Nicole Zhu
nicolez
Collaborators: Jess Peterson, Ahmad Wani
CS148, Fall 2014

1. **True / False Questions**

(a) **False:** The BRDF of the surface of a Lambertian object for a single wavelength can be specified as a one-dimensional function.
We have to consider other parameters, e.g. incoming and outgoing angles.

(b) **True:** The RGB and YUV color spaces are related by a linear transformation.

$$\begin{bmatrix} Y \\ U \\ V \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.148 & -0.289 & 0.437 \\ 0.615 & -0.515 & -0.100 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1.140 \\ 1 & -0.395 & -0.581 \\ 1 & 2.032 & 0 \end{bmatrix} \begin{bmatrix} Y \\ U \\ V \end{bmatrix}$$

(c) **True:** The ambient component of the Phong illumination model is equivalent to a constant BRDF.
$I_{ambient} = I_a k_a$, *with* I_a *being the ambient light in the scene.*

(d) **False:** For every visible wavelength, there exists a combination of RGB that can reproduce it.
We use 3 types of cones for vision. Each cone filters different colors due to their sensitivity curvature. Where it gets complicated is where these cones overlap, making it impossible for one single wavelength to map to just one cone.

So, whereas RGB does cover a large part of the color gamut, it doesn't cover all, e.g. saturated cyan and yellow. To date, it has not scaled when we tried adding a 4th and 5th primary color to an additive color reproduction system.

(e) **True:** A red flashlight appears red because it absorbs all colors other than red.
Newton observed that color is not inherent in objects. Rather, the surface of an object reflects some colors and absorbs all the others. We perceive only the reflected colors. So since red flashlights usually use a red filter to turn white light into red light, the filter effectively absorbs all colors other than red.

(f) **False** Any image that can be produced by an ideal CMYK printer can also be produced by an ideal CMY printer.
For instance, 100% C, 100% M and 100% Y do not give perfect black

(g) **False** The Gauss Seidel method can diverge when solving the discretized rendering equation while the Gauss Jacobi method always converges.
Both can converge and diverge.

(h) **False:** In order to capture the refraction of light in water, one needs to compute the refraction of light over 3D space as opposed to only at the surface of the water.
Only critical angle measurements are necessary

(i) **True:** Irradiance increases under a magnifying glass.
Irradiance measures watt/meter squared. Placing a magnifying glass between the paper and the sun does not increase the apparent brightness (radiance) of the sun as seen from the paper at the focal point. It does, however, increase the apparent size of the sun, which leads to more deposition of energy (irradiance) on the paper.

(j) **False:** The reflection of light in a room full of mirrors can be accurately modeled using the radiosity approach.
It cannot be perfectly modelled. According to current radiosity models for specular reflection, if there are many mirror reflections, a ray can bounce n times among the mirrors. So the computation for the first pass may increase by a factor of n. In this case, the amount of computation and memory required may be too large for a regular graphics workstation.

Furthermore, in the second pass, computation time is also increased by a factor of n using the ray tracing method. There is a proposed frame buffer to frame buffer copying scheme to avoid the tedious ray tracing. However when there are multiple mirror reflections, the problem of copying images in the proper sequence and pasting at the proper place becomes difficult.

(k) **False:** Color-blindness results from a lack of cones in the human eye.
It can also be due to physical or chemical damage to the eye, the optic nerve, or the brain. For instance, cerebral achromatopsia is an autosomal recessive congenital color vision disorder, the inability to perceive color at high light levels, i.e. during the day. In this case, it is a cerebral disorder.

(l) **False:** Tone mapping is applied to the gamma encoded images to convert them back to the original scene luminance.
The tone map is applied ***before*** *the gamma encoding to produce a scene-linear image for display. We apply an* ***inverse tone map*** *to convert back the luminance of the original scene, in addition to removing the gamma encoding.*

(m) **False** For frequencies below the flicker fusion rate, an intermittent light stimulus appears to be completely steady to the observer.

