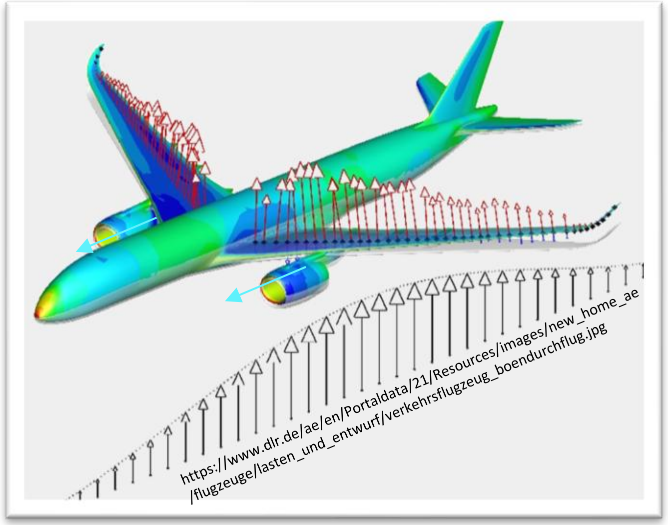
Chapter 3:

External Loads

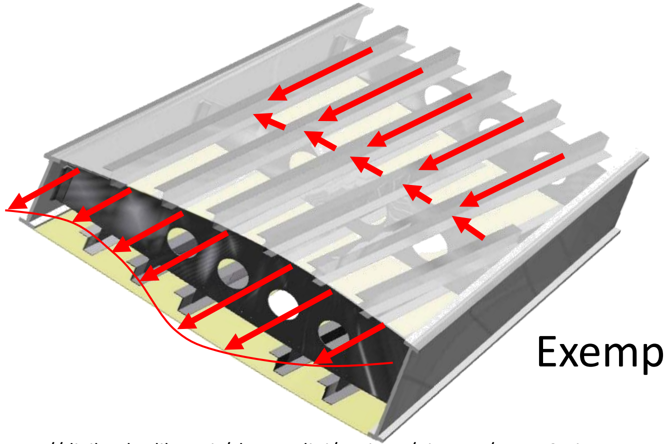
Aerospace Structure Loads

In the course of Loads Analysis, it is important to distinguish between

- External loads:
  - Are the applied loads like aerodynamics, inertia, engine thrust, etc.
- Internal loads $\longrightarrow$ Finite element method CL 6
  - Counter react the external loads,
  - I.e., internal and external loads are in equilibrium

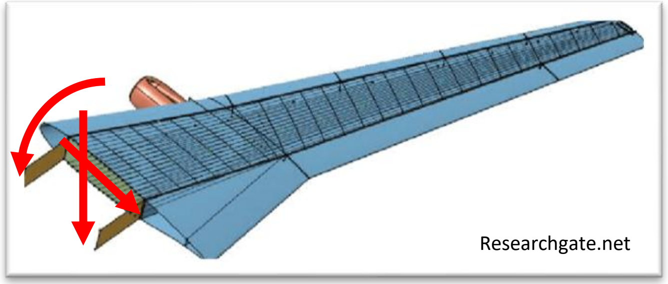


Exemplary external loads



Exemplary internal loads

https://digilander.libero.it/sbernardini/projects/pictures/Image21.jpg

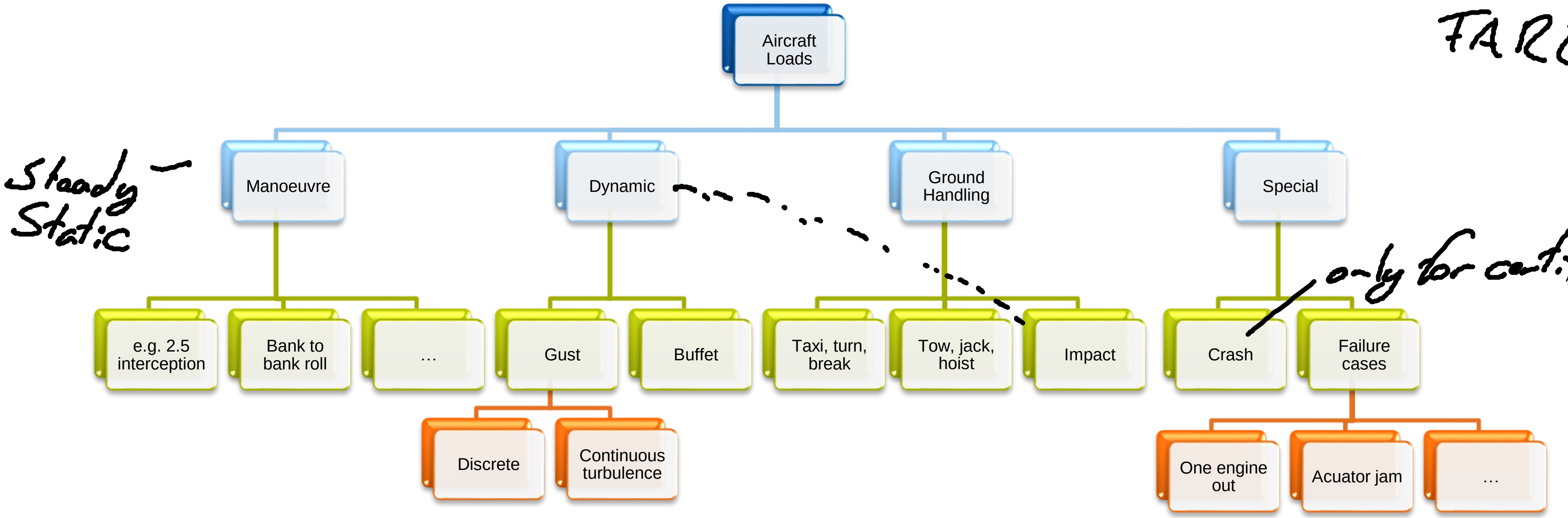


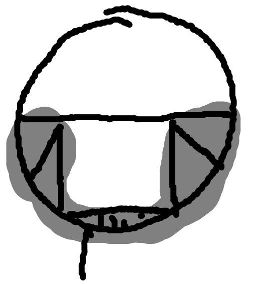
Aerospace Structure Loads – Aircraft

In the context of aerospace structure design loads are defined as

- Those forces applied to the structure components to establish the strength safety level of the complete aerospace vehicle
- Aircraft loads are organised in the following categories:

CS25 /23
FAR25 /25

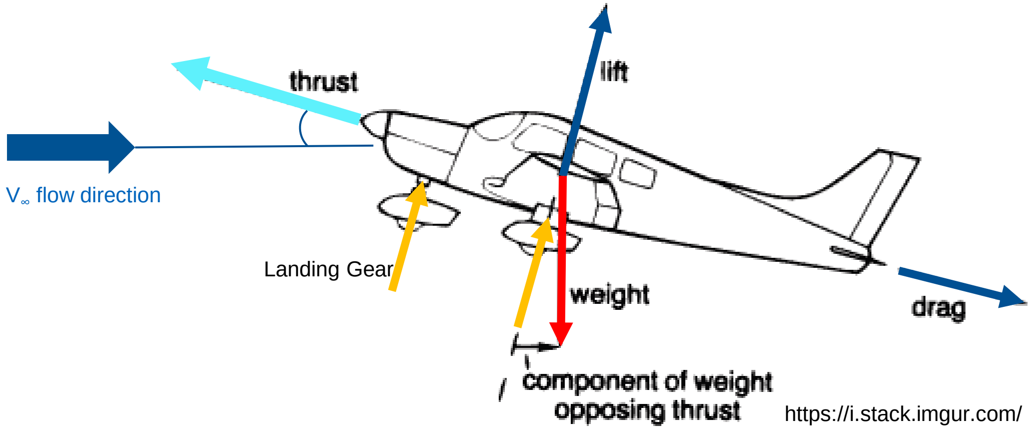




Aircraft Load Sources

The aircraft structure (Airframe) is exposed to following load sources:

- Aerodynamics: Lift and drag depend on flow conditions
  - Speed (Mach number M_a)
  - Altitude (air density)
  - Airfoil
  - Trimming conditions, mainly angle of attack
- Inertia:
  - Mass distribution, Centre of Gravity (CoG)
  - Accelerations
- Engine
  - Thrust including installation angle
  - Offset wrt. CoG
- Landing gear
  - Aircraft mass
  - Sink rate
  - Type of landing gear
- Special loads:
  - Engine and control surface faults

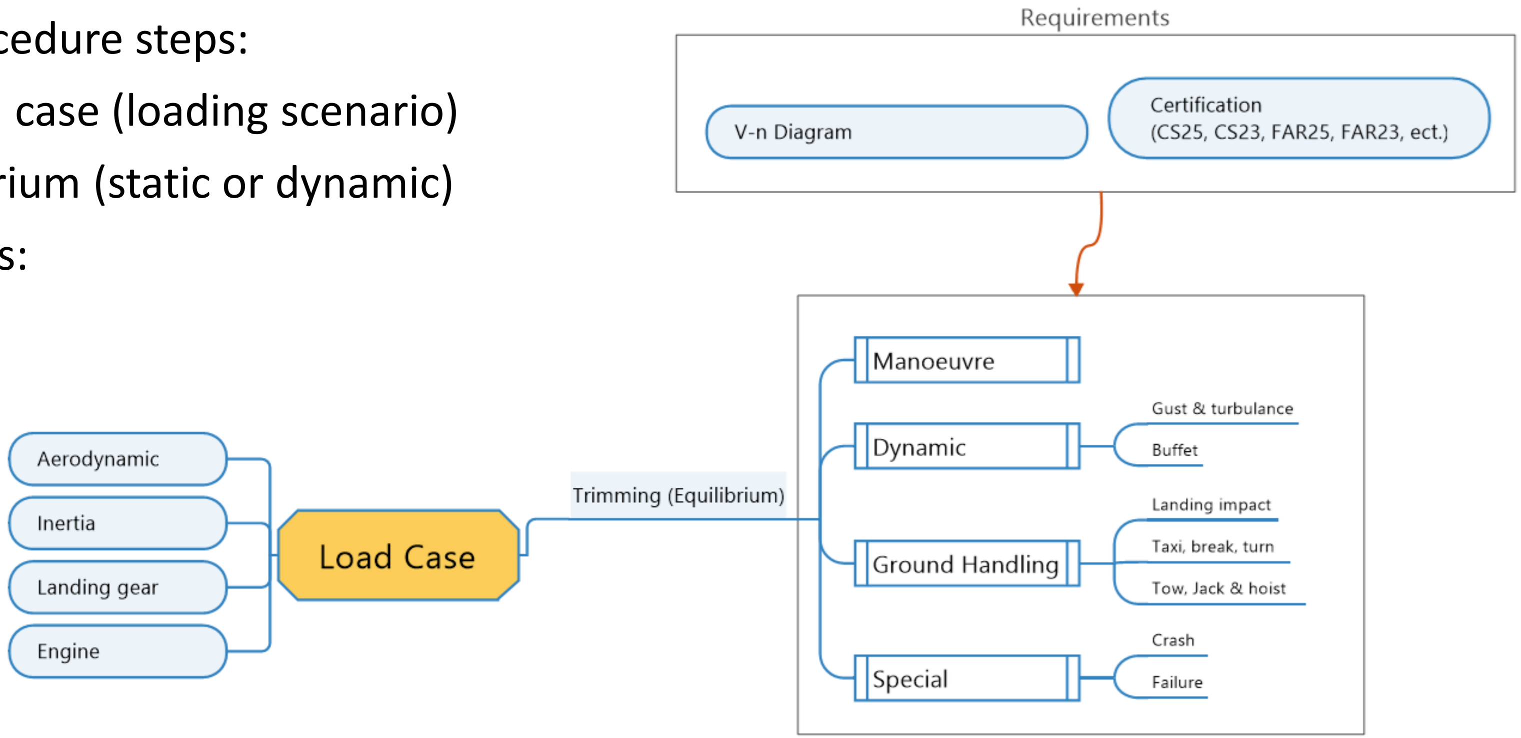


https://i.stack.imgur.com/

Aircraft Loads Analysis

The aircraft loads analysis has the procedure steps:

- Load sources are combined in a load case (loading scenario)
- All load cases are brought to equilibrium (static or dynamic)
- Load cases are categorised as follows:
  - Manoeuvre loads
  - Dynamic loads
  - Ground handling loads
  - Special loads



Aircraft load sources in more detail

- Aerodynamic loads
- Inertia loads
- Landing loads
- Special loads (Crash, engine failure, actuator jam, etc.)

