BM20A9200 Mathematics A – Exercise set 13

To be done by 11.–15.12.2023

Text in blue or red is not part of the problem or its solution. It's there as extra information to help you learn.

Exercise 1. Each of the following three set descriptions are wrong (the notation has one of more problems). Figure out what they try to mean and rewrite them using correct mathematical notation.

a) $\{n \in \mathbb{Z}, 5n\}$

b) $\{\emptyset, \{\}, 0, 1, 2, 1$

c) $\{x^2 \geq 2\}$

Exercise 2. a) Prove that $\neg P \Rightarrow Q$ is logically equivalent to $P \vee Q$.

b) Let A and B be non-empty sets. What is the contrapositive of the proposition "If there is no surjection $A \to B$ then there is no injection $B \to A$"?

c) Assume that $f\colon B \to A$ is an injection. Using it, define a surjection from A to B. Prove that it is a function and a surjection.

Exercise 3. Prove the following claim by induction:

$$3 + 2 \cdot 3 + 2 \cdot 3^2 + 2 \cdot 3^3 + \ldots + 2 \cdot 3^n = 3^{n+1}$$

for all $n \in \{1, 2, 3, 4, \ldots\}$.

Exercise 4. Recall the different formulas for calculating the number of ways of selecting k items from a set of n options and in which situations you can use them. There are 30 students. Each of them can get a grade 0,1,2,3,4 or 5 from the course.

a) In how many different ways the teacher can assign the grades to the 30 students?

b) After the semester, the teacher must report how many students got a 0, how many got a 1, etc. How many different such reports is it possible to make?

Exercise 5. Recall that one way to solve such an equation is either to reduce it to a Diophantine equation.

a) Find all $x \in \mathbb{Z}$ such that $6x \equiv 1 \pmod{70}$.

b) Find all $y \in \mathbb{Z}$ such that $6y \equiv 2 \pmod{70}$.

Exercise 6. Consider the following system of equations:

$$\begin{aligned} 3a - 3b + c &= 7 \\ -2a - 3b - 3c &= 9 \\ 2a + 3b + 4c &= -3 \end{aligned}$$

a) Write the system first as a matrix equation $\mathbf{Ax} = \mathbf{c}$ where $\mathbf{A}$ is the coefficient matrix, $\mathbf{x}$ the unknown vector and $\mathbf{c}$ the constant vector. Then write it as an augmented matrix.

b) Use the Gauss–Jordan method on the augmented matrix.

c) Rewrite the reduced row-echelon form augmented matrix into a system. Then read the values of a, b, c from the reduced row-echelon form, and verify that they solve the system above.