Appendix: Toward a Taylor Rule for Fiscal Policy

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Contents

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A Model solution

A.1 First-order conditions

All variables are denoted in real terms, a line over a variable indicates its steady state value, and a hat over a variable indicates its log-linearization around the steady-state.

Households

Maximization problem: The maximization problem of household i:

$$\begin{aligned}
\underset{b,c,I,k,u}{max} \quad & E_t \sum_{t=1}^{\infty} \beta^t \left[\varepsilon_{q,t} \frac{\left(c_t - hc_{t-1}\right)^{1-\sigma_c}}{1-\sigma_c} - \psi_l \frac{l_t\left(i\right)^{1+\sigma_l}}{1+\sigma_l} \right] \\
s.t. \quad & c_t + I_t + b_t = \left(1-\tau_t^w\right) \frac{W_t\left(i\right)}{P_t} l_t\left(i\right) + \left(\left(1-\tau_t^k\right) r_t^k u_t - \phi\left(u_t\right)\right) k_{t-1} \\
& + \frac{R_{t-1} b_{t-1}}{\pi_t} + (1-\tau_t^k) d_t + \iota_t\left(i\right) + \tau_t^T \qquad (1) \\
& k_t = \left(1-\delta\right) k_{t-1} + \varepsilon_{i,t} \left[1 - s\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \qquad (2)
\end{aligned}$$

Since the first-order conditions for household i are identical to the first-order conditions after aggregation, we report the aggregated first-order conditions.
First-order conditions:

$$\chi_t = \varepsilon_{q,t} \left(c_t - hc_{t-1}\right)^{-\sigma_c} - h\beta E_t \left[\varepsilon_{q,t+1} \left(c_{t+1} - hc_t\right)^{-\sigma_c}\right] \qquad (3)$$

$$\frac{1}{R_t} = \beta E_t \left[\frac{\chi_{t+1}}{\chi_t \pi_{t+1}}\right] \qquad (4)$$

$$q_t = \frac{1 - \beta E_t \left[\frac{\chi_{t+1}}{\chi_t} q_{t+1} s'\left(\frac{I_{t+1}}{I_t}\right) \varepsilon_{i,t+1} \left(\frac{I_{t+1}}{I_t}\right)^2\right]}{\varepsilon_{i,t} \left(1 - s\left(\frac{I_t}{I_{t-1}}\right) - s'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}}\right)} \qquad (5)$$

$$q_t = \beta E_t \left[\frac{\chi_{t+1}}{\chi_t} \left(\phi'\left(u_{t+1}\right) u_{t+1} - \phi\left(u_{t+1}\right) + q_{t+1}\left(1-\delta\right)\right)\right] \qquad (6)$$

$$\phi'\left(u_t\right) = r_t^k \left(1 - \tau_t^k\right) \qquad (7)$$

Functional forms:

$$s\left(\frac{I_t}{I_{t-1}}\right) = \frac{\nu}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 \qquad (8)$$

$$s'\left(\frac{I_t}{I_{t-1}}\right) = \nu\left(\frac{I_t}{I_{t-1}} - 1\right) \qquad (9)$$

$$\phi\left(u_t\right) = \frac{\bar{r}^k\left(1-\bar{\tau}^k\right)}{\sigma_u}\left(\exp\left(\sigma_u\left(u_t - 1\right)\right) - 1\right) \qquad (10)$$

$$\phi'\left(u_t\right) = \bar{r}^k\left(1-\bar{\tau}^k\right)\exp\left(\sigma_u\left(u_t - 1\right)\right) \qquad (11)$$

Wage setting

Each household supplies a differentiated type of labor service, $l_t(i)$, which is aggregated into a homogenous labor good by a representative competitive firm according to a Dixit-Stiglitz aggregator with $\theta_w > 1$ as the elasticity of substitution:

$$l_t^d = \left[\int_0^1 l_t\left(i\right)^{\frac{\theta_w - 1}{\theta_w}}\right]^{\frac{\theta_w}{\theta_w - 1}}. \tag{12}$$

Minimizing costs $W_t l_t^d$ taken individual wage costs of household i, $W_t(i)$, as given yields the demand for labor of type i as:

$$l_t\left(i\right) = \left(\frac{W_t\left(i\right)}{W_t}\right)^{-\theta_w} l_t^d \tag{13}$$

and the definition of the wage index W_t as

$$W_t = \left[\int_0^1 W_t\left(i\right)^{\theta_w - 1}\right]^{\frac{1}{\theta_w - 1}}. \tag{14}$$

For any wage rate, each household supplies as many labor services as demanded.

$$\max_{W_t(i)} \quad E_t\left[\sum_{k=0}^{\infty}\left(\gamma_w\beta\right)^k\left[\chi_{t+k}W_t\left(i\right)l_{t+k}\left(i\right) - U\left(l_{t+k}\left(i\right), c_{t+k}\left(i\right)\right)\right]\right] \tag{15}$$

The first-order conditions in recursive form are:

$$K_t^w = \left(\frac{l_t}{w_t^+}\right)^{1+\sigma_l} + \beta\gamma_w\left(\frac{\bar{\pi}}{\pi_{t+1}^w}\right)^{-\theta_w(1+\sigma_l)} K_{t+1}^w \tag{16}$$

$$F_t^w = \frac{(\theta_w - 1)}{\theta_w}\left(1 - \tau_t^w\right)\frac{l_t}{w_t^+}\chi_t + \beta\gamma_w\left(\frac{\pi_{t+1}}{\pi_{t+1}^w}\right)^{-\theta_w}\left(\frac{\bar{\pi}}{\pi_{t+1}}\right)^{1-\theta_w} F_{t+1}^w \tag{17}$$

$$\frac{K_t^w}{F_t^w} = \frac{1}{\psi_l}\left(w_t^*\right)^{1+\theta_w\sigma_l} w_t \tag{18}$$

Real wage inflation π^w:

$$\pi_t^w = \frac{w_t}{w_{t-1}}\pi_t \tag{19}$$

The law of motion for $w_t^* = \frac{W_t^\star}{W_t}$ is given by:

$$1 = \gamma_w\left(\frac{\bar{\pi}}{\pi_t^w}\right)^{1-\theta_w} + \left(1 - \gamma_w\right)\left(w_t^*\right)^{1-\theta_w} \tag{20}$$

Firm sector: first-order conditions

Final good production function:

$$y_t = \left[\int_0^1 y_t\left(j\right)^{\frac{\theta_p-1}{\theta_p}}\right]^{\frac{\theta_p}{\theta_p-1}} \tag{21}$$

Demand for good j:

$$y_t\left(j\right) = \left(\frac{P_t\left(j\right)}{P_t}\right)^{-\theta_p} y_t \tag{22}$$

with P_t defined as:

$$P_t = \left(\int_0^1 P_t(j)^{1-\theta_p} dj\right)^{1-\theta_p} \tag{23}$$

The production function of the intermediate good firm is:

$$y_t\left(j\right) = \left(u_t k_{t-1}\left(j\right)\right)^{\alpha}\left(l_t^d\left(j\right)\varepsilon_{z,t}\right)^{1-\alpha} - \Omega, \tag{24}$$

Intermediate-good firms maximize profits:

$$\max_{\left(u_t \cdot k_{t-1}(j)\right), l_t^d(j)} \left[\left[\frac{P_t\left(j\right)}{P_t}\right]^{-\theta_p}\left(y_t\left(j\right) - w_t l_t^d\left(j\right) - r_t^k u_t k_{t-1}\left(j\right)\right)\right] \tag{25}$$

Marginal costs are denoted by z. The first-order conditions of (25) are given by:

$$z_t\left(1-\alpha\right)\varepsilon_{z,t}^{(1-\alpha)}\left(u_t k_{t-1}\right)^{\alpha}\left(\frac{l_t}{w_t^+}\right)^{-\alpha} = w_t \tag{26}$$

$$z_t \alpha\left(u_t k_{t-1}\right)^{\alpha-1}\left(\left(\frac{l_t}{w_t^+}\right)\varepsilon_{z,t}\right)^{1-\alpha} = r_t^k \tag{27}$$

Price setting

Price-resetting firms choose $P_t^\star = P_t\left(j\right)$ to maximize the expected sum of discounted future profits:

$$\max_{P_t(j)} \quad E_t \sum_{k=0}^{\infty} \gamma_p^k m_{t+k}\left[P_t\left(j\right) y_{t+k}\left(j\right) - Z_{t+k} y_{t+k}\left(j\right)\right] \ , \tag{28}$$

where future profits are discounted by the stochastic discount factor $m_{t+k} = \beta^j \frac{\chi_{t+j} P_t}{\chi_t P_{t+j}}$.

Define $p_t^* = \frac{P_t^\star}{P_t}$. Using the demand for firm j and the aggregate price index, we rewrite the first-order condition to the maximization problem (28) and the law of motion for p_t^* as:

$$F_t^p = y_t \chi_t + \gamma_p \beta\left(\frac{\bar{\pi}}{\pi_{t+1}}\right)^{1-\theta_p} F_{t+1}^p \tag{29}$$

$$K_t^p = \frac{\theta_p}{\theta_p - 1} y_t \chi_t z_t + \gamma_p \beta\left(\frac{\bar{\pi}}{\pi_{t+1}}\right)^{-\theta_p} K_{t+1}^p \tag{30}$$

$$\frac{K_t^p}{F_t^p} = p_t^* \tag{31}$$

$$1 = \gamma_p \left(\frac{\bar{\pi}}{\pi_t}\right)^{1-\theta_p} + (1-\gamma_p)\left(p_t^*\right)^{1-\theta_p} \tag{32}$$

Aggregation

We use the variable p_t^+ to capture the resource costs induced by inefficient price dispersion:

$$p_t^+ = (1-\gamma_p)\left(p_t^*\right)^{-\theta_p} + \gamma_p \left(\frac{\bar{\pi}}{\pi_t}\right)^{-\theta_p} p_{t-1}^+ \tag{33}$$

w_t^+ captures the loss in output caused by taking wage dispersion into account, we use the variable:

$$w_t^+ = (1-\gamma_w)\left(w_t^*\right)^{-\theta_w} + \gamma_w \left(\frac{\bar{\pi}}{\pi_t^w}\right)^{-\theta_w} w_{t-1}^+ \tag{34}$$

The equilibrium condition of the labor market then becomes:

$$l_t = w_t^+ l_t^d \tag{35}$$

The dispersion of wages causes a dispersion in utility across households. This dispersion is measured by the variable $\tilde{w}_t^+$:

$$\tilde{w}_t^+ = (1-\gamma_w)\left(w_t^*\right)^{-\theta_w(1+\sigma_l)} + \gamma_w \left(\frac{\bar{\pi}}{\pi_t^w}\right)^{-\theta_w(1+\sigma_l)} \tilde{w}_{t-1}^+ \tag{36}$$

Dividends are defined as:

$$d_t = y_t - r_t^k u_t k_{t-1} - w_t \frac{l_t}{w_t^+} \tag{37}$$

The resource constraint of the economy is given by

$$\frac{\left(\left(u_t k_{t-1}\right)^{\alpha}\left(l_t^d \varepsilon_{z,t}\right)^{1-\alpha} - \Omega\right)}{p_t^+} = c_t + I_t + c_t^g + \phi\left(u_t\right) k_{t-1} \tag{38}$$

We define the part of y measured in the data as:

$$y_t^m = y_t - \phi\left(u_t\right) k_{t-1} \tag{39}$$

The aggregated utility across households is given by:

$$U_t = \frac{\epsilon_{q,t}\left(c_t - hc_{t-1}\right)^{1-\sigma_c}}{1-\sigma_c} - \psi_l \frac{\tilde{w}_t^+ \left(\frac{l_t}{w_t^+}\right)^{1+\sigma_l}}{1+\sigma_l} \tag{40}$$

welfare is defined as:

$$\mathcal{W}_t = U_t + \beta \mathcal{W}_{t+1}. \tag{41}$$

Government and monetary policy

Fiscal policy is described by the following tax revenues x_t and the budget constraint.

$$x_t = \tau_t^w w_t l_t + \tau_t^k \left[y_t - w_t \frac{l_t}{w_t^+} \right] \tag{42}$$

$$b_t - \frac{R_{t-1} b_{t-1}}{\pi_t} = c_t^g + \tau_t^T - x_t \tag{43}$$

The Monetary policy is described by the following feedback rule written in log-deviation from steady state

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \left(\rho_\pi \hat{\pi}_t + \rho_y \hat{y}_t^M \right) + \hat{\epsilon}_t^m \tag{44}$$

Shock processes

The following shock process are written in log-deviations from steady state

$$\hat{\tau}_t^T = \rho_T \hat{\tau}_{t-1}^T + \hat{\epsilon}_t^T \quad \hat{\epsilon}^T \ i.i.d. \tag{45}$$

$$\hat{c}_t^g = \rho_{cg} \hat{c}_{t-1}^g + \hat{\epsilon}_t^{cg} \quad \hat{\epsilon}^{cg} \ i.i.d \tag{46}$$

$$\hat{\varepsilon}_{i,t} = \rho_i \hat{\varepsilon}_{i,t-1} + \hat{\epsilon}_t^i \quad \hat{\epsilon}^i \ i.i.d. \tag{47}$$

$$\hat{\varepsilon}_{z,t} = \rho_z \hat{\varepsilon}_{z,t-1} + \hat{\epsilon}_t^z \quad \hat{\epsilon}^z \ i.i.d. \tag{48}$$

$$\hat{\varepsilon}_{q,t} = \rho_q \hat{\varepsilon}_{q,t-1} + \hat{\epsilon}_t^q \quad \hat{\epsilon}^q \ i.i.d. \tag{49}$$

To solve the competitive equilibrium of the model we need policy rules for the tax rates on capital income τ_t^k and labor income τ_t^w.

