BM20A9301 Statistics – Exercise set 2

To be done by 15.–19.1.2024

Text in blue or red is not part of the problem or its solution. It's there as extra information to help you learn.

To get points, you need to participate to exercise sessions, and have something to show for each exercise you have marked. If you cheat (mark a problem done but don't show anything when called), you don't get points for the session and get -5 points on your score.

Exercise 1 (Counting elements)**.** Denote by $|A|$ the number of elements of the set A.

(a) If $|A| = 20$, $|B| = 40$ and $|A \cap B| = 5$, what is $|A \cup B|$?

(b) If $|A| = 200$, $|B| = 23$ and $|A \cup B| = 203$, what is $|A \cap B|$?

(c) If $|A| = 34$, $|A \cap B| = 10$ and $|A \cup B| = 37$, what is $|B|$?

(d) If $|A| = 10$, $|A \cap B| = 4$, what is $|A - B|$?

Exercise 2 (Calculating probabilities)**.** Let A and B be events such that $A \subseteq B$. We know that $P(A) = 0.3$ and $P(B) = 0.5$. Calculate the following probabilities.

(a) $P(A \cup B)$

(b) $P(A \cap B)$

(c) $P(B - A)$

Exercise 3 (Probability modelling)**.** Two fair 8-sided dice are rolled. Model this random experiment mathematically and answer the following questions.

(a) Describe the sample space Ω of your model.

(b) What are the probabilities of each $s \in \Omega$?

(c) What is the probability that the sum of dices is greater than 13?

(d) What is the probability the sum is an even number?

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Exercise 4 (Combinatorial probability)**.** A fair coin is flipped **10 times** and it lands on heads or its opposite side tails.

(a) Describe the sample space.

(b) What is the probability to get heads every flip?

(c) What is the probability to get exactly one tails?

(d) What is the probability to get exactly two tails?

Exercise 5 (Independence and conditional probability)**.** Someone throws two fair dice, a red and a blue one. Consider the events B_6 = "the blue dice threw a 6", R_3 = "the red dice threw a 3" and $S_{\leq 9}$ = "the sum of the dice is at most 9".

(a) Are B_6 and R_3 idenpendent events?

(b) Draw the sample space for this problem and colour the occurences in which $S_{\leq 9}$ happens.

(c) Calculate $P(B_6 \mid S_{\leq 9})$ and $P(R_3 \mid S_{\leq 9})$.

(d) Are B_6 and R_3 still independent if you know for sure that $S_{\leq 9}$ happens? In other words are they conditionally independent conditioned on $S_{\leq 9}$, meaning $P(B_6 \cap R_3 \mid S_{\leq 9}) = P(B_6 \mid S_{\leq 9}) \cdot P(R_3 \mid S_{\leq 9})$?

Exercise 6 (Law of total probability)**.** The numbers in this exercise are completely fictional. Do not try to deduce how well people actually did in the Mathematics A exam from this exercise!

Students take two maths classes: Mathematics A and Statistics. A student gets a high score in Mathematics A with probability 40%. Those students that did well in that course have an 80% change of getting a high score in Statistics. Those that did not get a high score in Mathematics A have only a 30% chance to get a high score in Statistics. What is the probability that a student will get a high score in Statistics?