rule $D \to B$.

From earlier, $D^+ = \{B, D\}$.

Pick R_3 to have attributes BD and R_4 to have attributes $DEGH$.

Decomposition of $R_3(B, D)$:

Note that since R_3 is a two-attribute relation, it is already in BCNF and thus doesn't need to be further decomposed. ☺

Projecting the FDs of R onto R_3: $D \to B$.

Decomposition of $R_4(D, E, G, H)$:

Need to find the closure for each subset of the attributes of R_4:

D	E	G	H	closure	FDs
✓				$D^+ = D$	nothing
	✓			$E^+ = E$	nothing
		✓		$G^+ = G$	nothing
			✓	$H^+ = DEGH$	$H \to DEG$; H is a superkey of R_4
supersets of H				psh, irrelevant	can only generate weaker FDs
✓	✓			$DE^+ = DE$	nothing
✓		✓		$DG^+ = DG$	nothing
	✓	✓		$EG^+ = EG$	nothing
✓	✓	✓		$DEG^+ = DEG$	nothing

None of these rules violate BCNF; hence, R_4 is in BCNF!

R_4 has one FD: $H \to DEG$.

Decomposition of $R_2(A, C, F, H)$:

Need to find the closure for each subset of the attributes of R_2:

A	C	F	H	closure	FDs
✓				$A^+ = ACFH$	$A \to CFH$; A is a superkey of R_2
supersets of A				psh, irrelevant	can only generate weaker FDs
	✓			$C^+ = C$	nothing
		✓		$F^+ = F$	nothing
			✓	$H^+ = H$	nothing
	✓	✓		$CF^+ = ACFH$	$CF \to AH$; CF is a superkey of R_2
supersets of CF				psh, irrelevant	can only generate weaker FDs
	✓		✓	$CH^+ = CH$	nothing
		✓	✓	$FH^+ = FH$	nothing

None of these rules violate BCNF; hence, R_2 is in BCNF!

R_2 has the following FDs: $A \to CFH$ and $CF \to AH$.

Therefore...

According to the algorithm, R should be split into relations having the following schemas:

$$\boxed{R_2(A, C, F, H) \qquad R_3(B, D) \qquad R_4(D, E, G, H)}$$

... where the FDs of each relation are:

- For R_2: $A \to CFH$, $CF \to AH$
- For R_3: $D \to B$
- For R_4: $H \to DEG$

Question 2

Relation R with attributes $ABCDEF$ and functional dependencies S:

$$S = \{AB \to EF, \quad B \to CEF, \quad BCD \to AF, \quad BCDE \to A, \quad BCE \to D, \quad DF \to C\}$$

(a)

Compute all keys for R.

Need to calculate the closure for each subset of the set of all attributes of R.

X	X_S^+
A	A
B	$ABCDEF$
C	C
D	D
E	E
F	F
AB	$ABCEDF$
AC	AC
AD	AD
AE	AE
AF	$ACFH$
BC	$ABCEDF$
BD	$ABCDEF$
BE	$ABCDEF$
BF	$ABCDEF$
CD	CD

X	X_S^+
CE	CE
CF	CF
DE	DE
DF	CDF
EF	EF
ABC	$ABCDEF$
ABD	$ABCDEF$
ABE	$ABCDEF$
ABF	$ABCDEF$
ACD	ACD
ACE	ACE
ACF	ACF
ADE	ADE
ADF	$ACDF$
AEF	AEF
BCD	$ABCDEF$

X	X_S^+
BCE	$ABCDEF$
BCF	$ABCDEF$
BDE	$ABCDEF$
BDF	$ABCDEF$
BEF	$ABCDEF$
CDE	CDE
CDF	CDF
CEF	CEF
DEF	$CDEF$
$ABCD$	$ABCDEF$
$ABCE$	$ABCDEF$
$ABCF$	$ABCDEF$
$ABDE$	$ABCDEF$
$ABDF$	$ABCDEF$
$ABEF$	$ABCDEF$
$ACDE$	$ACDE$

X	X_S^+
$ACDF$	$ACDEF$
$ACEF$	$ACEF$
$ADEF$	$ACDEF$
$BCDE$	$ABCDEF$
$BCDF$	$ABCDEF$
$BCEF$	$ABCDEF$
$BDEF$	$ABCDEF$
$CDEF$	$CDEF$
$ABCDE$	$ABCDEF$
$ABCDF$	$ABCDEF$
$ABCEF$	$ABCDEF$
$ABDEF$	$ABCDEF$
$ACDEF$	$ABCDEF$
$BCDEF$	$ABCDEF$
$ACBDEF$	$ABCDEF$

Eliminating trivial and redundant FDs: B, $\cancel{AB}$, $\cancel{BC}$, $\cancel{ABC}$, $\cancel{ABD}$, $\cancel{ABE}$, $\cancel{ABF}$, $\cancel{BCD}$, $\cancel{BCE}$, $\cancel{BCF}$, $\cancel{BDE}$, $\cancel{BEF}$, $\cancel{ABCD}$, $\cancel{ABCE}$, $\cancel{ABCF}$, $\cancel{ABDE}$, $\cancel{ABDF}$, $\cancel{ABEF}$, $\cancel{BCDE}$, $\cancel{BCDF}$, $\cancel{BCEF}$, $\cancel{BDEF}$, $\cancel{ABCDE}$, $\cancel{ABCEF}$, $\cancel{ABDEF}$, $\cancel{ACDEF}$, $\cancel{BCDEF}$, $\cancel{ABCDEF}$

$\boxed{\therefore \text{ The keys of } R \text{ are: } B.}$

(b)

Compute a minimal basis for S.