A.2 Steady-state

To solve for the steady state we take the following as given: $\bar{\tau}^k$, $\bar{\tau}^w$, $\bar{R}$, $\bar{c}^g/\bar{y}$, $\bar{\tau}^T/\bar{y}$, $\bar{\varepsilon}_i = 1$, $\bar{\varepsilon}_q = 1$, and $\bar{\varepsilon}_z = 1$. It is straightforward to see that:

$$\bar{u} = 1 \text{ and } \bar{s} = \bar{s}' = 0, \tag{50}$$

that the Tobin's q condition is satisfied for:

$$\bar{q} = 1, \tag{51}$$

and the capital adjustment cost equations can be solved for:

$$\bar{\phi} = 0 \quad \bar{\phi}' = \bar{r}^k \left(1 - \bar{\tau}^k \right) \tag{52}$$

Given these conditions we can solve for the steady state in the following way:

$$\bar{\pi} = \bar{R}\beta \tag{53}$$

$$\bar{\pi}^w = \bar{\pi} \tag{54}$$

$$\bar{z} = \frac{\theta_p - 1}{\theta_p} \tag{55}$$

$$\bar{r}^k = \frac{1-\beta\left(1-\delta\right)}{\beta\left(1-\bar{\tau}^k\right)} \tag{56}$$

$$\bar{\phi}' = \bar{r}^k\left(1-\bar{\tau}^k\right) \tag{57}$$

$$\bar{w} = \left(1-\alpha\right)\left(\bar{r}^k\right)^{\frac{-\alpha}{(1-\alpha)}}\left(\alpha^\alpha\bar{z}\right)^{\frac{1}{1-\alpha}} \tag{58}$$

$$\frac{\bar{l}}{\bar{k}} = \frac{1-\alpha}{\alpha}\frac{\bar{r}^k}{w} \tag{59}$$

$$\frac{\bar{c}}{\bar{k}} = \left(1-\frac{\bar{c}^g}{\bar{y}}\right)\bar{z}\left(\frac{\bar{l}}{\bar{k}}\right)^{1-\alpha} - \delta \tag{60}$$

Now, we assume that labor supply in steady state is $\bar{l} = 1/3$ and solve for the corresponding scaling parameter:

$$\psi_l = \frac{\bar{w}\frac{\theta_w-1}{\theta_w}\left(1-h\right)^{-\sigma_c}\left(1-\bar{\tau}^w\right)\left(1-\beta h\right)\frac{\bar{c}}{\bar{k}}^{-\sigma_c}\frac{\bar{l}}{\bar{k}}^{\sigma_c}}{l^{\sigma_c+\sigma_l}} \tag{61}$$

$$\bar{k} = \bar{l}\frac{\bar{k}}{\bar{l}} \tag{62}$$

Now, we solve for the fixed costs of the firm to ensure that dividends in the steady state are $\bar{d} = 0$.

$$\Omega = \left(1-\bar{z}\right)\bar{k}^\alpha\bar{l}^{1-\alpha}; \tag{63}$$

$$\bar{y} = \bar{k}^\alpha\bar{l}^{1-\alpha} - \Omega \tag{64}$$

$$\bar{c} = \frac{\bar{c}}{\bar{k}}\bar{k} \tag{65}$$

$$\bar{I} = \delta\bar{k}; \tag{66}$$

$$\bar{c}^g = \frac{\bar{c}^g}{\bar{y}}\bar{y} \tag{67}$$

$$\bar{\tau}^T = \frac{\bar{\tau}^T}{\bar{y}}\bar{y} \tag{68}$$

$$\bar{x} = \bar{\tau}^w\bar{w}\bar{l} + \bar{\tau}^k\left(\bar{y}-\bar{w}\bar{l}\right) \tag{69}$$

$$\bar{b} = \frac{\left(\bar{\tau}^T+\bar{c}^g-\bar{x}\right)}{1-1/\beta} \tag{70}$$

General problem: Deflator not consistent

B Estimation

GDPCTPI

B.1 Data

All data are in levels and nominal values and the frequency is quarterly. Nominal data are converted to real values by dividing by the GDP deflator (*BEA NIPA table 1.1.4 line 1*). The calculation of the fiscal data is following Leeper, Plante, and Traum (2010).

gdp_rgd_obs - mean(gdp_rgd_obs) (!! per capita here)

Alfred: GDP

Nominal Output: This series is (*BEA NIPA table 1.1.5 line 1*).

Alfred: PCND

cnds_rim_obs - mean(cnds_rim_obs) (!! per capita here)

Private consumption: This series is defined as private consumption of non-durable goods (*BEA NIPA table 1.1.5 line 5*) and private consumption of services (*BEA NIPA table 1.1.5 line 6*).

Alfred: PCES (M)/ PCESV (Q)

igid_rim_obs - mean(igid_rim_obs) (!! per capita here)

Alfred: GPDI

Private investment: This series is gross private domestic investment (*BEA NIPA table 1.1.5 line 7*) plus private consumption of durable goods (*BEA NIPA table 1.1.5 line 4*).

Alfred: PCEDG (M)/ PCDG (Q)

Use existing obs - pi_dm_obs

Alfred: GDPCTPI

Inflation: The gross inflation rate is defined as the change in the implicit GDP deflator.

Use existing obs - ffr_obs

Nominal interest rate: The quarterly nominal interest rate is defined as the averages of daily figures of the effective fed funds rate obtained from the Board of Governors of the Federal Reserve System.

Alfred: COMPNFB

Use existing obs - wage_rgd_demean_obs

Nominal Wage: This series is defined as nonfarm business, all persons, hourly compensation duration index (2005=100), seasonally adjusted provided by the U.S. Department of Labor (PRS85006103).

Alfred: PRS85006023

(!! per capita here)

Hours worked: This series is defined as product of: nonfarm business, all persons, average weekly hours duration index (2005=100), seasonally adjusted (PRS85006023) and civilian employment, 16 years and over, measured in thousands and seasonally adjusted (CE16OV). Both time series are are provided by U.S. Department of Labor. The latter one is transformed into an index where 2005:4=1.

Alfred: A957RC1Q027SBEA

Alfred: A787RC1Q027SBEA

Government expenditure: This series is defined as the sum of government consumption expenditures (*BEA NIPA table 3.2 line 21*), government gross investment (*BEA NIPA table 3.2 line 42*), and government net purchases of non-produced assets (*BEA NIPA table 3.2 line 43*), minus government consumption of fixed capital (*BEA NIPA table 3.2 line 44*).

Alfred: A918RC1Q027SBEA

Alfred: AD08RC1Q027SBEA

Tax rates: Capital and labor tax rates are calculated following Leeper et al. (2010), where the labor tax rate is computed as:

$$\tau^w = \frac{IT + SIT}{W + PRI/2 + CI} \cdot \frac{(W + PRI/2)}{EC + PRI/2} + \frac{CSI}{EC + PRI/2} ,$$

Alfred: W780RC1Q027SBEA

where CSI denotes total contributions to social insurance (*BEA NIPA table 3.2 line 11*), EC denotes compensation of employees (*BEA NIPA table 1.12 line 2*), IT denotes

Alfred: COE

Alfred: WASCUR

Alfred: A074RC1Q027SBEA

Alfred: W071RC1Q027SBEA

personal current taxes (*BEA NIPA table 3.2 line 3*), SIT denotes state and local personal current taxes (*BEA NIPA table 3.3 line 3*), PRI denotes proprietors' income (*BEA NIPA table 1.12 line 9*), W denotes wage and salary accruals (*BEA NIPA table 1.12 line 3*), and CI is capital income. Capital income is defined as rental income (*BEA NIPA table 1.12 line 12*), corporate profits (*BEA NIPA table 1.12 line 13*), interest income (*BEA NIPA table 1.12 line 18*), and $PRI/2$. The average capital income tax rate is computed as:

Alfred: CPROFIT

Alfred: PROPINC

Alfred: BOGZ1FA086130003Q

Alfred: RENTIN

$$\tau^k = \frac{IT}{W + PRI/2 + CI} \cdot \frac{CI}{CI + PT} + \frac{CT + PT}{CI + PT} \, ,$$

where CT denotes taxes on corporate income (*BEA NIPA table 3.2 line 7*) and PT denotes property taxes (*BEA NIPA table 3.3 line 8*).

Alfred: B075RC1Q027SBEA

Updated until only 2018Q1: B249RC1Q027SBEA

Government tax revenues: Tax revenues, x, are defined as the sum of capital income taxes and taxes on labor. They are computed as:

$$x = \tau^w \cdot (EC + PRI/2) + \tau^k \cdot (CI + PT) \, .$$

Alfred: B096RC1Q027SBEA

Alfred: W014RC1Q027SBEA

Government transfers: This series is defined as the sum of net current transfers, net capital transfers, and subsidies (*BEA NIPA table 3.2 line 32*) minus the tax residual. Moreover, net current transfers are current transfer payments (*BEA NIPA table 3.1 line 22*) minus current transfer receipts (*BEA NIPA table 3.2 line 16*), net capital transfers are defined as the difference between capital transfer payments (*BEA NIPA table 3.2 line 43*) and capital transfer receipts (*BEA NIPA table 3.2 line 39*). The tax residual is defined as the sum of current tax receipts (*BEA NIPA table 3.2 line 2*), contributions of government social insurance (*BEA NIPA table 3.2 line 11*), income receipts on assets (*BEA NIPA table 3.2 line 12*), and the current surplus of government enterprises (*BEA NIPA table 3.2 line 19*), minus government tax revenues (as defined above).

Alfred: W020RC1Q027SBEA

Alfred: W011RC1Q027SBEA

Alfred: B232RC1Q027SBEA

Alfred: W780RC1Q027SBEA

Alfred: W006RC1Q027SBEA

Alfred: W009RC1Q027SBEA

Alfred: B097RC1Q027SBEA

Government debt: The change in government debt in the model is given by the net borrowing of the government. The net borrowing is computed using the NIPA deficits concept: *government expenditure + transfers - tax revenues* (all variables as defined above) plus interest payments (*BEA NIPA table 3.2 line 29*). As initial value for our debt series we use the market value of gross federal debt from December 1946 provided by Cox and Hirschhorn (1983).

Alfred: A091RC1Q027SBEA

The observable variables output, consumption, investment, wages, hours worked, government debt, and tax revenues are converted to per capita values by dividing them by the population index.

Population Index: index of population constructed so that 2005:4=1. The population series is is defined as the civilian non-institutional population aged 16 and over provided by the U.S. Department of Labor.

B.2 System of log-linearized equations

In the following we list the final set of log-linearized equations used to estimate the baseline as well as the extended model. In comparison to the model solution section, we include additional observable equation and shocks.

First-order conditions of the households':

$$(1-\beta h)\,\hat{\chi}_t = \hat{\varepsilon}_{q,t} - \frac{\sigma_c}{1-h}\left(\hat{c}_t - h\hat{c}_{t-1}\right) - h\beta\hat{\varepsilon}_{q,t+1} + \frac{h\beta\sigma_c}{1-h}\left(\hat{c}_{t+1} - h\hat{c}_t\right) \tag{71}$$

$$0 = \hat{\chi}_{t+1} - \hat{\chi}_t - \hat{\pi}_{t+1} + \hat{R}_t \tag{72}$$

$$\hat{k}_t = (1-\delta)\,\hat{k}_{t-1} + \delta\hat{I}_t + \delta\hat{\varepsilon}_{i,t}; \tag{73}$$

$$\hat{I}_t = \frac{\hat{I}_{t-1}}{(1+\beta)} + \frac{\beta\hat{I}_{t+1}}{(1+\beta)} + \frac{\hat{q}_t}{\nu\,(1+\beta)} + \frac{\hat{\varepsilon}_{i,t}}{\nu\,(1+\beta)} \tag{74}$$

$$\hat{\chi}_t + \hat{q}_t = \hat{\chi}_{t+1} + \beta\left[(1-\delta)\,\hat{q}_{t+1} + \bar{r}^k\left(1-\bar{\tau}^k\right)\hat{r}^k_{t+1} - \bar{r}^k\bar{\tau}^k\hat{\tau}^k_{t+1}\right] \tag{75}$$

$$\sigma_u\hat{u}_t = \hat{r}^k - \frac{\bar{\tau}^k}{1-\bar{\tau}^k}\hat{\tau}^k_t \tag{76}$$

Staggered prices and wages:

$$\hat{\pi}^w_t = \beta\hat{\pi}^w_{t+1} + \frac{(1-\gamma_w)\,(1-\beta\gamma_w)}{\gamma_w\,(1+\theta_w\sigma_l)}\left(\sigma_l\hat{l}_t - \hat{\chi}_t - \hat{w}_t + \frac{\bar{\tau}^w}{(1-\bar{\tau}^w)}\hat{\tau}^w\right) + \hat{\epsilon}^l_t \tag{77}$$

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \frac{(1-\gamma_p)\,(1-\beta\gamma_p)}{\gamma_p}\left(\hat{z}_t + \hat{\varepsilon}_{p,t}\right) \tag{78}$$

$$\hat{\pi}^w_t = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t \tag{79}$$

Firm:

$$\hat{z}_t + (1-\alpha)\,\hat{\varepsilon}_{z,t} + \alpha\left(\hat{u}_t + \hat{k}_{t-1}\right) - \alpha\hat{l}_t = \hat{w}_t \tag{80}$$

$$\hat{z}_t + (1-\alpha)\,\hat{\varepsilon}_{z,t} + (\alpha-1)\left(\hat{u}_t + \hat{k}_{t-1}\right) + (1-\alpha)\,\hat{l}_t = \hat{r}^k_t \tag{81}$$

Supply and demand:

$$\bar{y}\hat{y}_t = \bar{k}^\alpha\bar{l}^{1-\alpha}\left(\alpha\hat{k}_{t-1} + (1-\alpha)\left(\hat{l}_t + \hat{\varepsilon}_{z,t}\right) + \alpha\hat{u}_t\right) \tag{82}$$

$$\bar{y}\hat{y}_t = \bar{c}\hat{c}_t + \bar{I}\hat{I}_t + \bar{c}^g\hat{c}^g_t + \bar{r}^k\left(1-\bar{\tau}^k\right)\bar{k}\hat{u}_t + \hat{\epsilon}^y_t \tag{83}$$

$$\bar{y}\hat{y}^m = \bar{y}\hat{y}_t - \bar{r}^k\left(1-\bar{\tau}^k\right)\bar{k}\hat{u}_t \tag{84}$$

Government:

$$\bar{b}\hat{b}_t - \frac{\bar{b}}{\beta}\left(\hat{R}_{t-1} + \hat{b}_{t-1} - \hat{\pi}_t\right) = \bar{c}^g\hat{c}^g_t + \bar{\tau}^T\hat{\tau}^T_t - \bar{x}\hat{x}_t \tag{85}$$

$$\bar{x}\hat{x}_t = \bar{\tau}^w \bar{w}\bar{l}\left(\hat{\tau}_t^w + \hat{w}_t + \hat{l}_t\right) + \bar{\tau}^k \bar{r}^k \bar{k}\hat{\tau}_t^k + \bar{\tau}^k \left(\bar{y}\hat{y}_t - \bar{w}\bar{l}\left(\hat{w}_t + \hat{l}_t\right)\right) \tag{86}$$

Policy rules:

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left(\rho_\pi \hat{\pi}_t + \rho_y \hat{y}_t^m\right) + \hat{\epsilon}_t^m \tag{87}$$

$$\hat{\tau}_t^w = \rho_w \hat{\tau}_{t-1}^w + (1-\rho_w)\left(\eta_{wb}\hat{b}_{t-1} + \eta_{wy}\hat{y}_t^m + \eta_{wh}\hat{l}_t\right) + \hat{\epsilon}_t^{\tau^w} \tag{88}$$

$$\hat{\tau}_t^k = \rho_k \hat{\tau}_{t-1}^k + (1-\rho_k)\left(\eta_{kb}\hat{b}_{t-1} + \eta_{ky}\hat{y}_t^m + \eta_{kI}\hat{I}_t\right) + \hat{\epsilon}_t^{\tau^k} \tag{89}$$

