

BM20A9200 Mathematics A – Exercise set 5

To be done by 9.–13.10.2023

Text in blue or red is not part of the problem or its solution. It's there as extra information to help you learn.

Exercise 1. Are the following statements true for all sets A , B , C and D ?

a) $A \times (B - D) \subseteq (A \times B) - (C \times D)$.

b) $A \times (B - D) = (A \times B) - (C \times D)$.

Exercise 2. Prove by contrapositive the following claim for sets A and B :

$$\text{If } (A \cup B) \setminus B = A \text{ then } A \cap B = \emptyset.$$

Exercise 3. Denote $X = \{0, 1, 2, 3\}$. Which of the following rules define a function? Why?

a) $f: X \rightarrow X, f(n) = n^2 + n \cdot (-1)^{n+1}$

b) $g: X \rightarrow X, g(x) = 3$

c) $\sigma: \mathbb{R} \rightarrow \mathbb{R}, \sigma(x) = \sqrt{x-3}$

d) $\tau: \mathbb{Q} \rightarrow \mathbb{Q}, \tau(x) = \frac{2a-b}{a^2+2b^2}$, when the rational number x is written in the form $x = \frac{a}{b}$, where $a, b \in \mathbb{Z}$.

Info: The letters σ and τ are the Greek letters sigma and tau.

Hint: Recall that the definition of a function/mapping (Jouni's notes PDF number 4, page 1) has actually two conditions. We associate to every element in the domain (first set) an element of the codomain (second set). And we must not associate more than one element of the codomain to any element of the domain.

Exercise 4. Consider the function $g: \mathbb{R} \rightarrow \mathbb{R}$ for which $g(x) = 4 - x^2$ for all $x \in \mathbb{R}$.

Is the mapping g an injection? Is it a surjection?

Explain your answer in detail using the definitions of injections and surjections.

Exercise 5. Solve x in the following equations:

(a) $2^{3x-2} = 16$

(b) $3 \log_6 x = 21$

(c) $\log_2(3x-4) = 5$

(d) $\log_4 x + \log_4(x-6) = 2$

Exercise 6. Prove:

$$\log_6(15) \text{ is irrational.}$$

Hint: You probably need the fact that if n is odd, then n^k is odd for any $k \in \mathbb{N}$.