

BM20A9200 Mathematics A – Exercise set 4

To be done by 2.–6.10.2023

Text in blue or red is not part of the problem or its solution. It's there as extra information to help you learn.

Exercise 1. Let P , Q and R be logical propositions. Write the truth table of $P \Rightarrow (Q \Leftrightarrow R)$.

Exercise 2. Binary numbers:

- (a) How do you write the decimal number 69 in binary form? Use the methods described in the lecture notes.
- (b) What decimal number corresponds to the binary number 101 0101 0101?

Exercise 3. Write the interpretation of the following propositions about integers into English:

- 1. $\exists n(n = -n)$
- 2. $\exists n(n^2 = 2)$
- 3. $\forall n(3n \leq 4n)$
- 4. $\forall n(n^2 \geq n)$

Which of the statements are true and which false? Explain your answer.

Exercise 4 (Source: Advanced Mathematics Matriculation Exam 25.9.2017). Juha¹ tries to prove the following proposition: “If a positive integer is divisible by 3 then it is divisible by 6.” He suggests the following proof:

Step 1: Assume that a is divisible by 6.

Step 2: Then there is an integer b such that $a = 6b$.

Step 3: Now $a = 3 \cdot 2b$.

Step 4: Therefore a is divisible by 3.

Show that Juha's proposition is false. What is wrong with Juha's proof? Which statement does Juha's “proof” show, if any?

Exercise 5. Let $a, b \in \mathbb{R}$. Show that $a^2 - b^2 = 0$ if and only if² $a = b$ or $a + b = 0$.

Exercise 6. Consider a right triangle whose legs have lengths a and b and the hypotenuse has length c . The goal is to show $a + b > c$.

- 1. What is the counter-assumption of the claim?
- 2. By squaring both sides of the inequality, show that the counter-assumption leads to a contradiction with the Pythagorean theorem.
- 3. Using the previous, prove the claim using an indirect proof. You can assume the Pythagorean theorem to be true.

¹A common Finnish man's name

²Proving “ A if and only if B ” means proving both $A \Rightarrow B$ and $B \Rightarrow A$.