Exogenous variables:

$$\hat{c}_t^g = \rho_{cg}\hat{c}_{t-1}^g + \hat{\epsilon}_t^{cg} \tag{90}$$

$$\hat{\tau}_t^T = \rho_{\tau_L}\hat{\tau}_{t-1}^T + \hat{\epsilon}_T^{\tau^L} \tag{91}$$

$$\hat{\varepsilon}_{z,t} = \rho_z \hat{\varepsilon}_{z,t-1} + \hat{\epsilon}_t^z \tag{92}$$

$$\hat{\varepsilon}_{i,t} = \rho_i \hat{\varepsilon}_{i,t-1} + \hat{\epsilon}_t^i \tag{93}$$

$$\hat{\varepsilon}_{q,t} = \rho_q \hat{\varepsilon}_{q,t-1} + \hat{\epsilon}_t^q \tag{94}$$

$$\hat{\varepsilon}_{p,t} = \rho_P \hat{\varepsilon}_{p,t-1} + \frac{\hat{\epsilon}_t^p}{((1-\gamma_p)(1-\gamma_p\beta)/\gamma_p)} \tag{95}$$

Observable variables:

$$\hat{y}_t^{obs} = \hat{y}_t^m - \hat{y}_{t-1}^m \tag{96}$$

$$\hat{c}_t^{obs} = \hat{c}_t - \hat{c}_{t-1} \tag{97}$$

$$\hat{I}_t^{obs} = \hat{I}_t - \hat{I}_{t-1} \tag{98}$$

$$\hat{w}_t^{obs} = \hat{w}_t - \hat{w}_{t-1} \tag{99}$$

$$\hat{b}_t^{obs} = \hat{b}_t - \hat{b}_{t-1} \tag{100}$$

$$\hat{g}_t^{obs} = \hat{c}_t^g - \hat{c}_{t-1}^g \tag{101}$$

$$\hat{x}_t^{obs} = \hat{x}_t - \hat{x}_{t-1} \tag{102}$$

B.3 Prior and calibration

Description	Symbol	Value
Discount factor	β	0.9935
Capital share	α	0.36
Depreciation rate	δ	0.025
Price markup	$\theta_p/(\theta_p - 1)$	1.2
Wage markup	$\theta_w/(\theta_w - 1)$	1.1
Annualized nominal interest rate	$\bar{R}^4$	1.0530
Ratio of government consumption to output	$\bar{c}^g/\bar{y}$	0.085
Ratio of government transfers to output	$\bar{\tau}^T/\bar{y}$	0.105
Steady-state capital tax rate	$\bar{\tau}_k$	0.1929
Steady-state labor tax rate	$\bar{\tau}_w$	0.2088

Table 1: Parameter calibration.

Parameter	Symbol	Domain	Density	Para(1)	Para(2)
Preference parameter	σ_c	$\mathbb{R}^+$	Gamma	1.75	0.5
Inverse Frisch elasticity	σ_l	$\mathbb{R}^+$	Gamma	2.0	1
Habit persistence	h	$[0,1)$	Beta	0.5	0.15
Price stickiness	γ_p	$[0,1)$	Beta	0.5	0.1
Wage stickiness	γ_w	$[0,1)$	Beta	0.5	0.1
Investment adjustment cost	ν	$\mathbb{R}^+$	Gamma	4	0.75
Capital utilization cost	σ_u	$\mathbb{R}^+$	Gamma	2	0.5
Taylor-rule smoothing	ρ_R	$[0,1)$	Beta	0.8	0.1
Taylor-rule inflation	ρ_π	$\mathbb{R}^+$	Gamma	1.7	0.1
Taylor-rule output	ρ_y	$\mathbb{R}$	Gamma	0.125	0.05
Labor-tax smoothing	ρ_w	$[0,1)$	Beta	0.85	0.1
Labor-tax debt	η_{wb}	$\mathbb{R}$	Normal	0	0.5
Labor-tax output	η_{wy}	$\mathbb{R}$	Normal	0	0.5
Labor-tax hours worked	η_{wh}	$\mathbb{R}$	Normal	0	0.5
Capital-tax smoothing	ρ_k	$[0,1)$	Beta	0.85	0.1
Capital-tax debt	η_{kb}	$\mathbb{R}$	Normal	0	0.5
Capital-tax output	η_{ky}	$\mathbb{R}$	Normal	0	0.5
Capital-tax investment	η_{kI}	$\mathbb{R}$	Normal	0	0.5
AR transfers	ρ_{τ^T}	$[0,1)$	Beta	0.85	0.1
AR investment specific	ρ_i	$[0,1)$	Beta	0.85	0.1
AR technology	ρ_z	$[0,1)$	Beta	0.85	0.1
AR public consumption	ρ_{cg}	$[0,1)$	Beta	0.85	0.1
AR price mark-up	ρ_p	$[0,1)$	Beta	0.85	0.1
S.d. investment specific	ϵ_i	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. technology	ϵ_z	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. preference	ϵ_q	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. monetary policy	ϵ_m	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. labor tax	ϵ_{τ^w}	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. capital tax	ϵ_{τ^k}	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. transfer	ϵ_{τ^T}	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. public consumption	ϵ_{cg}	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. price mark-up shock	ϵ_{τ^k}	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. wage mark-up shock	ϵ_{τ^l}	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. resource constraint	ϵ_{cg}	$\mathbb{R}^+$	InvGam	0.01	4.0
S.d. measurement error taxes	ϵ_{tax}	$\mathbb{R}^+$	InvGam	0.01	4.0

Table 2: Prior distribution of model parameters. Para(1) and Para(2) correspond to means and standard deviations for the Beta, Gamma, Inverted Gamma, and Normal distribution.

B.4 Results

Parameter	Symbol	Baseline Model Mode	Median	[5 95]	Recommended Model Mode	Median	[5 95]
Preference parameter	σ_c	1.718	1.932	[1.22,2.69]	1.742	1.927	[1.23,2.67]
Inverse Frisch elasticity	σ_l	3.430	3.507	[1.65,5.58]	3.305	3.405	[1.57,5.44]
Habit persistence	h	0.567	0.564	[0.42,0.69]	0.561	0.566	[0.42,0.70]
Price stickiness	γ_p	0.677	0.684	[0.64,0.72]	0.678	0.687	[0.64,0.73]
Wage stickiness	γ_w	0.659	0.665	[0.59,0.73]	0.657	0.663	[0.59,0.73]
Investment adjustment cost	ν	2.961	3.270	[2.26,4.24]	2.906	3.207	[2.25,4.23]
Capital utilization cost	σ_u	1.753	1.798	[1.10,2.49]	1.682	1.746	[1.05,2.09]
Taylor-rule smoothing	ρ_R	0.832	0.833	[0.80,0.86]	0.832	0.833	[0.80,0.86]
Taylor-rule inflation	ρ_π	1.928	1.936	[1.77,2.10]	1.932	1.933	[1.77,2.09]
Taylor-rule output	ρ_y	0.028	0.033	[0.01,0.05]	0.029	0.035	[0.01,0.05]
Labor-tax smoothing	ρ_w	0.848	0.863	[0.79,0.94]	0.802	0.826	[0.73,0.92]
Labor-tax debt	η_{wb}	0.120	0.138	[-0.00,0.28]	0.074	0.097	[-0.01,0.24]
Labor-tax output	η_{wy}	0.417	0.385	[-0.28,1.02]	-	-	-
Labor-tax hours worked	η_{wh}	-	-	-	0.858	0.760	[0.03,1.43]
Capital-tax smoothing	ρ_k	0.868	0.875	[0.81,0.93]	0.831	0.849	[0.77,0.92]
Capital-tax debt	η_{kb}	0.447	0.463	[0.17,0.79]	0.428	0.454	[0.19,0.71]
Capital-tax output	η_{ky}	0.008	0.040	[-0.76,0.81]	-	-	-
Capital-tax investment	η_{kI}	-	-	-	0.496	0.498	[0.04,0.95]
AR transfers	ρ_{τ^T}	0.869	0.866	[0.79,0.94]	0.869	0.866	[0.79,0.94]
AR investment specific	ρ_i	0.878	0.863	[0.79,0.92]	0.873	0.857	[0.79,0.91]
AR technology	ρ_z	0.979	0.971	[0.94,0.99]	0.977	0.970	[0.94,0.99]
AR preference	ρ_q	0.885	0.879	[0.77,0.96]	0.884	0.873	[0.76,0.96]
AR public consumption	ρ_{cg}	0.963	0.961	[0.93,0.98]	0.962	0.961	[0.93,0.98]
AR price mark-up	ρ_p	0.964	0.933	[0.86,0.99]	0.960	0.928	[0.85,0.99]
S.d. investment specific x100	ϵ_i	2.605	2.903	[2.15,3.66]	2.719	3.030	[2.28,3.82]
S.d. technology x100	ϵ_z	0.637	0.642	[0.56,0.71]	0.638	0.643	[0.57,0.71]
S.d. preference x100	ϵ_q	1.891	2.169	[1.38,3.03]	1.886	2.158	[1.47,2.97]
S.d. monetary policy x100	ϵ_m	0.157	0.158	[0.14,0.18]	0.157	0.158	[0.14,0.17]
S.d. labor tax x100	ϵ_{τ^w}	2.172	2.214	[1.95,2.46]	2.123	2.167	[1.92,2.42]
S.d. capital tax x100	ϵ_{τ^k}	3.822	3.884	[3.45,4.34]	3.733	3.789	[3.36,4.23]
S.d. transfer x100	ϵ_{τ^T}	4.942	5.008	[4.43,5.62]	4.941	4.990	[4.44,5.57]
S.d. public consumption x100	ϵ_{cg}	2.631	2.662	[2.36,2.96]	2.632	2.670	[2.37,2.98]
S.d. price mark-up shock x100	ϵ_p	0.148	0.149	[0.12,0.17]	0.147	0.148	[0.12,0.17]
S.d. wage mark-up shock x100	ϵ_l	0.714	0.727	[0.63,0.82]	0.716	0.729	[0.63,0.82]
S.d. resource constraint x100	ϵ_y	1.110	1.119	[0.99,1.25]	1.110	1.116	[0.99,1.24]
S.d. measurement error taxes x100	ϵ_{tax}	0.533	0.540	[0.48,0.60]	0.533	0.538	[0.47,0.60]
Log data density			4163.99			4166.24	

Table 3: Posterior mode and posterior distribution of model's parameters.

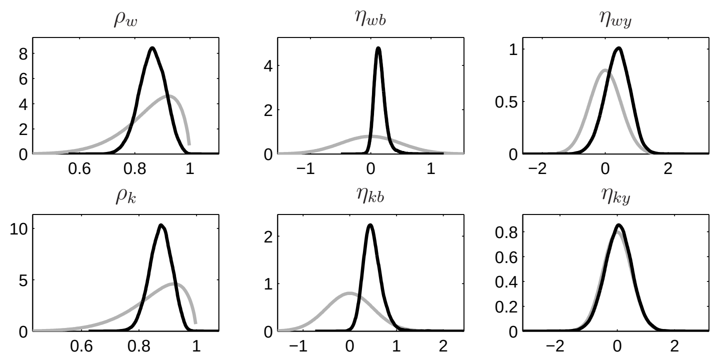


Figure 1: Posterior versus prior distribution for tax feedback rule parameters of the baseline model.

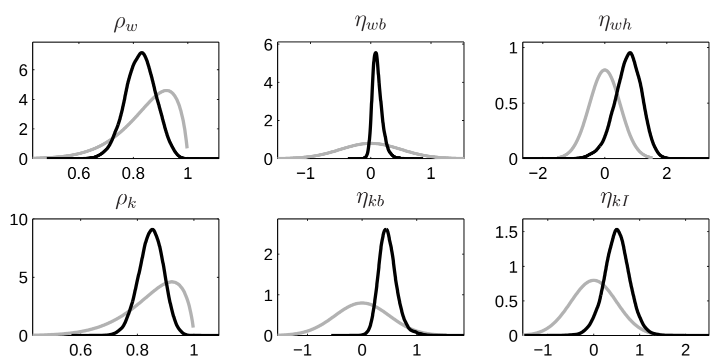


Figure 2: Posterior versus prior distribution for tax feedback rule parameters of the recommended model.

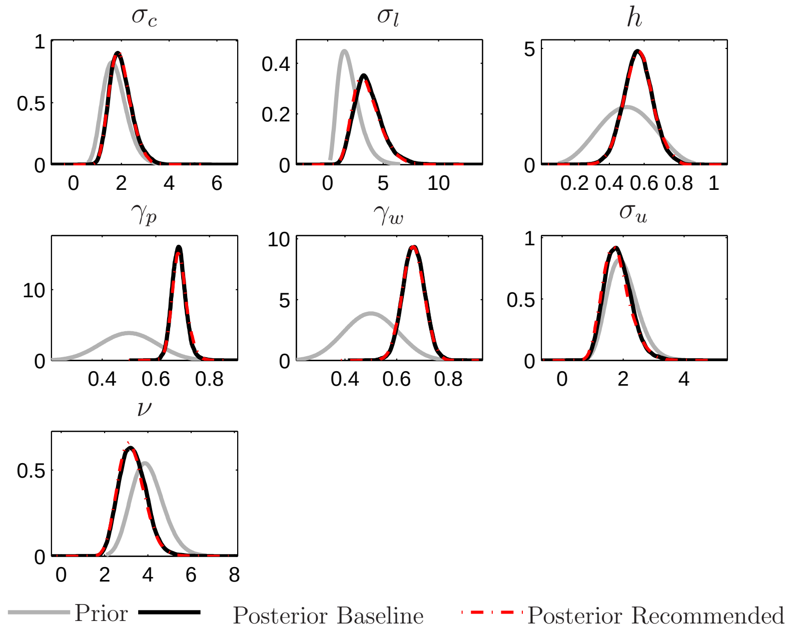


Figure 3: Posterior versus prior distribution for both model estimations.

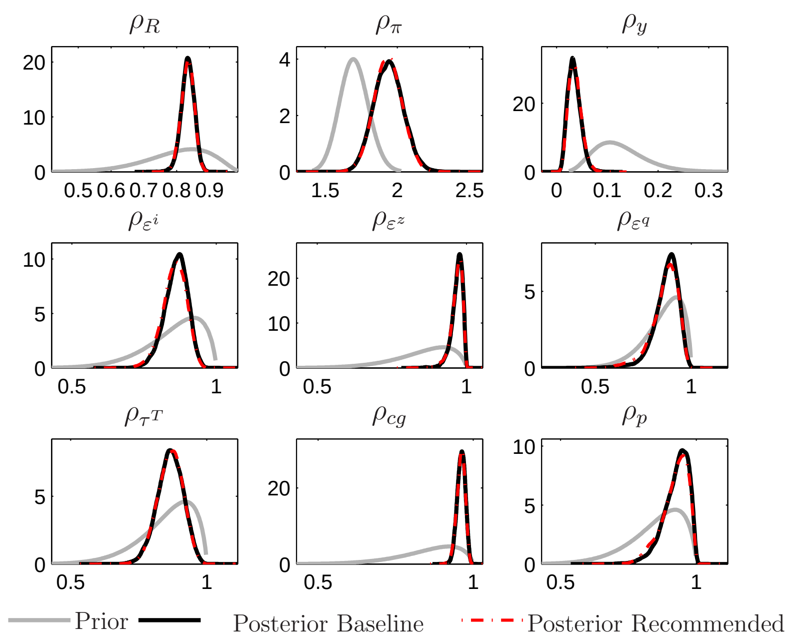


Figure 4: Posterior versus prior distribution for both model estimations.