Aerodynamics Loads

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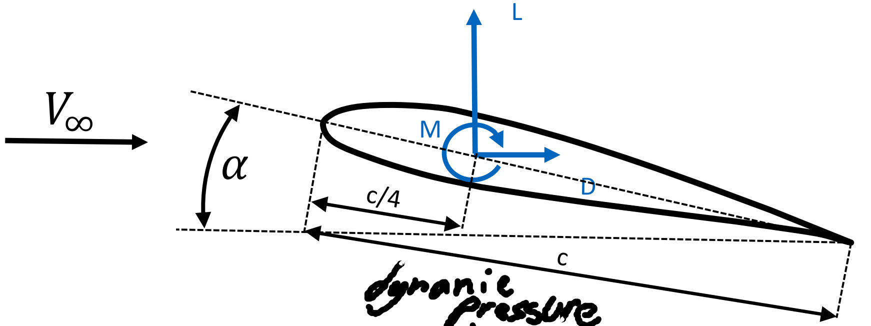
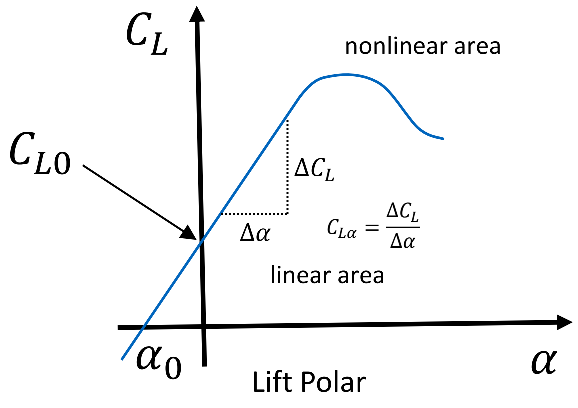
$\rho_{air\ sea\ level} \approx 1{,}225 \frac{kg}{m^3}$

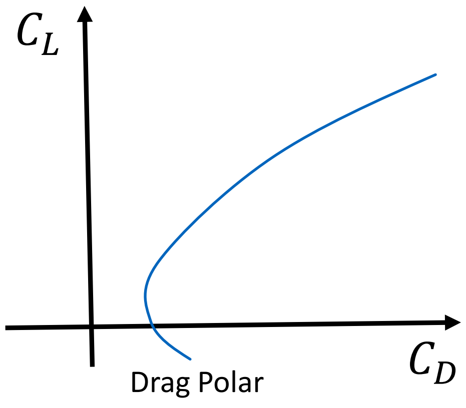
Aircraft Loads - Fundamentals of Aerodynamics

Lift and drag coefficients



resulting forces acting in the aerodynamic center





lift: $$L = \overbrace{\frac{\rho}{2} V_\infty^2}^{\text{dynamic pressure}} \cdot S \cdot C_L$$

drag: $$D = \frac{\rho}{2} V_\infty^2 \cdot S \cdot C_D$$

moment: $$M = \frac{\rho}{2} V^2 \cdot S \cdot \bar{c} \cdot C_M$$

lift curve slope: $$C_{L\alpha} = \frac{\partial C_L}{\partial \alpha}$$

dynamic pressure: $q = \frac{\rho}{2} V_\infty^2$

angle of attack: α

reference wing area: S

mean aerodynamic chord length: $\bar{c}$

Lift coefficient: C_L

Drag coefficient: C_D

Moment coefficient: C_M

Influence Parameters: altitude ($\boldsymbol{\rho}$) / speed ($\boldsymbol{V_\infty}$)

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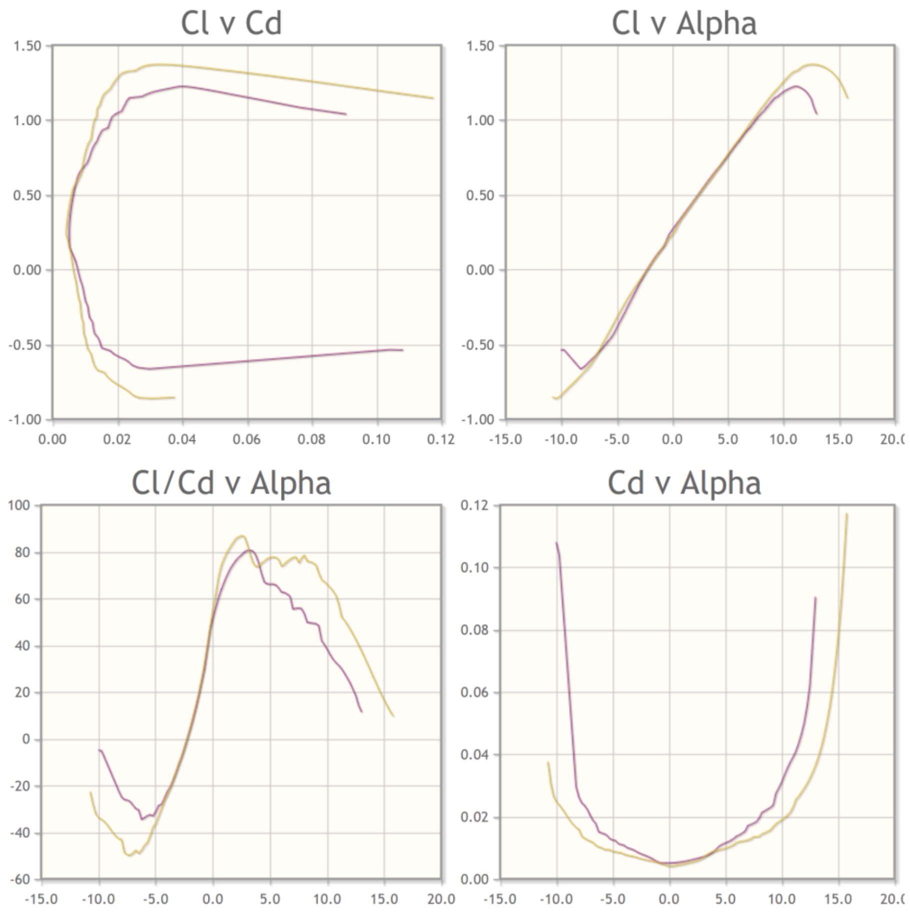
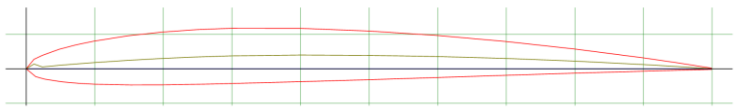
Aircraft Loads - Fundamentals of Aerodynamics

learn what numbers stand for

Lift and drag Polars – NACA 2408



NACA 2408 - NACA 2408 airfoil



Polars for NACA 2408 (naca2408-il)

Plot	Airfoil	Reynolds #	Ncrit	Max Cl/Cd	Description	Source	
☐	naca2408-il	50,000	9	37.4 at α=5.5°	Mach=0 Ncrit=9	Xfoil prediction	Details
☐	naca2408-il	50,000	5	36.7 at α=5°	Mach=0 Ncrit=5	Xfoil prediction	Details
☐	naca2408-il	100,000	9	52.6 at α=5°	Mach=0 Ncrit=9	Xfoil prediction	Details
☐	naca2408-il	100,000	5	48.9 at α=4.5°	Mach=0 Ncrit=5	Xfoil prediction	Details
☐	naca2408-il	200,000	9	66.6 at α=4°	Mach=0 Ncrit=9	Xfoil prediction	Details
☐	naca2408-il	200,000	5	58.9 at α=3.75°	Mach=0 Ncrit=5	Xfoil prediction	Details
☑	naca2408-il	500,000	9	80.6 at α=3.25°	Mach=0 Ncrit=9	Xfoil prediction	Details
☐	naca2408-il	500,000	5	67.5 at α=2.75°	Mach=0 Ncrit=5	Xfoil prediction	Details
☑	naca2408-il	1,000,000	9	86.8 at α=2.5°	Mach=0 Ncrit=9	Xfoil prediction	Details
☐	naca2408-il	1,000,000	5	82.1 at α=7.5°	Mach=0 Ncrit=5	Xfoil prediction	Details

Update plots   Reynolds number calculator

Set Reynolds number and Ncrit range		Low	High
Update Range	Reynolds Number	500,000	1,000,000
	NCrit	7	9

Polars depend on
- Airfoil
- Reynold’s number
- Mach number

http://airfoiltools.com/airfoil/details?airfoil=naca2408-il#polars

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Aircraft Loads - Fundamentals of Aerodynamics

Mach Number M:

- The Mach number is a quantity defined as the ratio of local flow velocity to local speed of sound and is given by

$$M = \frac{V}{a}$$

V - velocity
a - speed of air

with speed of sound: $a = \sqrt{\gamma R T}$

$\gamma = 1.4$ … adiabatic constant

$R = 287.052874\ J \cdot kg^{-1} \cdot K^{-1}$ … molar gas constant

T … absolute temperature K

- Mach number is a non-dimensional (i.e., unitless) parameter
- Mach number indicates the flow compressibility
  - Subsonic flow: $M < 1$ — 0.3 can ignore compressibility
  - Supersonic flow: $M > 1$
  - Sonic flow: $M = 1$
  - Transonic flow: $0.8 < M < 1.2$ - care about compressibility

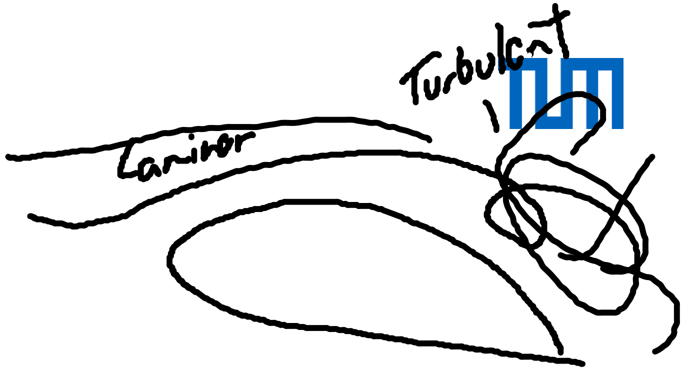
for drag lin. is fine for lift calcul.

Air at standard sea level:

$T = 15°C = 288.15\ K$

$$a = \sqrt{\gamma R T} = \sqrt{1.4 \cdot 287 \cdot 288.15} = 340.4\ \frac{m}{s}$$

http://airfoiltools.com/airfoil/details?airfoil=naca2408-il#polars



Aircraft Loads - Fundamentals of Aerodynamics

Reynold's Number:

- The Reynolds number (R_e) helps to predict the flow patterns in different fluid flow situations
  - Laminar (small R_e), low drag
  - Turbulent (large R_e), high drag

$$R_e = \frac{\rho \cdot V \cdot L}{\mu}$$

- ρ is the density of the fluid $(\frac{Kg}{m^3})$
- V is the speed $(\frac{m}{s})$
- L is a characteristic length, usually the Airfoil chord (m)
- μ is the dynamic viscosity of the fluid $(\frac{Kg}{m \cdot s} = Pa \cdot s)$ (depend on temperature, constant with pressure)
  - Air under standard atmospheric conditions (25 °C and pressure of 1 bar): $18.5\ \mu Pa \cdot s$
- Reynold's number represents the importance of viscosity i.e., friction (drag)

- Standard Day Conditions (ISA): Primarily used in aerospace and atmospheric sciences, with a reference temperature of 15°C and pressure of 1 atm.
- Standard Ambient Conditions (SATP): More commonly found in laboratory and chemical contexts, defined as 25°C and 1 atm.

http://airfoiltools.com/airfoil/details?airfoil=naca2408-il#polars

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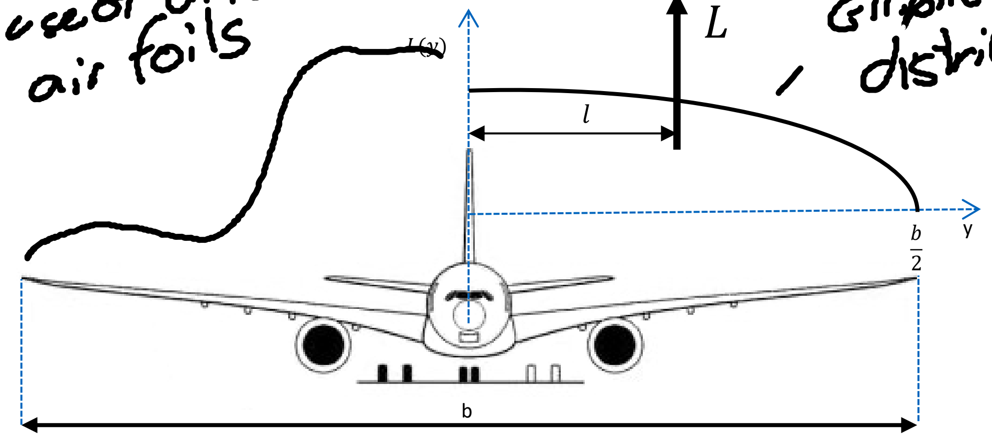
Aircraft Loads - Fundamentals of Aerodynamics

The lift distribution can be influenced by adjusting the airfoils, planform and twist/incidence angle of the wing.

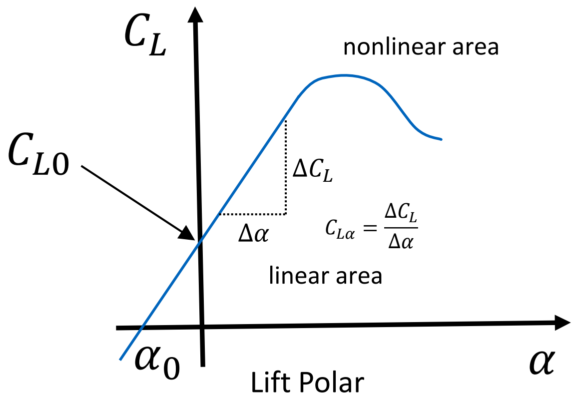
nowadays: use of diff. air foils

≈42%

assume Elliptic distribution



assumption: elliptic lift distribution



lift distribution: $L(y) = L_0 \cdot \sqrt{1 - \left(\frac{y}{b/2}\right)^2}$

lever arm of effective lift force: $l = \frac{4}{3\pi}\frac{b}{2} \approx 42\% \frac{b}{2}$

$$L = \frac{\rho}{2} V_\infty^2 \cdot S \cdot C_L$$

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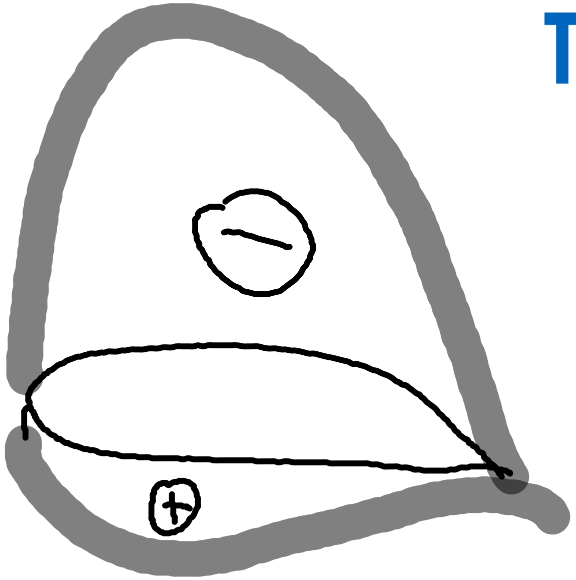
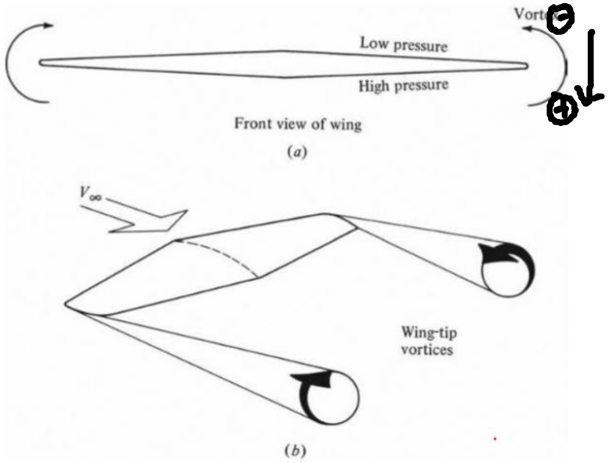


Aircraft Loads - Fundamentals of Aerodynamics

Difference between an infinite wing (wing section) and a finite wing

- The lift and drag polars are determined for infinite wings
- According to the tip vortices of finite wings:
  - Lift slope curve is reduced (down wash effect)
  - Induced drag is generated



Introduction to Flight, John D. Anderson Jr.