Frequencies that are $\geq$ the intermittent light stimulus appears to more steady to the observer. The flicker fusion rate is that threshold above which images/media appears flickers are more imperceivable.

Multiple Choice Questions

2. What does the global illumination model approximate?

B. Diffuse reflection in Phong illumination model

Firstly, ambient lighting is provided by a method of approximation, not the global illumination model. Global illumination on the other hand, simulates diffuse inter-reflection and caustics.

3. Radiant intensity is properly defined for

A. A point light source

By definition, radiant intensity measures the brilliance of a theoretical point source that radiates in all directions uniformly.

4. An HDR image is suitable for

C. A scene where some objects are brightly lit and the other objects are in shadow

HDR aims to add more "dynamic range" (light to dark ratio) to images. One way to think about it that our many images evaluated at different exposures are then stitched into one. The result is more true to the human eye.

5. When a light beam is transmitted through a material

C. Light energy is lost if there is absorption of light

Electrons vibrate at particular frequencies. When light of a frequency strikes a material with same frequency electrons, then the electrons will absorb the energy of the light wave. This energy is converted into vibrations, exciting electron neighbors, ending in thermal energy. This conversion means that the original light energy is lost when light is absorbed.

6. Which of the following statements are true?

D. In the Phong reflection model, the diffuse coefficient dependent on the radiance of the incoming light.

D is true bc the phong model is an approximation to reflected randiance of a surface. It takes into account the diffuse spectral, spectral reflectance distribution, ambient/diffuse/spectral constants, etc.

Note: B isn't true because irradiance is measured is per unit area. its a unit measurement, so increasing surface area overall doesn't change this unit measurement

7. OpenGL shading can model

A. Emission

D. Diffuse reflection of light from the light source

We have GL_EMISSON and diffuse, but nothing for absorption.

8. Which of the following statements is true regarding gamma encoding and human vision?

A. Human eyes are more sensitive to changes in darker tones than brighter tones.

C. Gamma encoding allocates more bits to the darker tones in the image.

We are more sensitive to darker tone changes in order to view a broader range of luminance. If not, the typical range in brightness would be too overwhelming.
Gamma encoding compensates by maximizing the use of the bits relative to how humans perceive light and color. More bits are allocated to the darker regions of the image than to the lighter regions.

9. Which color space(s) is/are cylindrical?

C. HSV

The hue varies by radians around the circular plan, saturation radiates from center of circular plan to edge, and the value increases from the bottom to top of the length of the cylindar.

10. Which statement is incorrect regarding light distribution functions?

A. BTDFs and BSSRDFs can both be used to model transparency.

For transparent objects you will have total internal reflection and transmission. The first will be modelled using bssrdf.

B. BRDFs, BTDFs, and BSSRDFs are all used to decide how much light is reflected, transmitted and absorbed on the surface.

BRDF is reflectance, BTDF is transmittance, and BSSRDF scattering-surface reflectance. As the names would suggest, they do not ALL calculate all reflectance, transmittance, etc.

C. BRDFs accurately account for the reflection, absorption, transmittance of light.

would only account for reflection… doesn't account for transmittance, etc.

D. Both True/False. BRDFs always depend on the wavelength of the light.

I posted on Piazza about this. The can be both true and false. $f_r(\lambda_i, \omega_i, \lambda_r, \omega_r)$
False case: In the real world, BRDF does always depend on wavelength of light, when wavelengths aren't binned in RGB, i.e. $f_r(\lambda_i, \omega_i, \lambda_r, \omega_r)$
True case: The standard industry function, BRDF, doesn't always depend on wavelength of light in its calculations. $f_r(\omega_i, \omega_r)$

11. Which of these statements is incorrect?

A. The Phong model can be approximated by a BRDF.

In very specific cases. A small set of materials can be approximated, but for the most part, the phong model is not reciprocal nor energy conserving, thus not approximable by BRDF.

