

## Week 12

Write and run your programs with IDLE editor. Submit finished programs to CodeGrade. Note that some tasks have several steps (A, B, C, ...) in CodeGrade.

**IMPORTANT:** End each input-command string with a newline symbol `\n`. For example:

```
variable = input("Some text:\n")
```

For all the following functions, write a main program that calls the recursive function in question according to the given examples.

**Task 1.** Write a recursive function `integer_sum(n)` to calculate the sum of the positive integers  $n + (n-2) + (n-4) \dots$  until  $n - i < 1$  (terms below 1 are not counted).

**Example run 1.**

```
Give a non-negative integer n:
```

```
10
```

```
n + (n-2) + (n-4) + ... = 30
```

**Example run 2.**

```
Give a non-negative integer n:
```

```
751
```

```
n + (n-2) + (n-4) + ... = 141376
```

**Task 2:** Implement a recursive function `power(x, n)` to calculate  $x$  power to  $n$ , where  $n$  is a non-negative integer and  $x$  is a floating point number (can be negative).

**Example run 1.**

```
Give a float x:
```

```
2.2
```

```
Give a non-negative integer n:
```

```
0
```

```
2.2 power to 0 is 1
```

**Example run 2.**

```
Give a float x:
```

```
2.0
```

```
Give a non-negative integer n:
```

```
20
```

```
2.0 power to 20 is 1048576.0
```

### Example run 3.

```
Give a float x:  
-3.0  
Give a non-negative integer n:  
15  
-3.0 power to 15 is -14348907.0
```

**Task 3:** Write a recursive algorithm `reverse_string(S)` which reverses the given string `S`.

### Example run 1.

```
Give a string to reverse:  
supercalifragilisticexpialidocious  
Original String: supercalifragilisticexpialidocious  
Reversed String: suoicodilaipxecitsiligarfilacrepu
```

**Task 4.** Create a recursive function `gcd(a, b)` to compute the greatest common divisor (GCD) of two positive integers `a` and `b`. GCD is the largest positive integer that divides two integers without leaving a remainder. You should use the idea that if  $a > b$ , then

$$\text{gcd}(a, b) = \text{gcd}(b, a - b).$$

This is the way how the parameters always get smaller. When `a` and `b` become equal, then `a` is the GCD.

The method is based on the fact that if  $d = \text{gcd}(a, b)$ , it means that  $d$  is the largest integer that divides both  $a$  and  $b$ . Therefore,  $a = k \cdot d$  and  $b = l \cdot d$ , for some integers  $k$  and  $l$ , where  $l > d$ . Now  $\text{gcd}(b, a - b) = \text{gcd}(ld, kd - ld) = \text{gcd}(ld, (k - 1)d) = d$ .

### Example run 1.

```
Give two positive integers separated by comma:  
121, 220  
gcd(121, 220) = 11
```