# APPENDIX: TOWARD A TAYLOR RULE FOR FISCAL POLICY

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## A Model solution

## A.1 First-order conditions

All variables are denoted in real terms, a line over a variable indicates its steady state value, and a hat over a variable indicates its log-linearization around the steady-state.

## Households

Maximization problem: The maximization problem of household i:

$$\max_{b,c,I,k,u} E_{t} \sum_{t=1}^{\infty} \beta^{t} \left[ \varepsilon_{q,t} \frac{(c_{t} - hc_{t-1})^{1-\sigma_{c}}}{1 - \sigma_{c}} - \psi_{l} \frac{l_{t}(i)^{1+\sigma_{l}}}{1 + \sigma_{l}} \right] 
s.t. c_{t} + I_{t} + b_{t} = (1 - \tau_{t}^{w}) \frac{W_{t}(i)}{P_{t}} l_{t}(i) + ((1 - \tau_{t}^{k}) r_{t}^{k} u_{t} - \phi(u_{t})) k_{t-1} 
+ \frac{R_{t-1}b_{t-1}}{\pi_{t}} + (1 - \tau_{t}^{k}) d_{t} + \iota_{t}(i) + \tau_{t}^{T}$$

$$(1)$$

$$k_{t} = (1 - \delta) k_{t-1} + \varepsilon_{i,t} \left[ 1 - s \left( \frac{I_{t}}{I_{t-1}} \right) \right] I_{t}$$

Since the first-order conditions for household i are identical to the first-order conditions after aggregation, we report the aggregated first-order conditions.

First-order conditions:

$$\chi_t = \varepsilon_{q,t} \left( c_t - h c_{t-1} \right)^{-\sigma_c} - h \beta E_t \left[ \varepsilon_{q,t+1} \left( c_{t+1} - h c_t \right)^{-\sigma_c} \right]$$
(3)

$$\frac{1}{R_t} = \beta E_t \left[ \frac{\chi_{t+1}}{\chi_t \pi_{t+1}} \right] \tag{4}$$

$$q_{t} = \frac{1 - \beta E_{t} \left[ \frac{\chi_{t+1}}{\chi_{t}} q_{t+1} s' \left( \frac{I_{t+1}}{I_{t}} \right) \varepsilon_{i,t+1} \left( \frac{I_{t+1}}{I_{t}} \right)^{2} \right]}{\varepsilon_{i,t} \left( 1 - s \left( \frac{I_{t}}{I_{t-1}} \right) - s' \left( \frac{I_{t}}{I_{t-1}} \right) \frac{I_{t}}{I_{t-1}} \right)}$$

$$(5)$$

$$q_{t} = \beta E_{t} \left[ \frac{\chi_{t+1}}{\chi_{t}} \left( \phi'(u_{t+1}) u_{t+1} - \phi(u_{t+1}) + q_{t+1} (1 - \delta) \right) \right]$$
 (6)

$$\phi'(u_t) = r_t^k \left(1 - \tau_t^k\right) \tag{7}$$

Functional forms:

$$s\left(\frac{I_t}{I_{t-1}}\right) = \frac{\nu}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2 \tag{8}$$

$$s'\left(\frac{I_t}{I_{t-1}}\right) = \nu\left(\frac{I_t}{I_{t-1}} - 1\right) \tag{9}$$

$$\phi(u_t) = \frac{\bar{r}^k \left(1 - \bar{\tau}^k\right)}{\sigma_u} \left(\exp\left(\sigma_u \left(u_t - 1\right)\right) - 1\right) \tag{10}$$

$$\phi'(u_t) = \bar{r}^k \left(1 - \bar{\tau}^k\right) \exp\left(\sigma_u \left(u_t - 1\right)\right) \tag{11}$$

## Wage setting

Each household supplies a differentiated type of labor service,  $l_t(i)$ , which is aggregated into a homogenous labor good by a representative competitive firm according to a Dixit-Stiglitz aggregator with  $\theta_w > 1$  as the elasticity of substitution:

$$l_t^d = \left[ \int_0^1 l_t \left( i \right)^{\frac{\theta_w - 1}{\theta_w}} \right]^{\frac{\theta_w}{\theta_w - 1}}.$$
 (12)

Minimizing costs  $W_t l_t^d$  taken individual wage costs of household i,  $W_t(i)$ , as given yields the demand for labor of type i as:

$$l_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\theta_w} l_t^d \tag{13}$$

and the definition of the wage index  $W_t$  as

$$W_{t} = \left[ \int_{0}^{1} W_{t} (i)^{\theta_{w}-1} \right]^{\frac{1}{\theta_{w}-1}}.$$
 (14)

For any wage rate, each household supplies as many labor services as demanded.

$$\max_{W_{t}(i)} E_{t} \left[ \sum_{k=0}^{\infty} (\gamma_{w}\beta)^{k} \left[ \chi_{t+k}W_{t}(i) l_{t+k}(i) - U(l_{t+k}(i), c_{t+k}(i)) \right] \right]$$
(15)

The first-order conditions in recursive form are:

$$K_t^w = \left(\frac{l_t}{w_t^+}\right)^{1+\sigma_l} + \beta \gamma_w \left(\frac{\bar{\pi}}{\pi_{t+1}^w}\right)^{-\theta_w(1+\sigma_l)} K_{t+1}^w$$
 (16)

$$F_t^w = \frac{(\theta_w - 1)}{\theta_w} (1 - \tau_t^w) \frac{l_t}{w_t^+} \chi_t + \beta \gamma_w \left(\frac{\pi_{t+1}}{\pi_{t+1}^w}\right)^{-\theta_w} \left(\frac{\bar{\pi}}{\pi_{t+1}}\right)^{1 - \theta_w} F_{t+1}^w$$
(17)

$$\frac{K_t^w}{F_t^w} = \frac{1}{\psi_l} (w_t^*)^{1+\theta_w \sigma_l} w_t$$
 (18)

Real wage inflation  $\pi^w$ :

$$\pi_t^w = \frac{w_t}{w_{t-1}} \pi_t \tag{19}$$

The law of motion for  $w_t^* = \frac{W_t^*}{W_t}$  is given by:

$$1 = \gamma_w \left(\frac{\bar{\pi}}{\pi_t^w}\right)^{1-\theta_w} + (1 - \gamma_w) (w_t^*)^{1-\theta_w}$$
 (20)

### Firm sector: first-order conditions

Final good production function:

$$y_t = \left[ \int_0^1 y_t(j)^{\frac{\theta_p - 1}{\theta_p}} \right]^{\frac{\theta_p}{\theta_p - 1}} \tag{21}$$

Demand for good j:

$$y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta_p} y_t \tag{22}$$

with  $P_t$  defined as:

$$P_t = \left(\int_0^1 P_t(j)^{1-\theta_p} dj\right)^{1-\theta_p} \tag{23}$$

The production function of the intermediate good firm is:

$$y_t(j) = (u_t k_{t-1}(j))^{\alpha} \left(l_t^d(j) \varepsilon_{z,t}\right)^{1-\alpha} - \Omega, \tag{24}$$

Intermediate-good firms maximize profits:

$$\max_{\left(u_{t}\cdot k_{t-1}(j)\right),l_{t}^{d}(j)}\left[\left[\frac{P_{t}\left(j\right)}{P_{t}}\right]^{-\theta_{p}}\left(y_{t}\left(j\right)-w_{t}l_{t}^{d}\left(j\right)-r_{t}^{k}u_{t}k_{t-1}\left(j\right)\right)\right]$$
(25)

Marginal costs are denoted by z. The first-order conditions of (25) are given by:

$$z_t \left(1 - \alpha\right) \varepsilon_{z,t}^{(1-\alpha)} \left(u_t k_{t-1}\right)^{\alpha} \left(\frac{l_t}{w_t^+}\right)^{-\alpha} = w_t \tag{26}$$

$$z_t \alpha \left( u_t k_{t-1} \right)^{\alpha - 1} \left( \left( \frac{l_t}{w_t^+} \right) \varepsilon_{z,t} \right)^{1 - \alpha} = r_t^k$$
 (27)

## Price setting

Price-resetting firms choose  $P_{t}^{\star}=P_{t}\left(j\right)$  to maximize the expected sum of discounted future profits:

$$\max_{P_{t}(j)} E_{t} \sum_{k=0}^{\infty} \gamma_{p}^{k} m_{t+k} \left[ P_{t}(j) y_{t+k}(j) - Z_{t+k} y_{t+k}(j) \right] , \qquad (28)$$

where future profits are discounted by the stochastic discount factor  $m_{t+k} = \beta^j \frac{\chi_{t+j} P_t}{\chi_t P_{t+j}}$ .

Define  $p_t^* = \frac{P_t^*}{P_t}$ . Using the demand for firm j and the aggregate price index, we rewrite the first-order condition to the maximization problem (28) and the law of motion for  $p_t^*$  as:

$$F_t^p = y_t \chi_t + \gamma_p \beta \left(\frac{\bar{\pi}}{\pi_{t+1}}\right)^{1-\theta_p} F_{t+1}^p \tag{29}$$

$$K_t^p = \frac{\theta_p}{\theta_p - 1} y_t \chi_t z_t + \gamma_p \beta \left(\frac{\bar{\pi}}{\pi_{t+1}}\right)^{-\theta_p} K_{t+1}^p$$
(30)

$$\frac{K_t^p}{F_t^p} = p_t^* \tag{31}$$

$$1 = \gamma_p \left(\frac{\bar{\pi}}{\pi_t}\right)^{1-\theta_p} + (1 - \gamma_p) (p_t^*)^{1-\theta_p}$$
 (32)

## Aggregation

We use the variable  $p_t^+$  to capture the resource costs induced by inefficient price dispersion:

$$p_t^+ = (1 - \gamma_p) (p_t^*)^{-\theta_p} + \gamma_p \left(\frac{\bar{\pi}}{\pi_t}\right)^{-\theta_p} p_{t-1}^+$$
(33)

 $\boldsymbol{w}_t^+$  captures the loss in output caused by taking wage dispersion into account, we use the variable:

$$w_t^+ = (1 - \gamma_w) (w_t^*)^{-\theta_w} + \gamma_w \left(\frac{\bar{\pi}}{\pi_t^w}\right)^{-\theta_w} w_{t-1}^+$$
(34)