Aircraft Loads - Fundamentals of Aerodynamics

Difference between an infinite wing (wing section) and a finite wing

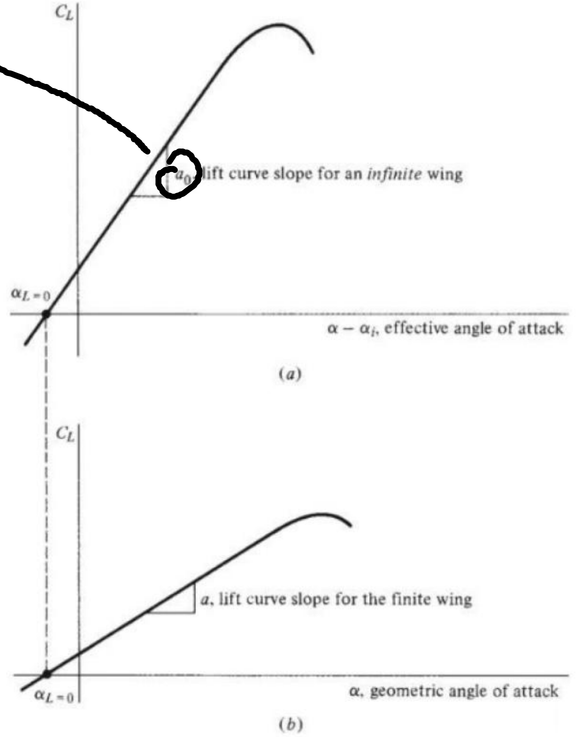
- Correction of finite wing lift polar slope:

$$\boxed{a = C_{L\alpha} = \frac{\partial C_L}{\partial \alpha} = \frac{a_0}{1 + 57{,}3\,\dfrac{a_0}{\pi \cdot e \cdot AR}}}$$

$C_{L\alpha}$

Infinite wing
(aerodynamic profile)

Finite wing



  - a: *slope of lift polar of finite wing*
  - a_0: *slope of lift polar of infinite wing*
  - $e \in [0.9 - 1.0]$: wing efficiency factor
  - $AR = \frac{b^2}{S}$: Wing aspect ratio
  - b: wing span length
  - S: wing area

Oswald
→ 1
→ 0,5

- Induced drag coefficient: $C_{D_i} = \frac{C_L^2}{\pi^2 \cdot AS \cdot e}$

elliptic

read

Introduction to Flight, John D. Anderson Jr.

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Aircraft Loads - Fundamentals of Aerodynamics

Estimation of aerodynamic lift

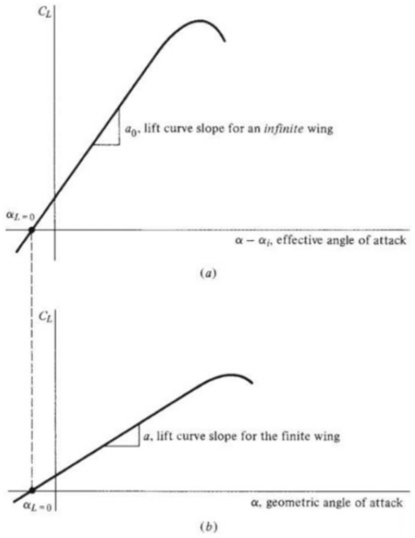
know what the numbers mean

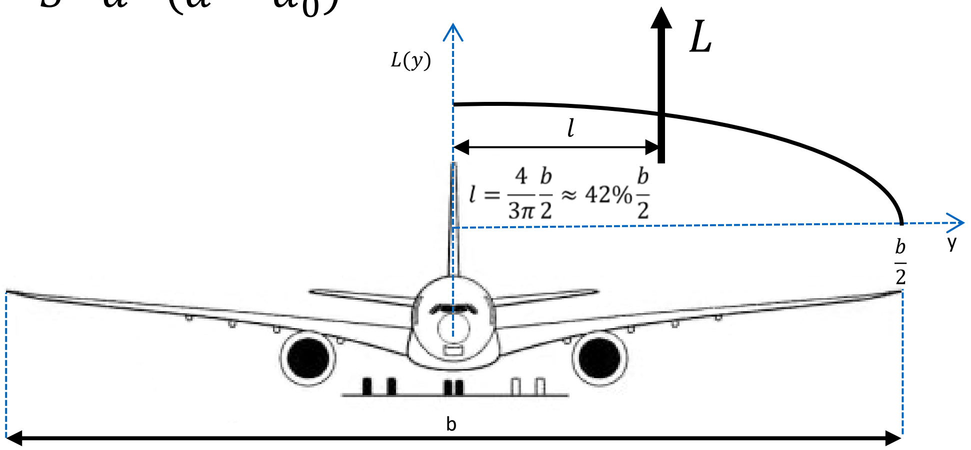
- Choose airfoil e.g., NACA0024
- Determine airfoil polars

- Measure polar lift curve slope $a_0 = C_{L\alpha,0} = \left(\frac{\partial C_L}{\partial \alpha}\right)_0$
- Correct polars to the finite wing:
  - Compute corrected lift curve slope $a = C_{L\alpha} = \frac{\partial C_L}{\partial \alpha}$
- Compute Lift: $L = \frac{\rho}{2} V_\infty^2 \cdot S \cdot C_L = \frac{\rho}{2} V_\infty^2 \cdot S \cdot a \cdot (\alpha - \alpha_0)$

Infinite wing (aerodynamic profile)

Finite wing





assumption: elliptic lift distribution

Inertia Loads

Aircraft Loads - Inertia

According to Newton's second law the inertia loads occur when the aircraft masses are accelerated

$$\vec{F} = M \cdot \vec{a}$$

Inertia loads

Aircraft mass (matrix)

accelerations

Moments
(in principal coordinate system)

Angular acceleration

$$\overrightarrow{M_o} = \frac{\overrightarrow{dL}}{dt} = \overrightarrow{I_0} \cdot \vec{\dot{\omega}}$$

Aircraft inertia tensor
(in principal coordinate system)

$\vec{L}$ Angular Momentum

Aircraft Loads - Inertia

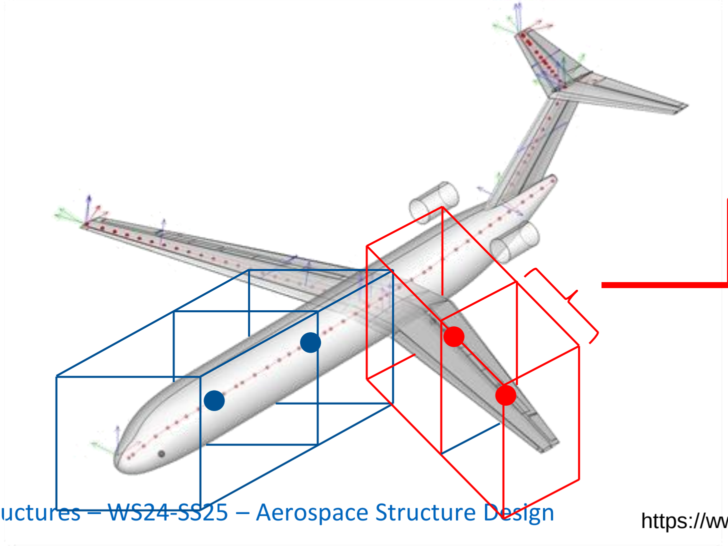
Mass distribution and mass model

- The aircraft masses are organised in following different categories
  - Non-structural mass
    - Engine, Fuel, Equipment (avionics, mission systems, hydraulic system, fuel system, etc.), painting, fairings, etc.
  - Structural mass
    - Primary structure (skin, stringers, spars, ribs, frames, floors, joints, etc.)
    - Secondary structure (pad-ups. Rivets, bolts and nuts, etc.)
- The mass model is usually integrated over boxes into lumped masses yielding the numerical mass model (for computing the loads)

Moment

Aircraft Inertia tensor

$$M, \overrightarrow{I_0}$$

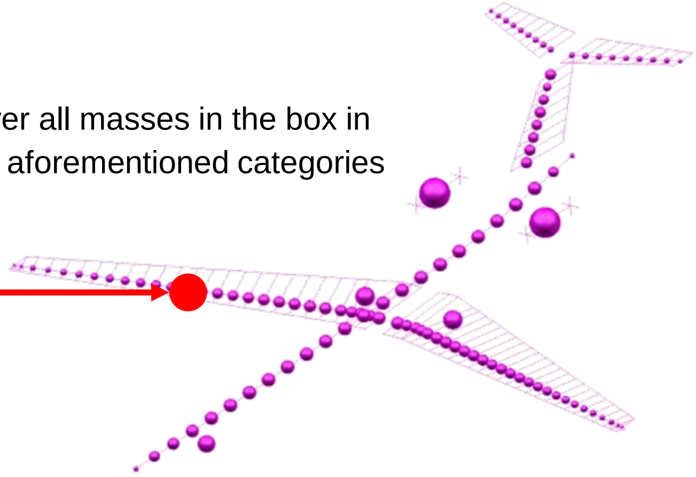


$$\sum M_i$$

$$\sum \vec{I_i} + M_i \cdot \vec{r_i}$$

Numerical Mass Model

Sum over all masses in the box in each of aforementioned categories

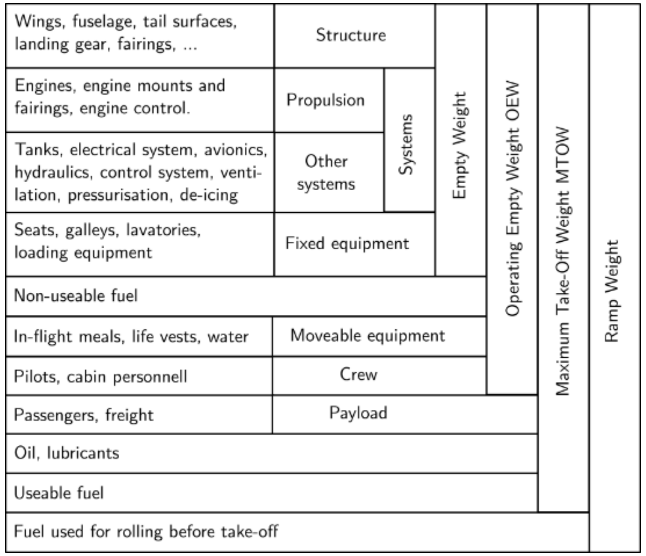


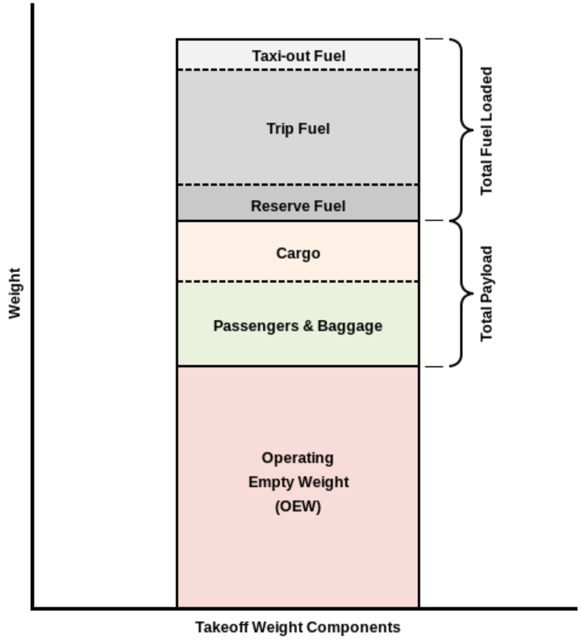
https://www.dlr.de/ae/en/desktopdefault.aspx/tabid-9639/16569_read-40470/gallery-1/216_read-1/

Aircraft Loads - Inertia

Mass conditions (configurations)

- The different aircraft mass conditions are arranged in so-called mass configurations
- The mass configurations play an important role in the computation of aircraft loads
- Most relevant mass configurations are:
  - Maximum Take-off Weight
  - Basic Flight Design Mass,
  - Maximum Wing Zero Fuel Mass „no fuel max payload"
  - Maximum Design Mass,
  - Minimum Flying Mass,
  - Landing Design Mass





https://commons.wikimedia.org/wiki/File:Takeoff_weight_diagram.svg

Aircraft Loads - Inertia

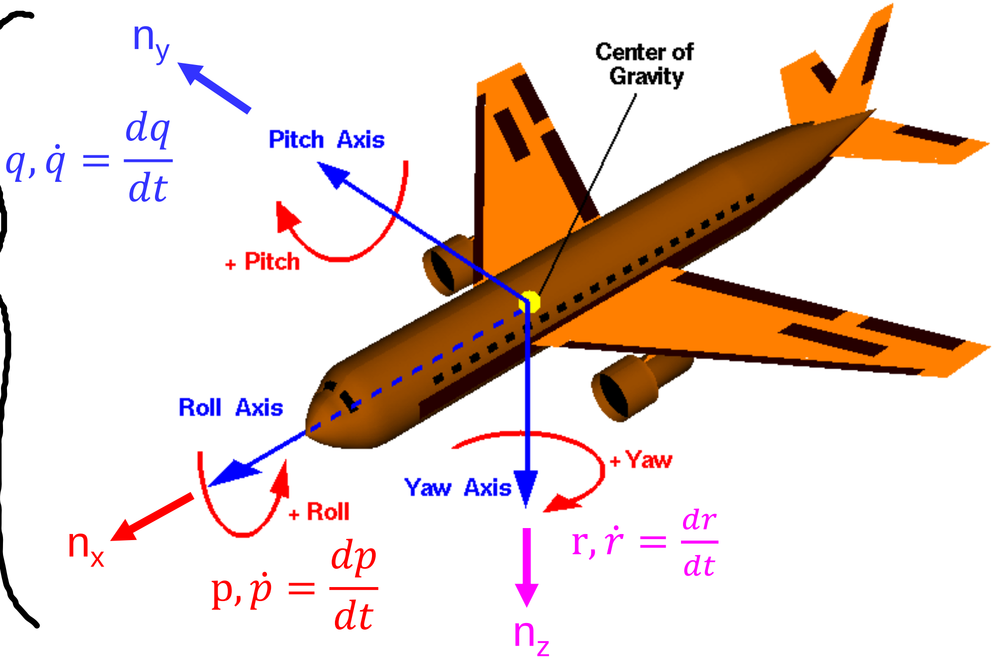
Accelerations and rates

- In flight the accelerations are defined in the so-called **aircraft body coordinate system**
- The rotational speed around an axis in flight is called “Rate”

