Technical Appendix for
"DSGE Models for Monetary Policy Analysis"

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A. Data Appendix

A.1. Data Sources

FRED2: Database of the Federal Reserve Bank of St. Louis available at:
http://research.stlouisfed.org/fred2/.
BLS: Database of the Bureau of Labor Statistics available at: http://www.bls.gov/.
FR: Fujita and Ramey (2006) data on job separations and job findings available at:
http://www.philadelphiafed.org/research-and-data/economists/fujita/transition_rates.xls.
SH: Data on job separations and job findings available at Robert Shimer's Homepage:
http://robert.shimer.googlepages.com/.
NIPA: Database of the National Income And Product Accounts available at:
http://www.bea.gov/national/nipaweb/index.asp
BGOV: Database of the Board of Governors of the Federal Reserve System available at:
http://www.federalreserve.gov/econresdata/default.htm.
CONFB: Database of the Conference Board available at:
http://www.conference-board.org/economics/HelpWanted.cfm

A.2. Raw Data

Nominal GDP (GDP)**:** nominal gross domestic product, billions of dollars, seasonally adjusted at annual rates, NIPA.
GDP Deflator (P) : price index of nominal gross domestic product, index numbers, 2005=100, seasonally adjusted, NIPA.
Nominal nondurable consumption ($\mathrm{C}^{nom}_{nondurables}$) : nominal personal consumption expenditures: nondurable goods, billions of dollars, seasonally adjusted at annual rates, NIPA.
Nominal durable consumption ($\mathrm{C}^{nom}_{durables}$) : nominal personal consumption expenditures: durable goods, billions of dollars, seasonally adjusted at annual rates, NIPA.
Nominal consumption services ($\mathrm{C}^{nom}_{services}$) : nominal personal consumption expenditures: services, billions of dollars, seasonally adjusted at annual rates, NIPA.
Nominal investment (I^{nom}) : nominal gross private domestic investment, billions of dollars, seasonally adjusted at annual rates, NIPA.
Price index: nominal durable consumption ($\mathrm{PC}^{nom}_{durables}$) : price index of durable goods, index numbers, 2005=100, seasonally adjusted at annual rates, NIPA.
Price index: nominal investment (PI^{nom}) : price index of nominal gross private domestic investment, index numbers, 2005=100, seasonally adjusted at annual rates, NIPA.
Employment (E)**:** civilian employment, CE16OV, seasonally adjusted, monthly, thousands, persons 16 years of age and older, FRED2.
Federal Funds Rate (FF)**:** effective federal funds rate, H.15 selected interest rates, monthly, percent, averages of daily figures, FRED2.

Treasury bill rate ($TBill$): 3-month treasury bill: secondary market rate, H.15 selected interest rates, monthly, percent, averages of business days, discount basis, FRED2.
Population (POP)**:** civilian noninstitutional population, not seasonally adjusted, monthly, thousands, FRED2.
Capacity utilization (CAP): capacity utilization, G.17 - industrial production and capacity utilization, UTL: manufacturing (SIC) G17/CAPUTL/CAPUTL.B00004.S.Q. , seasonally adjusted, percentage, BGOV.
Job separation rate (S)**:** separation rate: E to U, seasonally adjusted, monthly, 1976M1-2008M12, FR. Spliced with corresponding data from Robert Shimer for the sample before 1976, quarterly. SH.
Job finding rate (F)**:** Job finding rate: U to E, seasonally adjusted, monthly, 1976M1-2008M12, FR. Spliced with corresponding data from Robert Shimer for the sample before 1976, quarterly. SH.
Vacancies (V): index of help wanted advertising in newspapers, HELPWANT, The Conference Board, seasonally adjusted, monthly, index 1987=100,
Unemployment rate (U)**:** unemployment rate labor force status: unemployment rate, LNS14000000, seasonally adjusted, percent, 16 years and over, monthly frequency, BLS.
Nominal wage (W): nominal hourly compensation, PRS85006103, sector: nonfarm business, seasonally adjusted, index, 1992 = 100, BLS.
Average hours ($H^{avg.}$): average weekly hours, PRS85006023, sector: nonfarm business, seasonally adjusted, index, 1992 = 100, BLS.
Participation rate ($LabForce$) : civilian participation rate, CIVPART, the employment situation, seasonally adjusted, monthly, percent, BLS.

A.3. Data Transformations

Raw data are transformed as follows. POP is seasonally adjusted using the X12 (multiplicative) method. The indices for W and $H^{avg.}$ are normalized such that 2005=100. E, FF, $TBill$, POP, V, U and $LabForce$ are converted to quarterly frequencies by averaging monthly observations. For the job finding rate F, we compute the quarterly measure from monthly data as follows:

$$F_{q1} = F_{m1} + (1 - F_{m1})F_{m2} + (1 - (1 - F_{m1})F_{m2})F_{m3},$$

where F_{q1} denotes the finding rate of quarter 1 and F_{m1}, F_{m2}, F_{m3} are the corresponding monthly finding rates. The case for the separation rate, S, follows accordingly.

Due to missing data we use $TBill$ as a proxy for the FF prior 1954Q3. All data are available from 1948Q1 except for vacancies for which the first observation is 1951Q1.

We calculate the following time series which, among others, is used in the VAR:

$$\begin{aligned}
\text{real } GDP &= \frac{GDP}{P * POP} \\
\text{hours} &= \frac{H^{avg.} * E}{Pop} \\
\text{nominal consumption} &= C^{nom}_{nondurables} + C^{nom}_{services} \\
\text{nominal investment} &= I^{nom} + C^{nom}_{durables}
\end{aligned}$$

The price of investment is calculated as a Torn price index using PI^{nom}, $PC^{nom}_{durables}$, I^{nom} and $C^{nom}_{durables}$.The resulting price index PIT and quantity index QIT are used to calculate the relative price of investment as follows:

$$\text{relative price of investment}=\frac{PIT * I^{nom}}{P * QIT}.$$

B. Scaling of Variables in Medium-sized Model

We adopt the following scaling of variables. The neutral shock to technology is z_t and its growth rate is $\mu_{z,t}$:

$$\frac{z_t}{z_{t-1}} = \mu_{z,t}.$$

The variable, Ψ_t, is an investment specific technology shock and it is convenient to define the following combination of our two technology shocks:

$$\begin{aligned} z_t^+ &\equiv \Psi_t^{\frac{\alpha}{1-\alpha}} z_t, \\ \mu_{z^+,t} &\equiv \mu_{\Psi,t}^{\frac{\alpha}{1-\alpha}} \mu_{z,t}. \end{aligned} \tag{B.1}$$

Capital, $\bar{K}_t$, and investment, I_t, are scaled by $z_t^+\Psi_t$. Consumption goods C_t, government consumption G_t and the real wage, W_t/P_t are scaled by z_t^+. Also, υ_t is the multiplier on the nominal household budget constraint in the Lagrangian version of the household problem. That is, υ_t is the marginal utility of one unit of currency. The marginal utility of a unit of consumption is $\upsilon_t P_t$. The latter must be multiplied by z_t^+ to induce stationarity. Output, Y_t, is scaled by z_t^+. Optimal prices, $\tilde{P}_t$, chosen by intermediate good firms which are subject to Calvo price setting frictions are scaled by the price, P_t,of the homogeneous output good. Similarly, optimal wages, $\tilde{W}_t$, chosen by monopoly unions which are subject to Calvo wage setting frictions are scaled by the wage, W_t, of the homogenous labour input. Thus our scaled variables are:

$$\begin{aligned} k_{t+1} &= \frac{K_{t+1}}{z_t^+\Psi_t},\ \bar{k}_{t+1} = \frac{\bar{K}_{t+1}}{z_t^+\Psi_t},\ i_t = \frac{I_t}{z_t^+\Psi_t},\ c_t = \frac{C_t}{z_t^+}, \\ g_t &= \frac{G_t}{z_t^+},\ \psi_{z^+,t} = \upsilon_t P_t z_t^+,\ \bar{w}_t = \frac{W_t}{z_t^+ P_t},\ \tilde{y}_t = \frac{Y_t}{z_t^+}, \\ \tilde{p}_t &= \frac{\tilde{P}_t}{P_t}, w_t = \frac{\tilde{W}_t}{W_t}. \end{aligned} \tag{B.2}$$