C. Global Illumination can only model diffuse surfaces.

It can also model caustic surfaces, i.e. light from a curved surface, which are by definition, not diffuse surfaces.
D is true. Radiosity and the diffuse Phong model can be used together to get a better solution. This is true because the Phong model and radiosity can be combined to render a scene with ambient, diffuse, and specular illumination

Short Answer Questions

12. Describe the YUV and and HSV color spaces. What is a common application for each of these color spaces? How do you convert a color in HSV space to YUV space?

YUV

Y is brightness of color (luminance), and the U/V components make up a 2d gradient that determines the color itself (chroma). Y ranges from 0 to 1, and Y/U range from -.5 to .5, or any scaled representation of this metric. It gives us reduced bandwidths for chrominance components, bettering transmission and perceptibility than RGB, for instance.
Common applications involve the color image pipeline, encoding image/video for human perception.

HSV

Hue shifts with the radian of the circular dimension in a 360 degree range. Saturation increases linearly outward from the center of the circular plane in whichever unit the cylindar is defined as. And value of the color ranges from 0 to 1, with 0 being black and 1 being the full value of the color.
Common applications involve image processing. For example, instances where you want to separate color from intensity, e.g. histogram equalization of a color image. It is not used very often, however, in image analysis and computer vision.

To convert HSV to YUV, we use the intermediate RGB in the HSV -> RGB -> YUV sequence:

HSV -> RGB

1. *Hex = H (0 to 360) / 60*
2. *Calc primary (int component of Hex), secondary colors. = hex - primary color*
    - 2.1. *secondary colour = Hex - primary colour (45)*
    - 2.2. *a = (1 − S)V (46)*
    - 2.3. *b = (1 − (S ∗ secondary colour))V (47)*
    - 2.4. *c = (1 − (S ∗ (1 − secondary colour)))V*
3. *To find RGB, we use the following switch case:*
    if primary colour = 0
    R = V, G = c, B = a (49)
    if primary colour = 1 then
    R = b, G = V, B = a (50)
    if primary colour = 2 then
    R = a, G = V, B = c (51)
    if primary colour = 3 then
    R = a, G = b, B = V (52)
    if primary colour = 4 then

R = c, G = a, B = V (53)
if primary colour = 5 then
R = V, G = a, B = b

Now for RGB -> YUV, we simply solve the following matrix for YUV.

$$\begin{bmatrix} Y' \\ U \\ V \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.14713 & -0.28886 & 0.436 \\ 0.615 & -0.51499 & -0.10001 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

13. Consider a spherical environment map stored as an image. Why can distortion arise when the viewpoint differs significantly from the viewpoint in which the environment map was acquired? How can you reduce this distortion?

To transfer a spherical environment into a 2 dimentional image involves projection of the sphere onto a intermediary surface. Inevitably, different conversion paths result in distortion. To illustrate, typical viewpoints with with to acquire the environment map include normal, transverse and oblique that map onto azimuthal, cylindrical, conic, and arbitrary planes. These tangent projection surfaces lay the sphere onto the surface, in a way that isn't calculated.

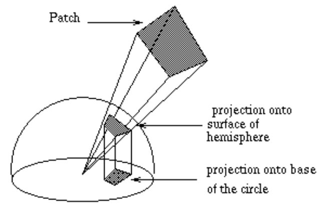
The distortions arise due to area, shape and distance inconsistencies btw the new viewpoint and the old one. (because when we acquire the environment map, we visually estimate by inspecting graticule patterns and the general shape of features on the sphere; we may also evaluate by measuring distances between selected sets of points).

We can reduce this distortion by deriving projects via perspective geometry, i.e. Gall's stereographic method.

14. Describe how you would combine the Phong model and radiosity to render a scene with ambient, diffuse, and specular illumination.
We break BRDF into specular and diffuse components with respective fractions for each that add up to 1. Then, we sum the original BRDF's ambient light with the the light generated by these two components (only specular is angle dependent) and get the total light.

