Lecture 14

# 寄存器分配

徐 辉

xuh@fudan.edu.cn

# 大纲

- 一、寄存器分配问题
- 二、着色算法
- 三、预分配和溢出

# 一、寄存器分配问题

# 非SSA指令翻译结果

- 虚拟寄存器只在当前代码块有效
  - 跨代码块可以重新编号
- 单个代码块内的寄存器编号递增

```
BB1:
    %a = alloca i32
    %b = alloca i32
    %r = alloca i32
    store i32 1, i32* %a
    store i32 1, i32* %b
    %a1 = load i32, i32* %a
    %r1 = icmp eq i32 %a1, 0
    br i1 %r1, label %BB2, label %BB3
BB2:
    %a2 = load i32, i32* %a
    %b1 = load i32, i32* %b
    %a3 = add i32 %a2, %b1
    store i32 %a3, i32* %a
    br label %BB2
BB3:
    %a4 = load i32, i32* %a
    %b2 = load i32, i32* %b
    %r2 = add i32 %a4, %b2
    store i32 %r2, i32* %r
```

```
main:
        sub     sp, sp, #16
        mov     %r1, #1
        str     %r1, [sp, #12]
        str     %r1, [sp, #8]
        ldr     %r2, [sp, #12]
        cbnz    %r2, .LBB0_2
.LBB0_1:
        ldr     %r1, [sp, #12]
        ldr     %r2, [sp, #8]
        add     %r3, %r1, %r2
        str     %r3, [sp, #12]
        b       .LBB0_1
.LBB0_2:
        ldr     %r1, [sp, #12]
        ldr     %r2, [sp, #8]
        add     %r3, %r1, %r2
        str     %r3, [sp, #4]
        add     sp, sp, #16
        ret
```

# deSSA指令翻译结果

- 虚拟寄存器可以在多个代码块之间共享
  - 跨代码块不能重新编号
- 代码块之间的寄存器编号递增

```
BB1:
    %a = alloca i32
    %b = alloca i32
    %r = alloca i32
    store i32 1, i32* %a
    store i32 1, i32* %b
    %a1 = load i32, i32* %a
    %b1 = load i32, i32* %b
    %r1 = icmp eq i32 %a1, 0
    br i1 %r1, label %BB2, label %BB3
BB2:
    %a2 = add i32 %a1, %b1
    store i32 %a2, i32* %a
    br label %BB2
BB3:
    %a3 = load i32, i32* %a
    %r2 = add i32 %a3, %b1
    store i32 %r2, i32* %r
    ret void
```

```
main:
        mov     %r1, #1
        str     %r1, [sp, #28]
        str     %r1, [sp, #24]
        ldr     %r2, [sp, #28]
        ldr     %r3, [sp, #24]
        cbnz    %r2, .LBB0_2
.LBB0_1:
        add     %r4, %r2, %r3
        str     %r4, [sp, #28]
        b       .LBB0_1
.LBB0_2:
        ldr     %r5, [sp, #28]
        add     %r6, %r3, %r5
        str     %r6, [sp, #20]
        add     sp, sp, #32
        ret
```