We define the scaled date t price of new installed physical capital for the start of period $t+1$ as $p_{k',t}$ and we define the scaled real rental rate of capital as $\bar{r}_t^k$:

$$p_{k',t} = \Psi_t P_{k',t},\ \bar{r}_t^k = \Psi_t r_t^k.$$

where $P_{k',t}$ is in units of the homogeneous good. The inflation rate is defined as:

$$\pi_t = \frac{P_t}{P_{t-1}}.$$

C. Equilibrium Conditions for the Medium-sized Model

C.1. Firms

We let s_t denote the firm's marginal cost, divided by the price of the homogeneous good. The standard formula, expressing this as a function of the factor inputs, is as follows:

$$s_t = \frac{\left(\frac{r_t^k P_t}{\alpha}\right)^{\alpha}\left(\frac{W_t R_t}{1-\alpha}\right)^{1-\alpha}}{P_t z_t^{1-\alpha}}.$$

When expressed in terms of scaled variables, this reduces to:

$$s_t = \left(\frac{\bar{r}_t^k}{\alpha}\right)^{\alpha}\left(\frac{\bar{w}_t R_t}{1-\alpha}\right)^{1-\alpha}. \tag{C.1}$$

Productive efficiency dictates that s_t is also equal to the ratio of the real cost of labor to the marginal product of labor:

$$s_t = \frac{\left(\mu_{\Psi,t}\right)^{\alpha}\bar{w}_t R_t}{(1-\alpha)\left(\frac{k_{i,t}}{\mu_{z^+,t}}/H_{i,t}\right)^{\alpha}}. \tag{C.2}$$

The only real decision taken by intermediate good firms is to optimize price when it is selected to do so under the Calvo frictions. The first order necessary conditions associated with price optimization are, after scaling:[1]

$$E_t\left[\psi_{z^+,t}y_t + \left(\frac{\tilde{\pi}_{f,t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_f}}\beta\xi_p F_{t+1}^f - F_t^f\right] = 0, \tag{C.3}$$

$$E_t\left[\lambda_f\psi_{z^+,t}y_t s_t + \beta\xi_p\left(\frac{\tilde{\pi}_{f,t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}} K_{t+1}^f - K_t^f\right] = 0, \tag{C.4}$$

$$\mathring{p}_t = \left[(1-\xi_p)\left(\frac{1-\xi_p\left(\frac{\tilde{\pi}_{f,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}\right)^{\lambda_f} + \xi_p\left(\frac{\tilde{\pi}_{f,t}}{\pi_t}\mathring{p}_{t-1}\right)^{\frac{\lambda_f}{1-\lambda_f}}\right]^{\frac{1-\lambda_f}{\lambda_f}}, \tag{C.5}$$

$$\left[\frac{1-\xi_p\left(\frac{\tilde{\pi}_{f,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}\right]^{(1-\lambda_f)} = \frac{K_t^f}{F_t^f}, \tag{C.6}$$

$$\tilde{\pi}_{f,t} \equiv \pi. \tag{C.7}$$

[1] When we log-linearize about the steady state, we obtain,

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\xi_p}\hat{s}_t,$$

where a hat indicates log-deviation from steady state.

C.2. Households

We now derive the equilibrium conditions associated with the household. We first consider the household's consumption saving decision. We then turn to its wage decision. The Lagrangian representation of the household's problem is:

$$
\begin{aligned}
&E_0^j \sum_{t=0}^{\infty} \beta^t \{ \left[\ln\left(C_t - bC_{t-1}\right) - A_L \frac{h_{j,t}^{1+\phi}}{1+\phi} \right] \\
&\upsilon_t \left[W_{t,j} h_{t,j} + X_t^k \bar{K}_t + R_{t-1} B_t + a_{t,j} - P_t \left(C_t + \frac{1}{\Psi_t} I_t \right) - B_{t+1} - P_t P_{k',t} \Delta_t \right] \\
&+\omega_t \left[\Delta_t + (1-\delta) \bar{K}_t + \left(1 - S\left(\frac{I_t}{I_{t-1}} \right) \right) I_t - \bar{K}_{t+1} \right] \}
\end{aligned}
$$

The first order condition with respect to C_t is:

$$
\frac{1}{C_t - bC_{t-1}} - E_t \frac{b\beta}{C_{t+1} - bC_t} = \upsilon_t P_t,
$$

or, after expressing this in scaled terms and multiplying by z_t^+:

$$
\psi_{z^+,t} = \frac{1}{c_t - b\frac{c_{t-1}}{\mu_{z^+,t}}} - \beta b E_t \frac{1}{c_{t+1}\mu_{z^+,t+1} - bc_t}. \tag{C.8}
$$

The first order condition with respect to Δ_t is, after rearranging:

$$
P_t P_{k',t} = \frac{\omega_t}{\upsilon_t}. \tag{C.9}
$$

The first order condition with respect to I_t is:

$$
\omega_t \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) - S'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] + E_t \beta \omega_{t+1} S'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 = \frac{P_t \upsilon_t}{\Psi_t}.
$$

Making use of (C.9), multiplying by $\Psi_t z_t^+$, rearranging and using the scaled variables,

$$
\begin{aligned}
&\psi_{z^+,t} p_{k',t} \left[1 - S\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}} \right) - S'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}} \right) \frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}} \right] \\
&+\beta \psi_{z^+,t+1} p_{k',t+1} S'\left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1} i_{t+1}}{i_t} \right) \left(\frac{i_{t+1}}{i_t} \right)^2 \mu_{z^+,t+1}\mu_{\Psi,t+1} \quad = \quad \psi_{z^+,t},
\end{aligned} \tag{C.10}
$$

Optimality of the choice of $\bar{K}_{t+1}$ implies the following first order condition:

$$
\omega_t = \beta E_t \upsilon_{t+1} X_{t+1}^k + \beta E_t \omega_{t+1} (1-\delta) = \beta E_t \upsilon_{t+1} \left[X_{t+1}^k + P_{t+1} P_{k',t+1} (1-\delta) \right].
$$

Using (C.9) again,

$$
\upsilon_t = E_t \beta \upsilon_{t+1} \left[\frac{X_{t+1}^k + P_{t+1} P_{k',t+1} (1-\delta)}{P_t P_{k',t}} \right] = E_t \beta \upsilon_{t+1} R_{t+1}^k, \tag{C.11}
$$

where R^k_{t+1} denotes the rate of return on capital:

$$R^k_{t+1} \equiv \frac{X^k_{t+1} + P_{t+1}P_{k',t+1}(1-\delta)}{P_t P_{k',t}}$$

Multiply (C.11) by $P_t z^+_t$ and express the results in scaled terms:

$$\psi_{z^+,t} = \beta E_t \psi_{z^+,t+1} \frac{R^k_{t+1}}{\pi_{t+1}\mu_{z^+,t+1}}. \tag{C.12}$$