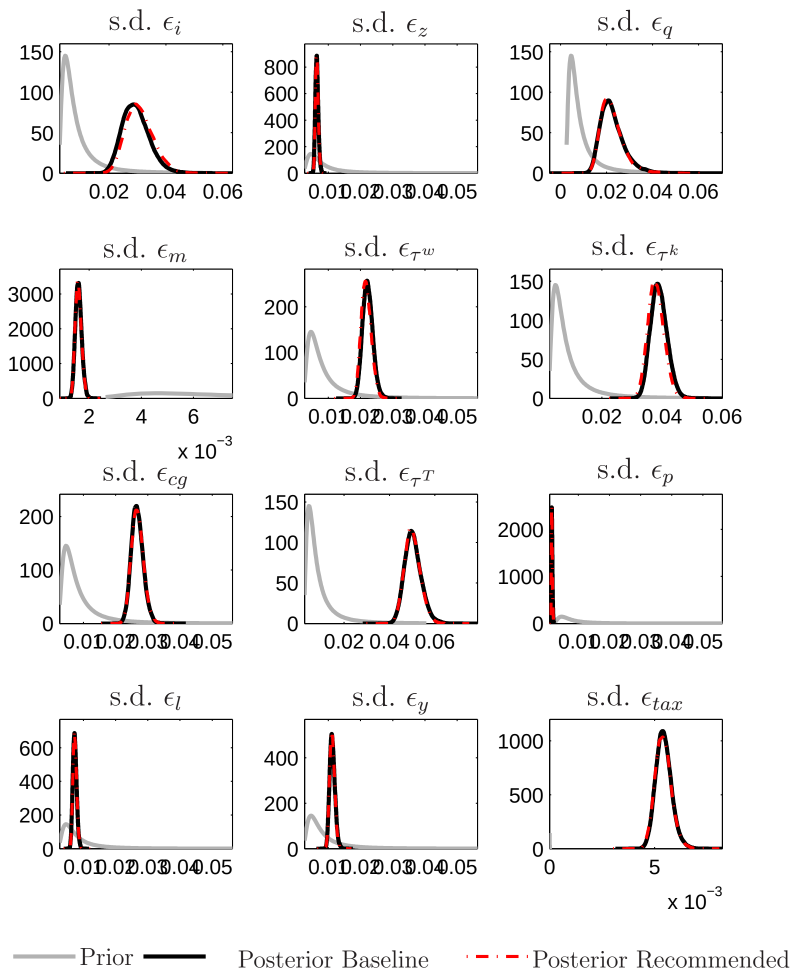


Figure 5: Posterior versus prior distribution for both model estimations.

B.5 Diagnostics

This section contains the Dynare diagnostic output for the estimations.

B.5.1 Baseline Model

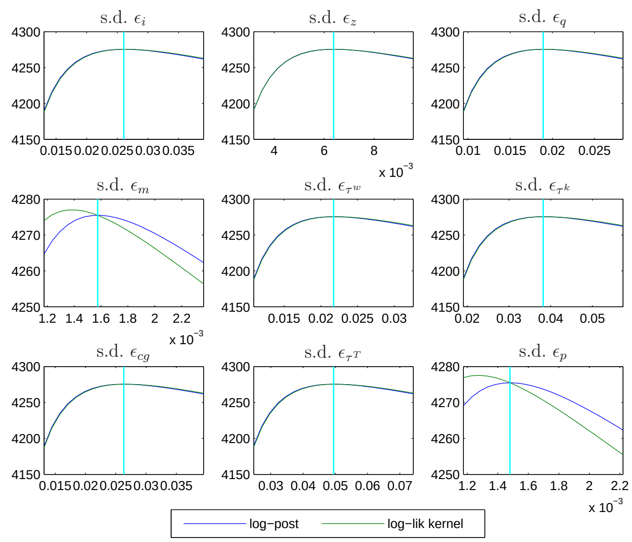


Figure 6: Check plots for posterior mode maximization of the baseline model.

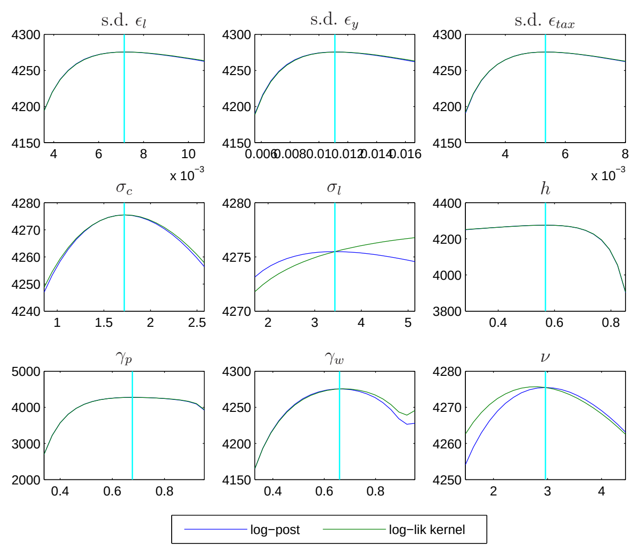


Figure 7: Check plots for posterior mode maximization of the baseline model.

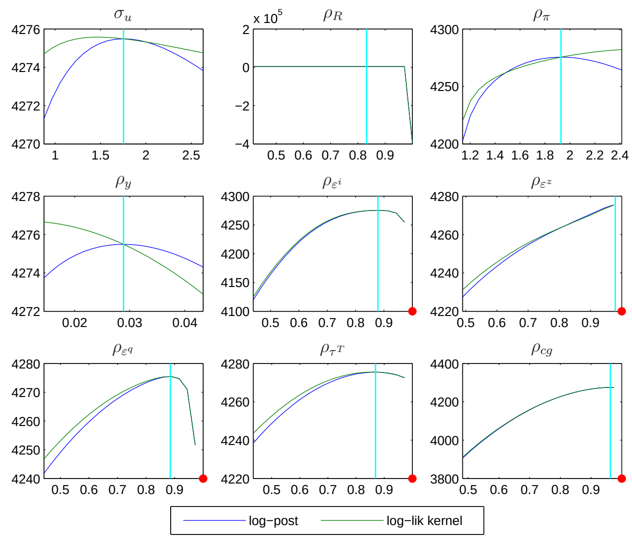


Figure 8: Check plots for posterior mode maximization of the baseline model.

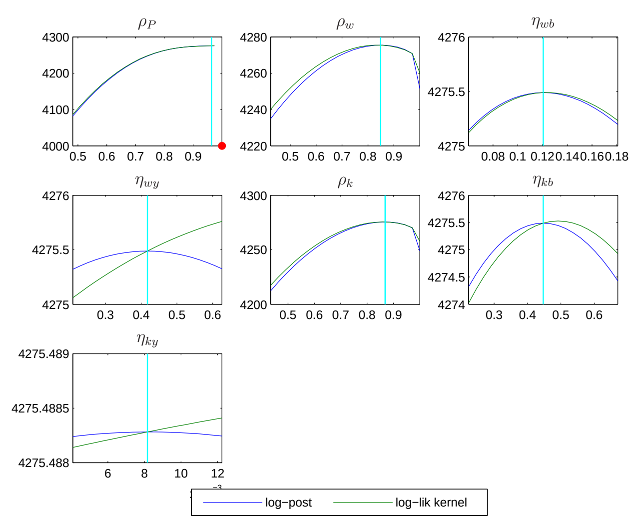


Figure 9: Check plots for posterior mode maximization of the baseline model.

