CS 357

Transformations

The View Transformation

$$V = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -a_v & -b_v & -c_v & 1 \end{bmatrix} \times \begin{bmatrix} a_x/r & -b_x/r & 0 & 0 \\ b_x/r & a_x/r & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} r/R & 0 & -c_x/R & 0 \\ 0 & 1 & 0 & 0 \\ c_x/R & 0 & r/R & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_z R/h & \dfrac{b_z a_x - a_z b_x}{h} & 0 \\ 0 & \dfrac{a_z b_x - b_z a_x}{h} & c_z R/h & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \pm 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $r = \sqrt{a_x^2 + b_x^2}$, $R = \sqrt{a_x^2 + b_x^2 + c_x^2}$,and $h = r\sqrt{a_z^2 + b_z^2 + c_z^2}$

The Perspective Transformation

$$P = \begin{bmatrix} D & 0 & 0 & 0 \\ 0 & D & 0 & 0 \\ 0 & 0 & \dfrac{Y}{Y-H} & 1 \\ 0 & 0 & \dfrac{-HY}{Y-H} & 0 \end{bmatrix}$$

where H, D, and Y are distances to the Hither plane, viewplane and Yon plane.

The Window Transformation

$$W = \begin{bmatrix} \dfrac{w_R - w_L}{v_R - v_L} & 0 & 0 & 0 \\ 0 & \dfrac{w_T - w_B}{v_T - v_B} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \dfrac{w_L v_R - v_L w_R}{v_R - v_L} & \dfrac{w_B v_T - v_B w_T}{v_T - v_B} & 0 & 1 \end{bmatrix}$$

where the v's and w's are respectively the boundaries of the viewport on the viewplane and the window on the screen.

The full view pipeline, which transforms object coordinates into screen coordinates, is
$[h, v, z, 1] = [x, y, z, 1]VPW < \text{homogenize.} > = [x, y, z, 1]VP < \text{homogenize} > W$

Example: Suppose the viewer is at (10, 10, 10) looking at point (0,0,0), with the viewer's up-vector <0,0,1>. The viewplane is z=10 (i.e, D =10); the hither and yon planes are defined by Y=15 and H = 5. We use a view angle of Θ=40 degrees, and use window boundaries 0 to 500 in each direction. Where on the screen is the point (2,5, 1) (specified in world coordinates) drawn?

$(a_v, b_v, c_v) = (10, 10, 10)$ $\quad <a_z, b_z, c_z> = <-10, -10, -10>$
$<a_x, b_x, c_x> = <a_z,b_z,c_z>$x **up**$=<b_z,-a_z,0>=<-10, 10, 0>$
$r = \sqrt{200} = R \quad h = r\sqrt{a_z^2 + b_z^2 + c_z^2} = 100\sqrt{6}$ This is enough to find V:

$$V = \begin{bmatrix} -0.707 & -0.408 & -0.578 & 0 \\ 0.707 & -0.408 & -0.578 & 0 \\ 0 & 0.818 & -0.578 & 0 \\ 0 & 0 & 17.33 & 1 \end{bmatrix}$$

We are given that D = 10, Y = 15 and H = 5, so $P = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 1.5 & 1 \\ 0 & 0 & -7.5 & 0 \end{bmatrix}$

$v_L = -D\tan(40) = -8.39$ Similarly, $v_R = v_T = 8.39$, v_B=-8.39.
The window boundaries are given as w_L=0, w_R = 500, w_T = 0, w_B = 500.

This gives $W = \begin{bmatrix} 29.79 & 0 & 0 & 0 \\ 0 & -29.79 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 250 & 250 & 0 & 1 \end{bmatrix}$

$[2 \quad 5 \quad 1 \quad 1]VPW = [2.12 \quad -2.04 \quad 12.71 \quad 1]PW = [21.2 \quad -20.4 \quad 11.56 \quad 12.71]W$
$= [3808.35 \quad 3783.62 \quad 11.56 \quad 12.71]$

This last result homogenizes to [299.7. 297.8, 0.91, 1]. Note that the first two coordinates are within our window boundaries and the z-coordinate is between 0 and 1. We would draw the point at pixel (300, 298).