Expressing the rate of return on capital, (**??**), in terms of scaled variables:

$$R^k_{t+1} = \frac{\pi_{t+1}}{\mu_{\Psi,t+1}} \frac{u_{t+1}\bar{r}^k_{t+1} - a(u_{t+1}) + (1-\delta)p_{k',t+1}}{p_{k',t}}. \tag{C.13}$$

The first order condition associated with capital utilization is:

$$\Psi_t r^k_t = a'(u_t),$$

or, in scaled terms,

$$\bar{r}^k_t = a'(u_t). \tag{C.14}$$

The first order condition with respect to B_{t+1} is:

$$\upsilon_t = \beta \upsilon_{t+1} R_t.$$

Multiply by $z^+_t P_t$:

$$\psi_{z^+,t} = \beta E_t \frac{\psi_{z^+,t+1}}{\mu_{z^+,t+1}\pi_{t+1}} R_t. \tag{C.15}$$

Finally, the law of motion for the capital stock, in terms of scaled variables is as follows:

$$\bar{k}_{t+1} = \frac{1-\delta}{\mu_{z^+,t}\mu_{\Psi,t}}\bar{k}_t + \left(1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t} i_t}{i_{t-1}}\right)\right) i_t. \tag{C.16}$$

C.3. Resource Constraint

The resource constraint after scaling by z^+_t is given by:

$$y_t = g_t + c_t + i_t + a(u_t)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}. \tag{C.17}$$

In appendix D we derive a relationship between total output of the homogeneous good, Y_t, and aggregate factors of production which in scaled form looks as follows:

$$y_t = (\mathring{p}_t)^{\frac{\lambda_f}{\lambda_f - 1}} \left[\left(\frac{1}{\mu_{\Psi,t}} \frac{1}{\mu_{z^+,t}} k_t\right)^{\alpha} H_t^{1-\alpha} - \varphi\right], \tag{C.18}$$

where

$$k_t = \bar{k}_t u_t. \tag{C.19}$$

Finally, GDP is given by:

$$gdp_t = g_t + c_t + i_t. \tag{C.20}$$

C.4. Wage Setting by the Monopoly Union

We turn now to the equilibrium conditions associated with the household wage-setting decision. Consider the j^{th} household that has an opportunity to reoptimize its wage at time t. We denote this wage rate by $\tilde{W}_t$. This is not indexed by j because the situation of each household that optimizes its wage is the same. In choosing $\tilde{W}_t$, the household considers the discounted utility (neglecting currently irrelevant terms in the household objective) of future histories when it cannot reoptimize:

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \left[-A_L \frac{(h_{j,t+i})^{1+\phi}}{1+\phi} + \upsilon_{t+i} W_{j,t+i} h_{j,t+i} \right],$$

where υ_t is the multiplier on the household's period t budget constraint. The demand for the j^{th} household's labor services, conditional on it having optimized in period t and not again since, is:

$$h_{j,t+i} = \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}.$$

Here, it is understood that $\tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \equiv 1$ when $i = 0$. Substituting this into the objective function and optimizing (see appendix F for details) yields the following equilibrium equations associated with wage setting:

$$\pi_{w,t+1} = \frac{W_{t+1}}{W_t} = \frac{\bar{w}_{t+1} z_{t+1}^{+} P_{t+1}}{\bar{w}_t z_t^{+} P_t} = \frac{\bar{w}_{t+1} \mu_{z^+,t+1} \pi_{t+1}}{\bar{w}_t}, \tag{C.21}$$

$$h_t = \mathring{w}_t^{\frac{\lambda_w}{1-\lambda_w}} H_t, \tag{C.22}$$

$$\mathring{w}_t = \left[(1-\xi_w) \left(\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w} \right)^{\lambda_w} + \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \mathring{w}_{t-1} \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}}. \tag{C.23}$$

In addition to (C.23), we have following equilibrium conditions associated with sticky wages[2]:

[2] Log−linearizing these equations about the nonstochastic steady state we obtain,

$$E_t \left[\begin{array}{c} \eta_0 \hat{\bar{w}}_{t-1} + \eta_1 \hat{\bar{w}}_t + \eta_2 \hat{\bar{w}}_{t+1} + \eta_3 \hat{\pi}_t + \eta_4 \hat{\pi}_{t+1} + \eta_5 \hat{\pi}_{t-1} \\ + \eta_6 \hat{\psi}_{z^+,t} + \eta_7 \hat{H}_t + \eta_8 \hat{\mu}_{z^+,t} + \eta_9 \hat{\mu}_{z^+,t+1} \end{array} \right] = 0,$$

where

$$\begin{aligned} b_w &= \frac{[\lambda_w \sigma_L - (1-\lambda_w)]}{[(1-\beta\xi_w)(1-\xi_w)]}, \eta_0 = b_w \xi_w, \eta_1 = \sigma_L \lambda_w - b_w \left(1 + \beta \xi_w^2\right), \eta_2 = b_w \beta \xi_w, \\ \eta_3 &= -b_w \xi_w (1 + \beta \kappa_w), \eta_4 = b_w \beta \xi_w, \eta_5 = b_w \xi_w \kappa_w, \eta_6 = (1-\lambda_w), \\ \eta_7 &= -(1-\lambda_w)\sigma_L, \eta_8 = -b_w \xi_w, \eta_9 = b_w \beta \xi_w. \end{aligned}$$

$$F_{w,t} = \frac{\psi_{z^+,t}}{\lambda_w} \mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t + \beta\xi_w E_t \left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1} \tag{C.24}$$

$$K_{w,t} = \left(\mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t\right)^{1+\phi} + \beta\xi_w E_t \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\phi)} K_{w,t+1} \tag{C.25}$$

$$\frac{1}{A_L}\left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w(1+\phi)} \bar{w}_t F_{w,t} = K_{w,t} \tag{C.26}$$

$$\tilde{\pi}_{w,t+1} = \pi_t^{\kappa_w} \pi^{(1-\kappa_w)} \mu_{z^+}. \tag{C.27}$$

C.5. Equilibrium Equations

The equilibrium conditions of the model correspond to the following 28 equations,

$$\begin{gathered}(C.1),(C.2),(C.3),(C.4),(C.5),(C.6),(C.7),(C.16),(C.8),(C.10),(C.14),\\(C.15),(C.22),(4.22),(C.17),(C.18),(4.24),(C.19),(4.25),(C.13),\\(C.21),(C.24),(C.25),(C.26),(C.23),(C.27),(C.20),(C.12),\end{gathered}$$

which can be used to solve for the following 28 unknowns:

$$\begin{gathered}\bar{r}_t^k, \bar{w}_t, R_t, s_t, \pi_t, p_{k',t}, k_{t+1}, \bar{k}_{t+1}, u_t, h_t, H_t, i_t, c_t, \psi_{z^+,t}, y_t,\\ K_t^f, F_t^f, \tilde{\pi}_{f,t}, \mathring{p}_t, K_{w,t}, F_{w,t}, \tilde{\pi}_t^w, R_t^k, S_t, a(u_t), \mathring{w}_t, \pi_{w,t}, gdp_t.\end{gathered}$$

