BM20A9200 Mathematics A – Exercise set 9

To be done by 13.–17.11.2023

Text in blue or red is not part of the problem or its solution. It's there as extra information to help you learn.

Exercise 1. Prove the following for all positive integers n:

a) if n is odd then $8 \mid n^2 - 1$,

b) if $3 \nmid n$ and n is odd then $24 \mid n^2 - 1$. **Hint:** $24 = 3 \cdot 8$ and $\gcd(3, 8) = 1$.

Exercise 2. Find some integer solution (where possible):

a) $3x - 5y = 7$

b) $21x - 35y = 24$

c) $97x + 127y = 1$

Exercise 3. Simplify the fraction

$$\frac{260\,712}{561\,752}$$

Exercise 4. A number l is called a common multiple of m and n if both m and n divide l. There are many such l. The smallest positive one is called the **least common multiple** of m and n and is denoted by $\operatorname{lcm}(m, n)$. For example $30 = \operatorname{lcm}(10, 6)$ because $30 = 3 \cdot 10 = 5 \cdot 6$ (so it's a common multiple), and any smaller multiple of 10 is not a multiple of 6.

(a) Find $\operatorname{lcm}(8, 12)$, $\operatorname{lcm}(20, 30)$, $\operatorname{lcm}(51, 68)$, $\operatorname{lcm}(23, 18)$.

(b) Compare the value of $\operatorname{lcm}(m, n)$ with the values of m, n and $\gcd(m, n)$. In what way are they related? No need to prove this. Just describe it.

(c) Compute $\operatorname{lcm}(301337, 307829)$ using the formula you found in (b). You probably need a calculator for this.

Exercise 5. What is the last digit of 7^{2023}?

Exercise 6. Find all integer solutions (x, y) to $6x - 13y = 5$.