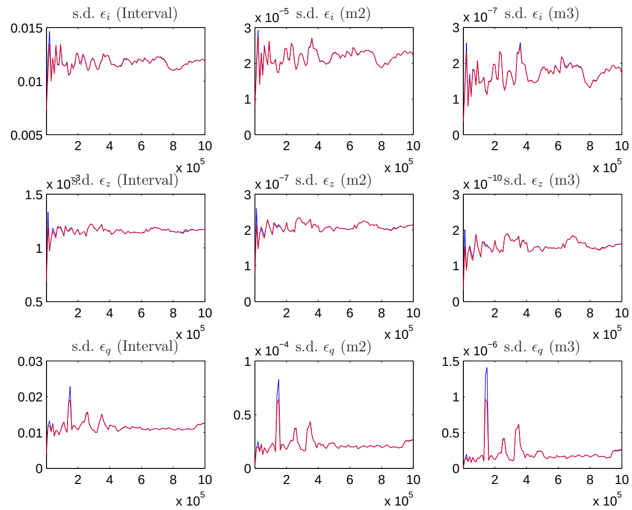


Figure 10: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

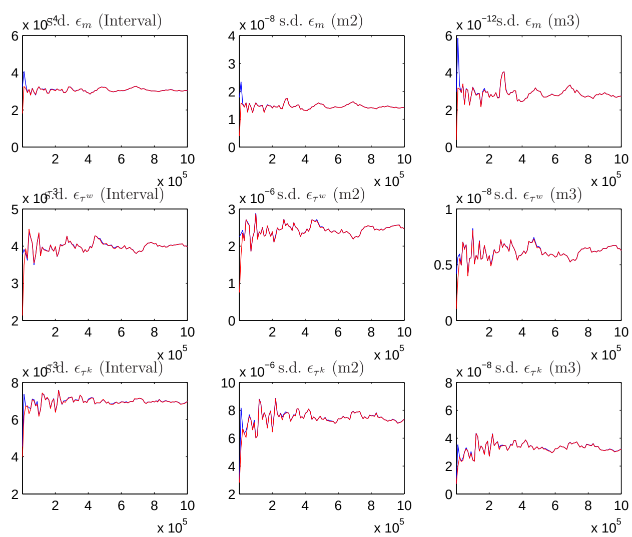


Figure 11: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

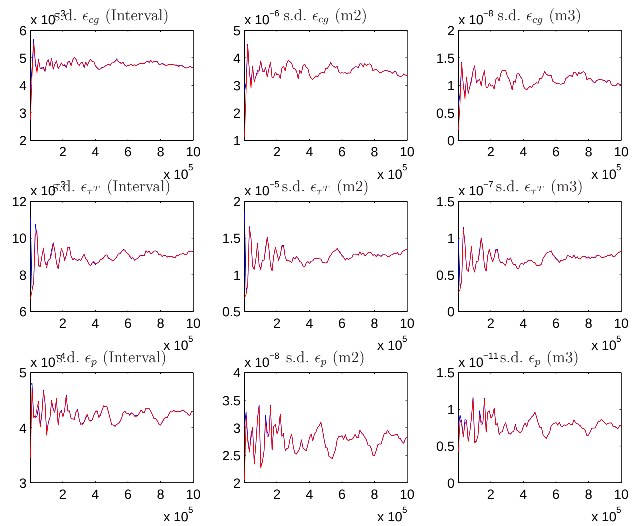


Figure 12: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

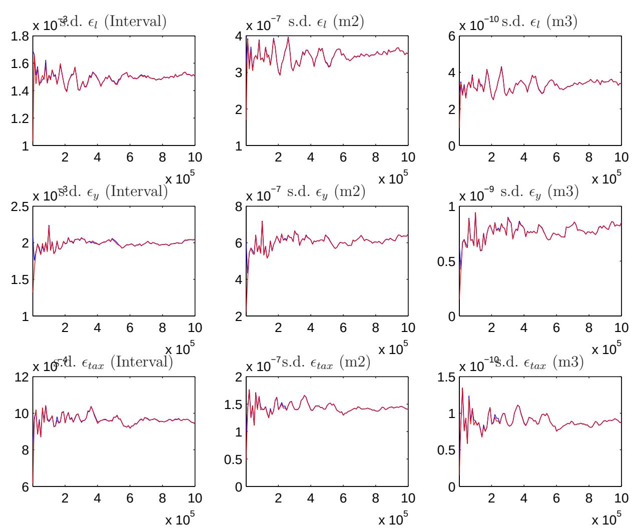


Figure 13: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

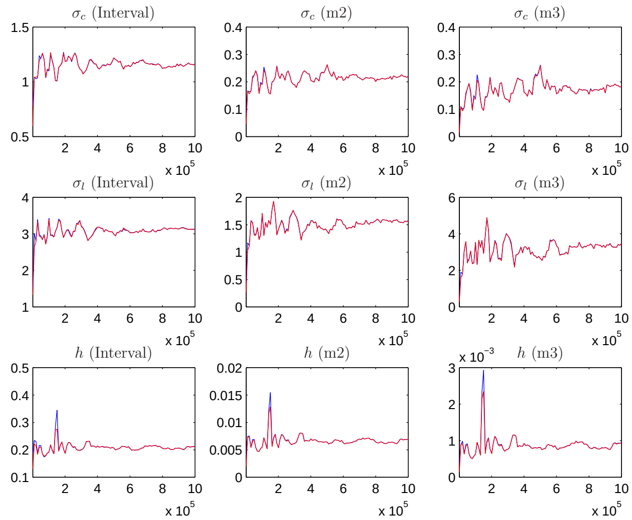


Figure 14: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

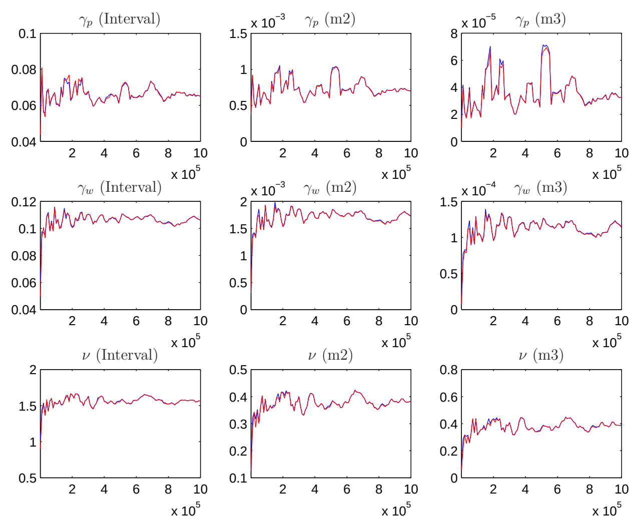


Figure 15: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

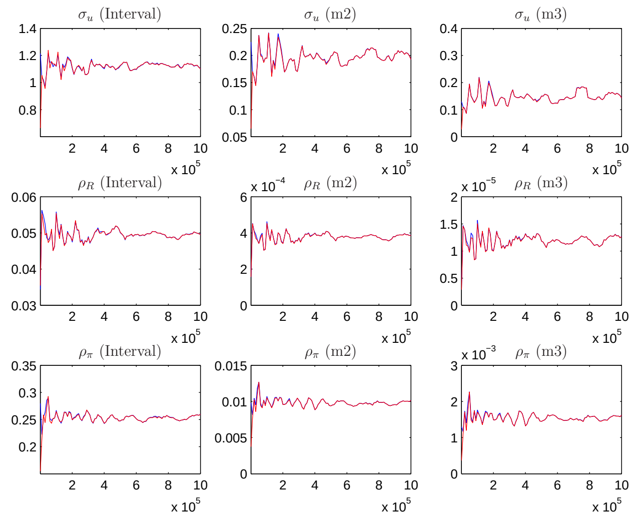


Figure 16: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

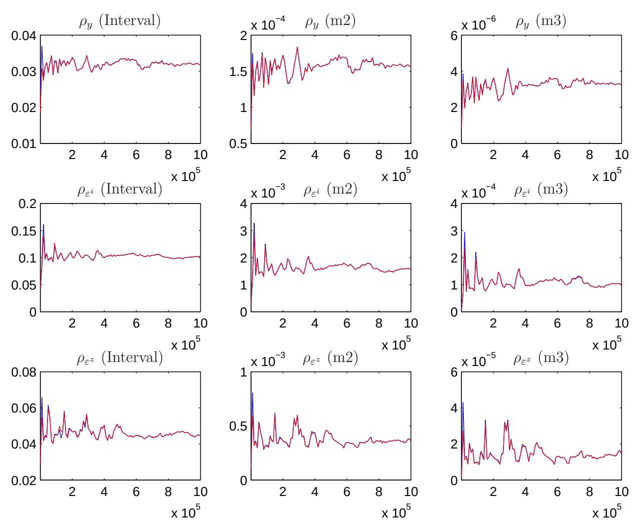


Figure 17: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

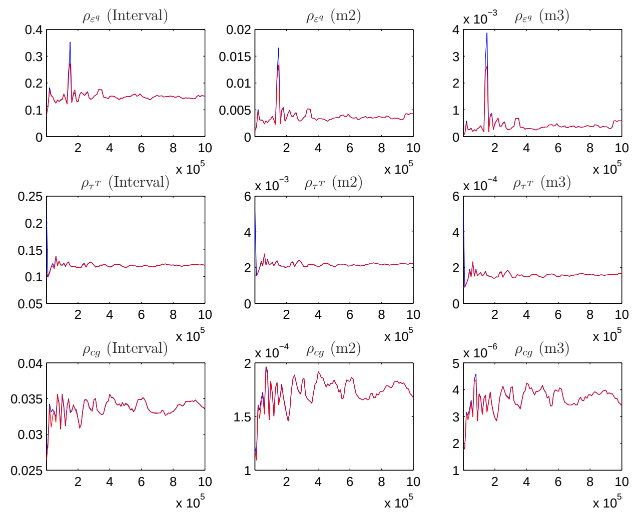


Figure 18: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

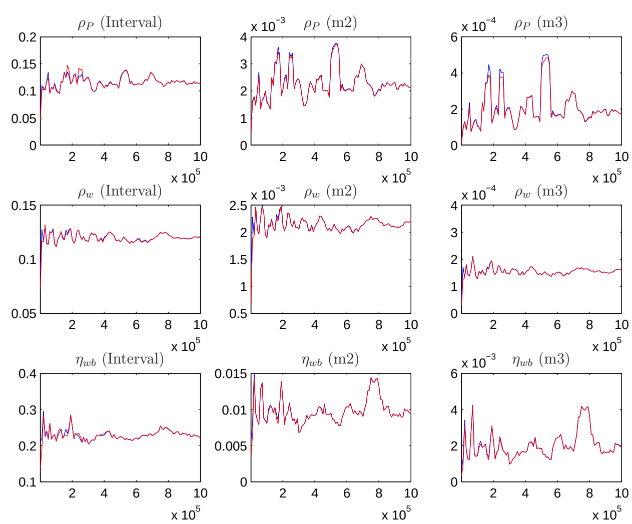


Figure 19: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

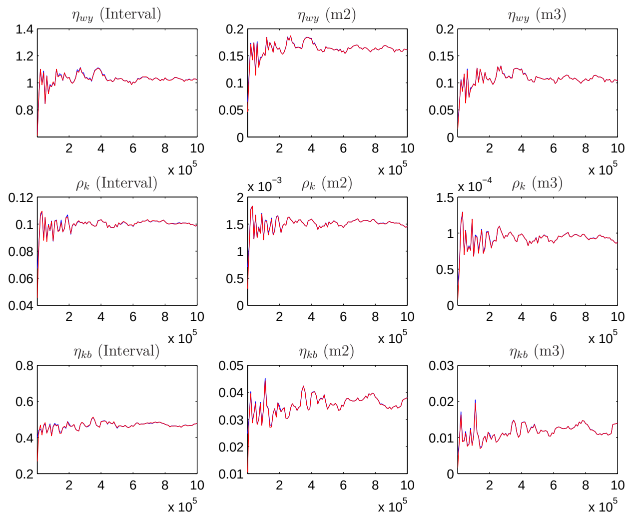


Figure 20: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

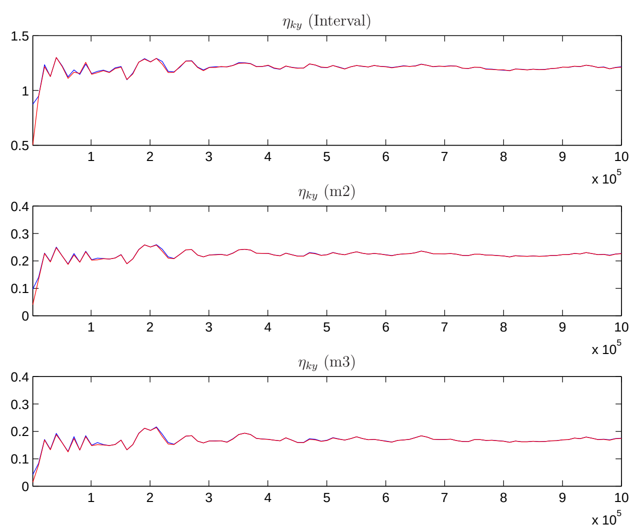


Figure 21: Univariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

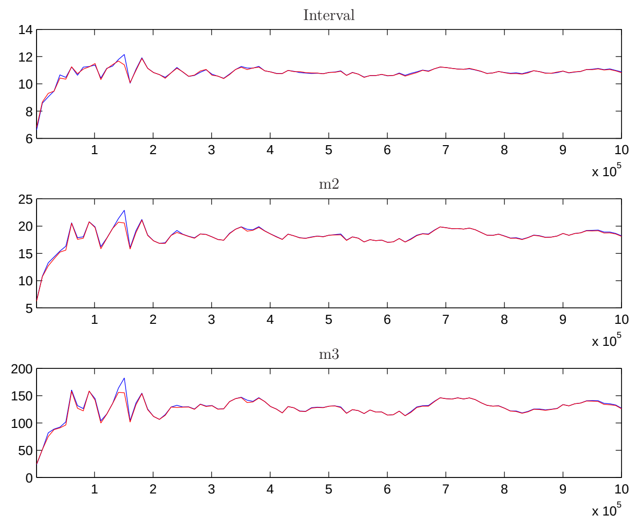


Figure 22: Multivariate convergence diagnostics for the Metropolis-Hastings of baseline model estimation. The first, second and third rows are respectively the criteria based on the eighty percent interval, the second and third moments. The different parameters are aggregated using the posterior kernel.

B.5.2 Recommended Model

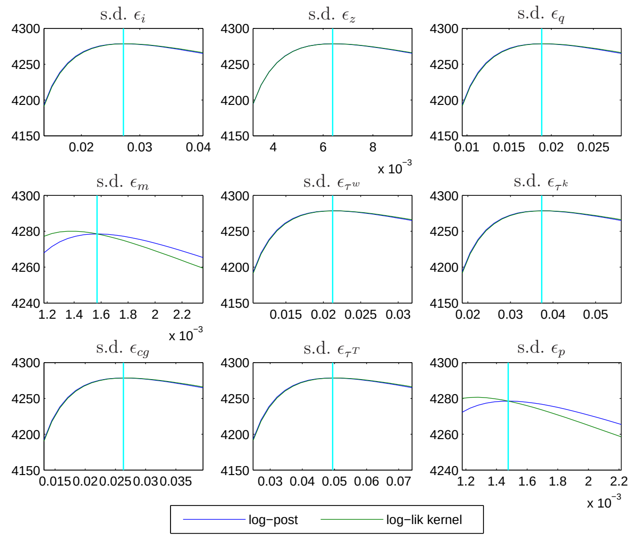


Figure 23: Check plots for posterior mode maximization of the recommended model.

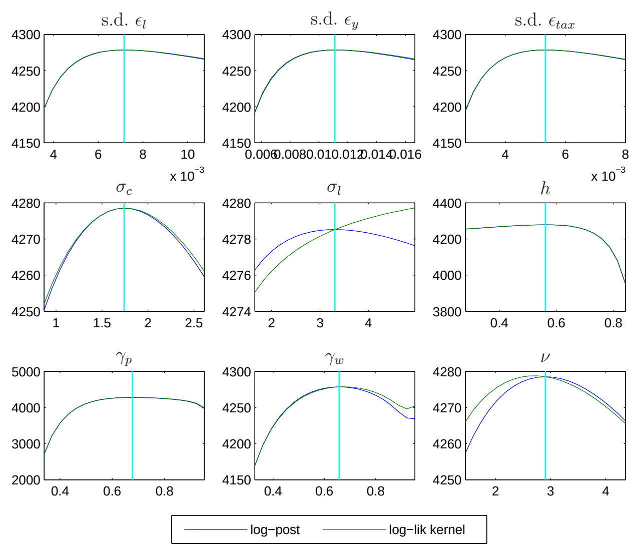


Figure 24: Check plots for posterior mode maximization of the recommended model.

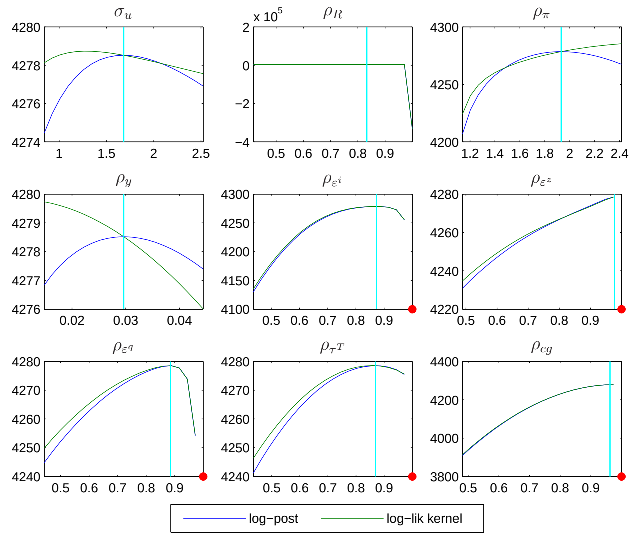


Figure 25: Check plots for posterior mode maximization of the recommended model.

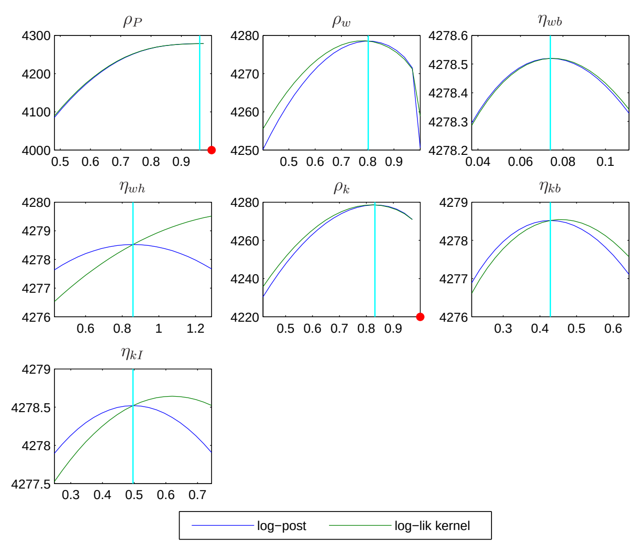


Figure 26: Check plots for posterior mode maximization of the recommended model.