D. Resource Constraint in the Medium-sized Model

We begin by deriving a relationship between total output of the homogeneous good, Y_t, and aggregate factors of production. We first consider the production of the homogenous output good:

$$\begin{aligned}Y_t^{sum} &= \int_0^1 Y_{i,t} di\\&= \int_0^1 \left[(z_t H_{i,t})^{1-\alpha} K_{i,t}^\alpha - z_t^+ \varphi\right] di\\&= \int_0^1 \left[z_t^{1-\alpha}\left(\frac{K_{i,t}}{H_{it}}\right)^\alpha H_{it} - z_t^+\varphi\right] di\\&= z_t^{1-\alpha}\left(\frac{K_t}{H_t}\right)^\alpha \int_0^1 H_{it} di - z_t^+\varphi,\end{aligned}$$

where K_t is the economy-wide average stock of capital services and H_t is the economy-wide average of homogeneous labor. The last expression exploits the fact that all intermediate good firms confront the same factor prices, and so they adopt the same capital services to homogeneous labor ratio. This follows from cost minimization, and holds for all firms, regardless whether or not they have an opportunity to reoptimize. Then,

$$Y_t^{sum} = z_t^{1-\alpha} K_t^{\alpha} H_t^{1-\alpha} - z_t^{+} \varphi.$$

The demand for $Y_{j,t}$ is

$$\left(\frac{P_t}{P_{i,t}}\right)^{\frac{\lambda_f}{\lambda_f - 1}} = \frac{Y_{i,t}}{Y_t},$$

so that

$$\mathring{Y}_t \equiv \int_0^1 Y_{i,t} di = \int_0^1 Y_t \left(\frac{P_t}{P_{i,t}}\right)^{\frac{\lambda_f}{\lambda_f - 1}} di = Y_t P_t^{\frac{\lambda_f}{\lambda_f - 1}} \left(\mathring{P}_t\right)^{\frac{\lambda_f}{1-\lambda_f}},$$

say, where

$$\mathring{P}_t = \left[\int_0^1 P_{i,t}^{\frac{\lambda_f}{1-\lambda_f}} di\right]^{\frac{1-\lambda_f}{\lambda_f}}. \tag{D.1}$$

Dividing by P_t,

$$\mathring{p}_t = \left[\int_0^1 \left(\frac{P_{it}}{P_t}\right)^{\frac{\lambda_f}{1-\lambda_f}} di\right]^{\frac{1-\lambda_f}{\lambda_f}},$$

or,

$$\mathring{p}_t = \left[(1-\xi_p)\left(\frac{1-\xi_p\left(\frac{\tilde{\pi}_t}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}\right)^{\lambda_f} + \xi_p\left(\frac{\tilde{\pi}_t}{\pi_t}\mathring{p}_{t-1}\right)^{\frac{\lambda_f}{1-\lambda_f}}\right]^{\frac{1-\lambda_f}{\lambda_f}}. \tag{D.2}$$

The preceding discussion implies:

$$Y_t = (\mathring{p}_t)^{\frac{\lambda_f}{\lambda_f - 1}} \mathring{Y}_t = (\mathring{p}_t)^{\frac{\lambda_f}{\lambda_f - 1}} \left[z_t^{1-\alpha} K_t^{\alpha} H_t^{1-\alpha} - z_t^{+}\varphi\right],$$

or, after scaling by z_t^{+},

$$y_t = (\mathring{p}_t)^{\frac{\lambda_f}{\lambda_f - 1}} \left[\left(\frac{1}{\mu_{\Psi,t}} \frac{1}{\mu_{z^+,t}} k_t\right)^{\alpha} H_t^{1-\alpha} - \varphi\right],$$

where

$$k_t = \bar{k}_t u_t.$$

Finally, we adjust hours worked in the resource constraint so that it corresponds to the total number of people working, as in (F.6):

$$y_t = (\mathring{p}_t)^{\frac{\lambda_f}{\lambda_f - 1}} \left[\left(\frac{1}{\mu_{\Psi,t}} \frac{1}{\mu_{z^+,t}} k_t\right)^{\alpha} \left[\mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t\right]^{1-\alpha} - \varphi\right].$$

It is convenient to also have an expression that exhibits the uses of the homogeneous output,

$$z_t^+ y_t = G_t + C_t + \tilde{I}_t,$$

or, after scaling by z_t^+:

$$y_t = g_t + c_t + i_t + a\left(u_t\right) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}.$$

E. Optimal Price Setting in the Medium-sized Model

The profit function of the i^{th} intermediate good firm with the substituted demand function is given by,

$$E_t \sum_{j=0}^{\infty} \beta^j \upsilon_{t+j} P_{t+j} Y_{t+j} \{ \left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{1-\frac{\lambda_f}{\lambda_f - 1}} - s_{t+j} \left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{\frac{-\lambda_f}{\lambda_f - 1}} \},$$

or,

$$E_t \sum_{j=0}^{\infty} \beta^j \upsilon_{t+j} P_{t+j} Y_{t+j} \{ \left(X_{t,j} \tilde{p}_t\right)^{1-\frac{\lambda_f}{\lambda_f - 1}} - s_{t+j} \left(X_{t,j} \tilde{p}_t\right)^{\frac{-\lambda_f}{\lambda_f - 1}} \},$$

where

$$\frac{P_{i,t+j}}{P_{t+j}} = X_{t,j} \tilde{p}_t,\ X_{t,j} \equiv \begin{cases} \frac{\tilde{\pi}_{t+j} \cdots \tilde{\pi}_{t+1}}{\pi_{t+j} \cdots \pi_{t+1}},\ j > 0 \\ 1,\ j = 0 \end{cases}.$$

The i^{th} firm maximizes profits by choice of $\tilde{p}_t$. The fact that this variable does not have an index, i, reflects that all firms that have the opportunity to reoptimize in period t solve the same problem, and hence have the same solution. Differentiating its profit function, multiplying the result by $\tilde{p}_t^{\frac{\lambda_f}{\lambda_f - 1}+1}$, rearranging, and scaling we obtain:

$$E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j A_{t+j} \left[\tilde{p}_t X_{t,j} - \lambda_f s_{t+j}\right] = 0,$$

where A_{t+j} is exogenous from the point of view of the firm:

$$A_{t+j} = \psi_{z^+,t+j} y_{t+j} X_{t,j}.$$

After rearranging the optimizing intermediate good firm's first order condition for prices, we obtain,

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j A_{t+j} \lambda_f s_{t+j}}{E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j A_{t+j} X_{t,j}} = \frac{K_t^f}{F_t^f},$$

say, where

$$\begin{aligned} K_t^f &\equiv E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j A_{t+j} \lambda_f s_{t+j} \\ F_t^f &= E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j A_{t+j} X_{t,j}. \end{aligned}$$

These objects have the following convenient recursive representations:

$$E_t\left[\psi_{z^+,t}y_t+\left(\frac{\tilde{\pi}_{,t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_f}}\beta\xi_p F_{t+1}^f-F_t^f\right] = 0$$

$$E_t\left[\lambda_f\psi_{z^+,t}y_t s_t+\beta\xi_p\left(\frac{\tilde{\pi}_{,t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}}K_{t+1}^f-K_t^f\right] = 0.$$

Turning to the aggregate price index:

$$\begin{aligned} P_t &= \left[\int_0^1 P_{it}^{\frac{1}{1-\lambda_f}}di\right]^{(1-\lambda_f)} \qquad \text{(E.1)}\\ &= \left[\left(1-\xi_p\right)\tilde{P}_t^{\frac{1}{1-\lambda_f}}+\xi_p\left(\tilde{\pi}_t P_{t-1}\right)^{\frac{1}{1-\lambda_f}}\right]^{(1-\lambda_f)}. \end{aligned}$$

After dividing by P_t and rearranging:

$$\frac{1-\xi_p\left(\frac{\tilde{\pi}_{,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}=\left(\tilde{p}_t\right)^{\frac{1}{1-\lambda_f}}. \qquad \text{(E.2)}$$

This completes the derivations of optimal decisions with respect to firms price setting.