The equilibrium condition of the labor market then becomes:

$$l_t = w_t^+ l_t^d \tag{35}$$

The dispersion of wages causes a dispersion in utility across households. This dispersion is measured by the variable  $\tilde{w}_t^+$ :

$$\tilde{w}_{t}^{+} = (1 - \gamma_{w}) (w_{t}^{*})^{-\theta_{w}(1+\sigma_{l})} + \gamma_{w} \left(\frac{\bar{\pi}}{\pi_{t}^{w}}\right)^{-\theta_{w}(1+\sigma_{l})} \tilde{w}_{t-1}^{+}$$
(36)

Dividends are defined as:

$$d_t = y_t - r_t^k u_t k_{t-1} - w_t \frac{l_t}{w_t^+}$$
(37)

The resource constraint of the economy is given by

$$\frac{\left(\left(u_{t}k_{t-1}\right)^{\alpha}\left(l_{t}^{d}\varepsilon_{z,t}\right)^{1-\alpha}-\Omega\right)}{p_{t}^{+}}=c_{t}+I_{t}+c_{t}^{g}+\phi\left(u_{t}\right)k_{t-1}$$
(38)

We define the part of y measured in the data as:

$$y_t^m = y_t - \phi(u_t) k_{t-1}$$
 (39)

The aggregated utility across households is given by:

$$U_{t} = \frac{\epsilon_{q,t} \left( c_{t} - h c_{t-1} \right)^{1 - \sigma_{c}}}{1 - \sigma_{c}} - \psi_{l} \frac{\tilde{w}_{t}^{+} \left( \frac{l_{t}}{w_{t}^{+}} \right)^{1 + \sigma_{l}}}{1 + \sigma_{l}}$$

$$\tag{40}$$

welfare is defined as:

$$W_t = U_t + \beta W_{t+1}. \tag{41}$$

## Government and monetary policy

Fiscal policy is described by the following tax revenues  $x_t$  and the budget constraint.

$$x_t = \tau_t^w w_t l_t + \tau_t^k \left[ y_t - w_t \frac{l_t}{w_t^+} \right]$$

$$\tag{42}$$

$$b_t - \frac{R_{t-1}b_{t-1}}{\pi_t} = c_t^g + \tau_t^T - x_t \tag{43}$$

The Monetary policy is described by the following feedback rule written in log-deviation from steady state

$$\hat{R}_{t} = \rho_{R} \hat{R}_{t-1} + (1 - \rho_{R}) \left( \rho_{\pi} \hat{\pi}_{t} + \rho_{y} \hat{y}_{t}^{M} \right) + \hat{\epsilon}_{t}^{m}$$
(44)

## Shock processes

The following shock process are written in log-deviations from steady state

$$\hat{\tau}_t^T = \rho_T \hat{\tau}_{t-1}^T + \hat{\epsilon}_t^T \quad \hat{\epsilon}^T \ i.i.d. \tag{45}$$

$$\hat{c}_t^g = \rho_{cg}\hat{c}_{t-1}^g + \hat{\epsilon}_t^{cg} \quad \hat{\epsilon}^{cg} \ i.i.d \tag{46}$$

$$\hat{\varepsilon}_{i,t} = \rho_i \hat{\varepsilon}_{i,t-1} + \hat{\epsilon}_t^i \quad \hat{\epsilon}^i \ i.i.d. \tag{47}$$

$$\hat{\varepsilon}_{z,t} = \rho_z \hat{\varepsilon}_{z,t-1} + \hat{\epsilon}_t^z \quad \hat{\epsilon}^z \ i.i.d. \tag{48}$$

$$\hat{\varepsilon}_{q,t} = \rho_q \hat{\varepsilon}_{q,t-1} + \hat{\epsilon}_t^q \quad \hat{\epsilon}^q \ i.i.d. \tag{49}$$

To solve the competitive equilibrium of the model we need policy rules for the tax rates on capital income  $\tau_t^k$  and labor income  $\tau_t^w$ .

## A.2 Steady-state

To solve for the steady state we take the following as given:  $\bar{\tau}^k$ ,  $\bar{\tau}^w$ ,  $\bar{R}$ ,  $\bar{c}^g/\bar{y}$ ,  $\bar{\tau}^T/\bar{y}$ ,  $\bar{\varepsilon}_i = 1$ ,  $\bar{\varepsilon}_q = 1$ , and  $\bar{\varepsilon}_z = 1$ . It is straightforward to see that:

$$\bar{u} = 1 \text{ and } \bar{s} = \bar{s}' = 0,$$
 (50)

that the Tobin's q condition is satisfied for:

$$\bar{q} = 1, \tag{51}$$

and the capital adjustment cost equations can be solved for:

$$\bar{\phi} = 0 \quad \bar{\phi}' = \bar{r}^k \left( 1 - \bar{\tau}^k \right) \tag{52}$$

Given these conditions we can solve for the steady state in the following way:

$$\bar{\pi} = \bar{R}\beta \tag{53}$$

$$\bar{\pi}^w = \bar{\pi} \tag{54}$$

$$\bar{z} = \frac{\theta_p - 1}{\theta_p} \tag{55}$$

$$\bar{r}^k = \frac{1 - \beta \left(1 - \delta\right)}{\beta \left(1 - \bar{\tau}^k\right)} \tag{56}$$

$$\bar{\phi}' = \bar{r}^k \left( 1 - \bar{\tau}^k \right) \tag{57}$$

$$\bar{w} = (1 - \alpha) \left(\bar{r}^k\right)^{\frac{-\alpha}{(1 - \alpha)}} \left(\alpha^{\alpha} \bar{z}\right)^{\frac{1}{1 - \alpha}} \tag{58}$$

$$\frac{\bar{l}}{\bar{k}} = \frac{1 - \alpha}{\alpha} \frac{\bar{r}^k}{w} \tag{59}$$

$$\frac{\bar{c}}{\bar{k}} = \left(1 - \frac{\bar{c}^g}{\bar{y}}\right) \bar{z} \left(\frac{\bar{l}}{\bar{k}}\right)^{1-\alpha} - \delta \tag{60}$$

Now, we assume that labor supply in steady state is  $\bar{l} = 1/3$  and solve for the corresponding scaling parameter:

$$\psi_l = \frac{\bar{w}\frac{\theta_w - 1}{\theta_w} (1 - h)^{-\sigma_c} (1 - \bar{\tau}^w) (1 - \beta h) \frac{\bar{c}}{k}^{-\sigma_c} \frac{\bar{l}}{k}^{\sigma_c}}{l^{\sigma_c + \sigma_l}}$$
(61)

$$\bar{k} = \bar{l}\frac{\bar{k}}{\bar{l}} \tag{62}$$

Now, we solve for the fixed costs of the firm to ensure that dividends in the steady state are  $\bar{d} = 0$ .

$$\Omega = (1 - \bar{z}) \,\bar{k}^{\alpha} \bar{l}^{1-\alpha};\tag{63}$$

$$\bar{y} = \bar{k}^{\alpha} \bar{l}^{1-\alpha} - \Omega \tag{64}$$

$$\bar{c} = \frac{\bar{c}}{\bar{k}}\bar{k} \tag{65}$$

$$\bar{I} = \delta \bar{k};$$
 (66)

$$\bar{c}^g = \frac{\bar{c}^g}{\bar{y}}\bar{y} \tag{67}$$

$$\bar{\tau}^T = \frac{\bar{\tau}^T}{\bar{y}} \bar{y} \tag{68}$$

$$\bar{x} = \bar{\tau}^w \bar{w} \bar{l} + \bar{\tau}^k \left( \bar{y} - \bar{w} \bar{l} \right) \tag{69}$$

$$\bar{b} = \frac{\left(\bar{\tau}^T + \bar{c}^g - \bar{x}\right)}{1 - 1/\beta} \tag{70}$$

#### Estimation В

#### B.1 Data

All data are in levels and nominal values and the frequency is quarterly. Nominal data are converted to real values by dividing by the GDP deflator (BEA NIPA table 1.1.4 line 1). The calculation of the fiscal data is following Leeper, Plante, and Traum (2010).

mean(gdp\_rgd\_obs) (!! per capita here

gdp\_rgd\_obs -

**Nominal Output:** This series is (BEA NIPA table 1.1.5 line 1).

Alfred: PCND

**Private consumption:** This series is defined as private consumption of non-durable goods cnds\_rim\_obs -(BEA NIPA table 1.1.5 line 5) and private consumption of services (BEA NIPA table mean(cnds\_rim\_obs) (!! per capita here) Alfred: PCES (M)/  $1.1.5 \ line \ 6$ ). PCESV (Q)

Alfred: GPDI

**Private investment:** This series is gross private domestic investment (BEA NIPA table igid\_rim\_obs\_ 1.1.5 line 7) plus private consumption of durable goods (BEA NIPA table 1.1.5 line mean(igid\_rim\_obs) Alfred: PCEDG (M)/ PCDG (Q)

(!! per capita here)

Alfred: GDPCTPI

Use existing obs Inflation: The gross inflation rate is defined as the change in the implicit GDP deflator. pi\_dm\_obs

Nominal interest rate: The quarterly nominal interest rate is defined as the averages of Use existing obs daily figures of the effective fed funds rate obtained from the Board of Governors of ffr\_obs the Federal Reserve System.

