BM20A9200 Mathematics A – Exercise set 2

To be done by 18.–22.9.2022

Exercise 1. Let U be a set and let $\mathcal{F}_1$ and $\mathcal{F}_2$ be nonempty families of subsets of U such that $\mathcal{F}_1 \subseteq \mathcal{F}_2$. Show (=prove) that the following inclusions hold:

(a) $\bigcup \mathcal{F}_1 \subseteq \bigcup \mathcal{F}_2$

(b) $\bigcap \mathcal{F}_1 \supseteq \bigcap \mathcal{F}_2$

Exercise 2. Let U be a set and let $\emptyset \neq \mathcal{F} \subseteq \mathcal{P}(U)$ be a nonempty family of subsets of U. Prove the following equalities:

(a) $(\bigcap \mathcal{F})^C = \bigcup \{A^C \mid A \in \mathcal{F}\}$

(b) $(\bigcup \mathcal{F})^C = \bigcap \{A^C \mid A \in \mathcal{F}\}$

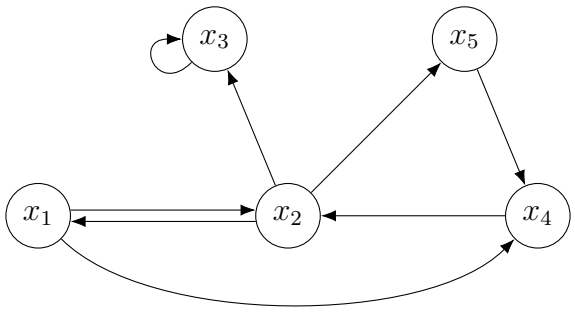
Recall that the complement of any $X \subseteq U$ is defined by $X^C = U \setminus X$.

Exercise 3. The courses taken by John, Mary, Paul and Sally are listed below:

John: MATH 212, CSIT 121, MATH 220
Mary: MATH 230, CSIT 121, MATH 211
Paul: CSIT 120, MATH 211, MATH 230
Sally: MATH 211, CSIT 120

Give a graphical representation of the relation R defined as aRb if student a is taking bourse b.

Exercise 4. Write the set of ordered pairs for the relation represented by the following directed graph:



Exercise 5. Let R be a binary relation on the set $\mathcal{P}(\{a, b\})$ with $a \neq b$ defined so that $(A, B) \in R$ holds if $A \,\Delta\, B = \emptyset$. Write out the relation R.

Exercise 6. Let A, B, C be sets. Prove the following equalities:

(a) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(b) $A \times (B \cup C) = (A \times B) \cup (A \times C)$