F. Optimal Wage Setting in the Medium-sized Model

The objective function with the substituted labor demand function looks as follows:

$$\begin{aligned} &E_t^j\sum_{i=0}^{\infty}\left(\beta\xi_w\right)^i[-A_L\frac{\left(\left(\frac{\tilde{W}_t\tilde{\pi}_{w,t+i}\cdots\tilde{\pi}_{w,t+1}}{W_{t+i}}\right)^{\frac{\lambda_w}{1-\lambda_w}}H_{t+i}\right)^{1+\phi}}{1+\phi}\\ &+\upsilon_{t+i}\tilde{W}_t\tilde{\pi}_{w,t+i}\cdots\tilde{\pi}_{w,t+1}\left(\frac{\tilde{W}_t\tilde{\pi}_{w,t+i}\cdots\tilde{\pi}_{w,t+1}}{W_{t+i}}\right)^{\frac{\lambda_w}{1-\lambda_w}}H_{t+i}]. \end{aligned}$$

Recalling the scaling of variables, (B.2), we have

$$\begin{aligned} \frac{\tilde{W}_t\tilde{\pi}_{w,t+i}\cdots\tilde{\pi}_{w,t+1}}{W_{t+i}} &= \frac{\tilde{W}_t\tilde{\pi}_{w,t+i}\cdots\tilde{\pi}_{w,t+1}}{\bar{w}_{t+i}z_{t+i}^+P_{t+i}}=\frac{\tilde{W}_t}{\bar{w}_{t+i}z_t^+P_t}X_{t,i} \qquad \text{(F.1)}\\ &= \frac{W_t\left(\tilde{W}_t/W_t\right)}{\bar{w}_{t+i}z_t^+P_t}X_{t,i}=\frac{\bar{w}_t\left(\tilde{W}_t/W_t\right)}{\bar{w}_{t+i}}X_{t,i}=\frac{w_t\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}, \end{aligned}$$

where

$$\begin{aligned} X_{t,i} &= \frac{\tilde{\pi}_{w,t+i}\cdots\tilde{\pi}_{w,t+1}}{\pi_{t+i}\pi_{t+i-1}\cdots\pi_{t+1}\mu_{z^+,t+i}\cdots\mu_{z^+,t+1}},\ i>0 \qquad \text{(F.2)}\\ &= 1,\ i=0. \end{aligned}$$

It is interesting to investigate the value of $X_{t,i}$ in steady state, as $i \to \infty$. Thus,

$$X_{t,i} = \frac{\left(\pi_t \cdots \pi_{t+i-1}\right)^{\kappa_w} \left(\pi^i\right)^{(1-\kappa_w)} \mu_{z^+}^i}{\pi_{t+i}\pi_{t+i-1} \cdots \pi_{t+1}\mu_{z^+,t+i} \cdots \mu_{z^+,t+1}}.$$

In steady state,

$$X_{t,i} = \frac{\left(\pi^i\right)^{\kappa_w} \left(\pi^i\right)^{(1-\kappa_w)} \mu_{z^+}^i}{\pi^i \mu_{z^+}^i} = 1.$$

Simplifying using the scaling notation,

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i [-A_L \frac{\left(\left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\phi}}{1+\phi}$$
$$+ \upsilon_{t+i} W_{t+i} \frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}],$$

or,

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i [-A_L \frac{\left(\left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\phi}}{1+\phi}$$
$$+ \psi_{z^+,t+i} w_t \bar{w}_t X_{t,i} \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}],$$

or,

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i [-A_L \frac{\left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\phi}}{1+\phi} w_t^{\frac{\lambda_w}{1-\lambda_w}(1+\phi)}$$
$$+ \psi_{z^+,t+i} w_t^{1+\frac{\lambda_w}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}].$$

Differentiating with respect to w_t,

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i [-A_L \frac{\left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\phi}}{1+\phi} \lambda_w (1+\phi) w_t^{\frac{\lambda_w}{1-\lambda_w}(1+\phi)-1}$$
$$+ \psi_{z^+,t+i} w_t^{\frac{\lambda_w}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}] = 0.$$

Dividing and rearranging,

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i [-A_L \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\phi} \tag{F.3}$$
$$+ \frac{\psi_{z^+,t+i}}{\lambda_w} w_t^{\frac{1-\lambda_w(1+\phi)}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}] = 0.$$

Solving for the wage rate:

$$\begin{aligned}
w_t^{\frac{1-\lambda_w(1+\phi)}{1-\lambda_w}} &= \frac{E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i A_L \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\phi}}{E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}} \\
&= \frac{A_L K_{w,t}}{\bar{w}_t F_{w,t}}
\end{aligned}$$

where

$$\begin{aligned}
K_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\phi} \\
F_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}.
\end{aligned}$$

Thus, the wage set by reoptimizing households is:

$$w_t = \left[\frac{A_L K_{w,t}}{\bar{w}_t F_{w,t}} \right]^{\frac{1-\lambda_w}{1-\lambda_w(1+\phi)}}.$$

We now express $K_{w,t}$ and $F_{w,t}$ in recursive form:

$$\begin{aligned}
K_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\phi} \\
&= H_t^{1+\phi} + \beta\xi_w \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{\pi_t^{\kappa_w} \pi^{(1-\kappa_w)} \mu_{z^+}}{\pi_{t+1}\mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+1} \right)^{1+\phi} \\
&\quad + (\beta\xi_w)^2 \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+2}} \frac{(\pi_t \pi_{t+1})^{\kappa_w} (\pi^2)^{(1-\kappa_w)} \mu_{z^+}^2}{\pi_{t+2}\pi_{t+1}\mu_{z^+,t+2}\mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+2} \right)^{1+\phi} \\
&\quad + ...
\end{aligned}$$

or,

$$\begin{aligned}
K_{w,t} &= H_t^{1+\phi} + E_t \beta\xi_w \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{\pi_t^{\kappa_w} \pi^{(1-\kappa_w)} \mu_{z^+}}{\pi_{t+1}\mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\phi)} \{ H_{t+1}^{1+\phi} \\
&\quad + \beta\xi_w \left(\left(\frac{\bar{w}_{t+1}}{\bar{w}_{t+2}} \frac{\pi_{t+1}^{\kappa_w} \pi^{(1-\kappa_w)} \mu_{z^+}}{\pi_{t+2}\mu_{z^+,t+2}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+2} \right)^{1+\phi} + ... \} \\
&= H_t^{1+\phi} + \beta\xi_w E_t \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{\pi_t^{\kappa_w} \pi^{(1-\kappa_w)} \mu_{z^+}}{\pi_{t+1}\mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\phi)} K_{w,t+1} \\
&= H_t^{1+\phi} + \beta\xi_w E_t \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\phi)} K_{w,t+1}, \qquad \text{(F.4)}
\end{aligned}$$

using,

$$\pi_{w,t+1} = \frac{W_{t+1}}{W_t} = \frac{\bar{w}_{t+1} z_{t+1}^{+} P_{t+1}}{\bar{w}_t z_t^{+} P_t} = \frac{\bar{w}_{t+1} \mu_{z^+,t+1} \pi_{t+1}}{\bar{w}_t}.$$