Alfred: COMPNFB

GDPCTPI

Nominal Wage: This series is defined as nonfarm business, all persons, hourly compensa-Use existing obs tion duration index (2005=100), seasonally adjusted provided by the U.S. Department wage\_rgd\_demean\_obs of Labor (PRS85006103). Alfred: PRS85006023

(!! per capita here

Hours worked: This series is defined as product of: nonfarm business, all persons, average weekly hours duration index (2005=100), seasonally adjusted (PRS85006023) and civilian employment, 16 years and over, measured in thousands and seasonally adjusted (CE16OV). Both time series are are provided by U.S. Department of Labor. The latter one is transformed into an index where 2005:4=1. Alfred:

Alfred: A957RC1Q027SBEA

A787RC1Q027SBEA Government expenditure: This series is defined as the sum of government consumption expenditures (BEA NIPA table 3.2 line 21), government gross investment (BEA NIPA table 3.2 line 42), and government net purchases of non-produced assets (BEA NIPA $\sim$ table 3.2 line 43), minus government consumption of fixed capital (BEA NIPA table Alfred: A918RC1Q027SBEA 3.2 line 44). Alfred:

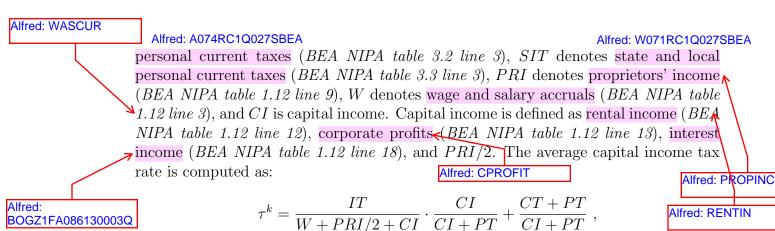
AD08RC1Q027SBEA

Lexites: Capital and labor tax rates are calculated following Leeper et al. (2010), where Me labor tax rate is computed as:

$$\tau^w = \frac{IT + SIT}{W + PRI/2 + CI} \cdot \frac{(W + PRI/2)}{EC + PRI/2} + \frac{CSI}{EC + PRI/2} \; ,$$

where CSI denotes total contributions to social insurance (BEA NIPA table 3.2 line 11), EC denotes compensation of employees (BEA NIPA table 1.12 line 2), IT denotes

Alfred: COE



where CT denotes taxes on corporate income  $\langle BEA \ NIPA \ table \ 3.2 \ line \ 7 \rangle$  and PTAlfred: denotes property taxes (BEA NIPA table 3.3 line 8). B075RC1Q027SBEA Updated until only 2018Q1

Government tax revenues: Tax revenues, x, are defined as the sum of capital income taxes and taxes on labor. They are computed as:

Alfred:

Alfred:

Alfred:

Alfred:

W780RC1Q027SBEA

W009RC1Q027SBEA

 $x = \tau^w \cdot (EC + PRI/2) + \tau^k \cdot (CI + PT) \,.$  Alfred: W014RC1Q027SEA Government transfers: This series is defined as the sum of net current transfers, net W014RC1Q027SBEA Alfred: capital transfers, and subsidies (BEA NIPA table 3.2 line 32) minus the tax residual W020RC1Q02 7SBEA Moreover, net current transfers are current transfer payments (BEA NIPA table 3.1 W011RC1Q027SBEA Alfred:

B232RC1Q02

W006RC1Q0 27SBEA

**7SBEA** 

Alfred:

line 22) minus current transfer receipts (BEA NIPA table 3.2 line 16), net capital transfers are defined as the difference between capital transfer payments (BEA NIPA table 3.2 line 43) and capital transfer receipts (BEA NIPA table 3.2 line 39). The tax residual is defined as the sum of current tax receipts (BEA NIPA table 3.2 line 2), contributions of government social insurance (BEA NIPA table 3.2 line 11), income receipts on assets (BEA NIPA table 3.2 line 12), and the current surplus of government enterprises (BEA NIPA table 3.2 line 19), minus government tax revenues (as defined

above).

Alfred: B097RC1Q027SBEA

Government debt: The change in government debt in the model is given by the net borrowing of the government. The net borrowing is computed using the NIPA deficits concept: government expenditure + transfers - tax revenues (all variables as defined A091RC1Q027SBEA above) plus interest payments (BEA NIPA table 3.2 line 29). As initial value for our debt series we use the market value of gross federal debt from December 1946 provided by Cox and Hirschhorn (1983).

> The observable variables output, consumption, investment, wages, hours worked, government debt, and tax revenues are converted to per capita values by dividing them by the population index.

> **Population Index:** index of population constructed so that 2005:4=1. The population series is is defined as the civilian non-institutional population aged 16 and over provided by the U.S. Department of Labor.

## B.2 System of log-linearized equations

In the following we list the final set of log-linearized equations used to estimate the baseline as well as the extended model. In comparison to the model solution section, we include additional observable equation and shocks.

First-order conditions of the households':

$$(1 - \beta h)\,\hat{\chi}_t = \hat{\varepsilon}_{q,t} - \frac{\sigma_c}{1 - h}\,(\hat{c}_t - h\hat{c}_{t-1}) - h\beta\hat{\varepsilon}_{q,t+1} + \frac{h\beta\sigma_c}{1 - h}\,(\hat{c}_{t+1} - h\hat{c}_t) \tag{71}$$

$$0 = \hat{\chi}_{t+1} - \hat{\chi}_t - \hat{\pi}_{t+1} + \hat{R}_t \tag{72}$$

$$\hat{k}_t = (1 - \delta) \,\hat{k}_{t-1} + \delta \hat{I}_t + \delta \hat{\varepsilon}_{i,t}; \tag{73}$$

$$\hat{I}_{t} = \frac{\hat{I}_{t-1}}{(1+\beta)} + \frac{\beta \hat{I}_{t+1}}{(1+\beta)} + \frac{\hat{q}_{t}}{\nu (1+\beta)} + \frac{\hat{\varepsilon}_{i,t}}{\nu (1+\beta)}$$
(74)

$$\hat{\chi}_{t} + \hat{q}_{t} = \hat{\chi}_{t+1} + \beta \left[ (1 - \delta) \, \hat{q}_{t+1} + \bar{r}^{k} \, (1 - \bar{\tau}^{k}) \, \hat{r}_{t+1}^{k} - \bar{r}^{k} \bar{\tau}^{k} \hat{\tau}_{t+1}^{k} \right]$$
(75)

$$\sigma_u \hat{u}_t = \hat{r}^k - \frac{\bar{\tau}^k}{1 - \bar{\tau}^k} \hat{\tau}_t^k \tag{76}$$

Staggered prices and wages:

$$\hat{\pi}_{t}^{w} = \beta \hat{\pi}_{t+1}^{w} + \frac{(1 - \gamma_{w})(1 - \beta \gamma_{w})}{\gamma_{w}(1 + \theta_{w}\sigma_{l})} \left( \sigma_{l} \hat{l}_{t} - \hat{\chi}_{t} - \hat{w}_{t} + \frac{\bar{\tau}^{w}}{(1 - \bar{\tau}^{w})} \hat{\tau}^{w} \right) + \hat{\epsilon}_{t}^{l}$$
(77)

$$\hat{\pi}_t = \beta \hat{\pi}_{t+1} + \frac{(1 - \gamma_p)(1 - \beta \gamma_p)}{\gamma_p} (\hat{z}_t + \hat{\varepsilon}_{p,t})$$
(78)

$$\hat{\pi}_t^w = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t \tag{79}$$

Firm:

$$\hat{z}_t + (1 - \alpha)\hat{\varepsilon}_{z,t} + \alpha \left(\hat{u}_t + \hat{k}_{t-1}\right) - \alpha \hat{l}_t = \hat{w}_t$$
(80)

$$\hat{z}_t + (1 - \alpha)\,\hat{\varepsilon}_{z,t} + (\alpha - 1)\,\left(\hat{u}_t + \hat{k}_{t-1}\right) + (1 - \alpha)\,\hat{l}_t = \hat{r}_t^k \tag{81}$$

Supply and demand:

$$\bar{y}\hat{y}_t = \bar{k}^{\alpha}\bar{l}^{1-\alpha}\left(\alpha\hat{k}_{t-1} + (1-\alpha)\left(\hat{l}_t + \hat{\varepsilon}_{z,t}\right) + \alpha\hat{u}_t\right)$$
(82)

$$\bar{y}\hat{y}_t = \bar{c}\hat{c}_t + \bar{I}\hat{I}_t + \bar{c}^g\hat{c}_t^g + \bar{r}^k \left(1 - \bar{\tau}^k\right)\bar{k}\hat{u}_t + \hat{\epsilon}_t^y \tag{83}$$

$$\bar{y}\hat{y}^m = \bar{y}\hat{y}_t - \bar{r}^k \left(1 - \bar{\tau}^k\right) \bar{k}\hat{u}_t \tag{84}$$

Government:

$$\bar{b}\hat{b}_{t} - \frac{\bar{b}}{\beta} \left( \hat{R}_{t-1} + \hat{b}_{t-1} - \hat{\pi}_{t} \right) = \bar{c}^{g} \hat{c}_{t}^{g} + \bar{\tau}^{T} \hat{\tau}_{t}^{T} - \bar{x} \hat{x}_{t}$$
 (85)

$$\bar{x}\hat{x}_t = \bar{\tau}^w \bar{w}\bar{l}\left(\hat{\tau}_t^w + \hat{w}_t + \hat{l}_t\right) + \bar{\tau}^k \bar{r}^k \bar{k}\hat{\tau}_t^k + \bar{\tau}^k \left(\bar{y}\hat{y}_t - \bar{w}\bar{l}\left(\hat{w}_t + \hat{l}_t\right)\right)$$
(86)

Policy rules:

$$\hat{R}_{t} = \rho_{R} \hat{R}_{t-1} + (1 - \rho_{R}) \left( \rho_{\pi} \hat{\pi}_{t} + \rho_{y} \hat{y}_{t}^{m} \right) + \hat{\epsilon}_{t}^{m}$$
(87)

$$\hat{\tau}_t^w = \rho_w \hat{\tau}_{t-1}^w + (1 - \rho_w) \left( \eta_{wb} \hat{b}_{t-1} + \eta_{wy} \hat{y}_t^m + \eta_{wh} \hat{l}_t \right) + \hat{\epsilon}_t^{\tau^w}$$
(88)

$$\hat{\tau}_t^k = \rho_k \hat{\tau}_{t-1}^k + (1 - \rho_k) \left( \eta_{kb} \hat{b}_{t-1} + \eta_{ky} \hat{y}_t^m + \eta_{kI} \hat{I}_t \right) + \hat{\epsilon}_t^{\tau^k}$$
(89)