Rate

- According to flight mechanic fundamentals the accelerations and rates are
  - n_x acceleration along the x axis
  - n_y acceleration along the y axis
  - n_z acceleration along the z axis
  - p rate around the x axis
  - q rate around the x axis
  - r rate around the x axis
  - $\dot{p} = \frac{dp}{dt}$ acceleration around the x axis
  - $\dot{q} = \frac{dq}{dt}$ acceleration around the y axis
  - $\dot{r} = \frac{dr}{dt}$ acceleration around the z axis

aircraft body coordinate system



https://www.grc.nasa.gov/www/k-12/airplane/Images/rotations.gif

Trimming

Aerodynamic + inertia + thrust = 0

. adjust flight surfaces for steady flight

$$\left.\begin{aligned} M_x &: \text{ailerons} \\ M_y &: \text{elevators} \\ M_z &: \text{Rudder} \end{aligned}\right\} \text{deflection}$$

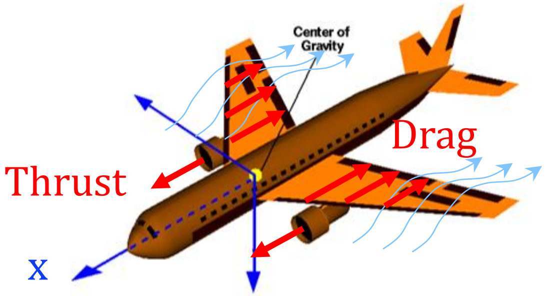
Aircraft Loads - Trimming

- Trimming the aircraft is the procedure of bringing all applied external loads into **equilibrium** to fly a **steady** manoeuvre
- Equilibrium is usually achieved in all six degrees of freedom (3 translational = Forces, 3 rotational = Moments):
  - Force equilibrium:

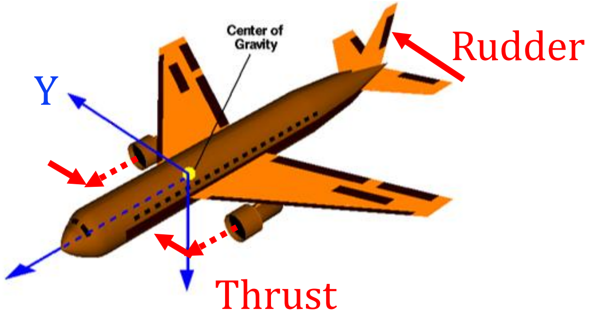
$\sum F_x = 0$: Thrust, drag and inertia

Sometimes the longitudinal x axis is omitted (when the aerodynamic data doesn't cover the drag properly)

thrust = drag

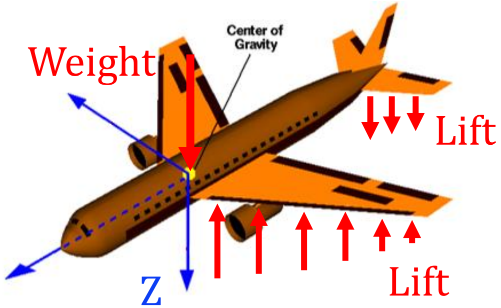


$\sum F_y = 0$: Rudder, lift, thrust and inertia



$\sum F_z = 0$: Lift and weight

lift = m·g



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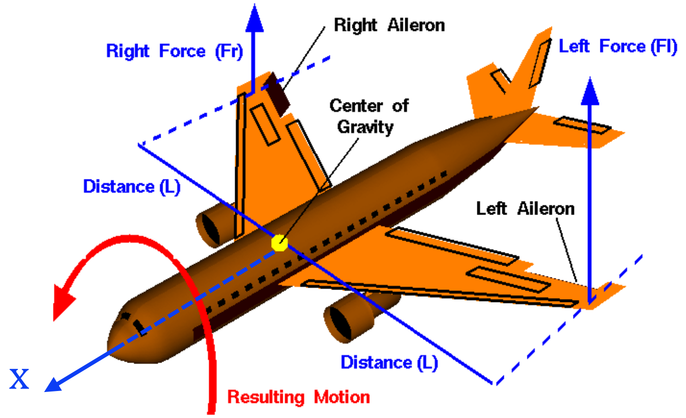
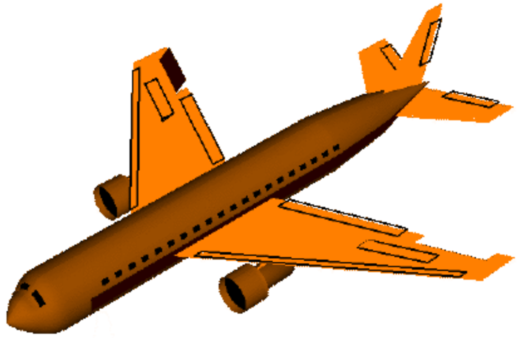
Aircraft Loads - Trimming

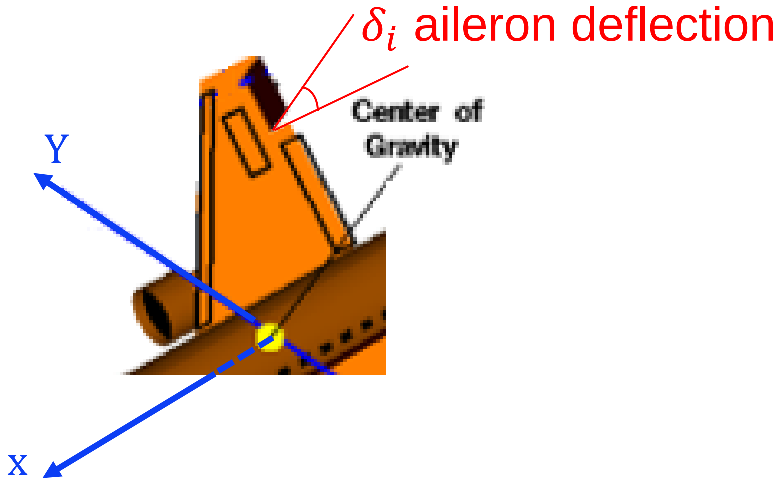


- Roll control:
  - Moment equilibrium around x axis:

    $\sum M_x = 0: Roll\ equilibrium\ (aileron, inertia)$

  - Unknown (trimming variable) is the antimetric deflection of the ailerons δ_i



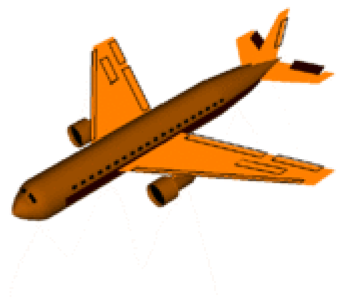


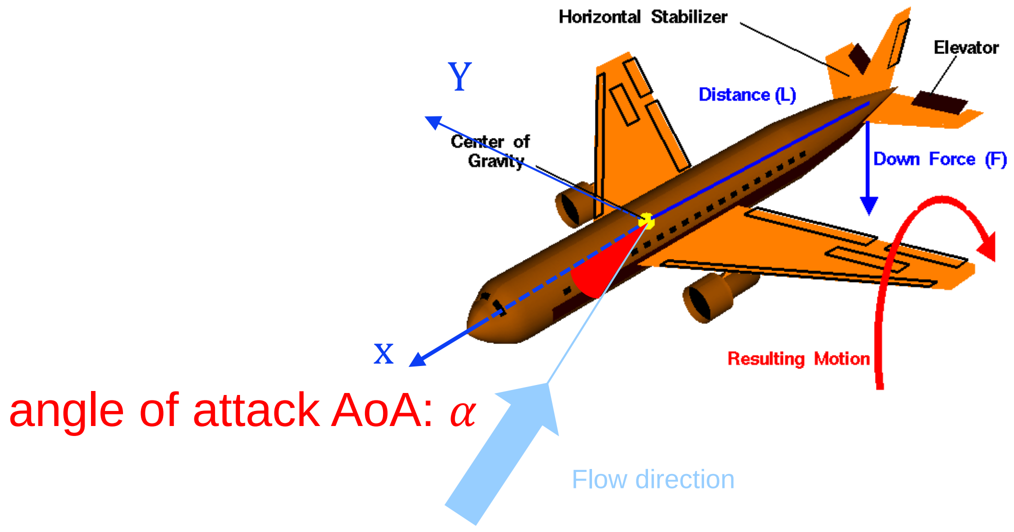
Aircraft Loads - Trimming

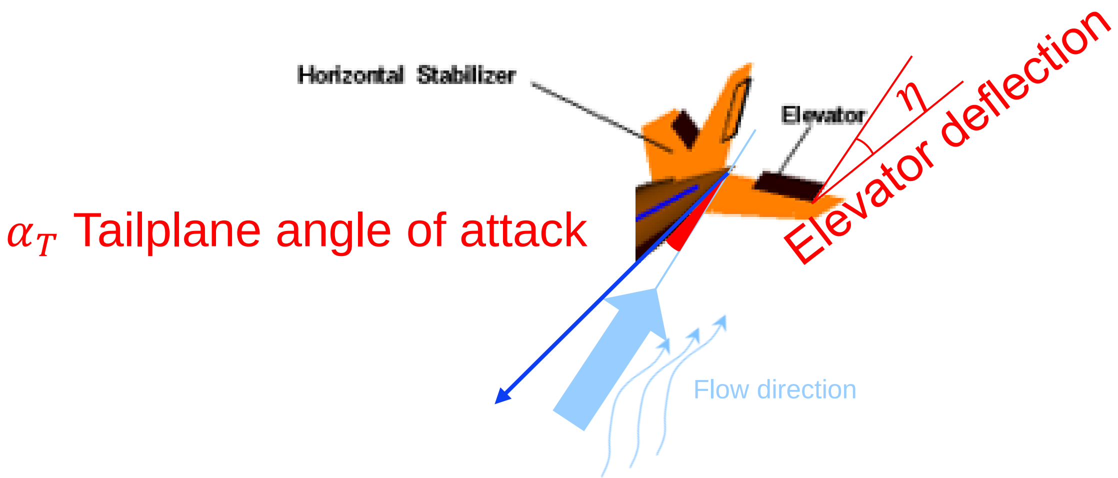
- Pitch control:
  - Moment equilibrium around y axis:

    $\sum M_y = 0 : Pitch\ equilibrium$

  - Unknowns (trimming variables) are
    - Angle of attack (AoA), α
    - Tailplane angle of attack α_T
    - Deflection of elevator η







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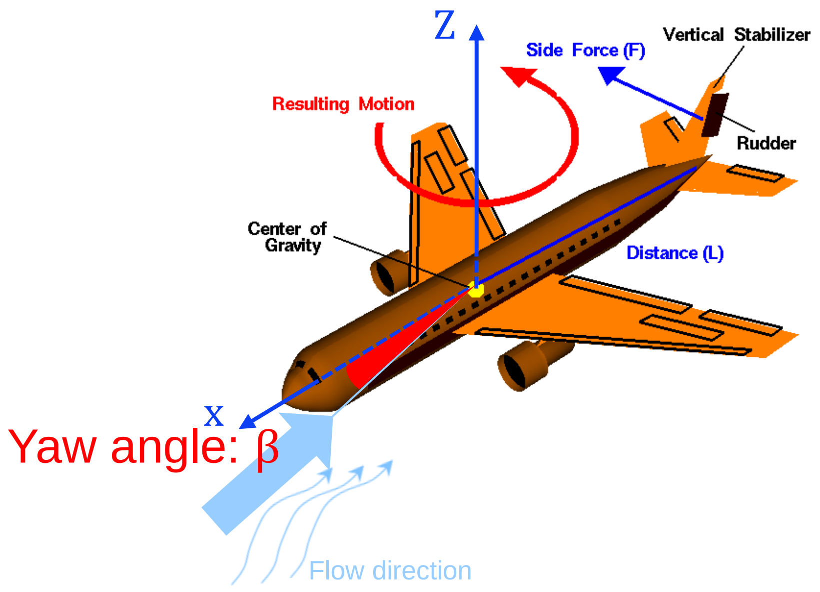
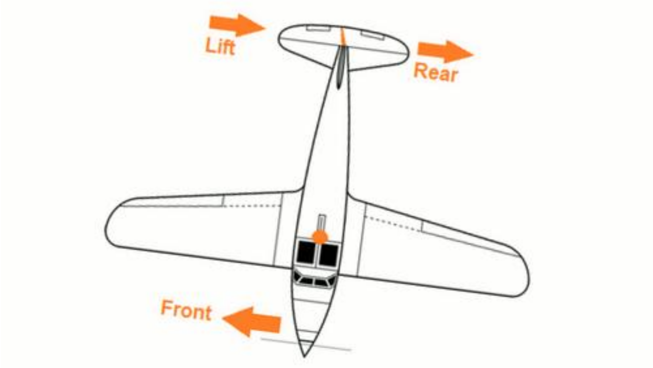
Aircraft Loads - Trimming

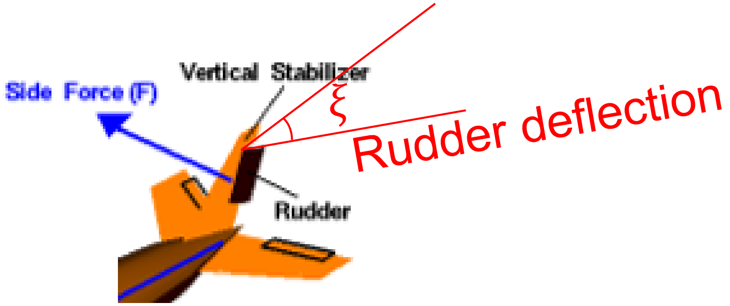
- Yaw control:
  - Moment equilibrium around z axis:

    $\sum M_z = 0 : Yaw\ equilibrium$

  - Unknowns (trimming variables) are
    - Yaw angle, β
    - Rudder deflection ξ







Aircraft Loads - Trimming

- Summary:
  - In order to trim the aircraft (for a steady manoeuvre load case) establish equilibrium in all degrees of freedom:

$$\sum F_x = 0 : \text{mainly, thrust, drag and inertia}$$

$$\sum F_y = 0 : \text{mainly, rudder force, lift, thrust and inertia}$$

$$\sum F_z = 0 : \text{mainly, lift and weight}$$

$$\sum M_x = 0 : Roll\ equilibrium,\ \ mainly\ aileron\ and\ inertia$$

$$\sum M_y = 0 : Pitch\ equilibrium,\ \ mainly\ elevator\ and\ inertia$$

$$\sum M_z = 0 : Yaw\ equilibrium, mainly\ rudderforce\ and\ inertia$$

  - Unknowns (trimming variable) are
    - Angle of attack (AoA), α
    - Yaw angle, β
    - Tailplane angle of attack α_T
    - Rudder deflection ξ
    - Deflection of elevator η
    - Deflection of any other control surface, δ
  - What do you think: Is the number of unknowns and equations identical in each load (trimming) case?