Also,

$$\begin{aligned}
F_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \\
&= \frac{\psi_{z^+,t}}{\lambda_w} H_t \\
&\quad + \beta\xi_w \frac{\psi_{z^+,t+1}}{\lambda_w} \left(\frac{\bar{w}_t}{\bar{w}_{t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{\pi_t^{\kappa_w} \pi^{1-\kappa_w} \mu_{z^+}}{\pi_{t+1} \mu_{z^+,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} H_{t+1} \\
&\quad + (\beta\xi_w)^2 \frac{\psi_{z^+,t+2}}{\lambda_w} \left(\frac{\bar{w}_t}{\bar{w}_{t+2}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \\
&\quad \times \left(\frac{(\pi_t \pi_{t+1})^{\kappa_w} \left(\pi^2\right)^{(1-\kappa_w)} \mu_{z^+}^2}{\pi_{t+2} \pi_{t+1} \mu_{z^+,t+2} \mu_{z^+,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} H_{t+2} \\
&\quad + \ldots
\end{aligned}$$

or,

$$\begin{aligned}
F_{w,t} &= \frac{\psi_{z^+,t}}{\lambda_w} H_t \\
&\quad + \beta\xi_w \left(\frac{\bar{w}_t}{\bar{w}_{t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{\pi_t^{\kappa_w} \pi^{1-\kappa_w} \mu_{z^+}}{\pi_{t+1} \mu_{z^+,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} \{ \frac{\psi_{z^+,t+1}}{\lambda_w} H_{t+1} \\
&\quad + \beta\xi_w \left(\frac{\bar{w}_{t+1}}{\bar{w}_{t+2}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{\pi_{t+1}^{\kappa_w} \pi^{1-\kappa_w} \mu_{z^+}}{\pi_{t+2} \mu_{z^+,t+2}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} \frac{\psi_{z^+,t+2}}{\lambda_w} H_{t+2} \\
&\quad + \ldots \} \\
&= \frac{\psi_{z^+,t}}{\lambda_w} H_t + \beta\xi_w \left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1},
\end{aligned}$$

so that

$$F_{w,t} = \frac{\psi_{z^+,t}}{\lambda_w} H_t + \beta\xi_w E_t \left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1}. \tag{F.5}$$

We obtain a second restriction on w_t using the relation between the aggregate wage rate and the wage rates of individual households:

$$W_t = \left[(1-\xi_w) \left(\tilde{W}_t\right)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\tilde{\pi}_{w,t} W_{t-1}\right)^{\frac{1}{1-\lambda_w}}\right]^{1-\lambda_w}.$$

Dividing both sides by W_t and rearranging,

$$w_t = \left[\frac{1-\xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w}.$$

Substituting, out for w_t from the household's first order condition for wage optimization:

$$\frac{1}{A_L}\left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w(1+\phi)} \bar{w}_t F_{w,t} = K_{w,t}.$$

We now derive the relationship between aggregate homogeneous hours worked, H_t, and aggregate household hours,

$$h_t \equiv \int_0^1 h_{j,t} dj.$$

Substituting the demand for $h_{j,t}$ into the latter expression, we obtain,

$$\begin{aligned} h_t &= \int_0^1 \left(\frac{W_{j,t}}{W_t}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_t dj \\ &= \frac{H_t}{(W_t)^{\frac{\lambda_w}{1-\lambda_w}}} \int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} dj \\ &= \mathring{w}_t^{\frac{\lambda_w}{1-\lambda_w}} H_t, \end{aligned} \tag{F.6}$$

where

$$\mathring{w}_t \equiv \frac{\mathring{W}_t}{W_t},\ \mathring{W}_t = \left[\int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} dj\right]^{\frac{1-\lambda_w}{\lambda_w}}.$$

Also,

$$\mathring{W}_t = \left[(1-\xi_w)\left(\tilde{W}_t\right)^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w\left(\tilde{\pi}_{w,t}\mathring{W}_{t-1}\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}}.$$

This completes the derivations of the optimal wage setting.

G. The Wage-Phillips Curve in the Medium-Sized Model

By (**??**),

$$h_{t+i}^t = \left(\frac{\tilde{W}_{t+i}^t}{W_{t+i}}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} = \left(\frac{w_t\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}. \tag{G.1}$$

Here, we have used,

$$w_t \equiv \frac{\tilde{W}_t}{W_t},\ \bar{w}_t \equiv \frac{W_t}{z_t^+ P_t}, \tilde{W}_{t+i}^t = \tilde{W}_t\tilde{\pi}_{w,t+1}\cdots\tilde{\pi}_{w,t+i}, \tag{G.2}$$

and

$$X_{t,i} = \begin{cases} \frac{\tilde{\pi}_{w,t+i}\cdots\tilde{\pi}_{w,t+1}}{\pi_{t+i}\pi_{t+i-1}\cdots\pi_{t+1}\mu_{z^+,t+i}\cdots\mu_{z^+,t+1}} & i>0 \\ 1 & i=0 \end{cases}.$$

Rewriting (**??**),

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \upsilon_{t+i} P_{t+i} z_{t+i}^+ \left[\frac{\tilde{W}_{t+i}^t h_{t+i}^t}{P_{t+i} z_{t+i}^+} - A_L \frac{\left(h_{t+i}^t\right)^{1+\phi}}{(1+\phi)\, \upsilon_{t+i} P_{t+i} z_{t+i}^+} \right].$$

or, using,

$$\psi_{z^+,t} = \upsilon_t P_t z_t^+,$$

we have

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \psi_{z^+,t+i} \left[\frac{\tilde{W}_{t+i}^t h_{t+i}^t}{P_{t+i} z_{t+i}^+} - A_L \frac{\left(h_{t+i}^t\right)^{1+\phi}}{(1+\phi)\, \psi_{z^+,t+i}} \right].$$

But,

$$\begin{aligned} \frac{\tilde{W}_{t+i}^t h_{t+i}^t}{P_{t+i} z_{t+i}^+} &= \frac{W_t \left(\tilde{W}_t / W_t\right) h_{t+i}^t}{P_t z_t^+} X_{t,i} = \bar{w}_t w_t h_{t+i}^t X_{t,i} \\ &= \bar{w}_t w_t \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} X_{t,i}. \end{aligned}$$

Then,

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \psi_{z^+,t+i} \left[\bar{w}_t w_t \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} X_{t,i} - A_L \frac{\left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\phi)} H_{t+i}^{1+\phi}}{(1+\phi)\, \psi_{z^+,t+i}} \right].$$

Differentiating with respect to w_t,

$$\begin{aligned} &E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \psi_{z^+,t+i} [\left(\frac{\lambda_w}{1-\lambda_w} + 1 \right) \bar{w}_t \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} X_{t,i} \\ &- \frac{\lambda_w}{1-\lambda_w} (1+\phi)\, A_L \frac{\left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\phi)} H_{t+i}^{1+\phi}}{w_t (1+\phi)\, \psi_{z^+,t+i}}]. \end{aligned}$$

or

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \psi_{z^+,t+i} h_{t+i}^t \left[\bar{w}_t w_t X_{t,i} - \lambda_w MRS_{t+i}^t \right], \tag{G.3}$$

Here, MRS_{t+i}^t in (G.3) denotes the (scaled) cost of working for the marginal worker in period $t+i$ whose wage was reoptimized in period t and not again reoptimized in periods $t+1, t+2, ..., t+i$:

$$MRS_{t+i}^t \equiv A_L \frac{\left(h_{t+i}^t\right)^{\phi}}{\psi_{z^+,t+i}}.$$

According to (G.3), the union seeks to set the wage to a markup, λ_w, over the cost of working of the marginal worker, on average.