Exogenous variables:

$$\hat{c}_t^g = \rho_{cg}\hat{c}_{t-1}^g + \hat{\epsilon}_t^{cg} \tag{90}$$

$$\hat{\tau}_t^T = \rho_{\tau_t} \hat{\tau}_{t-1}^T + \hat{\epsilon}_T^{\tau_L} \tag{91}$$

$$\hat{\varepsilon}_{z,t} = \rho_z \hat{\varepsilon}_{z,t-1} + \hat{\epsilon}_t^z \tag{92}$$

$$\hat{\varepsilon}_{i,t} = \rho_i \hat{\varepsilon}_{i,t-1} + \hat{\epsilon}_t^i \tag{93}$$

$$\hat{\varepsilon}_{q,t} = \rho_q \hat{\varepsilon}_{q,t-1} + \hat{\epsilon}_t^q \tag{94}$$

$$\hat{\varepsilon}_{p,t} = \rho_P \hat{\varepsilon}_{p,t-1} + \frac{\hat{\epsilon}_t^p}{((1 - \gamma_p)(1 - \gamma_p \beta)/\gamma_p)}$$
(95)

Observable variables:

$$\hat{y}_t^{obs} = \hat{y}_t^m - \hat{y}_{t-1}^m \tag{96}$$

$$\hat{c}_t^{obs} = \hat{c}_t - \hat{c}_{t-1} \tag{97}$$

$$\hat{I}_t^{obs} = \hat{I}_t - \hat{I}_{t-1} \tag{98}$$

$$\hat{w}_t^{obs} = \hat{w}_t - \hat{w}_{t-1} \tag{99}$$

$$\hat{b}_t^{obs} = \hat{b}_t - \hat{b}_{t-1} \tag{100}$$

$$\hat{g}_t^{obs} = \hat{c}_t^g - \hat{c}_{t-1}^g \tag{101}$$

$$\hat{x}_t^{obs} = \hat{x}_t - \hat{x}_{t-1} \tag{102}$$

# B.3 Prior and calibration

Description	Symbol	Value
Discount factor	β	0.9935
Capital share	$\alpha$	0.36
Depreciation rate	$\delta$	0.025
Price markup	$\theta_p/(\theta_p-1)$	1.2
Wage markup	$\theta_w/(\theta_w-1)$	1.1
Annualized nominal interest rate	$ar{R}^4$	1.0530
Ratio of government consumption to output	$ar{c}^g/ar{y}$	0.085
Ratio of government transfers to output	$ar{ au}^T/ar{y}$	0.105
Steady-state capital tax rate	$ar{ au}_k$	0.1929
Steady-state labor tax rate	$ar{ au}_w$	0.2088

Table 1: Parameter calibration.

Parameter	Symbol	Domain	Density	Para(1)	Para(2
Preference parameter	$\sigma_c$	$\mathbb{R}^+$	Gamma	1.75	0.5
Inverse Frisch elasticity	$\sigma_l$	$\mathbb{R}^+$	Gamma	2.0	1
Habit persistence	h	[0, 1)	Beta	0.5	0.15
Price stickiness	$\gamma_p$	[0, 1)	Beta	0.5	0.1
Wage stickiness	$\gamma_w$	[0, 1)	Beta	0.5	0.1
Investment adjustment cost	$\nu$	$\mathbb{R}^+$	Gamma	4	0.75
Capital utilization cost	$\sigma_u$	$\mathbb{R}^+$	Gamma	2	0.5
Taylor-rule smoothing	$ ho_R$	[0, 1)	Beta	0.8	0.1
Taylor-rule inflation	$ ho_{\pi}$	$\mathbb{R}^{+}$	Gamma	1.7	0.1
Taylor-rule output	$ ho_y$	$\mathbb{R}$	Gamma	0.125	0.05
Labor-tax smoothing	$ ho_w$	[0, 1)	Beta	0.85	0.1
Labor-tax debt	$\eta_{wb}$	$\mathbb{R}$	Normal	0	0.5
Labor-tax output	$\eta_{wy}$	$\mathbb{R}$	Normal	0	0.5
Labor-tax hours worked	$\eta_{wh}$	$\mathbb{R}$	Normal	0	0.5
Capital-tax smoothing	$\rho_k$	[0, 1)	Beta	0.85	0.1
Capital-tax debt	$\eta_{kb}$	$\mathbb{R}$	Normal	0	0.5
Capital-tax output	$\eta_{ky}$	$\mathbb{R}$	Normal	0	0.5
Capital-tax investment	$\eta_{kI}$	$\mathbb{R}$	Normal	0	0.5
AR transfers	$ ho_{ au^T}$	[0, 1)	Beta	0.85	0.1
AR investment specific	$ ho_i$	[0, 1)	Beta	0.85	0.1
AR technology	$\rho_z$	[0, 1)	Beta	0.85	0.1
AR public consumption	$ ho_{cg}$	[0, 1)	Beta	0.85	0.1
AR price mark-up	$ ho_p$	[0,1)	Beta	0.85	0.1
S.d. investment specific	$\epsilon_i$	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. technology	$\epsilon_z$	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. preference	$\epsilon_q$	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. monetary policy	$\epsilon_m^{^q}$	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. labor tax	$\epsilon_{ au^w}$	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. capital tax	$\epsilon_{ au^k}$	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. transfer	$\epsilon_{ au^T}$	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. public consumption	$\epsilon_{cg}^{'}$	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. price mark-up shock	$\epsilon_{ au^k}$	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. wage mark-up shock	$\epsilon_{ au^l}$	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. resource constraint	$\epsilon_{cg}$	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. measurement error taxes	$\epsilon_{tax}$	$\mathbb{R}^+$	InvGam	0.01	4.0

Table 2: Prior distribution of model parameters. Para(1) and Para(2) correspond to means and standard deviations for the Beta, Gamma, Inverted Gamma, and Normal distribution.

## B.4 Results

			Baseline N	aseline Model		Recommended Model		
Parameter	Symbol	Mode	Median	[5 95]	Mode	Median	[5 95]	
Preference parameter	$\sigma_c$	1.718	1.932	[1.22, 2.69]	1.742	1.927	[1.23,2.67	
Inverse Frisch elasticity	$\sigma_l$	3.430	3.507	[1.65, 5.58]	3.305	3.405	[1.57, 5.44]	
Habit persistence	h	0.567	0.564	[0.42, 0.69]	0.561	0.566	[0.42, 0.70]	
Price stickiness	$\gamma_p$	0.677	0.684	[0.64, 0.72]	0.678	0.687	[0.64, 0.73]	
Wage stickiness	$\gamma_w$	0.659	0.665	[0.59, 0.73]	0.657	0.663	[0.59, 0.73]	
Investment adjustment cost	$\nu$	2.961	3.270	[2.26, 4.24]	2.906	3.207	[2.25, 4.23]	
Capital utilization cost	$\sigma_u$	1.753	1.798	[1.10,2.49]	1.682	1.746	[1.05,2.09	
Taylor-rule smoothing	$ ho_R$	0.832	0.833	[0.80, 0.86]	0.832	0.833	[0.80,0.86	
Taylor-rule inflation	$ ho_{\pi}$	1.928	1.936	[1.77, 2.10]	1.932	1.933	[1.77,2.09	
Taylor-rule output	$ ho_y$	0.028	0.033	[0.01, 0.05]	0.029	0.035	[0.01,0.05	
Labor-tax smoothing	$ ho_w$	0.848	0.863	[0.79, 0.94]	0.802	0.826	[0.73,0.92	
Labor-tax debt	$\eta_{wb}$	0.120	0.138	[-0.00, 0.28]	0.074	0.097	[-0.01,0.24	
Labor-tax output	$\eta_{wy}$	0.417	0.385	[-0.28, 1.02]	-	-	-	
Labor-tax hours worked	$\eta_{wh}$	-	-	-	0.858	0.760	[0.03, 1.43]	
Capital-tax smoothing		0.868	0.875	[0.81, 0.93]	0.831	0.849	[0.77, 0.92]	
Capital-tax debt	$ ho_k$	0.303 $0.447$	0.463	[0.31, 0.33] $[0.17, 0.79]$	0.331 $0.428$	0.349 $0.454$	[0.17, 0.32]	
Capital-tax debt Capital-tax output	$\eta_{kb}$	0.008	0.403 $0.040$	[-0.76, 0.81]	0.420	0.404	[0.13,0.11	
Capital-tax investment	$\eta_{ky}$	-	-	[-0.70,0.01]	0.496	0.498	[0.04, 0.95]	
Capital-tax investment	$\eta_{kI}$				0.450	0.430	[0.04,0.33	
AR transfers	$ ho_{ au^T}$	0.869	0.866	[0.79, 0.94]	0.869	0.866	[0.79, 0.94]	
AR investment specific	$ ho_i$	0.878	0.863	[0.79, 0.92]	0.873	0.857	[0.79, 0.91]	
AR technology	$ ho_z$	0.979	0.971	[0.94, 0.99]	0.977	0.970	[0.94, 0.99]	
AR preference	$ ho_q$	0.885	0.879	[0.77, 0.96]	0.884	0.873	[0.76, 0.96]	
AR public consumption	$ ho_{cg}$	0.963	0.961	[0.93, 0.98]	0.962	0.961	[0.93, 0.98]	
AR price mark-up	$ ho_p$	0.964	0.933	[0.86, 0.99]	0.960	0.928	[0.85,0.99	
S.d. investment specific x100	$\epsilon_i$	2.605	2.903	[2.15, 3.66]	2.719	3.030	[2.28,3.82	
S.d. technology x100	$\epsilon_z$	0.637	0.642	[0.56, 0.71]	0.638	0.643	[0.57, 0.71]	
S.d. preference x100	$\epsilon_q$	1.891	2.169	[1.38, 3.03]	1.886	2.158	[1.47,2.97	
S.d. monetary policy x100	$\epsilon_m^{'}$	0.157	0.158	[0.14, 0.18]	0.157	0.158	[0.14, 0.17]	
S.d. labor tax x100	$\epsilon_{ au^w}$	2.172	2.214	[1.95, 2.46]	2.123	2.167	[1.92, 2.42]	
S.d. capital tax x100	$\epsilon_{ au^k}$	3.822	3.884	[3.45, 4.34]	3.733	3.789	[3.36, 4.23]	
S.d. transfer x100	$\epsilon_{ au^T}$	4.942	5.008	[4.43, 5.62]	4.941	4.990	[4.44, 5.57]	
S.d. public consumption x100	$\epsilon_{cg}$	2.631	2.662	[2.36, 2.96]	2.632	2.670	[2.37,2.98	
S.d. price mark-up shock x100	$\epsilon_p$	0.148	0.149	[0.12, 0.17]	0.147	0.148	[0.12, 0.17]	
S.d. wage mark-up shock x100	$\epsilon_l$	0.714	0.727	[0.63, 0.82]	0.716	0.729	[0.63, 0.82]	
S.d. resource constraint x100	$\epsilon_y$	1.110	1.119	[0.99, 1.25]	1.110	1.116	[0.99, 1.24]	
S.d. measurement error taxes x100	$\epsilon_{tax}$	0.533	0.540	[0.48, 0.60]	0.533	0.538	[0.47,0.60	
Log data density			4163.99			4166.24		

Table 3: Posterior mode and posterior distribution of model's parameters.