Let S_1 be the set of FDs obtained by rewriting all FDs in S such that the right-hand sides of all rules are singletons. S_1 thus contains the following FDs:

1 $AB \to E$	3 $B \to C$	5 $B \to F$	7 $BCD \to F$	9 $BCE \to D$
2 $AB \to F$	4 $B \to E$	6 $BCD \to A$	8 $BCDE \to A$	10 $DF \to C$

Looking for redundant FDs in S_1 to eliminate:

FD	Exclude these from S_1 when computing closure	Closure	Decision
1	1	$AB^+ = ABFCED$	discard
2	1 2	$AB^+ = ABCEFD$	discard
3	1 2 3	$B^+ = BEF$	keep
4	1 2 4	$B^+ = BCF$	keep
5	1 2 5	$B^+ = BCEDAF$	discard
6	1 2 5 6	$BCD^+ = BCDEAF$	discard
7	1 2 5 6 7	$BCD^+ = BCDEA$	keep
8	1 2 5 6 8	$BCDE^+ = BCDEF$	keep
9	1 2 5 6 9	No other way to get D w/o this FD	keep
10	1 2 5 6 10	$DF^+ = DF$	keep

Let S_2 denote the set containing the remaining FDs:

3	$B \to C$
4	$B \to E$
7	$BCD \to F$
8	$BCDE \to A$
9	$BCE \to D$
10	$DF \to C$

We will now attempt to reduce the LHS's of the FDs in S_2, closing over the full set S_2:

7 $BCD \to F$
$B^+ = BCEDFA$, so we can reduce the LHS to B.
8 $BCDE \to A$
$B^+ = BCEDFA$, so we can reduce the LHS to B.
9 $BCE \to D$
$B^+ = BCEDFA$, so we can reduce the LHS to B.
10 $DF \to C$
$D^+ = D$, so we can't reduce the LHS to D.
$F^+ = F$, so we can't reduce the LHS to F.
So, this FD remains as it is.

Let S_3 denote the set of FDs obtained after reducing the LHS's in S_2:

3	$B \to C$
4	$B \to E$
7'	$B \to F$
8'	$B \to A$
9'	$B \to D$
10	$DF \to C$

Looking for redundant FDs in S_3 to eliminate:

FD	Exclude these from S_3 when computing closure	Closure	Decision
3	3	$B^+ = BEFADC$	discard
4	3 4	No other way to get E w/o this FD	keep
7'	3 7'	No other way to get F w/o this FD	keep
8'	3 8'	No other way to get A w/o this FD	keep
9'	3 9'	No other way to get D w/o this FD	keep
10'	3 10'	$DF^+ = DF$	keep

No further simplifications are possible.

Let S_4 denote the set of FDs obtained after eliminating redundant FDs from S_3:

4	$B \rightarrow E$
7'	$B \rightarrow F$
8'	$B \rightarrow A$
9'	$B \rightarrow D$
10	$DF \rightarrow C$

$\therefore$ A minimal basis for S is:

$$\boxed{B \rightarrow E, \qquad B \rightarrow F, \qquad B \rightarrow A, \qquad B \rightarrow D, \qquad DF \rightarrow C}$$

(c)

Using the minimal basis from part (b), employ the 3NF synthesis algorithm to obtain a lossless and dependency-serving decomposition of R into a collection of relations that are in 3NF.

Let S_5 denote the set obtained after merging the RHS's of the FDs in S_4:

$$S_5 = \{B \rightarrow ADEF, \quad DF \rightarrow C\}$$

The relations resulting from the FDs in S_5 are:

$$\boxed{R_1(A, B, D, E, F), \quad R_2(C, D, F)}$$

(d)

Does your schema allow redundancy?

We will show that R_1 is in BCNF by showing that there exists no non-trivial FD for $R_1 = ABDEF$ not having a superkey LHS:

X	X_S^+
A	A
B	$ABDEF$
D	D
E	E
F	F
AB	$ABDEF$
AD	AD
AE	AE
AF	AF
BD	$ABDEF$
BE	$ABDEF$
BF	$ABDEF$
DE	DE
DF	DF
EF	EF
ABD	$ABDEF$

X	X_S^+
ABE	$ABDEF$
ABF	$ABDEF$
ADE	ADE
ADF	ADF
AEF	AEF
BDE	$ABDEF$
BDF	$ABDEF$
BEF	$ABDEF$
DEF	DEF
$ABDE$	$ABDEF$
$ABDF$	$ABDEF$
$ABEF$	$ABDEF$
$ADEF$	$ADEF$
$BDEF$	$ABDEF$
$ABDEF$	$ABDEF$

All non-trivial FDs for R_1 have a superkey LHS.

We will do the same for $R_2 = CDF$:

X	X_S^+
C	C
D	D
F	F
CD	CD
CF	CF
DF	CDF
CDF	CDF

All non-trivial FDs for R_2 have a superkey LHS.

$\therefore$ No, the schema does not allow redundancy.