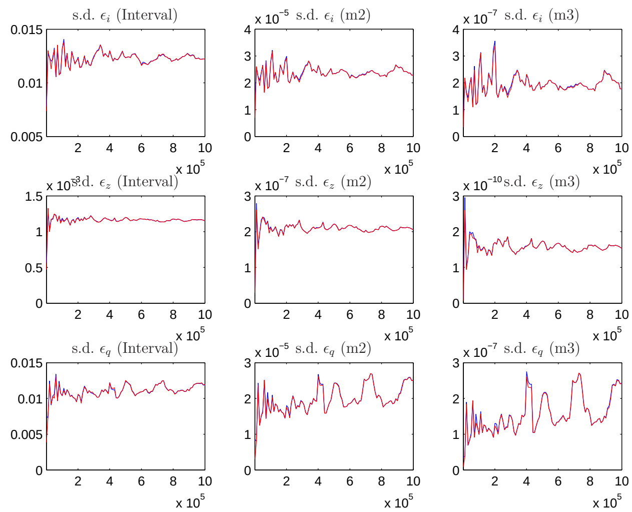


Figure 27: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

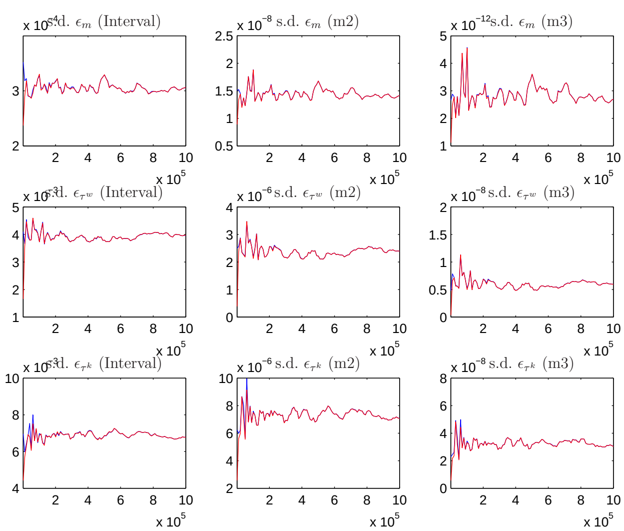


Figure 28: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

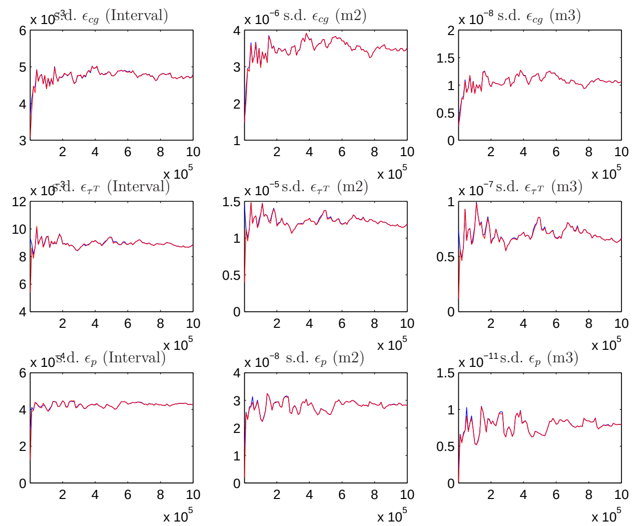


Figure 29: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

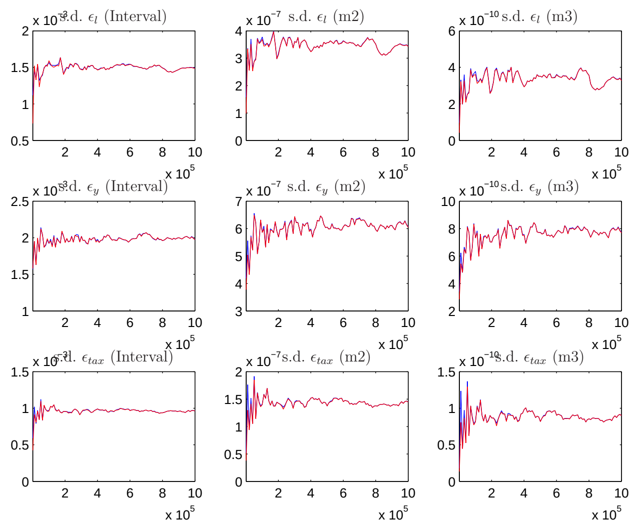


Figure 30: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

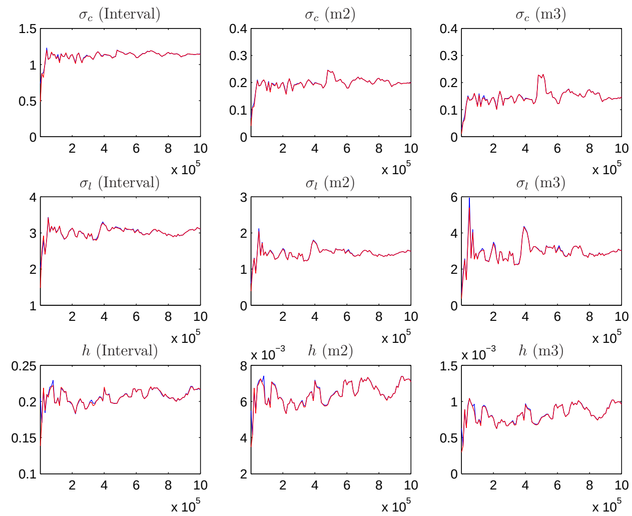


Figure 31: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

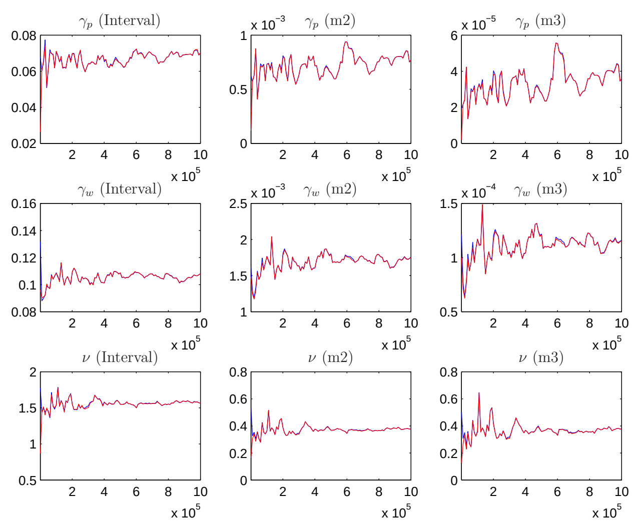


Figure 32: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

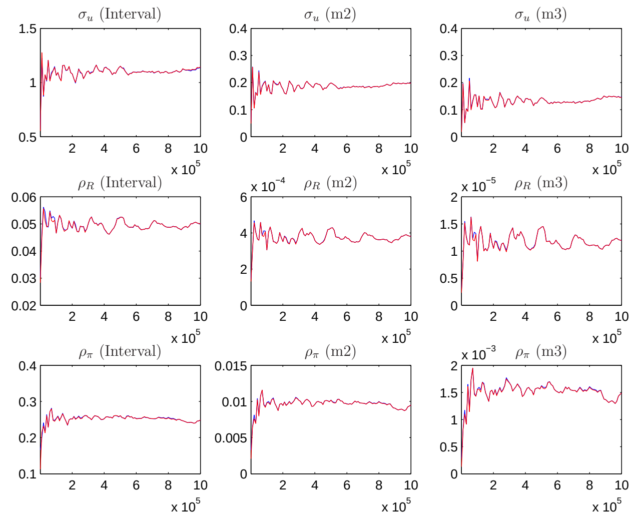


Figure 33: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

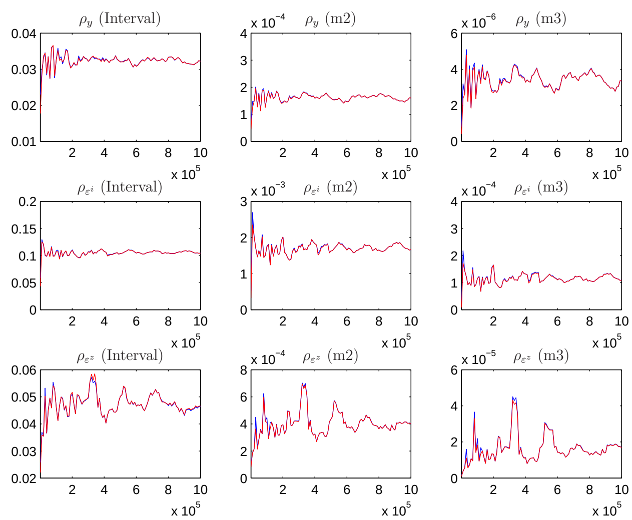


Figure 34: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

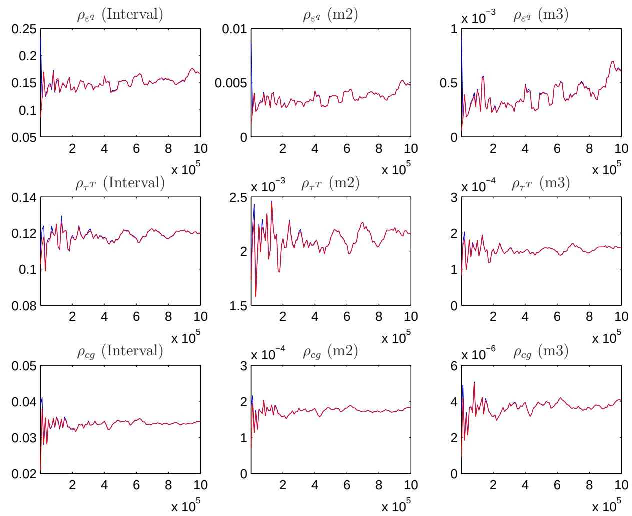


Figure 35: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

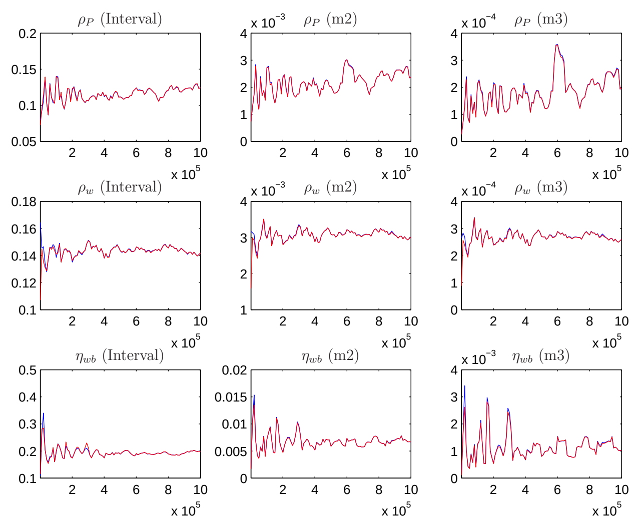


Figure 36: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

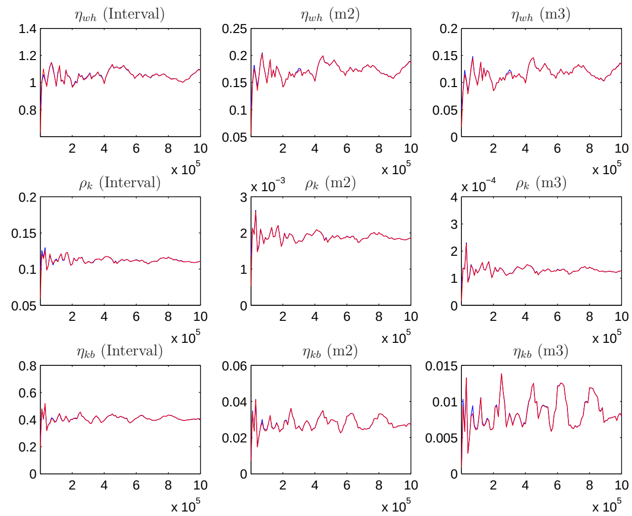


Figure 37: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

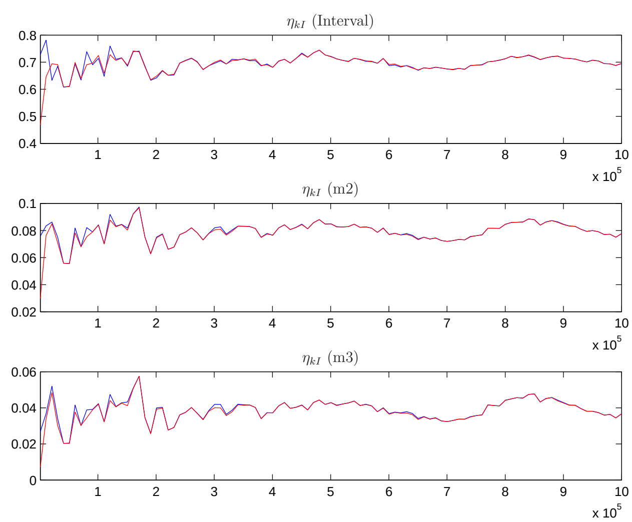


Figure 38: Univariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

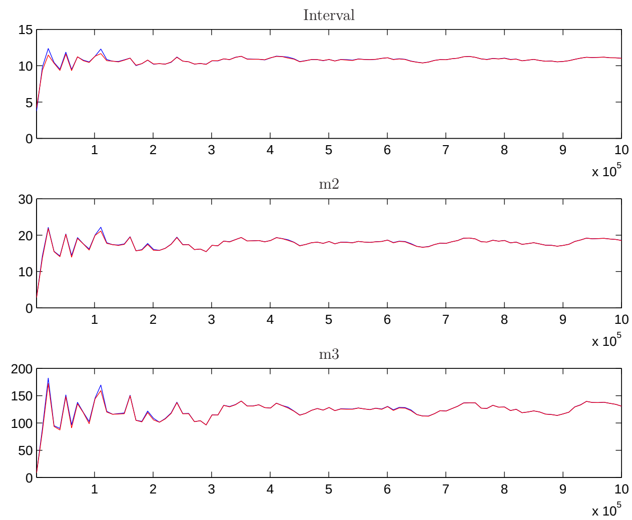


Figure 39: Multivariate convergence diagnostics for the Metropolis-Hastings of the recommended model estimation. The first, second and third rows are respectively the criteria based on the eighty percent interval, the second and third moments. The different parameters are aggregated using the posterior kernel.

C Variable selection

C.1 Optimal Policy

Let N be the number of endogenous variables. The optimal policy problem is defined as maximizing the life-time expected utility

$$E_0 \sum_{t=j}^{\infty} \beta^j U\left(c_{t+j} - hc_{t+j-1}, l_{t+j}\right), \tag{103}$$

where aggregate utility is defined by Equation (40), subject to the following $(N-2)$ equations: (2), (3)-(7), (16)-(20), (26), (27), (29)-(38), and (42)- (44).

The first-order conditions of the maximization problem yield $2N-2$ equations for the N endogenous variables and $N-2$ Lagrangian multipliers associated with the private sector equilibrium constraints.

The optimal equilibrium is then defined as a set of stationary variables F_t^w, F_t^p, K_t^w, K_t^p, p_t^*, w_t^*, d_t, p_t^+, w_t^+, π_t^w, π_t, w_t, y_t, l_t, l_t^d, k_t, z_t, $\varepsilon_{i,t}$, $\varepsilon_{z,t}$, $\varepsilon_{q,t}$, χ_t, I_t, c_t, u_t, r_t^k, b_t, x_t, R_t, τ_t^T, τ_t^w, τ_t^k, c_t^g, $\tilde{w}_t^+$, and $N-2$ Lagrangian multipliers satisfying the first-order conditions of the optimal policy problem, as well as the laws of motion for the autoregressive shock processes (45)-(49), given exogenous stochastic processes $\{\epsilon_t^i, \epsilon_t^q, \epsilon_t^z, \epsilon_t^{cg}, \epsilon_t^T, \epsilon_t^m\}_{t=0}^{\infty}$, values of the N endogenous variables dated $t<0$, and values of the $(N-2)$ Lagrangian multipliers dated $t<0$.