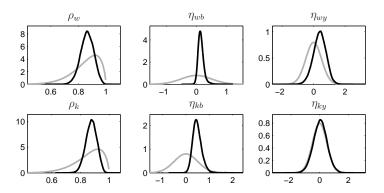


Figure 1: Posterior versus prior distribution for tax feedback rule parameters of the baseline model.

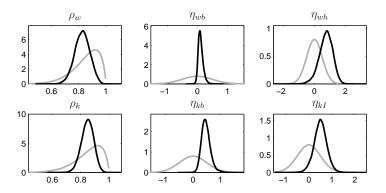


Figure 2: Posterior versus prior distribution for tax feedback rule parameters of the recommended model.

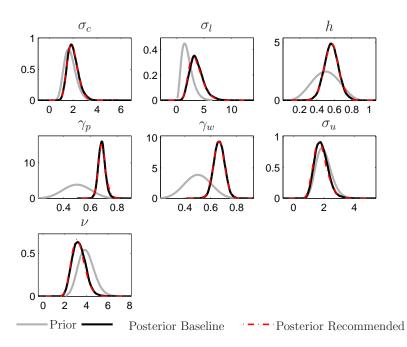


Figure 3: Posterior versus prior distribution for both model estimations.

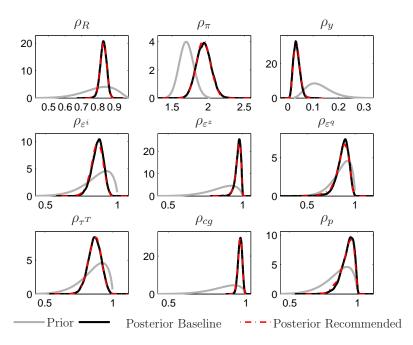


Figure 4: Posterior versus prior distribution for both model estimations.

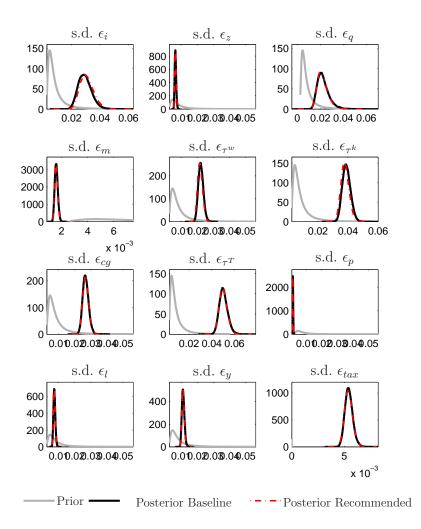


Figure 5: Posterior versus prior distribution for both model estimations.

## **B.5** Diagnostics

This section contains the Dynare diagnostic output for the estimations.

## B.5.1 Baseline Model

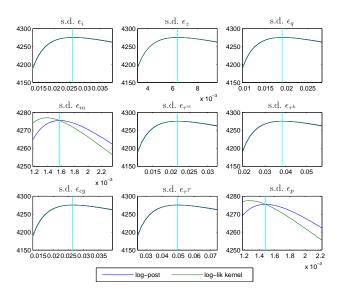


Figure 6: Check plots for posterior mode maximization of the baseline model.

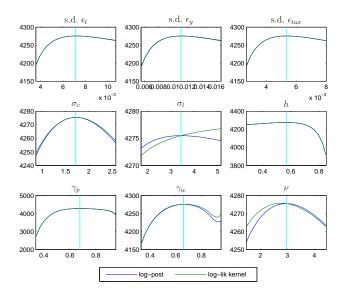


Figure 7: Check plots for posterior mode maximization of the baseline model.

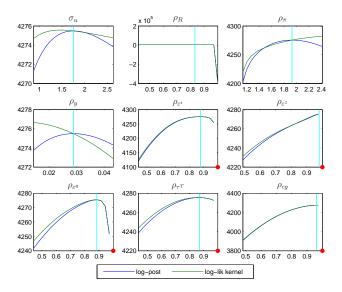


Figure 8: Check plots for posterior mode maximization of the baseline model.

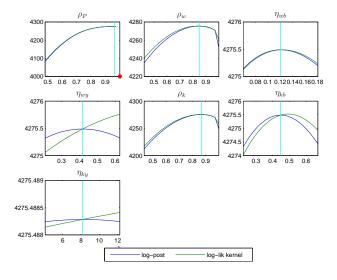


Figure 9: Check plots for posterior mode maximization of the baseline model.

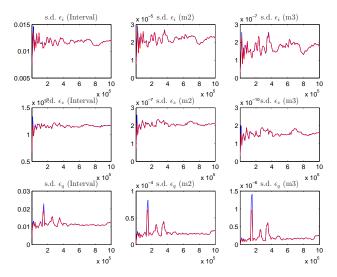


Figure 10: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

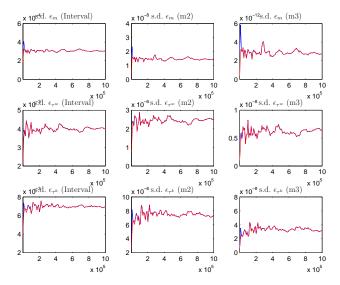


Figure 11: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

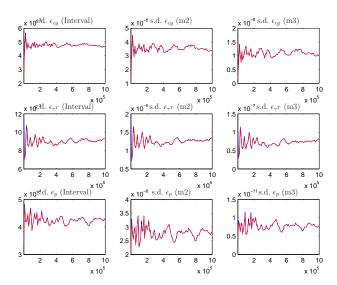


Figure 12: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

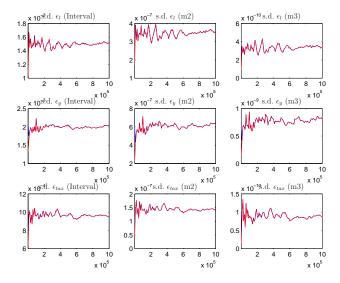


Figure 13: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

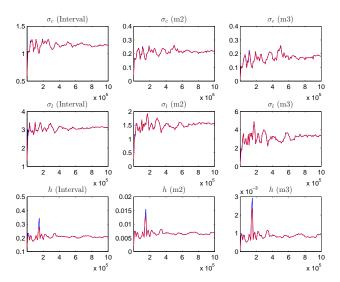


Figure 14: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

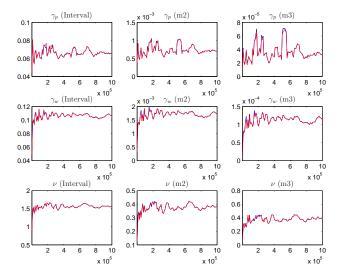


Figure 15: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

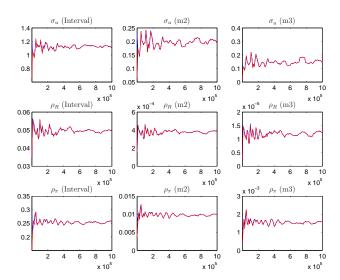


Figure 16: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

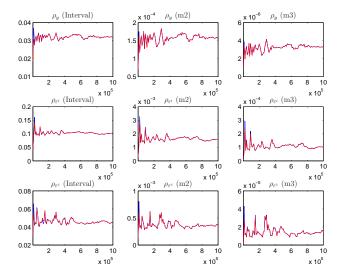


Figure 17: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

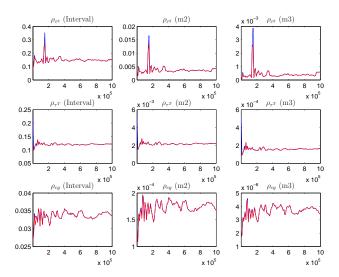


Figure 18: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

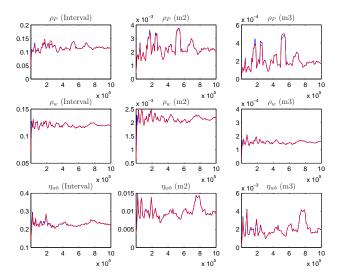


Figure 19: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

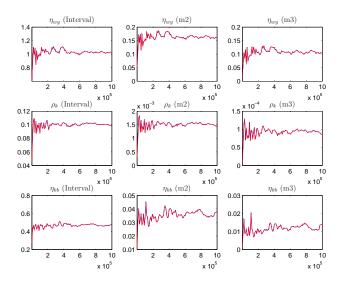


Figure 20: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

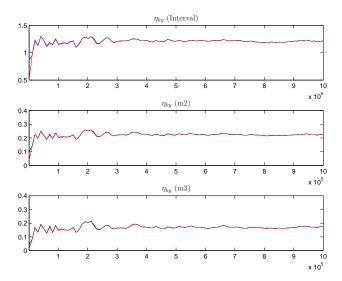


Figure 21: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

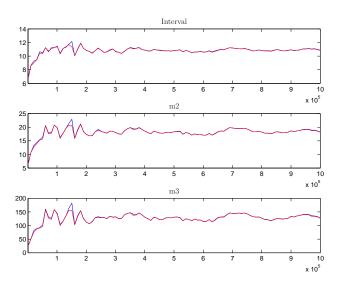


Figure 22: Multivariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third rows are respectively the criteria based on the eighty percent interval, the second and third moments. The different parameters are aggregated using the posterior kernel.

## **B.5.2** Recommended Model

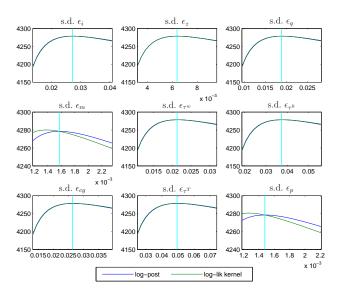


Figure 23: Check plots for posterior mode maximization of the recommended model.

