BM20A9200 Mathematics A – Exercise set 7

To be done by 30.10.–3.11.2023

Text in blue or red is not part of the problem or its solution. It's there as extra information to help you learn.

Recall that there is no lesson or exercise sessions during the week of 23.–27.10. We do not have a mid-term exam either; we only have a final exam in January and later.

Exercise 1. Let $D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be the set of digits.

(a) How many 4-digit pin codes are there?

(b) How many increasing sequences of 4 digits is there ($x_1 \leq x_2 \leq x_3 \leq x_4$)?

(c) How many subsets of 4 elements of D are there?

Be ready to explain how you arrived at your conclusion.

Exercise 2. Find the values of

$$\binom{70}{5} \qquad \text{and} \qquad \binom{121}{115}.$$

Explain how you calculated the values.

Exercise 3. You go to a sushi restaurant for lunch. You will buy 10 pieces of sushi. The restaurant has the following on offer:

- Anago (sea eel)
- Ebi (shrimp)
- Kappa maki (cucumber maki)
- Maguro (blue fin tuna)
- Sake (salmon)

In how many different ways is it possible to make the selection? (Only the final count of every type of sushi matters, not the order)

Exercise 4. Prove the following for any natural numbers n:

$$n^2 = \binom{n}{2} + \binom{n+1}{2}.$$

Exercise 5. Prove *Pascal's rule*. It states that for positive natural numbers n and k we have

$$\binom{n-1}{k} + \binom{n-1}{k-1} = \binom{n}{k}.$$

Exercise 6. Prove that for any positive integer n

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$