15. Explain Nusselt's analog and how it can be used to calculate form factors.

Nusselt's analog is a geometrical picture that can aid intuition about the view factor.



The Nusselt analogue.[Watt,92]

The view factor between a differential element and the patch can be obtained projecting onto the surface of a unit hemisphere, and then projecting that in turn onto a unit circle around the point of interest in the plane of the base. The view factor is then equal to the differential area times the proportion of the unit circle covered by this projection.

It can measure form factors for complicated surfaces, by photographing them through a suitable fish eyed lenses. In particular, Nusselt's analog helps to pre-computes the contribution to the form factor from each cell on the surface of a hemicube.

Then its surface patch is projected onto the hemicube, and the form factor is approximated using the pre-computed form factor values as follows.

$$\begin{bmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & \cdots & -\rho_2 F_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n F_{n1} & -\rho_n F_{n2} & \cdots & 1-\rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

16. Where are cones most densely packed on the retina? Is the same true for rods?
Cones are most densely packed in the fovea centralis, located in the center of the macula region of the retina. Rods are absent there but dense elsewhere.
The fc is responsible for both color vision and the highest visual acuity. On the other hand, the rods are absent from the fovea, but are largely spread over other parts of the retina. These rods are responsible for night vision, our most sensitive motion detection, and our peripheral vision.

17. Why is the RGB color model unsuitable for use in printing?
Printer ink doesn't emit light, it absorbs light. Ink colors are subtractive colours. Each ink blocks the other. But RGB relies on light as being additive. So when you mix RGB, you get white. But since ink is subtractive, we will get a near black color, or similarly dark color with RGB.

18. In subsurface scattering, light incident on a given location scatters internally and exits from another location on the surface. How would you extend the 4D BRDF to model subsurface scattering?
We use BDRF's approximation of a single scattering and make adjustments for multiple scattering through integration.

The same sampling in multiple scattering can be achieved by setting up various weight coefficient with respect to different positions of the reflection of the single incident flux. We apply the reversible optical circuit principle.

19. Many photo inkjet printers often have more than four cartridges required for the CMYK model. What is an advantage of the having more than four cartridges?

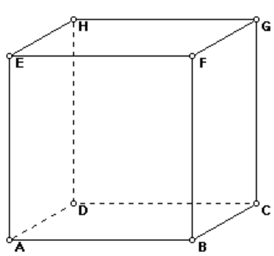
We use CcMmYK, as an example (It is a six color printing process used for photo printing). It extends the customary four color CMYK process, by adding light cyan and light magenta.

This allows us to remove harsh halftones / dots that appear in CMYK models. By using light cyan or magenta, we saturate areas that would typically use halftoning. CcMmYK also reduces graininess in the middle tone region, helping the appearance of things like blue skies and skin.

Long Answer Questions

20. Consider a cube ABCDEFGH with edge length R. A point light source with radiant intensity Il is located at point A, and seven tiny diffuse planar patches (with diffuse coefficient kd) are located at B, C, D, E, F, G, and H, respectively, all with normals straightly pointing to A. Each patch has an area of a and we assume that a << R2 . Derive an expression for the radiosity of any one of the patches. (done in collaboration with Jess Peterson)

We use the diffuse reflectance: B_{0} = pi k_{d} I_{l}/R^2.



let's use these point positions for the cube

Choosing point B, we then have to figure out the angles between other surfaces and the surface at B. Using the equation:
cos(outgoing angle from other surface) cos(incoming angle to B) / R^2,

we get the following fraction of light passed from other surface to B:

C-B: 1/2R^2
D-B: 0 (because at right angle to B and cos(90) = 0)
E-B: 0 (same)
F-B: 1/2R^2

G-B: 1/2R^2
H-B: 0

For the radiosity from C, we use the following light fractions:

B-C: 1/2R^2
D-C: 1/2R^2
E-C: 0
F-C: 1/2R^2
G-C: 1/2R^2
H-C: 1/2R^2

The radiosity from B, D, and E are symmetric to each other. Also, the radiosity from C, F, G, and H are symmetric to each other.