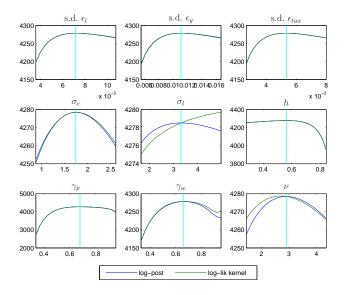


Figure 24: Check plots for posterior mode maximization of the recommended model.

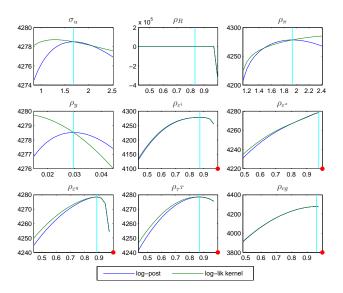


Figure 25: Check plots for posterior mode maximization of the recommended model.

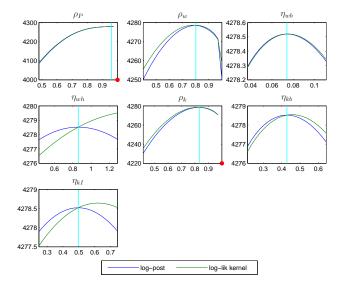


Figure 26: Check plots for posterior mode maximization of the recommended model.

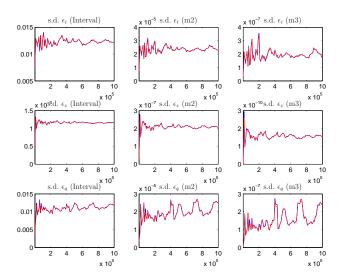


Figure 27: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

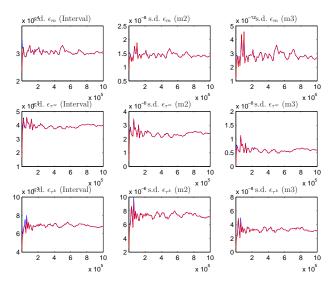


Figure 28: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

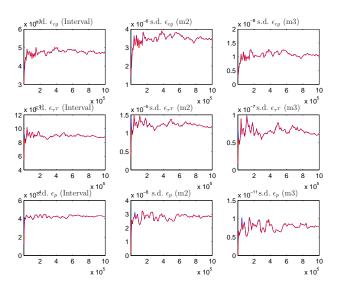


Figure 29: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

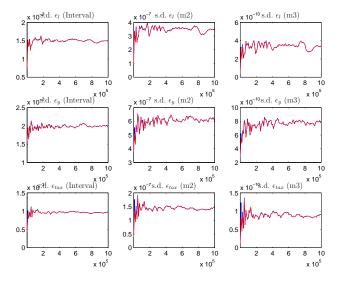


Figure 30: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

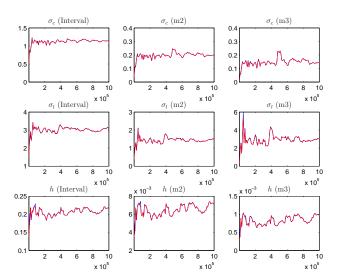


Figure 31: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

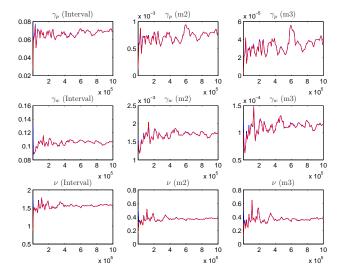


Figure 32: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

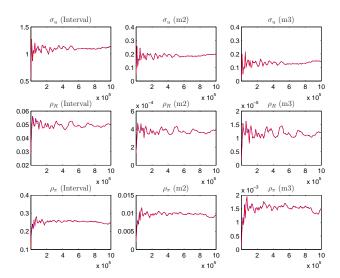


Figure 33: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

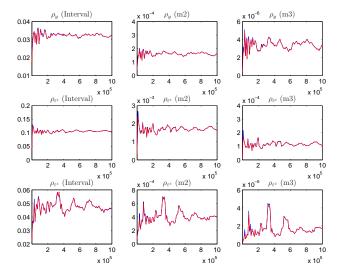


Figure 34: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

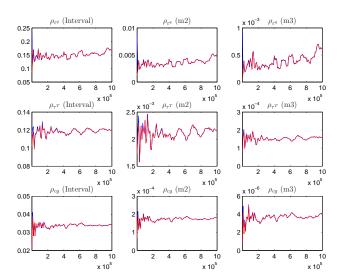


Figure 35: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

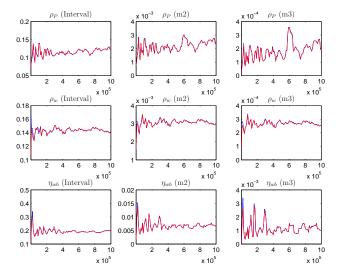


Figure 36: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

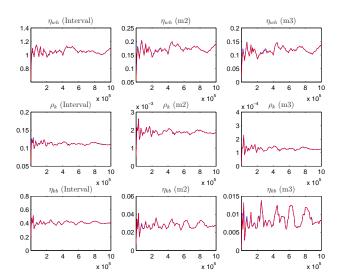


Figure 37: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

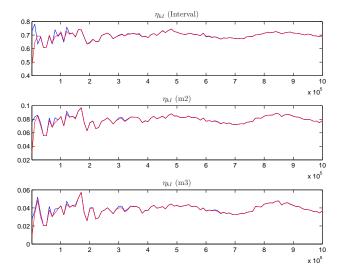


Figure 38: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

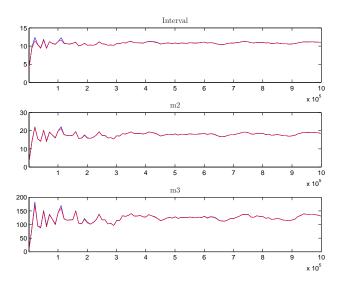


Figure 39: Multivariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third rows are respectively the criteria based on the eighty percent interval, the second and third moments. The different parameters are aggregated using the posterior kernel.

## C Variable selection

## C.1 Optimal Policy

Let N be the number of endogenous variables. The optimal policy problem is defined as maximizing the life-time expected utility

$$E_0 \sum_{t=j}^{\infty} \beta^j U \left( c_{t+j} - h c_{t+j-1}, l_{t+j} \right), \tag{103}$$

where aggregate utility is defined by Equation (40), subject to the following (N-2) equations: (2), (3)-(7), (16)-(20), (26), (27), (29)-(38), and (42)- (44).

The first-order conditions of the maximization problem yield 2N-2 equations for the N endogenous variables and N-2 Lagrangian multipliers associated with the private sector equilibrium constraints.

The optimal equilibrium is then defined as a set of stationary variables  $F_t^w$ ,  $F_t^p$ ,  $K_t^w$ ,  $K_t^p$ ,  $p_t^*$ ,  $w_t^*$ ,  $d_t$ ,  $p_t^+$ ,  $w_t^+$ ,  $\pi_t^w$ ,  $\pi_t$ ,  $w_t$ ,  $y_t$ ,  $l_t$ ,  $l_t^d$ ,  $k_t$ ,  $z_t$ ,  $\varepsilon_{i,t}$ ,  $\varepsilon_{z,t}$ ,  $\varepsilon_{q,t}$ ,  $\chi_t$ ,  $l_t$ ,  $c_t$ ,  $u_t$ ,  $r_t^k$ ,  $b_t$ ,  $x_t$ ,  $R_t$ ,  $\tau_t^T$ ,  $\tau_t^w$ ,  $\tau_t^k$ ,  $c_t^g$ ,  $\tilde{w}_t^+$ , and N-2 Lagrangian multipliers satisfying the first-order conditions of the optimal policy problem, as well as the laws of motion for the autoregressive shock processes (45)-(49), given exogenous stochastic processes  $\{\epsilon_t^i, \epsilon_t^q, \epsilon_t^z, \epsilon_t^{cg}, \epsilon_t^T, \epsilon_t^m\}_{t=0}^{\infty}$ , values of the N endogenous variables dated t < 0, and values of the (N-2) Lagrangian multipliers dated t < 0.

## C.2 Estimation general feedback rule

For the posterior mode estimation of the simple linear feedback rules, the DSGE model is closed with the following tax rules:

$$\hat{\tau}_{t}^{w} = \eta_{wk} \hat{k}_{t-1} + \eta_{wb} \hat{b}_{t-1} + \eta_{wy} \hat{y}_{t} + \eta_{wc} \hat{c}_{t} + \eta_{wh} \hat{l}_{t} + \eta_{ww} \hat{w}_{t} + \eta_{wI} \hat{I}_{t} + \eta_{w\pi} \hat{\pi}_{t} + \eta_{wR} \hat{R}_{t}$$
(104)

$$\hat{\tau}_{t}^{k} = \eta_{kk}\hat{k}_{t-1} + \eta_{kb}\hat{b}_{t-1} + \eta_{ky}\hat{y}_{t} + \eta_{kc}\hat{c}_{t} + \eta_{kh}\hat{l}_{t} + \eta_{kw}\hat{w}_{t} + \eta_{kI}\hat{I}_{t} + \eta_{k\pi}\hat{\pi}_{t} + \eta_{kR}\hat{R}_{t}$$
(105)

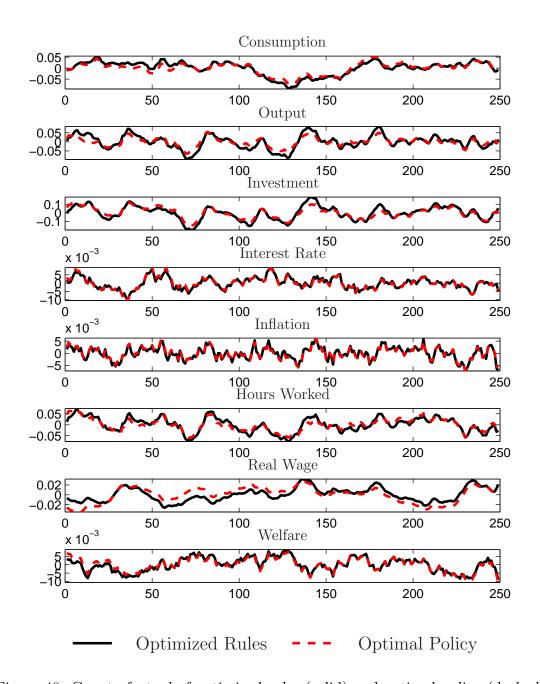


Figure 40: Counterfactual of optimized rules (solid) and optimal policy (dashed).