# 寄存器分配问题

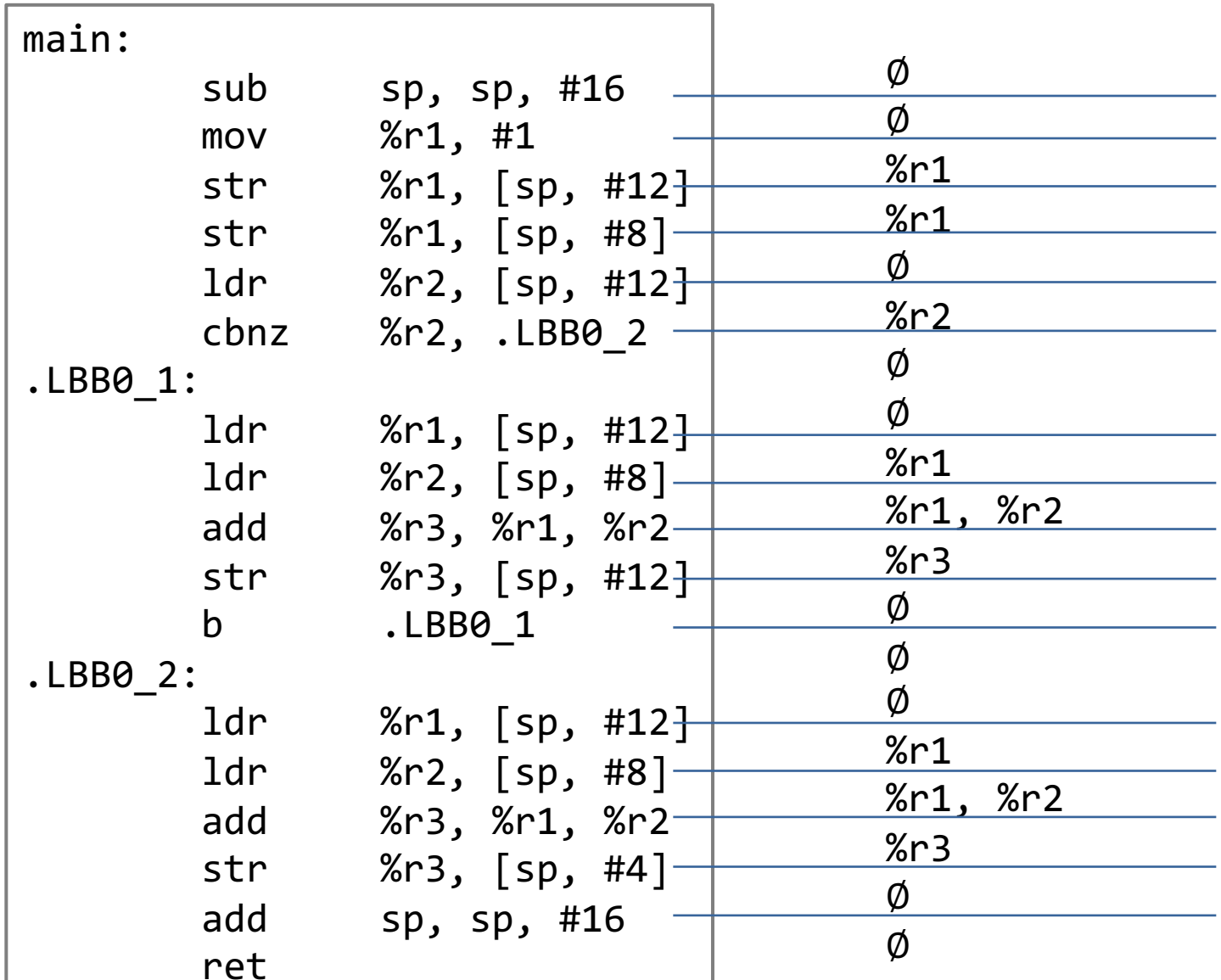
- 指令翻译的寄存器需遵循寄存器用法约定
- 如何为其它虚拟寄存器分配实际的物理寄存器?
  - 指令翻译没有限制虚拟寄存器的数量
  - 但物理寄存器的数量是有限的
  - 物理寄存器不足则将数据写入内存（spill），使用时再读取

| aarch64寄存器 | 调用规约 | 注释 |
|---|---|---|
| X0-X7 | 参数1-8 | Caller-saved |
| X0-X1 | 返回值 | Caller-saved |
| X8 | 特殊用途：间接调用返回地址 | Caller-saved |
| **X9-X15** | **临时寄存器** | **Caller-saved** |
| X16-X17 | 特殊用途：Intra-Procedure-Call | Caller-saved |
| X18 | 特殊用途：平台寄存器 | Caller-saved |
| **X19-X28** | **普通寄存器** | **Callee-saved** |
| X29 | 栈帧基指针 | Callee-saved |
| X30 | 返回地址 | Callee-saved |
| SP | 栈顶指针 | Callee-saved |