Therefore, we get two equations:

B_{B} = B_{0} + (B_{C} + B_{F} + B_{G})(k_{d} a / 2R^2) = B_{0} + 3 B_{C}(k_{d} a / 2R^2)

B_{C} = B_{0} + (B_{B} + B_{D})(k_{d} a / 2R^2) + (B_{F} + B_{G} + B_{H})(k_{d} a / 2R^2) = B_{0} + (2 B_{B} + 3B_{C})(k_{d} a / 2R^2)

Then we solve for B_{B} to yield the solution equation.

21. Explain how to approximate the form factor using a hemicube. Why is it advantageous to use a hemicube instead of a hemisphere?
Like in the Nussel analog, we place the center of a cube at a pt on the surface. The upper half of the cute is the projection body. Like the hemisphere, we divide the hemicube into discrete areas, with pre computed form factor values. When a surface is projected onto the hemicube, the sum of the form factor values of the discrete areas of the hemicube faces which are covered by the projection of the surface is the form factor between the point on the first surface (about which the cube is placed) and the second surface (the one which was projected).

As we have seen, the form factor is:

$$F_{A_i \to A_j} = F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos(\phi_i)\cos(\phi_j)}{\pi r^2} \, dA_j dA_i$$

eventually yielding:

$$= \frac{1}{\pi(x^2 + y^2 + z^2)^2} \Delta A$$

It is advantageous to use a hemicube instead of a hemisphere for various reasons. For one, the occluding or intervening patch problem is easily solved. Additionally, you can precisely unfold the hemicube into a planar image, unlike the hemisphere. So, you can easily produce each of these images by placing a camera on a patch, and render it pointing forwards, up, down, left and right. We save rendering time.

22. Give a 2x2 example (different from the one given in class) where the Jacobi approach converges, diverges and goes in an infinite cycle. Show that the iterative solution to radiosity can diverge in the case of perfectly reflective surfaces.
We use $y^{(n+1)} = q + py^{(n-1)}$ *for the Jacobi method*
for even values of $n + 1$*, this gives us*
$y^{n+1} = q + (pq + p^2q + \dots + p^{(n/2 - 1)}q) + p^{(n/2)}y^0$

and for odd values of $n+1$*,*
$y^{n+1} = q + (pq + p^2q + \dots + p^{((n-1)/2)}\ ) + p^{((n+1)/2)} y^{-1}$

The divergence, convergence, and cycle are thus:
divergence: $q \neq 0$, $|p| > 1$
convergence: $q \neq 0$, $|p| < 1$
cycle: $q \neq 0$, $p = -1$

for the second part of the question, we achieved the same result as last years video. The only difference is that substituting the radiosity equation with the Jacobi method used above, we get the following geometric series for matrices:

$B^{(n+1)} = E(I - PF)^{(-1)} (I - (PF)^{(n/2)}) + (PF)^{(n+1)} B^0$
the limit as n goes to infinity is still 0.

23. Consider performing radiosity computations with moving point lights. Assume that the motion of the point lights are known. Do you need to recompute the entire matrix for computing the radiosity as the lights move? Why or why not? Is it possible to perform this computation in parallel?
As we see from last years solution, radiance of outgoing light depends on G and a visibility term. We know that the form factor, two patches Ai and Ai, are purely geometrically indep, so we are just concerned with the form factor as things start moving.
With the moving light, we still don't need to recompute the entire matrix for computing the radiosity. We simply need to recalculate the form factor as the object moves, as the moving light changes the visibility between the patches. Like last years solution, we have algorithms that maximize reusing. We can back buffer, calculate the swept volume, check if occluded, then recalculate only when necessary. Additionally, we can even reuse form factors, when the geometric info is appropriate.