C.2 Estimation general feedback rule

For the posterior mode estimation of the simple linear feedback rules, the DSGE model is closed with the following tax rules:

$$\begin{aligned}\hat{\tau}_t^w =& \eta_{wk}\hat{k}_{t-1} + \eta_{wb}\hat{b}_{t-1} + \eta_{wy}\hat{y}_t + \eta_{wc}\hat{c}_t + \eta_{wh}\hat{l}_t + \eta_{ww}\hat{w}_t + \\ & \eta_{wI}\hat{I}_t + \eta_{w\pi}\hat{\pi}_t + \eta_{wR}\hat{R}_t\end{aligned} \tag{104}$$

$$\begin{aligned}\hat{\tau}_t^k =& \eta_{kk}\hat{k}_{t-1} + \eta_{kb}\hat{b}_{t-1} + \eta_{ky}\hat{y}_t + \eta_{kc}\hat{c}_t + \eta_{kh}\hat{l}_t + \eta_{kw}\hat{w}_t + \\ & \eta_{kI}\hat{I}_t + \eta_{k\pi}\hat{\pi}_t + \eta_{kR}\hat{R}_t\end{aligned} \tag{105}$$

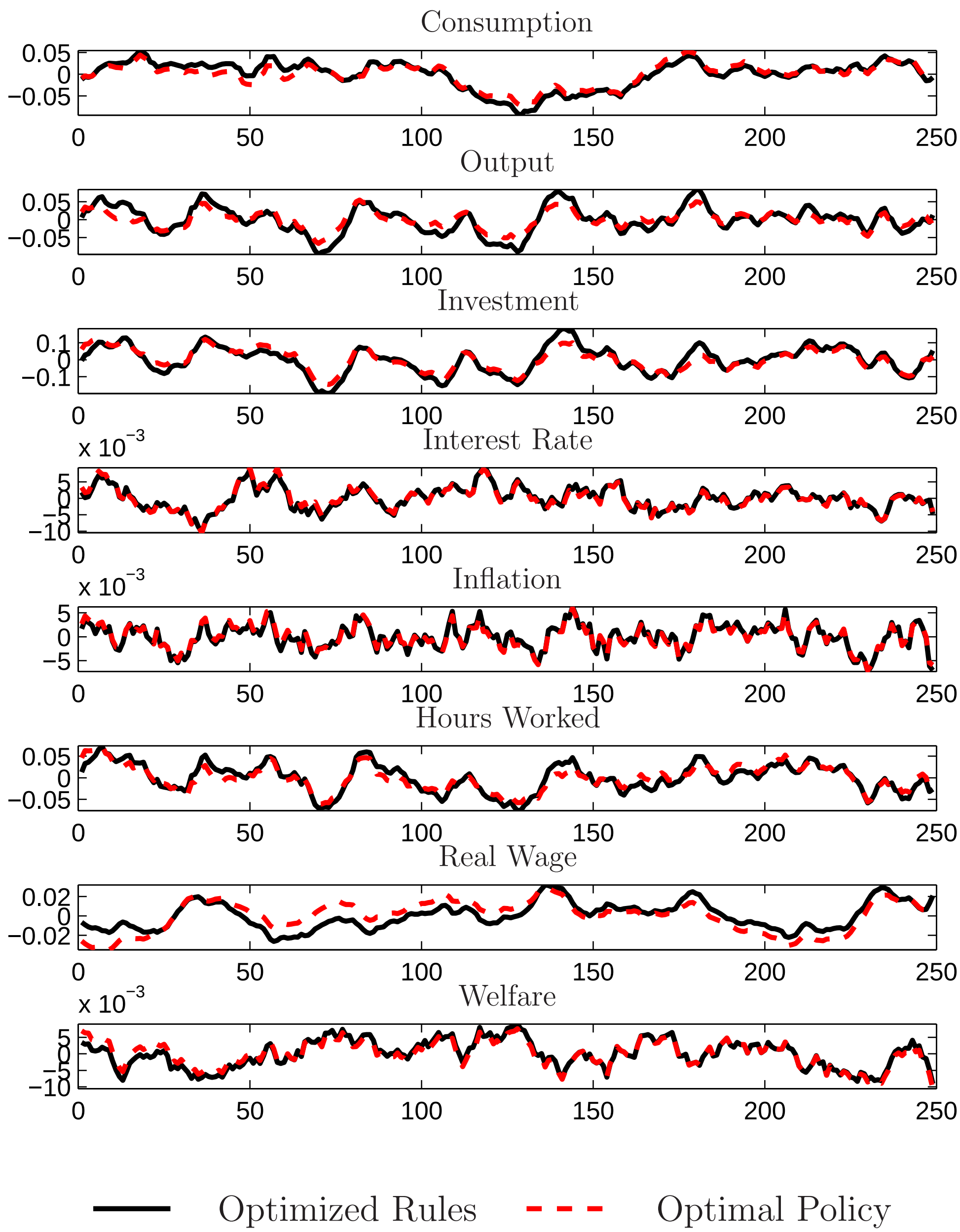


Figure 40: Counterfactual of optimized rules (solid) and optimal policy (dashed).

Feedback Parameter	Symbol	Mode	S.d.	T-value
		Tax Rate on Labor Income		
Capital	η_{wk}	-0.8091	0.1630	4.9641
Debt	η_{wb}	-0.0568	0.0486	1.1694
Output	η_{wy}	1.7889	0.5867	3.0490
Consumption	η_{wc}	-0.3455	0.3949	0.8749
Hours worked	η_{wh}	-6.1840	0.4780	12.9382
Wage rate	η_{ww}	-0.7528	0.6158	1.2224
Investment	η_{wI}	-0.2179	0.2582	0.8438
Inflation	$\eta_{w\pi}$	-5.2469	0.9003	5.8277
Nominal interest rate	η_{wR}	5.5651	0.8505	6.5433
		Tax Rate on Capital Income		
Capital	η_{kk}	-0.3715	0.6508	0.5709
Debt	η_{kb}	-2.3788	0.1330	17.8898
Output	η_{ky}	0.4596	0.5448	0.8436
Consumption	η_{kc}	1.7116	0.3849	4.4472
Hours worked	η_{kh}	0.9508	0.4208	2.2592
Wage rate	η_{kw}	3.7079	0.7443	4.9820
Investment	η_{kI}	3.5151	0.2365	14.8605
Inflation	$\eta_{k\pi}$	2.3441	0.8985	2.6090
Nominal interest rate	η_{kR}	-0.5434	0.6652	0.8168

Table 4: Posterior mode maximization of optimized feedback coefficients.

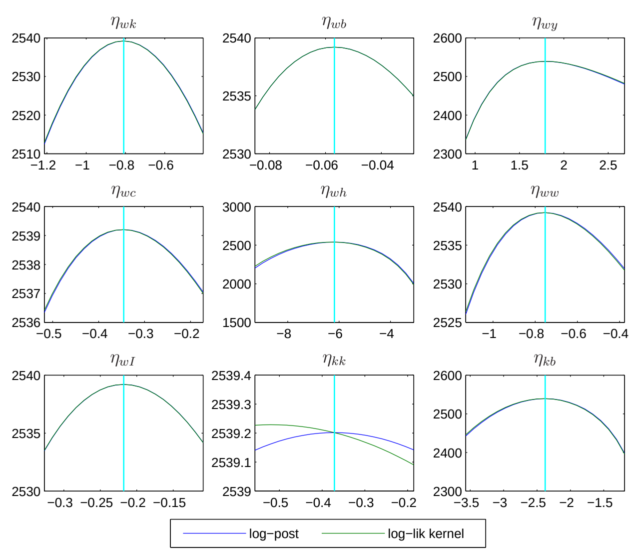


Figure 41: Check plots for posterior mode maximization of the model with optimized rules.

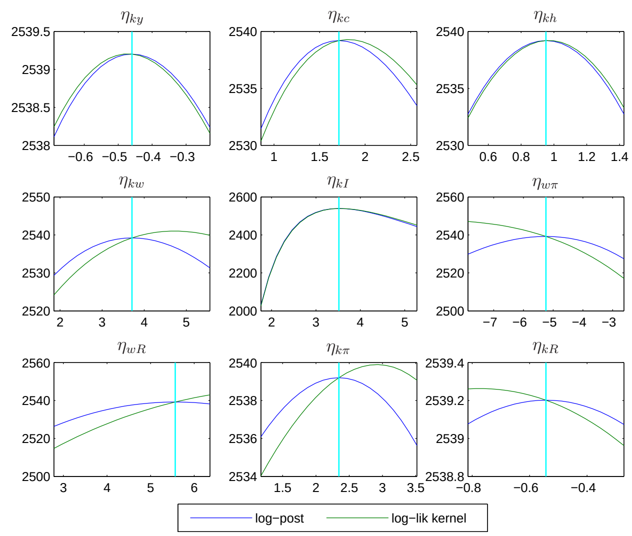


Figure 42: Check plots for posterior mode maximization of the model with optimized rules.

C.3 Identification

Feedback Parameter	Symbol	Percentile		
		50%	25%	75%
Tax Rate on Labor Income				
Capital	η_{wk}	0.2915	0.1649	0.6022
Output	η_{wy}	0.8026	0.4848	1.2752
Consumption	η_{wc}	0.1234	0.0414	0.3016
Hours worked	η_{wh}	2.9360	2.3006	3.4760
Wage rate	η_{ww}	0.1695	0.0630	0.3859
Investment	η_{wI}	0.2609	0.1020	0.4687
Inflation	$\eta_{w\pi}$	0.0835	0.0371	0.1956
Nominal interest rate	η_{wR}	0.1671	0.0971	0.2351
Tax Rate on Capital Income				
Capital	η_{kk}	0.0074	0.0024	0.0183
Output	η_{ky}	0.0260	0.0105	0.0491
Consumption	η_{kc}	0.0306	0.0147	0.0528
Hours worked	η_{kh}	0.0299	0.0174	0.0456
Wage rate	η_{kw}	0.0745	0.0463	0.1124
Investment	η_{kI}	0.4195	0.2762	0.5751
Inflation	$\eta_{k\pi}$	0.0023	0.0011	0.0039
Nominal interest rate	η_{kR}	0.0020	0.0007	0.0039

Table 5: Elasticity of income tax rate's variance w.r.t. corresponding feedback parameters of the tax rules.

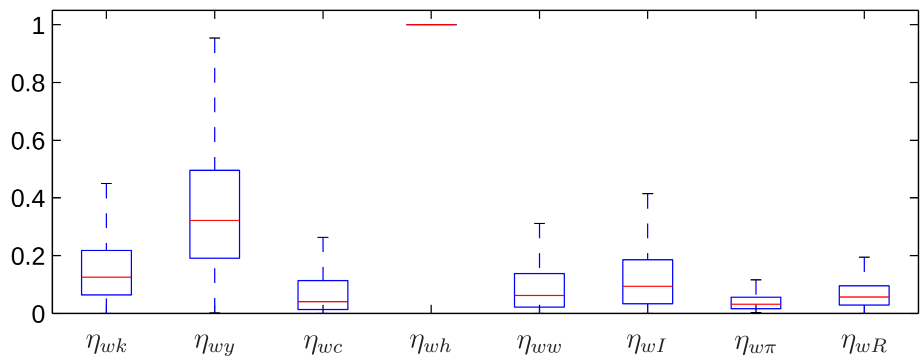


Figure 43: Relative elasticity of variables' variance w.r.t. feedback parameters of the labor income tax rule.

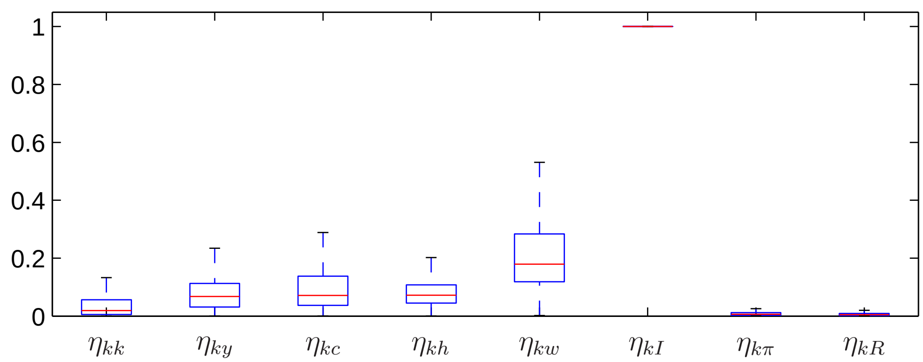


Figure 44: Relative elasticity of variables' variance w.r.t. feedback parameters of the capital income tax rule.

C.4 Robustness

Extended feedback rules For the first robustness check we extend the linear feedback rules by the additional contemporaneous variables, rental rate of capital (r^k), capacity utilization rate (u), and marginal costs (z):

$$\begin{aligned}\hat{\tau}_t^w = &\eta_{wk}\hat{k}_{t-1} + \eta_{wb}\hat{b}_{t-1} + \eta_{wy}\hat{y}_t + \eta_{wc}\hat{c}_t + \eta_{wh}\hat{l}_t + \eta_{ww}\hat{w}_t + \\ &\eta_{wI}\hat{I}_t + \eta_{w\pi}\hat{\pi}_t + \eta_{wR}\hat{R}_t + \eta_{wr^k}\hat{r}_t^k + \eta_{wu}\hat{u}_t + \eta_{wz}\hat{z}_t\end{aligned} \tag{106}$$

$$\begin{aligned}\hat{\tau}_t^k = &\eta_{kk}\hat{k}_{t-1} + \eta_{kb}\hat{b}_{t-1} + \eta_{ky}\hat{y}_t + \eta_{kc}\hat{c}_t + \eta_{kh}\hat{l}_t + \eta_{kw}\hat{w}_t + \\ &\eta_{kI}\hat{I}_t + \eta_{k\pi}\hat{\pi}_t + \eta_{kR}\hat{R}_t + \eta_{kr^k}\hat{r}_t^k + \eta_{ku}\hat{u}_t + \eta_{kz}\hat{z}_t\end{aligned} \tag{107}$$

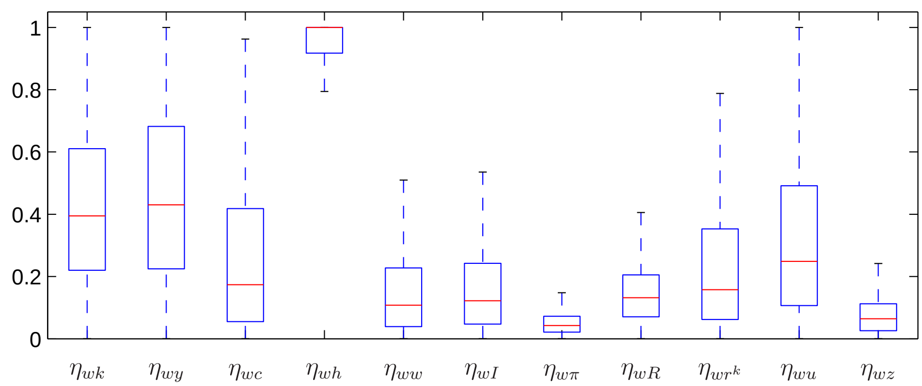


Figure 45: Relative elasticity of variables' variance w.r.t. feedback parameters of the labor income tax rule.

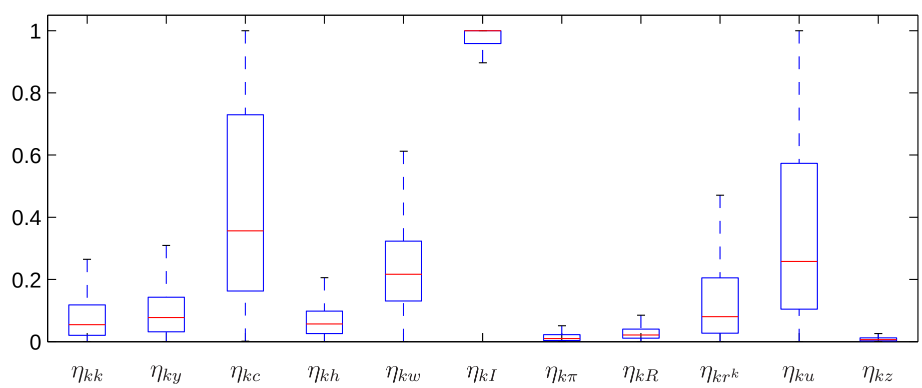
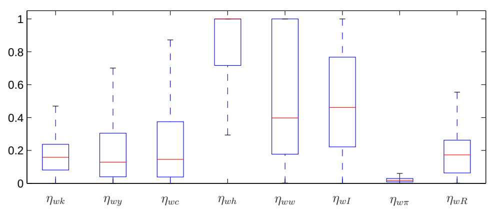


Figure 46: Relative elasticity of variables' variance w.r.t. feedback parameters of the capital income tax rule.

