

BM20A9301 Statistics – Exercise set 3

To be done by 22.–26.1.2024

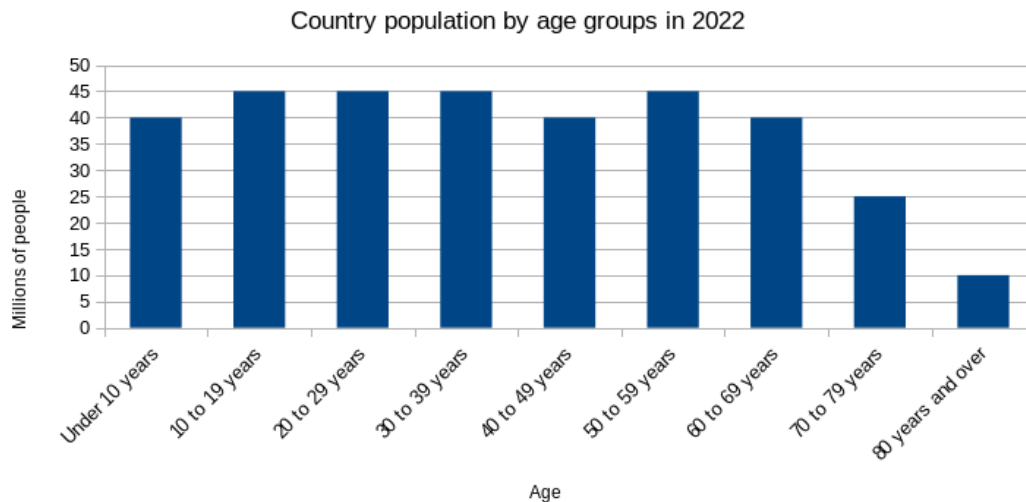
Text in blue or red is not part of the problem or its solution. It's there as extra information to help you learn.

Exercise 1 (Bayes' rule). 2% of men have awesome manicured nails. 20% of women have awesome manicured nails. You see a random adult with awesome nails. What is the probability that it's a man? **Hint:** unless otherwise stated, in this kind of problems you can assume there's just as many men as women.

Exercise 2 (Total probability). In a Never Have I Ever television game show, the host tells some statistics about 2022:

- 1% of people under 30 years old have ever shoplifted,
- 4% of people at least 30 but under 60 years old have ever shoplifted,
- 11% of people at least 60 years old have ever shoplifted.

The next contestant is going to be a randomly selected person from the county's population. See the chart below. What is the probability that they've **never** shoplifted?



Exercise 3 (Prosecutor problem). A hundred taxis operate in a city. One of them is orange and the rest are yellow. A pedestrian gets ran over by a taxi one a dark and foggy night. Based on earlier studies, people see an orange car as yellow with 90% probability, and a yellow one as orange with 8% probability in those circumstances.

- If the criminal taxi was yellow, what is the probability that a witness would see an orange taxi?
- If the criminal taxi was orange, what is the probability that a witness would see an orange taxi?

- (c) A-priori (before interviewing witnesses) we suspect that the taxi that ran over the pedestrian is orange with 1% probability and yellow with 99% probability. If we pick a witness at random, what is our subjective probability that the witness would claim to have seen an orange taxi?
- (d) A witness claims the taxi was orange, so the police arrest the driver. What is the probability that the taxi that ran over the person is actually orange considering the witness account?
- (e) Looking at the answer to (a) the prosecutor called for the conviction of the orange taxi's driver. Do you agree with that? Explain why.

Exercise 4 (Expected value). You can solve this by simulating (Python, R, Excel, whatever) or by calculating. Up to you.

When a pokémon uses Fury Attack, it hits 2–5 times according to the below probabilities. On average how many hits does the pokémon do per Fury Attack provided the attack succeeds?

number of hits	2	3	4	5
probability	3/8	3/8	1/8	1/8



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Exercise 5 (Valid distributions). Determine whether or not the following tables are valid probability distributions of some discrete random variable X . Explain why or why not.

(a)

x	0	1	2	3	4
$P(X = x)$	-0.25	0.5	0.35	0.1	0.3

(b)

x	home	draw	away
$P(X = x)$	0.325	0.406	0.164

(c)

x	25	26	27	28	29
$P(X = x)$	0.13	0.27	0.28	0.18	0.14

Exercise 6 (Distribution of a combined random variable). Suppose you have two “loaded” dice, meaning that the probability of rolling a six is twice as high as rolling any other number (the probabilities for 1, 2, 3, 4, 5, 6 are $\frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{2}{7}$). You roll them both and let X denote their sum.

- (a) What is the range (=possible values) of X ?
- (b) Write down (as a table) the probability density function (PDF) of X .