We can perform these calculations in parallel since every patch pair is only interdependent. Assuming we know the behavior of the moving light, we can delegate each patch pair to different processors, employing parallelization.

24. Rendering Equation in presence of a participating media: Consider a emitting participating medium e.g. fire with spatially constant emission throughout the space. When a ray is traced from point A to point B in this medium, light is accumulated along the ray based on the distance between A and B. How will the rendering equation change? What would change in the finite element method? Does anything change if the emission was spatially varying instead of constant?

As usual, our rendering function is:

$$L_o(x, \omega_o) = L_e(w, \omega_o) + \int L_i(x', \omega_i) f(x, \omega_j, \omega_o) cos\theta_i$$

where Lo, Le and Li represent total spectral, emitted, incoming radiances, respectively. And f is the BRDF of the surface. So, in the presence of fire, with constant spatial density, the light to the surface would be attenuated by some factors R', where R(|x'-x|) would be a function of the distance between x the source and x' the surface.
Thus the incoming radiance would be R times Lo. So the new rendering function:

$$L_o(x, \omega_o) = L_e(w, \omega_o) + \int L_i(x' - \omega_i) R(|x' - x||) f(x, \omega_j, \omega_o) cos\theta_i$$

We know that $cos\theta_i\, d\omega_i$ *make up the form factors. In this instance, we have R(x'-x), or R(X). Thus our new form factor is:*

$$\frac{1}{R_i} \int_{R_i}\int_{R_j} \frac{cos\theta_i cos\theta_i}{\pi \gamma^2} R(X) dR_i dR_j$$

Considering our situation where emission was not constant by spatial variance, R would depend on the specific path from x' to x. Then we would have to integrate from x' to x. This would be quite expensive, time and computation wise.

25. For an 10-bit display, denote its maximum luminance intensity by Imax and denote its minimum luminance intensity by Imin. Given that the just-noticeable relative difference of intensity is equal or larger than 2%, calculate the range of the 1023 steps where the difference in intensity values on one level are noticeable to the those two levels away. under the following conditions:

We have a gamma correction:
I(i) = A $i^\gamma + I_{min}$ *as* $(eqn\ 1)$
We also have relative diff by substituting eqn 1 as:

$$\frac{A(i+1)^{\gamma} - Ai^{\gamma}}{Ai^{\gamma} + I_{min}}$$

With a just noticeable difference 2%, we have:

$\frac{A(i+1)^{\gamma} - Ai^{\gamma}}{Ai^{\gamma} + I_{min}} \geq 2\%$, *which we save as (eqn 2)*

1. Imin = 0, with decoding gamma γ = 2

Substitution into eqn 2 gives us:

-0.498 $\leq i \leq$ 1004975, and since i is an integer in {0, 1023}, we have

$0 \leq i \leq 100$

2. Imin = 0.03Imax, with decoding gamma γ = 2

Substitution into into eqn 1 gives us:

Imin = A(1023)^2/.97

and eqn 2 gives us:

$\frac{A(i+1)^{\gamma} - Ai^{\gamma}}{Ai^{\gamma} + 0.03A(1023^2)/.97} \geq 2\%$, *which yield no solution.*

3. Imin = 0, with decoding gamma γ = 1

Substitution into eqn 2 gives us:

$(i+1) - i \geq 0.02i$, *and since i is an integer in {0, 1023}, we have*

$0 \leq i \leq 50$

4. Imin = 0.01Imax, with decoding gamma γ = 1

Substitutiing Imin into eqn 2 gives us:

$\frac{A(i+1-i)}{Ai^{\gamma} + I_{0.01min}} \geq 2\%$

From eqn 1 we have

*Imin = A*1023 / .99, so eqn 2 is now:*

$i \leq 39667$, *and since i is an int in* $\{0...1023\}$

$0 \leq i \leq 39$