Load Cases

Aircraft Loads – Definition of Load Cases

- Reminder: Load cases have the following categories:
  - Manoeuvre loads
  - Dynamic loads
  - Ground handling loads - gear loads are significant
  - Special loads - to get the certifications
- Load cases are defined in order to guaranty the safety and integrity of the airframe
- The load cases depend on the aircraft type and planned usage described in the **Structure Design Criteria (SDC)**:
  - The **SDC** define, among others, the maneuvers, speeds, useful load, and aircraft design weights which are to be considered for structural design and sizing
- The SDC include the V-n diagram (Flight Envelope)
  - V: aircraft speed
  - n: load factor (follows next)
- The V-n diagram is very important for both the design and the operation of an aircraft – explained next

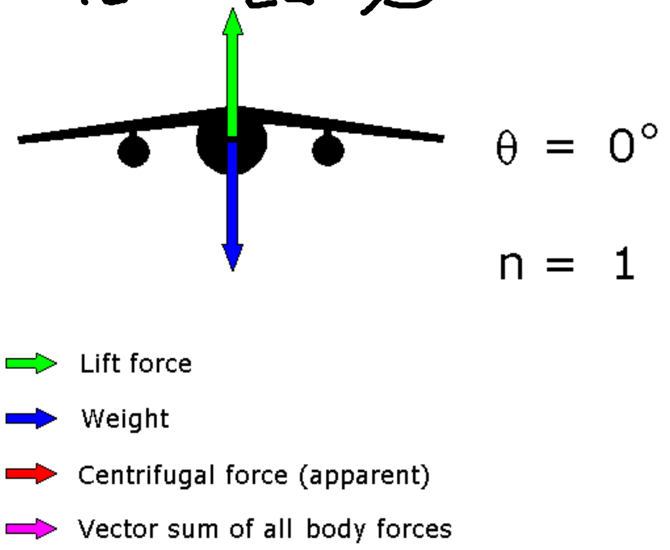
Aircraft Loads – V-n diagram – Load Factor n

- Definition: In aeronautics, the load factor is the ratio of the lift of an aircraft to its weight and represents a global measure of the ("load") to which the structure of the aircraft is subjected:

$$n_z = \frac{lift}{inertia\ force} = \frac{L}{m \cdot g}$$

- This principle is extended to the axes: $n_x = \frac{F_x}{m \cdot g}$, $n_y = \frac{F_y}{m \cdot g}$
- Terminology of Loads:
  - Limit Load: Maximum expected load during operation, no permanent structural deformation or damage allowed
  - Ultimate Load: Maximum structural load above which structural failure can occur UL = LL · design safety factor
  - Generally, **design safety factor = 1.5**
  - Between limit load and ultimate load local damages and permanent deformations are allowed

UL: Ultimate load
LL: Limit load
$UL \overset{often}{=} LL \cdot 1{,}5$



https://en.wikipedia.org/wiki/File:Load_factor_in_a_turn.gif

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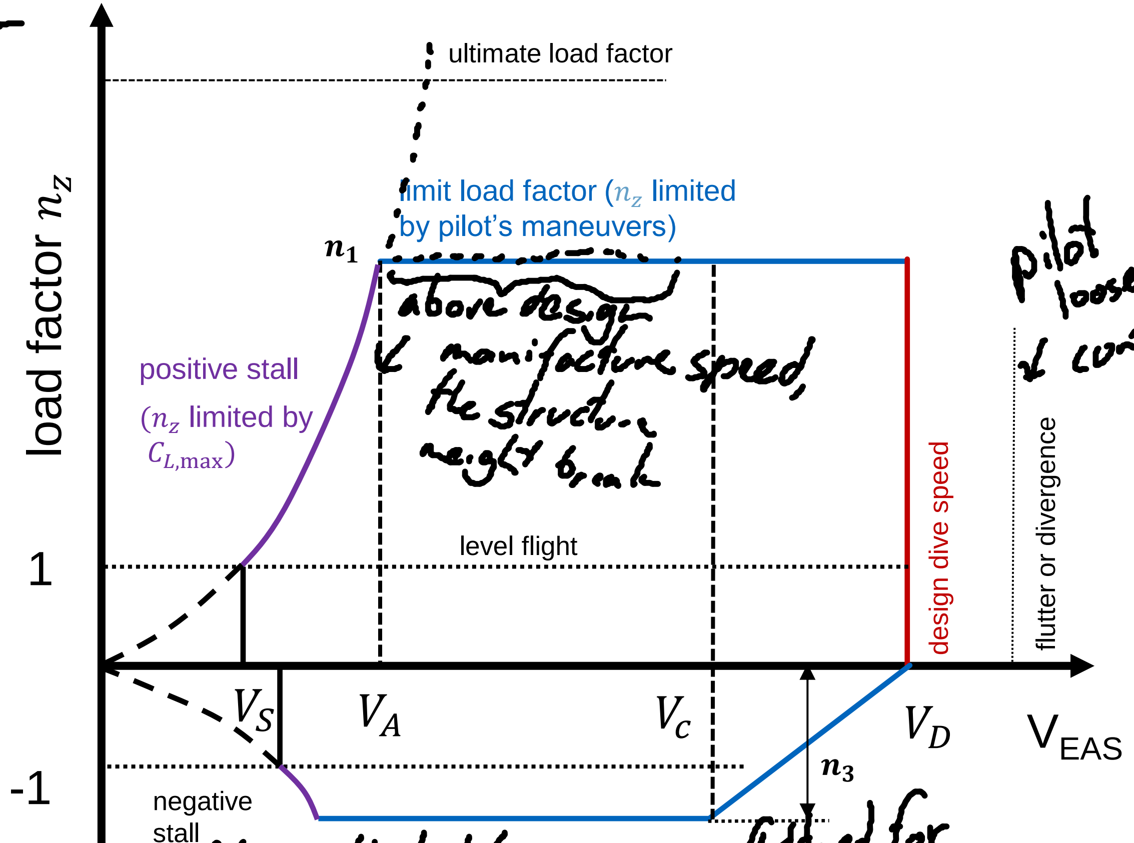
Aircraft Loads – V-n diagram

- In aerospace engineering V-n diagram, also called flight envelope, of an aircraft or spacecraft refers to the capabilities of a design in terms of airspeed and load factor.

only valid for symmetric maneuvers !!



above design manifacture speed, the structure might break

pilot looses control

limited by stall

limited by loads/design

flattened for pilots limitations

V_S: stall speed

V_A: design maneuver speed

American Airlines Flight 587

V_c: design cruise speed

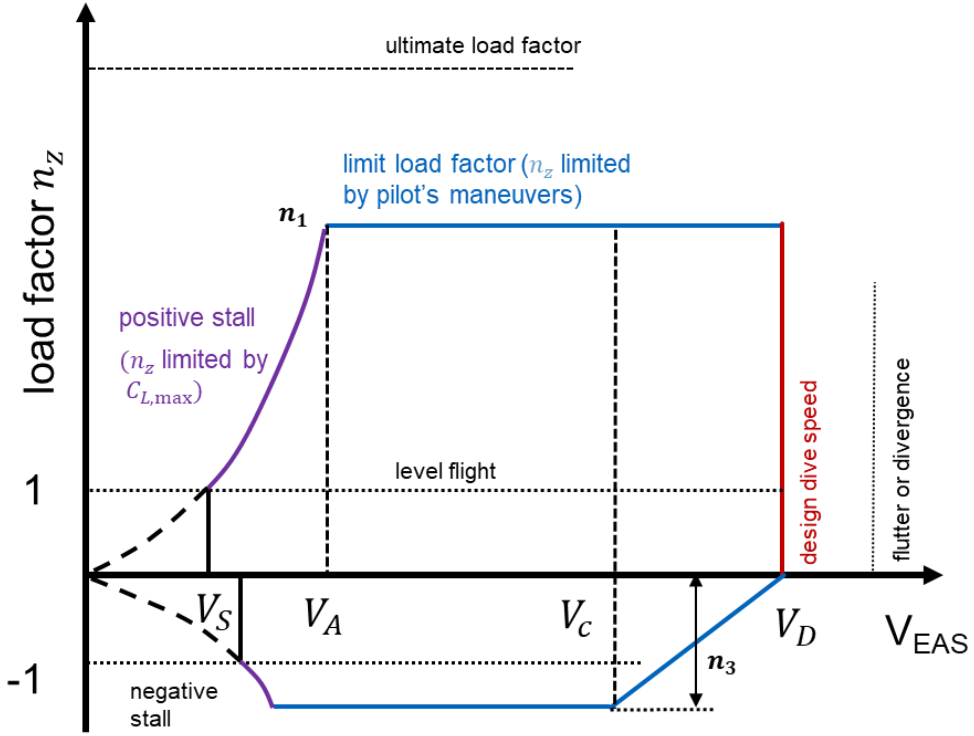
V_D: design dive speed

According to CS25.333:

$$V_c < 0.8\, V_D$$
$$V_D > V_c + 0.05\, M_a$$

Aircraft Loads – V-n diagram – Load Factor n

- Because the air density varies with altitude and compressibility the V-n diagram is defined for each critical combinations of altitude and weight



	normal	aerobatics
n_1	2.1 + 24000 / (W +10000)	6.0
n_3	-1.0	-3.0

- n_1 is dependent on weight (W in lb), the lighter the aircraft is the higher can be the *Limit Load Factor*.
- $n_1 \geq 2.5$
- $n_1 \leq 3.8$

- For each flap setting, a specific maneuver V-n diagram has to be created, to account for different speed and load factor limitations

- The structure has to be designed for all load cases within the boundary of the V-n diagrams

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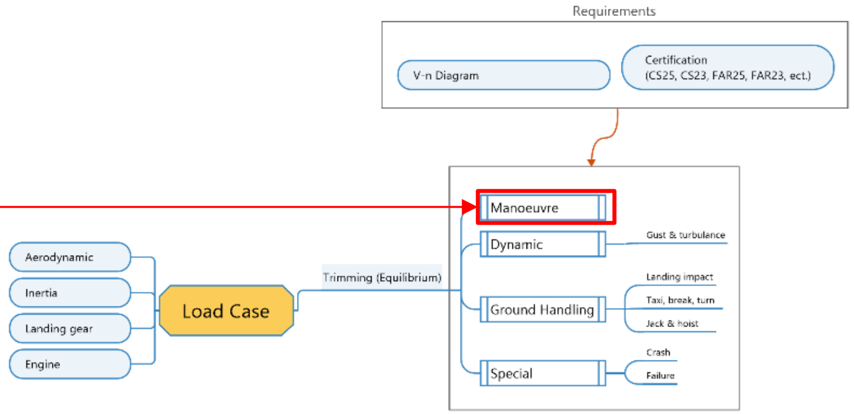
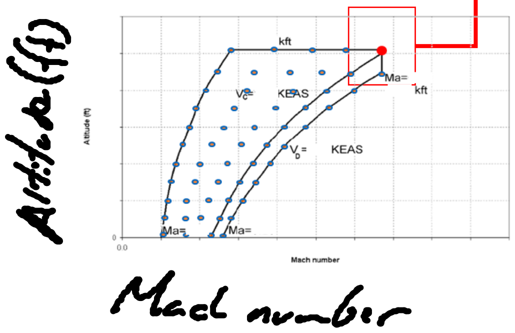
Aircraft Loads – Definition of Load Cases

Manoeuvre Load Cases (Steady)

- Theoretically, for each point in the V-n diagram all possible combinations of
  - Weight (mass configuration), speed and altitude (dynamic pressure) and load factor has to be examined
  - Additionally, to the symmetrical manoeuvres the combination with roll and yaw has to be examined as well
  - A huge amount of load cases is set up and analysed