aarch64寄存器用法约定

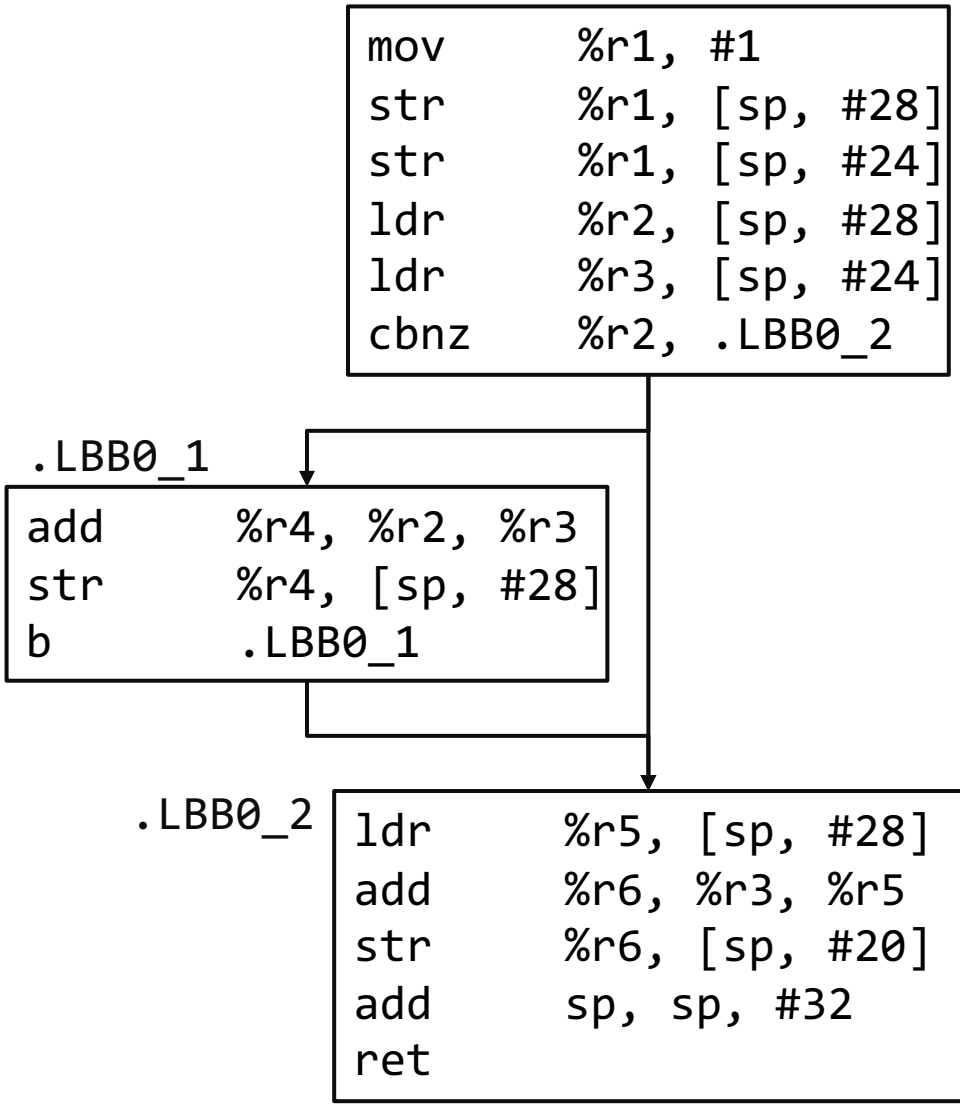
# 活跃性分析（非SSA）

- 如果一个寄存器还会被使用，则在当前节点是活跃的



# 活跃性分析（SSA）

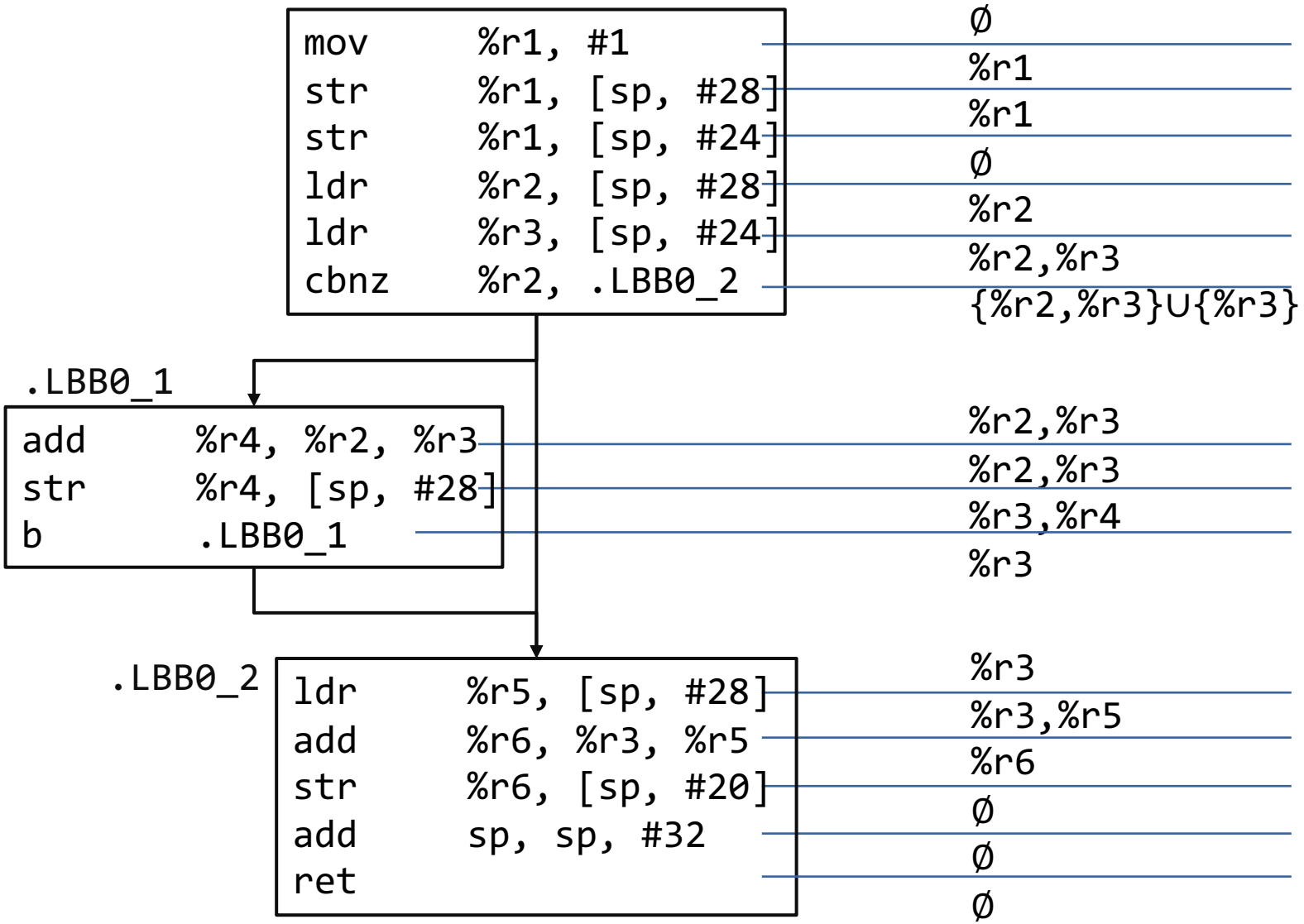
- 基于控制流图分析活跃性




- 反向遍历控制流图：
  - 如遇到指令：`add r1, r2, r3`
    - $\mathrm{Gen(n)} = \{\mathrm{r2}, \mathrm{r3}\}$
    - $\mathrm{KILL(n)} = \{\mathrm{r1}\}$
  - 如遇到指令：`ldr r, [mem]`
    - $\mathrm{KILL(n)} = \{\mathrm{r}\}$
  - 如遇到指令：`str r, [mem]`
    - $\mathrm{Gen(n)} = \{\mathrm{r}\}$
  - 如遇到指令：`op r, label`
    - $\mathrm{Gen(n)} = \{\mathrm{r}\}$
  - …
- $\mathrm{IN(n)} = (\mathrm{OUT(n)} - \mathrm{KILL(n)}) \cup \mathrm{Gen(n)}$

# 活跃性分析（SSA）

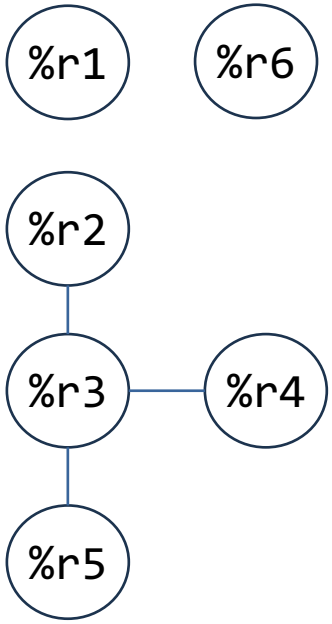
- 基于控制流图分析活跃性



# 干扰图（Interference Graph）

- 干扰：两个同时活跃的寄存器存在干扰关系
- 干扰图：连接所有存在干扰关系的寄存器节点
