BM20A9200 Mathematics A – Exercise set 12

To be done by 4.–8.12.2023

Text in blue or red is not part of the problem or its solution. It's there as extra information to help you learn.

The exercise sessions of Independence Day 6.12. have been moved to other days. Please check your schedule.

Exercise 1. Let $\mathbf{A}$ be the following augmented matrix

$$\mathbf{A} = \left[\begin{array}{ccc|c} -1 & -2 & 5 & 1 \\ -1 & 7 & 3 & 3 \\ 5 & 1 & 6 & 4 \end{array}\right]$$

Perform the following elementary row operations one after the other starting from $\mathbf{A}$.

a) Multiply the elements of row 2 by 6,

b) then interchange rows 1 and 3,

c) then multiply row 1 by two and add the result to row 2.

Exercise 2. Let

$$\mathbf{A} = \begin{bmatrix} 6 & -1 & 2 & 4 \\ 2 & -1 & 6 & 2 \\ -2 & 7 & -3 & 9 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 & -2 & 1 \\ 2 & 4 & 0 \\ 1 & -2 & 10 \\ 2 & 2 & -3 \end{bmatrix}, \qquad \mathbf{C} = \begin{bmatrix} 7 & 5 & 0 \\ 6 & 5 & -3 \\ 9 & 5 & -1 \end{bmatrix}$$

a) Calculate the matrix product $\mathbf{ABC}$.

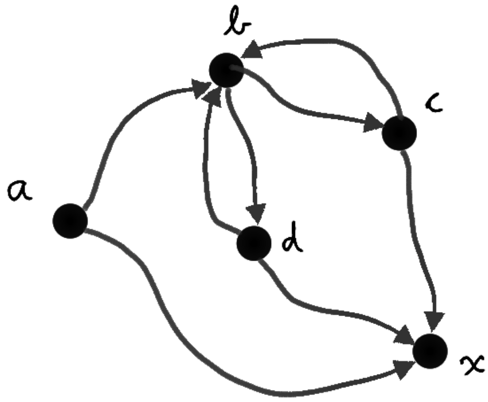
b) Find some other legal product where $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ each appear once and calculate it.

Exercise 3. Prove that if $\mathbf{A} = [a_{ij}]_{m\times n}$ and $\mathbf{B} = [b_{ij}]_{n\times p}$ then

$$(\mathbf{AB})^{\mathsf{T}} = \mathbf{B}^{\mathsf{T}}\mathbf{A}^{\mathsf{T}}.$$

Hint: It suffices to consider an element in general position (i, j) and show that this element is the same on both sides of the equation above. You will also need to use the general formula for the element of a matrix product.

Exercise 4. Consider the following directed graph:



(a) Write its adjacency matrix corresponding to the ordering of its elements (a, b, c, d, x).

(b) In how many different ways can one go from a to x in exactly 5 steps.

Exercise 5. Find the inverse $\mathbf{A}^{-1}$ of the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 2 & 3 & -4 \end{bmatrix}.$$

Verify your result.

Exercise 6. Matrices are useful in computer graphics. For instance, let a point (x, y) be represented by a column vector $\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$. Given an angle θ, the product

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

rotates the point (x, y) counterclockwise by θ with respect to the origin and the x-axis. Here sin and cos are trigonometric functions. See the following link for a reminder: https://en.wikipedia.org/wiki/Trigonometric_functions.

Let us consider a rectangle whose corners are at xy-coordinates $(0, 0), (0, 1), (3, 0), (3, 1)$. Describe what happens to these points in the rotation described above when $\theta = 60°$.