We now expand (G.3) about a steady state in which

$$w_t = 1,\ \chi_{t,i} = 1,\ X_{t,i} = 1,\ \text{for all } i,\ \bar{w}_t = \lambda_w MRS,\ \pi_t = \bar{\pi}, \pi_{w,t} = \pi\mu_{z^+}, \tilde{\pi}_{w,t+1} = \pi\mu_{z^+}$$

It is convenient to obtain some preliminary results. Note,

$$\begin{aligned}
\hat{X}_{t,i} &= \begin{cases} -\left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \cdots + \Delta_{\kappa_w}\hat{\pi}_{t+i}\right) - \left(\hat{\mu}_{z^+,t+i} + \cdots + \hat{\mu}_{z^+,t+1}\right) & i > 0 \\ 0 & i = 0 \end{cases} \\
\hat{\chi}_{t,i} &= \begin{cases} -\left(\Delta_{\kappa_w}\hat{\pi}_{w,t+1} + \cdots + \Delta_{\kappa_w}\hat{\pi}_{w,t+i}\right) & i > 0 \\ 0 & i = 0 \end{cases} \\
\widehat{MRS}^t_{t+i} &= -\hat{\psi}_{z^+,t+i} + \phi\hat{H}_{t+i} + \phi\left(\hat{h}^t_{t+i} - \hat{H}_{t+i}\right)
\end{aligned}$$

where

$$\Delta_{\kappa_w}\hat{\pi}_{t+1} \equiv \hat{\pi}_{t+1} - \kappa_w\hat{\pi}_t,\ \Delta_{\kappa_w}\hat{\pi}_{w,t+i} \equiv \hat{\pi}_{w,t+1} - \kappa_w\hat{\pi}_t$$

Also, from (G.1),

$$\hat{h}^t_{t+i} - \hat{H}_{t+i} = \begin{cases} \frac{\lambda_w}{1-\lambda_w}\left(\hat{w}_t - \Delta_{\kappa_w}\hat{\pi}_{w,t+1} \cdots - \Delta_{\kappa_w}\hat{\pi}_{w,t+i}\right) & i > 0 \\ \frac{\lambda_w}{1-\lambda_w}\hat{w}_t & i = 0 \end{cases}.$$

We have

$$\begin{aligned}
&\phi\frac{\lambda_w}{1-\lambda_w}\hat{w}_t + (\beta\xi_w)\phi\frac{\lambda_w}{1-\lambda_w}\left(\hat{w}_t - \Delta_{\kappa_w}\hat{\pi}_{w,t+1}\right) \\
&+(\beta\xi_w)^2\phi\frac{\lambda_w}{1-\lambda_w}\left(\hat{w}_t - \Delta_{\kappa_w}\hat{\pi}_{w,t+1} - \Delta_{\kappa_w}\hat{\pi}_{w,t+2}\right) \\
&+(\beta\xi_w)^3\phi\frac{\lambda_w}{1-\lambda_w}\left(\hat{w}_t - \Delta_{\kappa_w}\hat{\pi}_{w,t+1} - \Delta_{\kappa_w}\hat{\pi}_{w,t+2} - \Delta_{\kappa_w}\hat{\pi}_{w,t+3}\right) \\
&+ ... \\
= \ &\frac{1}{1-\beta\xi_w}\phi\frac{\lambda_w}{1-\lambda_w}\hat{w}_t - \phi\frac{\lambda_w}{1-\lambda_w}\frac{1}{1-\beta\xi_w}(\beta\xi_w)\Delta_{\kappa_w}\hat{\pi}_{w,t+1} \\
&-\phi\frac{\lambda_w}{1-\lambda_w}\frac{1}{1-\beta\xi_w}(\beta\xi_w)^2\Delta_{\kappa_w}\hat{\pi}_{w,t+2} - \phi\frac{\lambda_w}{1-\lambda_w}\frac{1}{1-\beta\xi_w}(\beta\xi_w)^3\Delta_{\kappa_w}\hat{\pi}_{w,t+3} \\
&- ...
\end{aligned}$$

Using the last expression, we can write the discounted sum of the marginal cost of working as follows:

$$S_{MRS,t} \equiv \sum_{i=0}^{\infty}(\beta\xi_w)^i\,\widehat{MRS}^t_{t+i} = S_{o,t} + \phi\frac{\lambda_w}{1-\lambda_w}\frac{1}{1-\beta\xi_w}\left[\hat{w}_t - S_{w,t}\right]. \tag{G.4}$$

Here,

$$S_{o,t} \equiv \sum_{i=0}^{\infty}(\beta\xi_w)^i\left[-\hat{\psi}_{z^+,t+i} + \phi\hat{H}_{t+i}\right] = -\hat{\psi}_{z^+,t} + \phi\hat{H}_t + \beta\xi_w S_{o,t+1} \tag{G.5}$$

$$S_{w,t} \equiv \sum_{i=1}^{\infty}(\beta\xi_w)^i\Delta_{\kappa_w}\hat{\pi}_{w,t+i} = \beta\xi_w\Delta_{\kappa_w}\hat{\pi}_{w,t+1} + \beta\xi_w S_{w,t+1}. \tag{G.6}$$

The following expression is also useful:

$$\begin{aligned} S_{X,t} &= \sum_{i=1}^{\infty} (\beta\xi_w)^i \hat{X}_{t,i} = \frac{1}{1-\beta\xi_w} \sum_{i=1}^{\infty} (\beta\xi_w)^i \left[-\hat{\mu}_{z^+,t+i} - \Delta_{\kappa_w} \hat{\pi}_{t+i}\right] \\ &= \frac{\beta\xi_w}{1-\beta\xi_w} \left[-\hat{\mu}_{z^+,t+1} - \Delta_{\kappa_w} \hat{\pi}_{t+1}\right] + \beta\xi_w S_{X,t+1}. \end{aligned} \tag{G.7}$$

Because the object in square brackets in (G.3) is zero in steady state, the expansion of (G.3) does not require expanding the expression outside the square bracket. Taking this and $\bar{w} = \lambda_w MRS$ into account, the expansion of (G.3) is:

$$0 = \frac{1}{1-\beta\xi_w} \left(\widehat{\bar{w}}_t + \hat{w}_t\right) + S_{X,t} - S_{MRS,t}. \tag{G.8}$$

We now deduce the restriction across wages implied by the aggregate wage index. Using the wage updating equation and the fact that non-optimizing unions are selected at random, the aggregate wage index reduces to:

$$W_t = \left[(1-\xi_w)\left(\tilde{W}_t\right)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\tilde{\pi}_{w,t} W_{t-1}\right)^{\frac{1}{1-\lambda_w}}\right]^{1-\lambda_w}$$

Divide by W_t and use (G.2):

$$1 = (1-\xi_w)(w_t)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}.$$

Log-linearize this expression about steady state, to obtain:

$$\hat{w}_t = \frac{\xi_w}{1-\xi_w} \Delta_{\kappa_w} \hat{\pi}_{w,t}.$$

Replace $S_{MRS,t}$ in (G.8) using (G.4) and then substitute out for $\hat{w}_t$ using the previous expression:

$$\begin{aligned} &\frac{1}{1-\beta\xi_w}\widehat{\bar{w}}_t + \frac{1}{1-\beta\xi_w}\frac{\xi_w}{1-\xi_w}\Delta_{\kappa_w}\hat{\pi}_{w,t} + S_{X,t} \\ = \; & S_{o,t} + \phi\frac{\lambda_w}{1-\lambda_w}\frac{1}{1-\beta\xi_w}\frac{\xi_w}{1-\xi_w}\Delta_{\kappa_w}\hat{\pi}_{w,t} - \phi\frac{\lambda_w}{1-\lambda_w}\frac{1}{1-\beta\xi_w}S_{w,t} \end{aligned} \tag{G.9}$$

