

BM20A9200 Mathematics A – Exercise set 6

To be done by 16.–20.10.2023

Text in blue or red is not part of the problem or its solution. It's there as extra information to help you learn.

This set (nbr 6) starts having topics from the week after (lesson 7). You can try to solve them already by reading the background material, or wait till lesson nbr 7 which is on 17.10.

Exercise 1. Solve the equation

$$2\sqrt{1-x^2} = x - 1$$

in the set of real numbers.

Exercise 2. Consider the mapping $f: \mathbb{R} \setminus \{2\} \rightarrow \mathbb{R}$ defined by $f(x) = x/(2-x)$.

a) Is it an injection? Prove it.

b) Is it a surjection? Prove it.

Exercise 3. Define the functions $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 5 - (x-3)^2$ and $h: \mathbb{R} \rightarrow \mathbb{R}$

$$h(x) = \begin{cases} 2-x, & \text{if } x < 2 \\ \frac{1}{2}x + 1, & \text{if } x \geq 2. \end{cases}$$

Draw the graphs of the functions f and h .

a) Denote $U = [1, 4]$. Determine the preimage $f^{\leftarrow}(U)$ using the graph of the function f . Mark clearly U and $f^{\leftarrow}(U)$.

b) Denote $V = [1, 3]$. Determine the preimage $h^{\leftarrow}(V)$ using the graph of h . Mark clearly V and $h^{\leftarrow}(V)$.

Exercise 4. Find a bijection between the sets $\{1, 2, 3, 4, \dots\}$ and $\{8, 13, 18, 23, \dots\}$ where the next number is the previous plus 5.

Exercise 5. a) Let $X = \{1, 2\}$. Find all possible functions/mappings $f: X \rightarrow X$. You can use a diagram, or tell how the elements of X map using some other way.

b) Let $Y = \{1, 2, 3\}$. How many different possible mappings $g: Y \rightarrow Y$ are there?

c) Let $Z = \{1, 2, \dots, n\}$. How many different possible mappings $h: Z \rightarrow Z$ do exist?

Exercise 6. Expand $(1+x)^7$.