Feedback Parameter	Symbol	Mode S.d.		T-value					
Tax Rate on Labor Income									
Capital	$\eta_{wk}$	-0.8091	0.1630	4.9641					
Debt	$\eta_{wb}$	-0.0568	0.0486	1.1694					
Output	$\eta_{wy}$	1.7889	0.5867	3.0490					
Consumption	$\eta_{wc}$	-0.3455	0.3949	0.8749					
Hours worked	$\eta_{wh}$	-6.1840	0.4780	12.9382					
Wage rate	$\eta_{ww}$	-0.7528	0.6158	1.2224					
Investment	$\eta_{wI}$	-0.2179	0.2582	0.8438					
Inflation	$\eta_{w\pi}$	-5.2469	0.9003	5.8277					
Nominal interest rate	$\eta_{wR}$	5.5651	0.8505	6.5433					
TAX RATE ON CAPITAL INCOME									
TAX I(AI	E ON CAL	FIIAL INC	OME						
Capital	$\eta_{kk}$	-0.3715	0.6508	0.5709					
Debt	$\eta_{kb}$	-2.3788	0.1330	17.8898					
Output	$\eta_{ky}$	0.4596	0.5448	0.8436					
Consumption	$\eta_{kc}$	1.7116	0.3849	4.4472					
Hours worked	$\eta_{kh}$	0.9508	0.4208	2.2592					
Wage rate	$\eta_{kw}$	3.7079	0.7443	4.9820					
Investment	$\eta_{kI}$	3.5151	0.2365	14.8605					
Inflation	$\eta_{k\pi}$	2.3441	0.8985	2.6090					
Nominal interest rate	$\eta_{kR}$	-0.5434	0.6652	0.8168					

Table 4: Posterior mode maximization of optimized feedback coefficients.

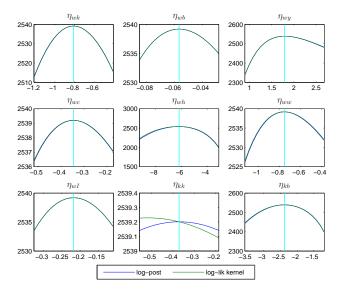


Figure 41: Check plots for posterior mode maximization of the model with optimized rules.

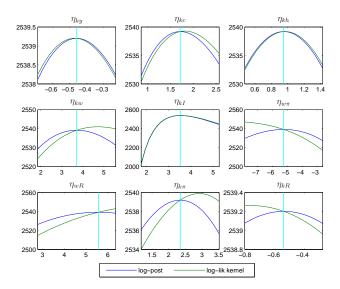


Figure 42: Check plots for posterior mode maximization of the model with optimized rules.

## C.3 Identification

Feedback Parameter	Symbol		Percentile					
	Symbol	50%	25%	75%				
Tax Rate on Labor Income								
Capital	$\eta_{wk}$	0.2915	0.1649	0.6022				
Output	$\dot{\eta}_{wy}$	0.8026	0.4848	1.2752				
Consumption	$\eta_{wc}$	0.1234	0.0414	0.3016				
Hours worked	$\eta_{wh}$	2.9360	2.3006	3.4760				
Wage rate	$\eta_{ww}$	0.1695	0.0630	0.3859				
Investment	$\eta_{wI}$	0.2609	0.1020	0.4687				
Inflation	$\eta_{w\pi}$	0.0835	0.0371	0.1956				
Nominal interest rate	$\eta_{wR}$	0.1671	0.0971	0.2351				
Tax Rate on Capital Income								
Capital	$\eta_{kk}$	0.0074	0.0024	0.0183				
Output	$\eta_{ky}$	0.0260	0.0105	0.0491				
Consumption	$\eta_{kc}$	0.0306	0.0147	0.0528				
Hours worked	$\eta_{kh}$	0.0299	0.0174	0.0456				
Wage rate	$\eta_{kw}$	0.0745	0.0463	0.1124				
Investment	$\eta_{kI}$	0.4195	0.2762	0.5751				
Inflation	$\eta_{k\pi}$	0.0023	0.0011	0.0039				
Nominal interest rate	$\eta_{kR}$	0.0020	0.0007	0.0039				

Table 5: Elasticity of income tax rate's variance w.r.t. corresponding feedback parameters of the tax rules.

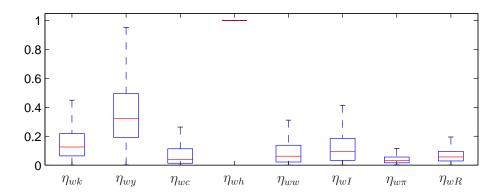


Figure 43: Relative elasticity of variables' variance w.r.t. feedback parameters of the labor income tax rule.

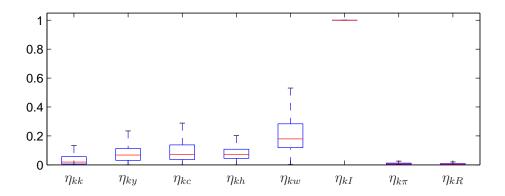


Figure 44: Relative elasticity of variables' variance w.r.t. feedback parameters of the capital income tax rule.

## C.4 Robustness

**Extended feedback rules** For the first robustness check we extend the linear feedback rules by the additional contemporaneous variables, rental rate of capital  $(r^k)$ , capacity utilization rate (u), and marginal costs (z):

$$\hat{\tau}_{t}^{w} = \eta_{wk}\hat{k}_{t-1} + \eta_{wb}\hat{b}_{t-1} + \eta_{wy}\hat{y}_{t} + \eta_{wc}\hat{c}_{t} + \eta_{wh}\hat{l}_{t} + \eta_{ww}\hat{w}_{t} + \eta_{wI}\hat{I}_{t} + \eta_{w\pi}\hat{\pi}_{t} + \eta_{wR}\hat{R}_{t} + \eta_{wr}^{k}\hat{r}_{t}^{k} + \eta_{wu}\hat{u}_{t} + \eta_{wz}\hat{z}_{t}$$

$$(106)$$

$$\hat{\tau}_{t}^{k} = \eta_{kk}\hat{k}_{t-1} + \eta_{kb}\hat{b}_{t-1} + \eta_{ky}\hat{y}_{t} + \eta_{kc}\hat{c}_{t} + \eta_{kh}\hat{l}_{t} + \eta_{kw}\hat{w}_{t} + \eta_{kI}\hat{I}_{t} + \eta_{k\pi}\hat{\pi}_{t} + \eta_{kR}\hat{R}_{t} + \eta_{kr}\hat{r}_{t}^{k} + \eta_{ku}\hat{u}_{t} + \eta_{kz}\hat{z}_{t}$$

$$(107)$$

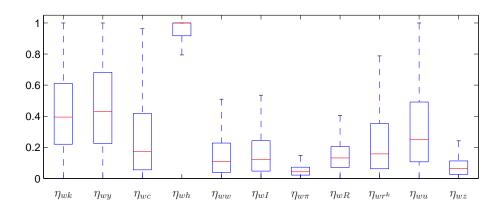


Figure 45: Relative elasticity of variables' variance w.r.t. feedback parameters of the labor income tax rule.

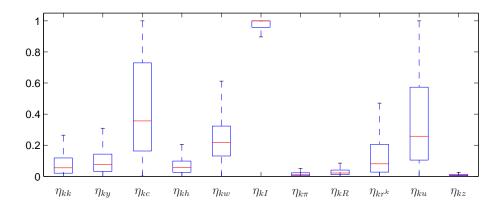


Figure 46: Relative elasticity of variables' variance w.r.t. feedback parameters of the capital income tax rule.

Feedback Parameter	Symbol	Mode	S.d.	T-value				
Tax Rate on Labor Income								
Capital	n ,	-1.3051	0.5459	2.3907				
Debt	$\eta_{wk}$	-0.2544	0.5459 $0.1389$	1.8321				
Output	$\eta_{wb}$	1.2004	0.1509 $0.6519$	1.8414				
Consumption	$\eta_{wy}$	-0.3576	0.0913 $0.3917$	0.9130				
Hours worked	$\eta_{wc}$	-4.0346	0.6091	6.6238				
Wage rate	$\eta_{wh} \ \eta_{ww}$	0.6934	0.6482	1.0698				
Investment	,	0.0334 $0.1518$	0.0462 $0.3168$	0.4791				
Inflation	$\eta_{wI} \ \eta_{w\pi}$	-3.1620	0.9463	3.3416				
Nominal interest rate	$\eta_{wR}$	3.9025	0.8686	4.4928				
Rental rate of capital	$\eta_{wr^k}$	-0.3056	0.8434	0.3624				
Utilization rate	$\eta_{wr^k} \ \eta_{wu}$	-1.7411	0.8756	1.9884				
Marginal cost	$\eta_{wz}$	-4.3520	0.7616	5.7141				
1710181101 0000	'Twz	1.0020	0.1010	0.1111				
Tax Rate on Capital Income								
Capital	$\eta_{kk}$	-0.4039	0.6827	0.5917				
Debt	$\eta_{kb}$	-1.7218	0.2127	8.0962				
Output	$\eta_{ky}$	-0.2347	0.6107	0.3844				
Consumption	$\eta_{kc}$	1.7596	0.3625	4.8547				
Hours worked	$\eta_{kh}$	-0.6248	0.6317	0.9890				
Wage rate	$\eta_{kw}$	2.6813	0.7686	3.4884				
Investment	$\eta_{kI}$	2.9619	0.3030	9.7739				
Inflation	$\eta_{k\pi}$	1.6159	0.9532	1.6951				
Nominal interest rate	$\eta_{kR}$	-1.5342	0.6032	2.5433				
Rental rate of capital	$\eta_{wr^k}$	-0.4046	0.8617	0.4695				
Utilization rate	$\eta_{wu}$	2.8653	0.9400	3.0482				
Marginal cost	$\eta_{wz}$	2.2522	0.7152	3.1491				

Table 6: Posterior mode maximization of optimized feedback coefficients.