Case		Mach[]	nx[g]	ny[g]	nz[g]	dp[rad/s^2]	dq[rad/s^2]	dr[rad/s^2]
10. 1G Roll	mtom	0.26	-0.5	0.3	-1.0	2.5	0.0	0.2
10. 1G Roll	mtom	0.26	1.0	-0.3	-1.0	-2.5	0.0	-0.2
11. 1G Roll	4223kg	0.26	-0.5	0.1	-1.0	0.3	0.0	-0.6
11. 1G Roll	4223kg	0.26	1.0	-0.1	-1.0	-0.3	0.0	0.6
11. 1G Roll	4223kg	0.26	1.0	-0.3	-1.0	0.3	0.0	-0.6
11. 1G Roll	mtom	0.26	-0.5	-0.3	-1.0	0.2	0.0	-0.4
11. 1G Roll	mtom	0.26	-0.5	0.0	-1.0	-0.3	0.0	0.2
11. 1G Roll	mtom	0.26	-0.5	-0.3	-1.0	0.3	0.0	-0.6
11. 1G Roll	mtom	0.26	-0.5	-0.2	-1.0	0.3	0.0	-0.6
11. 1G Roll	mtom	0.26	1.0	0.1	-1.0	-0.3	0.0	0.4
11. 1G Roll	mtom	0.26	1.0	0.3	-1.0	-0.3	0.0	0.6
12. 1G Roll	mtom	0.26	-0.5	-0.1	-1.0	0.3	0.0	-0.4
12. 1G Roll	mtom	0.26	1.0	0.2	-1.0	-0.3	0.0	0.6
13. lateral Gust	mtom	0.23	0.0	-0.2	1.0	0.0	0.0	0.0
13. vertical Gust	mtom	0.23	0.0	0.0	-0.9	0.0	0.0	0.0
13. vertical Gust	mtom	0.23	0.0	0.0	2.9	0.0	0.0	0.0
2. Pull Out Steady	mtom	0.26	-0.5	-0.4	2.8	0.8	0.0	-0.6
2. Pull Out Steady	mtom	0.26	1.0	0.3	2.8	0.8	0.0	0.5
2. Pull Out Steady	mtom	0.26	1.0	0.4	2.8	-0.8	0.0	0.6
3. Pull Out Response	mtom	0.26	-0.5	-0.1	2.8	0.0	-1.5	0.0
3. Pull Out Response	mtom	0.26	1.0	-0.1	2.8	0.0	0.6	0.0
4. Push Over Steady	mtom	0.26	-0.5	0.0	-1.0	0.3	0.0	-0.2
5. Push Over Steady	mtom	0.26	-0.5	-0.3	-1.0	0.4	0.0	-0.6
5. Push Over Steady	mtom	0.26	1.0	0.3	-1.0	-0.4	0.0	0.6
6. Push Over Response	mtom	0.26	1.0	-0.1	-1.0	0.0	1.5	0.0
7. 2G Rolling Pull Out	mtom	0.26	-0.5	-0.4	2.0	-2.5	0.0	-0.6
7. 2G Rolling Pull Out	mtom	0.26	1.0	-0.4	2.0	-2.5	0.0	-0.4
8. 2G Rolling Pull Out	4223kg	0.26	1.0	-0.1	2.0	-0.3	0.0	0.6
8. 2G Rolling Pull Out	mtom	0.26	1.0	-0.4	2.0	0.3	0.0	-0.6
8. 2G Rolling Pull Out	mtom	0.26	1.0	-0.4	2.0	0.3	0.0	-0.6
9. 2G Rolling Pull Out	4223kg	0.26	-0.5	0.1	2.0	0.3	0.0	-0.6
9. 2G Rolling Pull Out	[illegible]	0.26	-0.5	0.1	2.0	0.3	0.0	[illegible]

Case		Mach[]	nx[g]	ny[g]	nz[g]	dp[rad/s^2]	dq[rad/s^2]	dr[rad/s^2]
10. 1G Roll	mtom	0.26	-0.5	0.3	-1.0	2.5	0.0	0.2
10. 1G Roll	mtom	0.26	1.0	-0.3	-1.0	-2.5	0.0	-0.2
11. 1G Roll	4223kg	0.26	-0.5	0.1	-1.0	0.3	0.0	-0.6

Each load case is defined through 6 accelerations (sometimes also 3 rates)
These need to get trimmed!

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Aircraft Loads – Gust

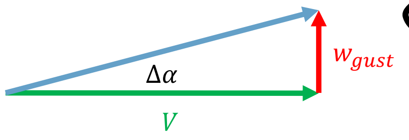
$\Delta C_L = C_{L\alpha} \Delta\alpha$

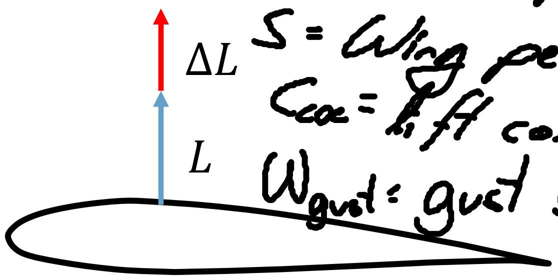
$L = \frac{1}{2} \rho V^2 S C_L$

ρ : Air density
V : true air speed
S = Wing perform area
$C_{L\alpha}$ = lift coefficient at α
W_{gust} : gust speed

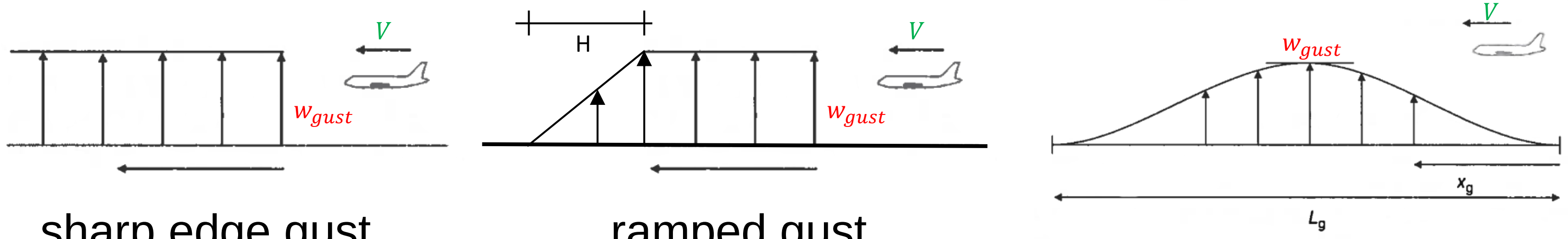
Change of the load factor due to vertical gust:

$$\Delta n_z = \frac{\Delta L}{m \cdot g}$$





Gust modelling:



sharp edge gust

ramped gust

1-cos gust

For a sharp edge gust:

$$\Delta L = \frac{\rho}{2} V^2 \cdot S \cdot C_{L\alpha} \cdot \Delta\alpha \Rightarrow \Delta\alpha = \frac{w_{gust}}{V} \; (small\ angles) \Rightarrow \Delta L = \frac{\rho}{2} \cdot S \cdot C_{L\alpha} \cdot V \cdot w_{gust}$$

$$n_z = n_{z,trim} + \Delta n_z \Rightarrow n_z = 1 + \Delta n_z = 1 + \frac{\frac{\rho}{2} \cdot w_{gust} \cdot S \cdot C_{L\alpha}}{W} V$$

Assuming level flight

Aircraft Loads – Gust V-n Diagram

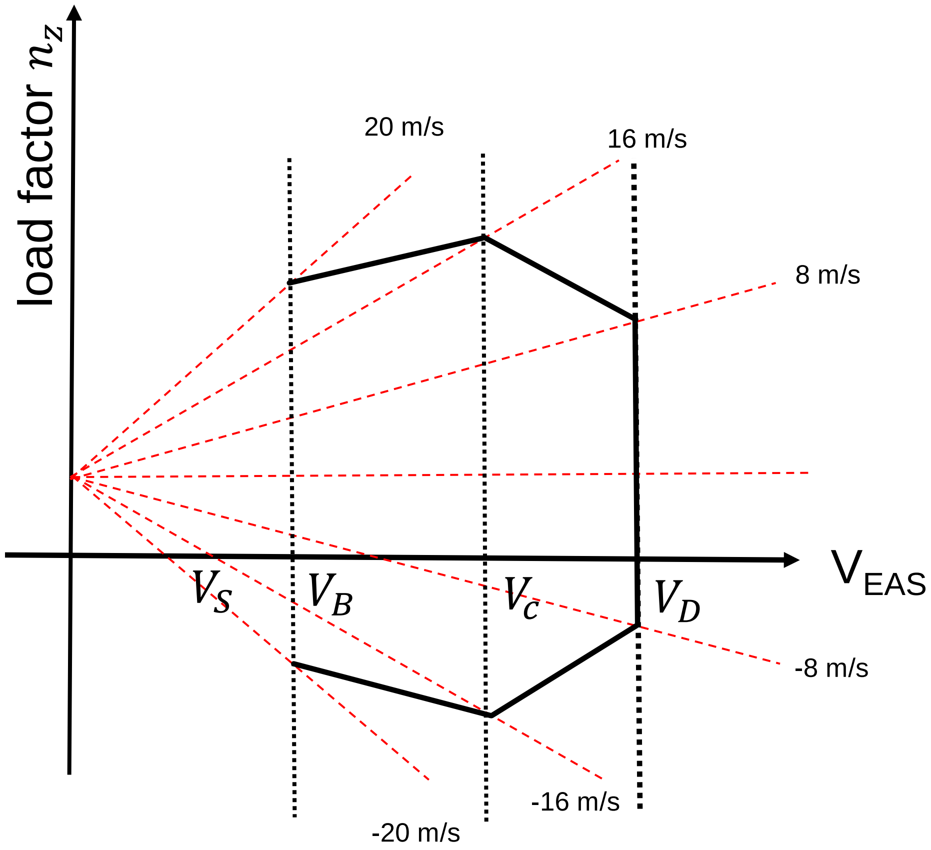
- Gust load factor is directly proportional to airspeed for a given gust speed

$$n_z(V) = 1 + \frac{\frac{\rho}{2} \cdot w_{Gust} \cdot S \cdot C_{L\alpha}}{W} V$$

- Leading to a speed limit for operation in rough air
- For each design weight W, a specific gust v-n diagram must be created
- With $w_L = \frac{W}{S}$ the wing loading:

$$n_z(V) = 1 + \frac{\frac{\rho}{2} \cdot w_{Gust} \cdot C_{L\alpha}}{w_L} V$$

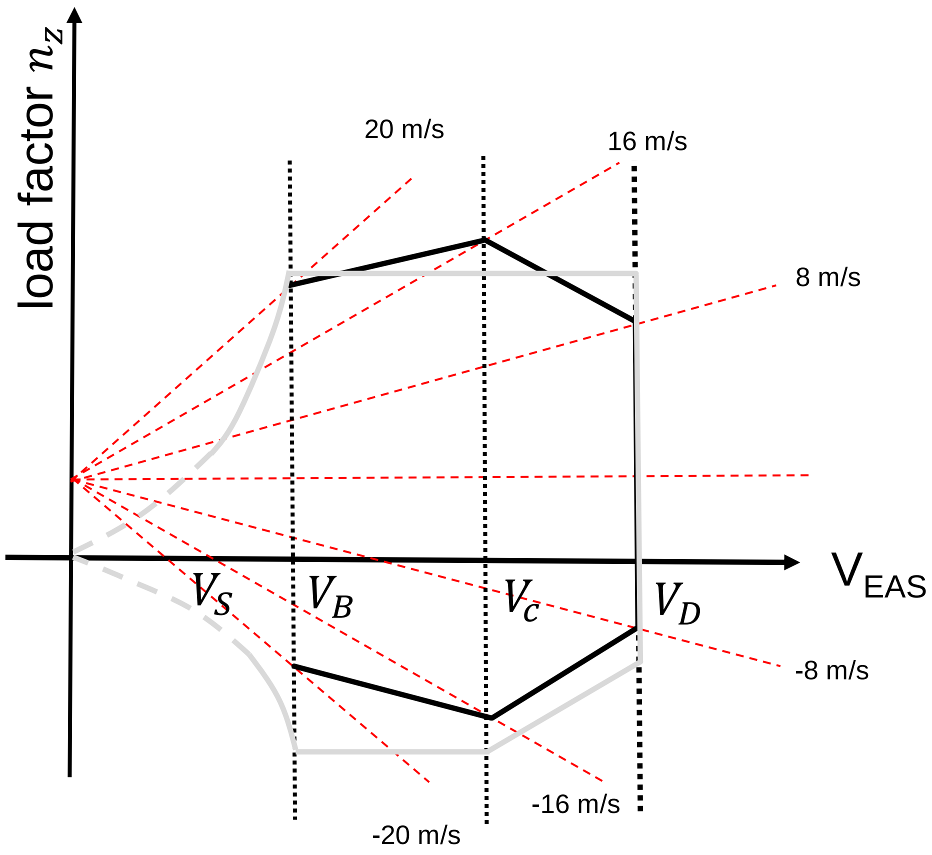
- The lower the wing loading (high aspect ratio wings) the higher the gust effect!



V_B: design speed for maximum gust intensity

Aircraft Loads – Gust V-n Diagram

Superimposing the gust and manoeuvre V-n diagrams reveals areas where gust is design-driving



Qualitative diagram only!