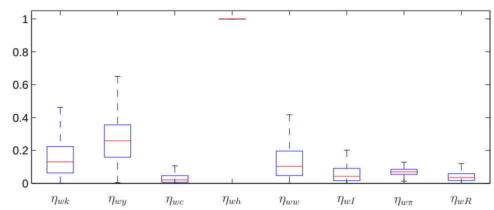
Feedback Parameter	Symbol	Mode	S.d.	T-value
Tax Rate on Labor Income				
Capital	η_{wk}	-1.3051	0.5459	2.3907
Debt	η_{wb}	-0.2544	0.1389	1.8321
Output	η_{wy}	1.2004	0.6519	1.8414
Consumption	η_{wc}	-0.3576	0.3917	0.9130
Hours worked	η_{wh}	-4.0346	0.6091	6.6238
Wage rate	η_{ww}	0.6934	0.6482	1.0698
Investment	η_{wI}	0.1518	0.3168	0.4791
Inflation	$\eta_{w\pi}$	-3.1620	0.9463	3.3416
Nominal interest rate	η_{wR}	3.9025	0.8686	4.4928
Rental rate of capital	η_{wr^k}	-0.3056	0.8434	0.3624
Utilization rate	η_{wu}	-1.7411	0.8756	1.9884
Marginal cost	η_{wz}	-4.3520	0.7616	5.7141
Tax Rate on Capital Income				
Capital	η_{kk}	-0.4039	0.6827	0.5917
Debt	η_{kb}	-1.7218	0.2127	8.0962
Output	η_{ky}	-0.2347	0.6107	0.3844
Consumption	η_{kc}	1.7596	0.3625	4.8547
Hours worked	η_{kh}	-0.6248	0.6317	0.9890
Wage rate	η_{kw}	2.6813	0.7686	3.4884
Investment	η_{kI}	2.9619	0.3030	9.7739
Inflation	$\eta_{k\pi}$	1.6159	0.9532	1.6951
Nominal interest rate	η_{kR}	-1.5342	0.6032	2.5433
Rental rate of capital	η_{wr^k}	-0.4046	0.8617	0.4695
Utilization rate	η_{wu}	2.8653	0.9400	3.0482
Marginal cost	η_{wz}	2.2522	0.7152	3.1491

Table 6: Posterior mode maximization of optimized feedback coefficients.

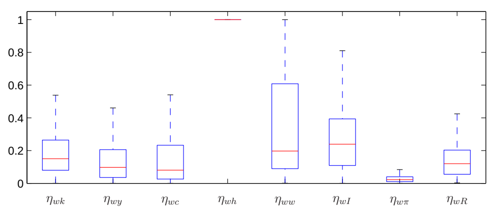
Key parameter variation We conduct robustness exercises with respect to key parameters for optimal policy dynamics as pointed out Schmitt-Grohé and Uribe (2006). We investigate different parameter combination by taken draws from the corresponding marginal posterior distribution. In particular, we focus on the 5th and 95th HPD.



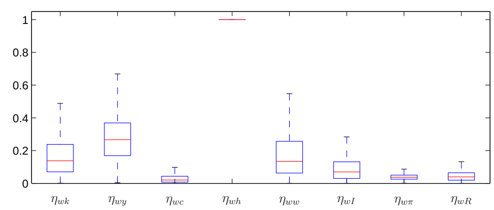
(a) $\gamma_p = 0.64$ $\gamma_w = 0.59$



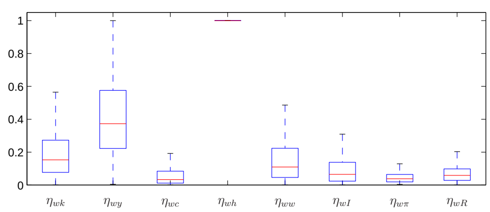
(b) $\gamma_p = 0.64$ $\gamma_w = 0.73$



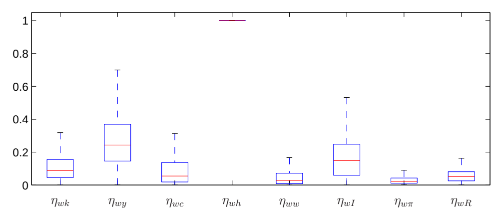
(c) $\gamma_p = 0.72$ $\gamma_w = 0.59$



(d) $\gamma_p = 0.72$ $\gamma_w = 0.73$

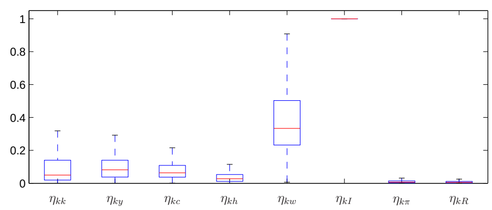


(e) $\nu = 2.26$

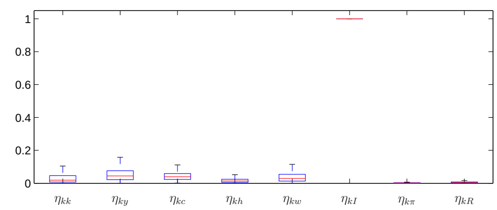


(f) $\nu = 4.24$

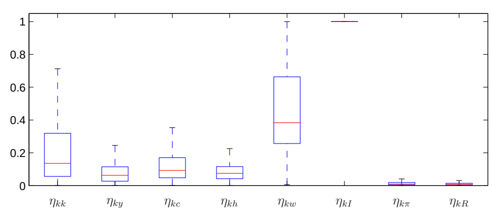
Figure 47: Relative elasticity of labor income tax rate's variance w.r.t. feedback parameters of the labor income tax rule for different degrees of stickiness and investment adjustment costs.



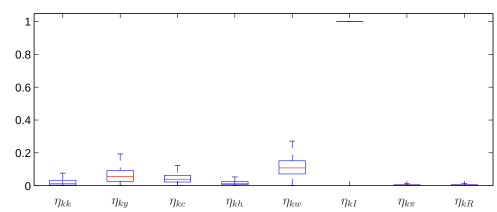
(a) $\gamma_p = 0.64$ $\gamma_w = 0.59$



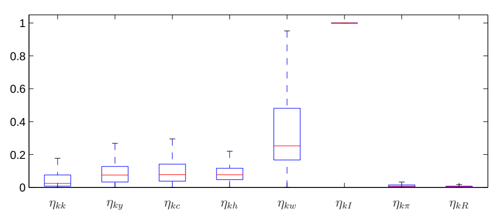
(b) $\gamma_p = 0.64$ $\gamma_w = 0.73$



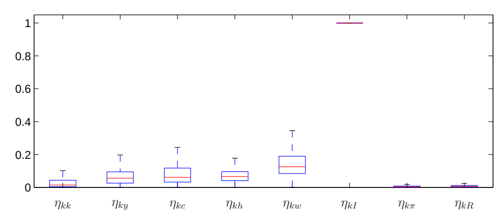
(c) $\gamma_p = 0.72$ $\gamma_w = 0.59$



(d) $\gamma_p = 0.72$ $\gamma_w = 0.73$



(e) $\nu = 2.26$



(f) $\nu = 4.24$

Figure 48: Relative elasticity of capital income tax rate's variance w.r.t. feedback parameters of the capital income tax rule for different degrees of stickiness and investment adjustment costs.

Feedback Parameter	Symbol	$\gamma_p = 0.64$ $\gamma_w = 0.59$	$\gamma_p = 0.64$ $\gamma_w = 0.73$	$\gamma_p = 0.72$ $\gamma_w = 0.59$	$\gamma_p = 0.72$ $\gamma_w = 0.73$	$\nu = 2.26$	$\nu = 4.24$
			Tax Rate on Labor Income				
Capital	η_{wk}	-0.7324	-1.7804	-0.7138	-1.8529	-0.9158	-0.6619
		(0.1324)	(0.2945)	(0.1357)	(0.3224)	(0.1797)	(0.1450)
Debt	η_{wb}	0.0057	0.0636	-0.0494	0.0599	-0.0875	-0.0303
		(0.0426)	(0.0746)	(0.0366)	(0.0719)	(0.0552)	(0.0396)
Output	η_{wy}	-0.4161	2.6418	0.0407	2.8082	1.9475	1.5068
		(0.5482)	(0.6897)	(0.5435)	(0.6917)	(0.6010)	(0.5663)
Consumption	η_{wc}	-0.7218	0.0661	-0.4923	0.1221	-0.1696	-0.5223
		(0.3243)	(0.5312)	(0.3272)	(0.5447)	(0.4078)	(0.3755)
Hours worked	η_{wh}	-2.9264	-10.2702	-3.5992	-11.1865	-6.4791	-5.8093
		(0.5529)	(0.5171)	(0.5048)	(0.4746)	(0.4797)	(0.4753)
Wage rate	η_{ww}	2.8712	-2.4406	1.8655	-3.2121	-1.1579	-0.2534
		(0.6250)	(0.7728)	(0.6175)	(0.7986)	(0.6335)	(0.5898)
Investment	η_{wI}	-0.6163	0.0828	-0.3780	0.4425	-0.0702	-0.3625
		(0.2131)	(0.3702)	(0.2100)	(0.3638)	(0.2603)	(0.2526)
Inflation	$\eta_{w\pi}$	-2.3878	-9.2705	-3.0121	-7.3200	-5.8391	-4.4779
		(0.8940)	(0.9096)	(0.9207)	(0.9535)	(0.9070)	(0.8929)
Interest rate	η_{wR}	7.9244	6.2028	7.4014	7.0912	5.0765	6.2683
		(0.6047)	(0.9679)	(0.6393)	(0.9680)	(0.8658)	(0.8209)
			Tax Rate on Capital Income				
Capital	η_{kk}	-1.7948	1.1275	-2.4998	0.5004	-0.3119	-0.6313
		(0.6858)	(0.6566)	(0.6395)	(0.6565)	(0.6701)	(0.6453)
Debt	η_{kb}	-2.4939	-2.3552	-2.1145	-2.0731	-2.5843	-2.0653
		(0.1394)	(0.1292)	(0.1348)	(0.1251)	(0.1353)	(0.1330)
Output	η_{ky}	0.4434	-0.0827	-0.2500	-0.3770	-0.4657	-0.3521
		(0.5849)	(0.5761)	(0.5513)	(0.5458)	(0.5509)	(0.5398)
Consumption	η_{kc}	1.8144	2.0510	1.6716	1.9963	1.4863	1.9642
		(0.4010)	(0.4366)	(0.3676)	(0.4118)	(0.3908)	(0.3799)
Hours worked	η_{kh}	0.3948	0.4377	0.7783	0.4121	0.9155	0.9548
		(0.4702)	(0.4736)	(0.4009)	(0.3988)	(0.4195)	(0.4237)
Wage rate	η_{kw}	5.2745	0.2039	6.6905	1.9955	4.3386	3.1701
		(0.7453)	(0.8184)	(0.7313)	(0.8025)	(0.7366)	(0.7515)
Investment	η_{kI}	3.1101	4.7290	2.7392	4.2606	3.1974	3.8879
		(0.2367)	(0.3002)	(0.2137)	(0.2831)	(0.2309)	(0.2492)
Inflation	$\eta_{k\pi}$	2.6401	0.0968	2.6780	0.8251	2.5235	2.0629
		(0.8758)	(0.8783)	(0.9206)	(0.9347)	(0.8963)	(0.9008)
Interest rate	η_{kR}	-0.0529	0.4807	-0.9840	0.3156	-0.1066	-1.1154
		(0.7899)	(0.7439)	(0.6833)	(0.6986)	(0.6684)	(0.6642)

Table 7: Posterior mode maximization of optimized feedback coefficients for different degrees of stickiness and investment adjustment costs. The values in parenthesis are the corresponding standard deviations.

D Conditional welfare costs

We follow Schmitt-Grohé and Uribe (2007) and calculate the conditional welfare cost in consumption units relative to the optimal policy solution by solving for Ψ the following equation

$$\mathcal{W}_t^O = E_t \sum_{j=0}^{\infty} \beta^j \left[\varepsilon_{q,t+j} \frac{\left(c_{t+j}^s - hc_{t+j-1}^s\right)^{1-\sigma_c} (1+\Psi)^{1-\sigma_c}}{1-\sigma_c} - \psi_l \frac{\tilde{w}_{t+j}^{+,s} \left(\frac{l_{t+j}^s}{w_{t+j}^{+,s}}\right)^{1+\sigma_l}}{1+\sigma_l} \right]. \tag{108}$$

This yields:

$$\Psi = \left[\frac{\mathcal{W}_t^O + \mathcal{W}_{L,t}^s}{\mathcal{W}_t^s + \mathcal{W}_{L,t}^s} \right]^{\frac{1}{1-\sigma_c}} - 1 \tag{109}$$

where variables with O represents variables under the optimal solution and variables with S indicates variables under the alternative fiscal policy regime with simple fiscal feedback rules. The variable $\mathcal{W}_{L,t}^s$ is defined as:

$$\mathcal{W}_{L,t}^s = E_t \sum_{j=0}^{\infty} \beta^j \left[\psi_l \frac{\tilde{w}_{t+j}^{+,s} \left(\frac{l_{t+j}^s}{w_{t+j}^{+,s}}\right)^{1+\sigma_l}}{1+\sigma_l} \right] \tag{110}$$

For the minimization of ψ we have to set bounds for the parameters to ensure theoretical meaningful results. More precisely we decide to restrict the coefficient in front of debt to be between $[0, 10]$ and $[-10, 0]$ for labor income tax rates and capital income tax rates respectively to ensure sustainability. The positive or negative range is related to the optimal steady-state. Moreover, we assume that all automatic stabilizers have to be between $[-20, 20]$.

Fiscal feedback rule with	Feedback parameter				Welfare cost Ψ x 100
No automatic stabilizer	η_{wb}	η_{kb}	-	-	
	2.8499	0.000			2.542
Automatic stabilizer	η_{wb}	η_{kb}	η_{wy}	η_{ky}	
(baseline model)	0.204	-1.664	-3.788	14.722	1.602
Automatic stabilizer	η_{wb}	η_{kb}	η_{wh}	η_{kI}	
(recommended model)	0.000	-3.186	-3.697	8.400	1.175

Table 8: Welfare costs under different optimized fiscal feedback rules.

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