**Key parameter variation** We conduct robustness exercises with respect to key parameters for optimal policy dynamics as pointed out Schmitt-Grohé and Uribe (2006). We investigate different parameter combination by taken draws from the corresponding marginal posterior distribution. In particular, we focus on the 5th and 95th HPD.

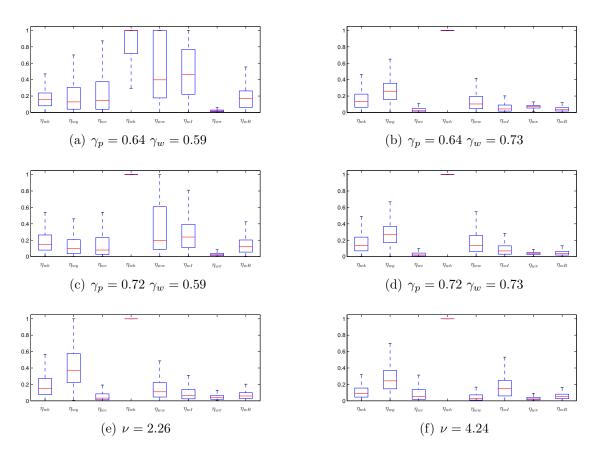


Figure 47: Relative elasticity of labor income tax rate's variance w.r.t. feedback parameters of the labor income tax rule for different degrees of stickiness and investment adjustment costs.

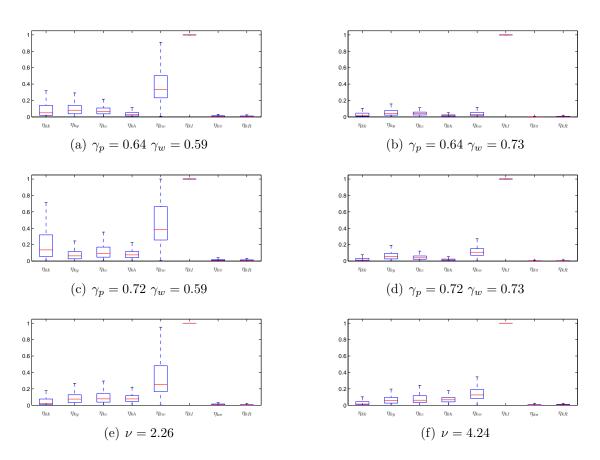


Figure 48: Relative elasticity of capital income tax rate's variance w.r.t. feedback parameters of the capital income tax rule for different degrees of stickiness and investment adjustment costs.

Feedback Parameter	Symbol	$\gamma_p = 0.64$ $\gamma_w = 0.59$	$\gamma_p = 0.64$ $\gamma_w = 0.73$	$\gamma_p = 0.72$ $\gamma_w = 0.59$	$\gamma_p = 0.72$ $\gamma_w = 0.73$	$\nu = 2.26$	$\nu = 4.24$
	Т	AX RATE ON	LABOR INC	COME			
Capital	$\eta_{wk}$	-0.7324 $(0.1324)$	-1.7804 (0.2945)	-0.7138 (0.1357)	-1.8529 (0.3224)	-0.9158 (0.1797)	-0.6619 (0.1450)
Debt	$\eta_{wb}$	0.0057 $(0.0426)$	0.0636 $(0.0746)$	-0.0494 (0.0366)	0.0599 $(0.0719)$	-0.0875 $(0.0552)$	-0.0303 (0.0396)
Output	$\eta_{wy}$	-0.4161 $(0.5482)$	2.6418 $(0.6897)$	0.0407 $(0.5435)$	2.8082 $(0.6917)$	1.9475 (0.6010)	1.5068 (0.5663)
Consumption	$\eta_{wc}$	-0.7218 $(0.3243)$	0.0661 $(0.5312)$	-0.4923 $(0.3272)$	0.1221 $(0.5447)$	-0.1696 $(0.4078)$	-0.5223 (0.3755)
Hours worked	$\eta_{wh}$	-2.9264 $(0.5529)$	-10.2702 $(0.5171)$	-3.5992 $(0.5048)$	-11.1865 (0.4746)	-6.4791 $(0.4797)$	-5.8093 (0.4753)
Wage rate	$\eta_{ww}$	2.8712 (0.6250)	-2.4406 (0.7728)	1.8655 (0.6175)	-3.2121 (0.7986)	-1.1579 (0.6335)	-0.2534 (0.5898)
Investment	$\eta_{wI}$	-0.6163 (0.2131)	0.0828 $(0.3702)$	-0.3780 (0.2100)	0.4425 $(0.3638)$	-0.0702 (0.2603)	-0.3625 (0.2526)
Inflation	$\eta_{w\pi}$	-2.3878 (0.8940)	-9.2705 (0.9096)	-3.0121 (0.9207)	-7.3200 (0.9535)	-5.8391 (0.9070)	-4.4779 (0.8929
Interest rate	$\eta_{wR}$	7.9244 (0.6047)	6.2028 $(0.9679)$	7.4014 (0.6393)	7.0912 (0.9680)	5.0765 (0.8658)	6.2683 (0.8209
	TA	X RATE ON	Capital In	COME			
Capital	$\eta_{kk}$	-1.7948 (0.6858)	1.1275 (0.6566)	-2.4998 (0.6395)	0.5004 $(0.6565)$	-0.3119 (0.6701)	-0.6313 (0.6453)
Debt	$\eta_{kb}$	-2.4939 $(0.1394)$	-2.3552 $(0.1292)$	-2.1145 $(0.1348)$	-2.0731 $(0.1251)$	-2.5843 $(0.1353)$	-2.0653 (0.1330
Output	$\eta_{ky}$	0.4434 $(0.5849)$	-0.0827 $(0.5761)$	-0.2500 $(0.5513)$	-0.3770 $(0.5458)$	-0.4657 $(0.5509)$	-0.3521 $(0.5398)$
Consumption	$\eta_{kc}$	1.8144 (0.4010)	2.0510 (0.4366)	1.6716 (0.3676)	1.9963 (0.4118)	1.4863 (0.3908)	1.9642 (0.3799
Hours worked	$\eta_{kh}$	0.3948 $(0.4702)$	0.4377 (0.4736)	0.7783 (0.4009)	0.4121 $(0.3988)$	0.9155 $(0.4195)$	0.9548 $(0.4237)$
Wage rate	$\eta_{kw}$	5.2745 (0.7453)	0.2039 $(0.8184)$	6.6905 $(0.7313)$	1.9955 $(0.8025)$	4.3386 $(0.7366)$	3.1701 (0.7515
Investment	$\eta_{kI}$	3.1101 (0.2367)	4.7290 (0.3002)	2.7392 (0.2137)	4.2606 (0.2831)	3.1974 $(0.2309)$	3.8879 $(0.2492)$
Inflation	$\eta_{k\pi}$	2.6401 $(0.8758)$	0.0968 (0.8783)	2.6780 $(0.9206)$	0.8251 $(0.9347)$	2.5235 (0.8963)	2.0629 (0.9008
Interest rate	$\eta_{kR}$	-0.0529 (0.7899)	0.4807 $(0.7439)$	-0.9840 (0.6833)	0.3156 $(0.6986)$	-0.1066 (0.6684)	-1.1154 (0.6642

Table 7: Posterior mode maximization of optimized feedback coefficients for different degrees of stickiness and investment adjustment costs. The values in parenthesis are the corresponding standard deviations.

## D Conditional welfare costs

We follow Schmitt-Grohé and Uribe (2007) and calculate the conditional welfare cost in consumption units relative to the optimal policy solution by solving for  $\Psi$  the following equation

$$W_{t}^{O} = E_{t} \sum_{j=0}^{\infty} \beta^{j} \left[ \varepsilon_{q,t+j} \frac{\left(c_{t+j}^{s} - hc_{t+j-1}^{s}\right)^{1-\sigma_{c}} \left(1 + \Psi\right)^{1-\sigma_{c}}}{1 - \sigma_{c}} - \psi_{l} \frac{\tilde{w}_{t+j}^{+,s} \left(\frac{l_{t+j}^{s}}{w_{t+j}^{+,s}}\right)^{1+\sigma_{l}}}{1 + \sigma_{l}} \right]. \quad (108)$$

This yields:

$$\Psi = \left[ \frac{\mathcal{W}_t^O + \mathcal{W}_{L,t}^s}{\mathcal{W}_t^s + \mathcal{W}_{L,t}^s} \right]^{\frac{1}{1-\sigma_c}} - 1 \tag{109}$$

where variables with O represents variables under the optimal solution and variables with S indicates variables under the alternative fiscal policy regime with simple fiscal feedback rules. The variable  $\mathcal{W}_{L,t}^s$  is defined as:

$$\mathcal{W}_{L,t}^{s} = E_{t} \sum_{j=0}^{\infty} \beta^{j} \left[ \psi_{l} \frac{\tilde{w}_{t+j}^{+,s} \left( \frac{l_{t+j}^{s}}{w_{t+j}^{+,s}} \right)^{1+\sigma_{l}}}{1+\sigma_{l}} \right]$$

$$(110)$$

For the minimization of  $\psi$  we have to set bounds for the parameters to ensure theoretical meaningful results. More precisely we decide to restrict the coefficient in front of debt to be between [0,10] and [-10,0] for labor income tax rates and capital income tax rates respectively to ensure sustainability. The positive or negative range is related to the optimal steady-state. Moreover, we assume that all automatic stabilizers have to be between [-20,20].

Fiscal feedback rule with	Feedback parameter				Welfare cost $\Psi \ge 100$
No automatic stabilizer	$\eta_{wb}$	$\eta_{kb}$	-	-	
	2.8499	0.000			2.542
Automatic stabilizer	$\eta_{wb}$	$\eta_{kb}$	$\eta_{wy}$	$\eta_{ky}$	
(baseline model)	0.204	-1.664	-3.788	14.722	1.602
Automatic stabilizer	$\eta_{wb}$	$\eta_{kb}$	$\eta_{wh}$	$\eta_{kI}$	
(recommended model)	0.000	-3.186	-3.697	8.400	1.175

Table 8: Welfare costs under different optimized fiscal feedback rules.

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