Landing Gear Loads

Landing Gear Loads

- In certification requirements the landing gear load cases are subdivided into the two categories:
  - Landing impact
  - Other ground handling: taxi, take-off and landing roll, braked roll, turning, jacking, towing
- In course of this lecture “Landing” load cases will be discussed in more detail
- European certification requirements can be found on the EASA (European Union Aviation Safety Agency) website:
  - CS 25 Large Aeroplanes: https://www.easa.europa.eu/en/document-library/certification-specifications/cs-25-amendment-27
  - CS-23 Normal, Utility, Aerobatic and Commuter Aeroplanes: https://www.easa.europa.eu/en/document-library/certification-specifications/cs-23-amendment-5-and-amc-gm-cs-23-issue-3

Landing Loads

- Landing conditions are described in certification requirements e.g., CS 25.471 to 25.487
  - Level landing conditions:
    - Three-point landing
    - Two-point landing
    - One-point landing
  - Taildown condition
- The design limit descent velocity (sink rate)
  - is 3.05m/s (10 fps) at landing weight or
  - 1.83 m/s (6 fps) at maximum take of weight
- Airplane lift, not exceeding airplane weight, may be assumed
- The method of analysis of landing gear loads must take into account
  - landing gear dynamic characteristics,
  - spin up and spring back,
  - the rigid-body response, and
  - the structural dynamic response of the airframe if significant
- The coefficient of friction between the tires and the ground may be considered 0.8

sd: standard day at sea level

Landing Speeds

- Landing speeds per CS 25.479(a) are specified to bound the possible operation of the aircraft as follows:
  - Landing speed at a standard day at sea level:

aircraft tail speed

$$V_{landing,sd} = VL_1 = V_{so} + V_{tw}$$

stall speed

Where,

V_{so} is the airplane stall speed for a standard day at sea level with the flaps in landing configuration

  - Landing speed at a hot day with 22.8°C above standard day temperature and the highest altitude the airplane will be certified to land at:

$$V_{landing,hd} = 1.25\, VL_2 + V_{tw}$$

Where,

V_{L2} is the airplane stall speed for a hot day (above mentioned conditions)

V_{tw} is the aircraft tail winds

- = 0 for aircraft certified for tail winds of 19km/h (10 kn) or less
- V_{tw} = when the airplane is to be certified for tail winds greater than 19km/h (10 kn)

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Landing Speeds

- Effect of hot day and altitude on landing speed:

$$V_{L2} = V_{so}\left(\frac{\rho_0}{\rho}\right)^{\frac{1}{2}}\left(\frac{T_{HD}}{T_0}\right)^{\frac{1}{2}}$$

With,

– V_{so} is stall speed at sea level and standard day

– ρ_0 is density at standard day conditions

– ρ is density at maximum altitude

– T_0 is temperature at standard conditions

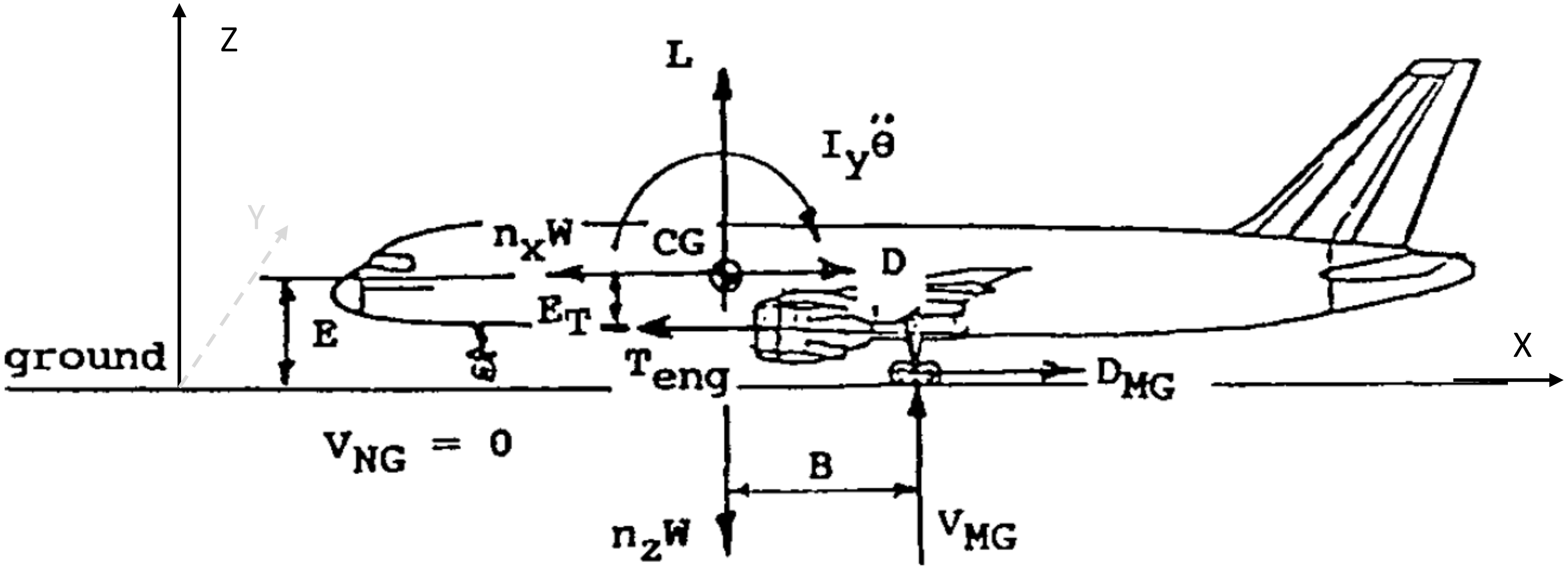
– T_{HD} is temperature at hot day conditions ($T_0 + 22.8°C$)

Standard day (air) parameters

- Temperature (T): 15 °C (59 °F)
- Density (ρ): 1.225 kg/m^3 (0.00237 slug/ft^3)
- Pressure (p): 101.325 kPa (14.7 lb/ in^2), 1 atm
- Viscosity (μ): 17.3 μN·s/m^2 (3.62 × 10−7 lb s/ft^2)

Two-Point Landing Conditions

- An attitude in which the main wheels are assumed to contact the ground with the nose wheel just clear of the ground
- Two-point landing provides the maximum loads at main landing gear



Ref: T. Lomax: Structural Loads Analysis for Commercial Transport Aircraft Theory and Practice

Two-Point Landing – Loads Analysis

- Equilibrium in Z

$$\sum F_z = 0:\ V_{MG} = n_z \cdot W - L$$

- Assuming $W = L$ (CS25.473(b))

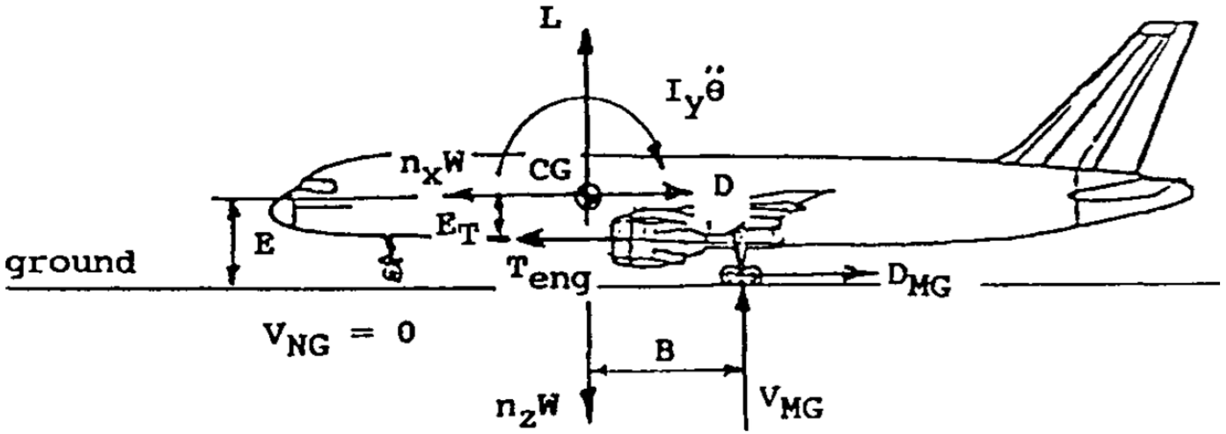
$$V_{MG} = W(n_z - 1) \xrightarrow{\text{yields}} \Delta n_z = \frac{V_{MG}}{W}$$

- Equilibrium in X

$$\sum F_x = 0:\ D_{MG} = n_x \cdot W - D - T_{eng}$$

- Assuming $T_{eng} = D$

$$n_x = \frac{D_{MG}}{W}$$



Ref: T. Lomax: Structural Loads Analysis for Commercial Transport Aircraft Theory and Practice

Two-Point Landing – Loads Analysis

- Moment equilibrium around centre of gravity:

  $$\sum M_{cg} = 0:\ I_y \cdot \ddot{\theta} = B \cdot V_{MG} + E_{ax} \cdot D_{MG} - E_T \cdot T_{eng}$$

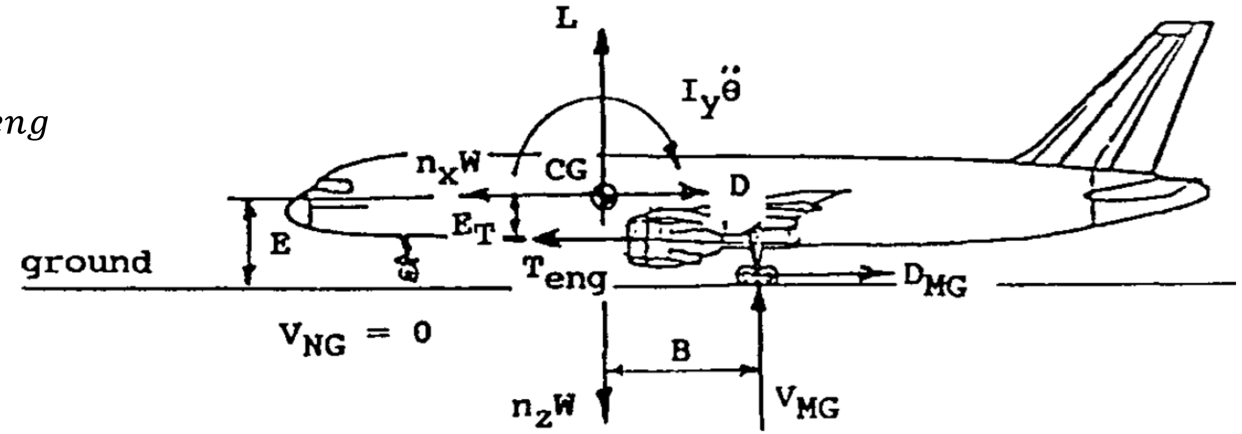
  $$\ddot{\theta} = \frac{B \cdot V_{MG} + E_{ax} \cdot D_{MG} - E_T \cdot T_{eng}}{I_y}$$

  with,

  $$E_{ax} = E - r$$

  r is the rolling radius of the wheels

  I_y is the mass rotational moment of inertia around Y



- By neglecting the engine thrust, the resulting pitching acceleration will be conservative

Ref: T. Lomax: Structural Loads Analysis for Commercial Transport Aircraft Theory and Practice

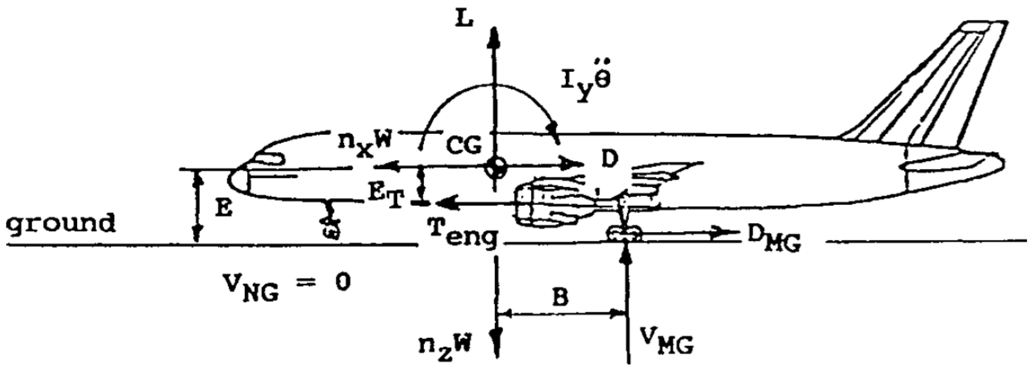
Two-Point Landing – Loads Analysis

- Vertical landing loads
  - $n_z = 1 + \frac{V_{MG}}{W}$

    V_{MG} the landing gear reaction force is obtained from [drop test] or dynamic analysis

Table 6.3 Two-point level landing load factors (design landing weights are shown, and limit descent velocity = 10 fps)

Airplane	Wheels per main gear	Max. landing weight, lb	V_{MG},[a] lb	n_z[b]
A	4	247,000	137,800	2.12
B	2	135,000	93,200	2.38
C	2	161,000	107,600	2.34
D	2	114,000	96,900	2.70
E	2	121,000	97,000	2.60
F	4	198,000	120,000	2.21



  - Through selection of landing gear, the design n_z can be chosen and accordingly the vertical landing loads V_{MG}

Ref: T. Lomax: Structural Loads Analysis for Commercial Transport Aircraft Theory and Practice

Two-Point Landing – Loads Analysis

- Horizontal landing loads:

  According to the certification requirements (CS 25.479 (d)(1)) the drag load acting on the main landing gear is equal to 0.25 of the maximum vertical ground reaction

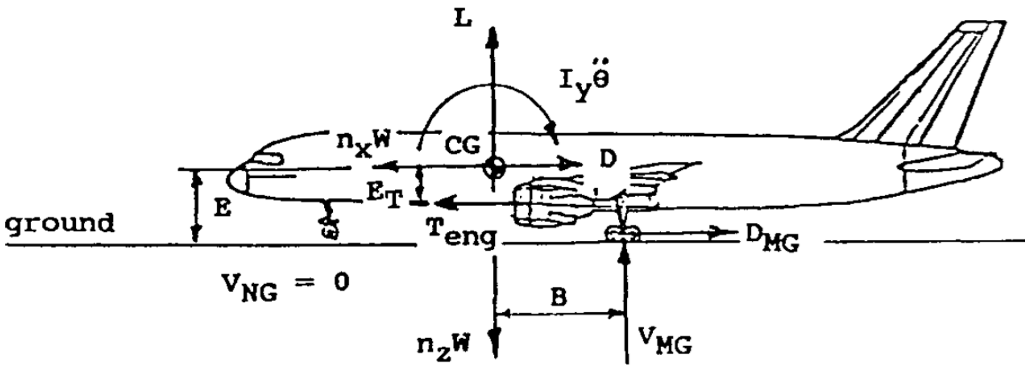
$$-n_x = \frac{D_{MG}}{W} = \frac{0.25 \cdot V_{MG}}{W} = 0.25 \cdot \Delta n_z$$

$$-n_x = 0.25(n_z - 1)$$

- Pitching acceleration:

  By neglecting the thrust term

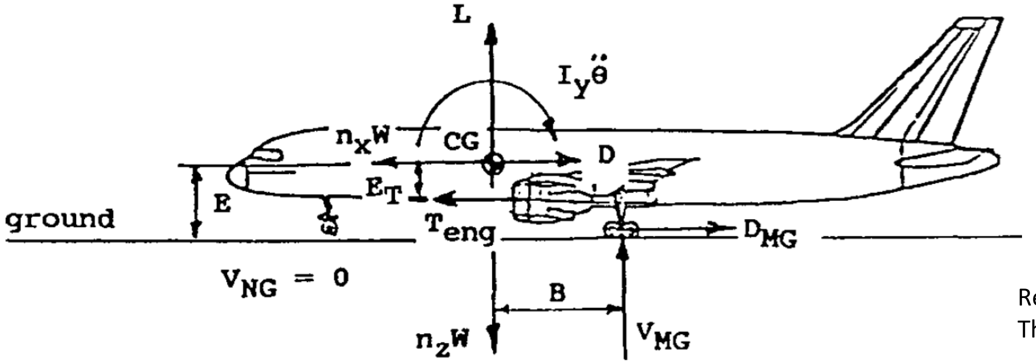
$$\ddot{\theta} = \frac{V_{MG}(B + 0.25 E_{ax})}{I_y}$$



Ref: T. Lomax: Structural Loads Analysis for Commercial Transport Aircraft Theory and Practice

Two-Point Landing – Loads Analysis

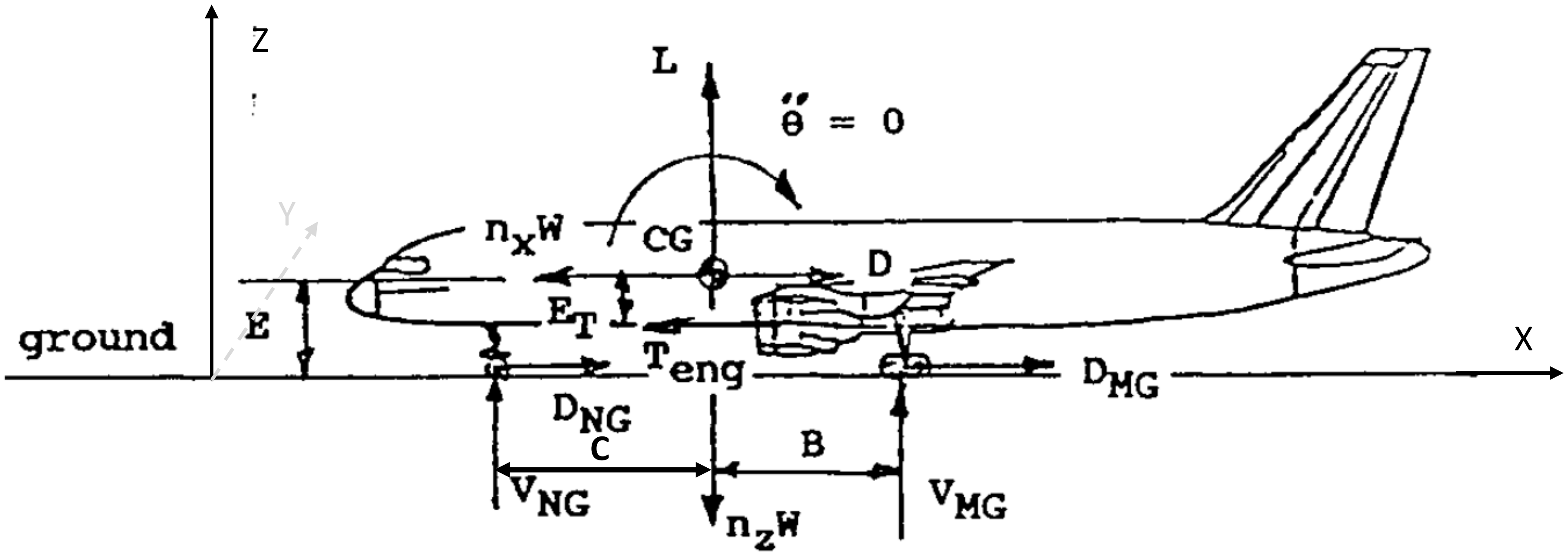
- Summary
  - Choose $n_z = 1 + \frac{V_{MG}}{W}$ depending on your landing gear
  - Compute V_{MG}
  - $n_x = 0.25(n_z - 1)$
  - $\ddot{\theta} = \frac{V_{MG}(B+0.25E_{ax})}{I_y}$
  - Lift (and drag if required) according to altitude (airport), landing speed and trimming
    - Landing at standard day conditions and sea level ($V_{landing,sd}$ based on VL1)
    - Landing at hot day conditions and highest altitude airport ($V_{landing,hd}$ based on VL2)



Ref: T. Lomax: Structural Loads Analysis for Commercial Transport Aircraft Theory and Practice

Three-Point Landing Conditions

- An attitude whereby the nose and main gears contact the runway simultaneously
- Three-point landing provides the maximum loads at the nose gear



Ref: T. Lomax: Structural Loads Analysis for Commercial Transport Aircraft Theory and Practice

Three-Point Landing – Loads Analysis

- Equilibrium in Z

  $\sum F_z = 0{:}\ V_{MG} + V_{NG} = n_z W - L$

- Assuming $W = L$ (CS25.473(b))

  $V_{MG} + V_{NG} = W(n_z - 1)$

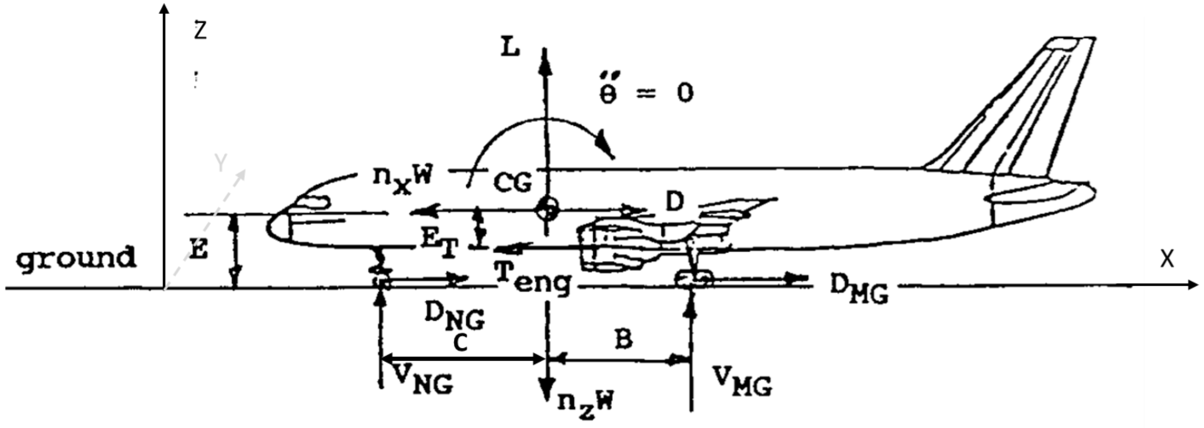
  $n_z = 1 + \frac{V_{MG}+V_{NG}}{W}$

- Equilibrium in X

  $\sum F_x = 0{:}\ D_{MG} + D_{NG} = n_x W - D - T_{eng}$

- Assuming $T_{eng} = D$

  $n_x = \frac{D_{MG}+D_{NG}}{W}$



Ref: T. Lomax: Structural Loads Analysis for Commercial Transport Aircraft Theory and Practice

Three-Point Landing – Loads Analysis

- Moment equilibrium around centre of gravity:

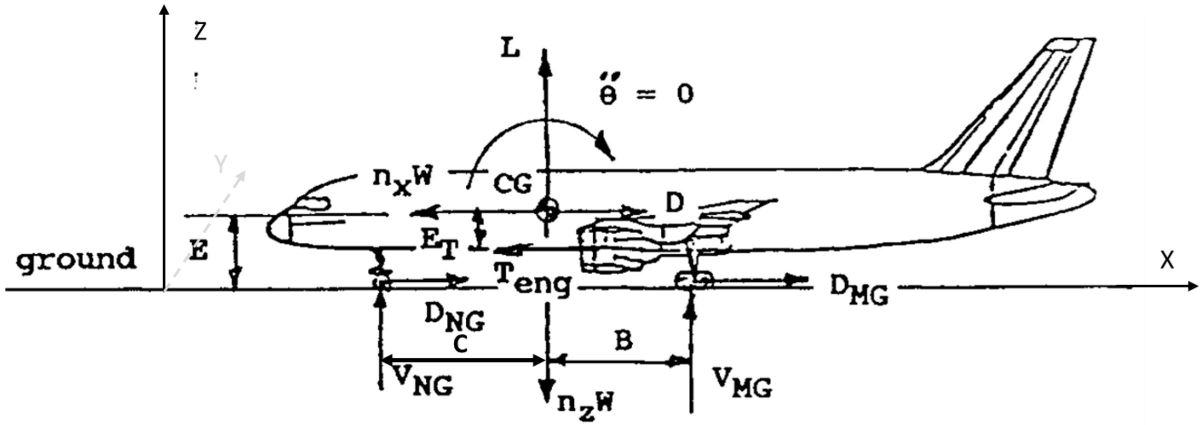
  $\sum M_{cg} = 0\text{:}\ V_{NG} \cdot C - D_{NG} \cdot E_{NGa} = B \cdot V_{MG} + E_{MGa} \cdot D_{MG} - E_T \cdot T_{eng}$

  with,

  $E_{NGa} = E - r_{NG}$

  $E_{MGa} = E - r_{MG}$

  r_{NG}, r_{MG} are the rolling radii of the wheels



- Engine thrust is assumed to be equal the drag D for landing conditions

Ref: T. Lomax: Structural Loads Analysis for Commercial Transport Aircraft Theory and Practice

Three-Point Landing – Loads Analysis

- In order to solve for the nose landing gear load, following assumptions are made:
  - During the landing, thrust equals drag
  - The pitching moment due to engine thrust may be neglected
  - According to certification (CS 25.479 (d)(1)) landing gear drag is assumed 0.25 the vertical loading

    $n_x = 0.25(n_z - 1)$

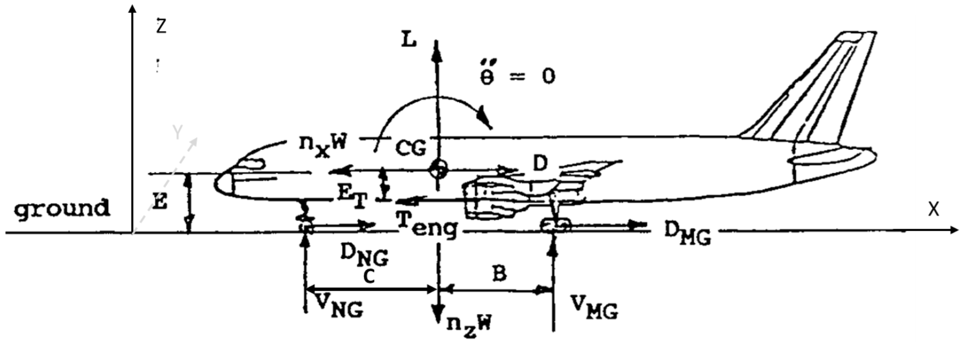
    $D_{NG} = 0.25 \cdot V_{NG}$

  - Inserting into the moment equilibrium equation leads to

    $V_{NG} \cdot (C - 0.25 \cdot E_{NGa}) = V_{MG} \cdot (B + 0.25 \cdot E_{MGa})$

    $V_{NG} = (n_z - 1) \cdot W \frac{F}{1+F}$

    $F = \frac{B+0.25 \cdot E_{MGa}}{C-0.25 \cdot E_{NGa}}$

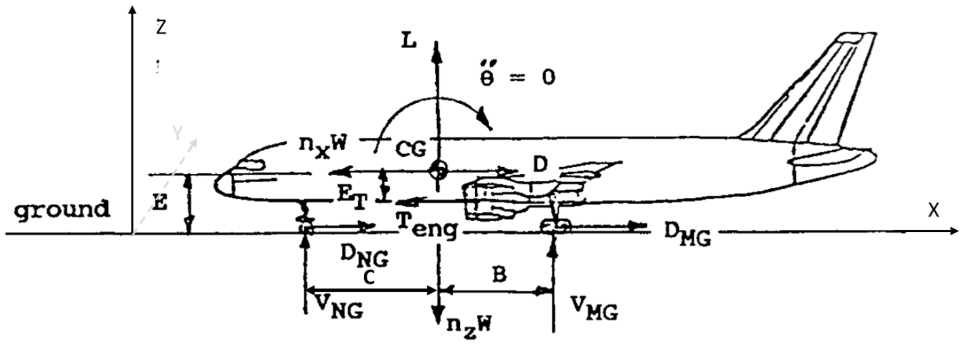


Ref: T. Lomax: Structural Loads Analysis for Commercial Transport Aircraft Theory and Practice

Three-Point Landing – Loads Analysis

- Summary
  - Chose n_z depending on your landing gear (see table above)
  - $n_x = 0.25(n_z - 1)$
  - $V_{NG} = (n_z - 1) \cdot W \frac{F}{1+F}$

    with $F = \frac{B+0.25 \cdot E_{MGa}}{C-0.25 \cdot E_{NGa}}$



  - Lift (and drag if required) according to altitude (airport), landing speed and trimming
    - Landing at standard day conditions and sea level ($V_{landing,sd}$ based on VL1)
    - Landing at hot day conditions and highest altitude airport ($V_{landing,hd}$ based on VL2)

Ref: T. Lomax: Structural Loads Analysis for Commercial Transport Aircraft Theory and Practice

Landing Gear Loads - Overview

- In the course of this lecture the level landing with two-point and three-point conditions has been discussed
- Analogue following loading conditions must be analysed for certification (out of scope of this lecture):
  - Taildown
  - Spin-up and spring-back
  - One-point (one-gear) landing
  - Side load
  - Rebound landing
  - Ground handling: taxi, take-off and landing roll, braked roll, turning, jacking, towing
- It is important to mention that the landing load calculations presented within this lecture are based on simplified methods and can be used in early design phases
- In later design phases more sophisticated simulations based on multi-body structure-aerodynamic coupled analysis are performed (with landing gear model in the loop)