Multiply (G.9) evaluated at $t+1$ by $\beta\xi_w$ and subtract the result from (G.9) evaluated at t to obtain:

$$\begin{aligned} &\frac{1}{1-\beta\xi_w}\left(\widehat{\bar{w}}_t - \beta\xi_w\widehat{\bar{w}}_{t+1}\right) + \frac{1}{1-\beta\xi_w}\frac{\xi_w}{1-\xi_w}\left(\Delta_{\kappa_w}\hat{\pi}_{w,t} - \beta\xi_w\Delta_{\kappa_w}\hat{\pi}_{w,t+1}\right) \\ &+ \left(S_{X,t} - \beta\xi_w S_{X,t+1}\right) \\ = \; & \left(S_{o,t} - \beta\xi_w S_{o,t+1}\right) + \phi\frac{\lambda_w}{1-\lambda_w}\frac{1}{1-\beta\xi_w}\frac{\xi_w}{1-\xi_w}\left(\Delta_{\kappa_w}\hat{\pi}_{w,t} - \beta\xi_w\Delta_{\kappa_w}\hat{\pi}_{w,t+1}\right) \\ &-\phi\frac{\lambda_w}{1-\lambda_w}\frac{1}{1-\beta\xi_w}\left(S_{w,t} - \beta\xi_w S_{w,t+1}\right) \end{aligned}$$

Simplify this expression using (G.5), (G.6) and (G.7):

$$\begin{aligned}
&\frac{1}{1-\beta\xi_w}\left(\widehat{\bar{w}}_t-\beta\xi_w\widehat{\bar{w}}_{t+1}\right)+\frac{1}{1-\beta\xi_w}\frac{\xi_w}{1-\xi_w}\left(\Delta_{\kappa_w}\hat{\pi}_{w,t}-\beta\xi_w\Delta_{\kappa_w}\hat{\pi}_{w,t+1}\right) \qquad (G.10)\\
&-\frac{\beta\xi_w}{1-\beta\xi_w}\left(\hat{\mu}_{z^+,t+1}+\Delta_{\kappa_w}\hat{\pi}_{t+1}\right)\\
= \;&-\hat{\psi}_{z^+,t}+\phi\hat{H}_t+\phi\frac{\lambda_w}{1-\lambda_w}\frac{1}{1-\beta\xi_w}\frac{\xi_w}{1-\xi_w}\left(\Delta_{\kappa_w}\hat{\pi}_{w,t}-\beta\xi_w\Delta_{\kappa_w}\hat{\pi}_{w,t+1}\right)\\
&-\phi\frac{\lambda_w}{1-\lambda_w}\frac{1}{1-\beta\xi_w}\beta\xi_w\Delta_{\kappa_w}\hat{\pi}_{w,t+1}
\end{aligned}$$

The relationship between wage and price inflation, the change in the real wage and technology growth is given by:

$$\widehat{\bar{w}}_t=\widehat{\bar{w}}_{t-1}+\hat{\pi}_{w,t}-\hat{\pi}_t-\hat{\mu}_{z^+,t}. \qquad (G.11)$$

Then,

$$\begin{aligned}
\hat{\pi}_t &= \widehat{\bar{w}}_{t-1}-\widehat{\bar{w}}_t+\hat{\pi}_{w,t}-\hat{\mu}_{z^+,t}\\
\Delta_{\kappa_w}\hat{\pi}_t &\equiv \hat{\pi}_t-\kappa_w\hat{\pi}_{t-1}\\
&= \widehat{\bar{w}}_{t-1}-\widehat{\bar{w}}_t+\hat{\pi}_{w,t}-\hat{\mu}_{z^+,t}-\kappa_w\hat{\pi}_{t-1}\\
&= \widehat{\bar{w}}_{t-1}-\widehat{\bar{w}}_t+\Delta_{\kappa_w}\hat{\pi}_{w,t}-\hat{\mu}_{z^+,t}
\end{aligned}$$

Use this to substitute out for $\Delta_{\kappa_w}\hat{\pi}_{t+1}$ in (G.10):

$$\begin{aligned}
&\frac{1}{1-\beta\xi_w}\left(\widehat{\bar{w}}_t-\beta\xi_w\widehat{\bar{w}}_{t+1}\right)+\frac{1}{1-\beta\xi_w}\frac{\xi_w}{1-\xi_w}\left(\Delta_{\kappa_w}\hat{\pi}_{w,t}-\beta\xi_w\Delta_{\kappa_w}\hat{\pi}_{w,t+1}\right)\\
&-\frac{\beta\xi_w}{1-\beta\xi_w}\left(\widehat{\bar{w}}_t-\widehat{\bar{w}}_{t+1}+\Delta_{\kappa_w}\hat{\pi}_{w,t+1}\right)\\
= \;&-\hat{\psi}_{z^+,t}+\phi\hat{H}_t+\phi\frac{\lambda_w}{1-\lambda_w}\frac{1}{1-\beta\xi_w}\frac{\xi_w}{1-\xi_w}\left(\Delta_{\kappa_w}\hat{\pi}_{w,t}-\beta\xi_w\Delta_{\kappa_w}\hat{\pi}_{w,t+1}\right)\\
&-\phi\frac{\lambda_w}{1-\lambda_w}\frac{1}{1-\beta\xi_w}\beta\xi_w\Delta_{\kappa_w}\hat{\pi}_{w,t+1}
\end{aligned}$$

Collecting terms in $\widehat{\bar{w}}_t$, $\widehat{\bar{w}}_{t+1}$, $\hat{\pi}_{w,t}$, $\hat{\pi}_{w,t+1}$:

$$\begin{aligned}
&\left[\frac{1}{1-\beta\xi_w}-\frac{\beta\xi_w}{1-\beta\xi_w}\right]\widehat{\bar{w}}_t-\left[\frac{\beta\xi_w}{1-\beta\xi_w}-\frac{\beta\xi_w}{1-\beta\xi_w}\right]\widehat{\bar{w}}_{t+1}\\
&+\left[\frac{1}{1-\beta\xi_w}\frac{\xi_w}{1-\xi_w}-\phi\frac{\lambda_w}{1-\lambda_w}\frac{1}{1-\beta\xi_w}\frac{\xi_w}{1-\xi_w}\right]\Delta_{\kappa_w}\hat{\pi}_{w,t}\\
&-[\frac{\beta\xi_w}{1-\beta\xi_w}\frac{\xi_w}{1-\xi_w}+\frac{\beta\xi_w}{1-\beta\xi_w}-\phi\frac{\lambda_w}{1-\lambda_w}\frac{\beta\xi_w}{1-\beta\xi_w}\frac{\xi_w}{1-\xi_w}\\
&-\phi\frac{\lambda_w}{1-\lambda_w}\frac{1}{1-\beta\xi_w}\beta\xi_w]\Delta_{\kappa_w}\hat{\pi}_{w,t+1}\\
= \;&-\hat{\psi}_{z^+,t}+\phi\hat{H}_t
\end{aligned}$$

or,

$$\begin{aligned}
&\frac{1}{1-\beta\xi_w}\frac{\xi_w}{1-\xi_w}\left[1-\phi\frac{\lambda_w}{1-\lambda_w}\right]\Delta_{\kappa_w}\hat{\pi}_{w,t} \\
= \quad & -\hat{\psi}_{z^+,t}+\phi\hat{H}_t-\widehat{\bar{w}}_t+\frac{\beta\xi_w}{1-\beta\xi_w}\frac{1}{1-\xi_w}\left[1-\phi\frac{\lambda_w}{1-\lambda_w}\right]\Delta_{\kappa_w}\hat{\pi}_{w,t+1}
\end{aligned}$$

Let

$$\kappa_w=\frac{(1-\xi_w)(1-\beta\xi_w)}{\xi_w}.$$

Multiply the previous expression by κ_w and rearrange:

$$\left(1-\phi\frac{\lambda_w}{1-\lambda_w}\right)\Delta_{\kappa_w}\hat{\pi}_{w,t}=\kappa_w\left(-\hat{\psi}_{z^+,t}+\phi\hat{H}_t-\widehat{\bar{w}}_t\right)+\left(1-\phi\frac{\lambda_w}{1-\lambda_w}\right)\beta\Delta_{\kappa_w}\hat{\pi}_{w,t+1}.$$