- 含义：存在干扰关系的寄存器在某一时刻同时存活，应分配不同的物理寄存器

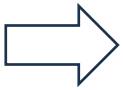
| | | |
|---|---|---|
| ∅ | ∅ | %r3 |
| %r1 | %r2,%r3 | %r3,%r5 |
| %r1 | %r3,%r4 | %r6 |
| ∅ | %r3 | ∅ |
| %r2 | | ∅ |
| %r2,%r3 | | ∅ |



# 翻译结果

w9: %r3
w8: %r1,%r2,%r3,%r4,%r5

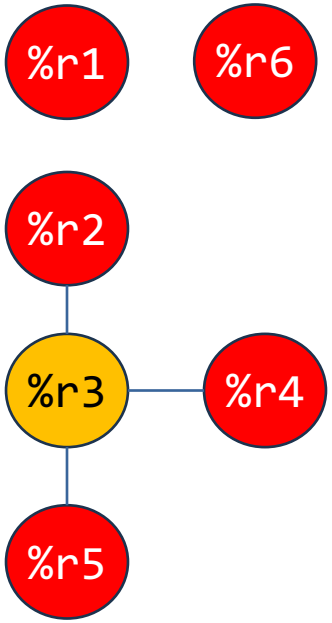
```
main:
        mov     %r1, #1
        str     %r1, [sp, #28]
        str     %r1, [sp, #24]
        ldr     %r2, [sp, #28]
        ldr     %r3, [sp, #24]
        cbnz    %r2, .LBB0_2
.LBB0_1:
        add     %r4, %r2, %r3
        str     %r4, [sp, #28]
        b       .LBB0_1
.LBB0_2:
        ldr     %r5, [sp, #28]
        add     %r6, %r3, %r5
        str     %r6, [sp, #20]
        add     sp, sp, #32
        ret
```

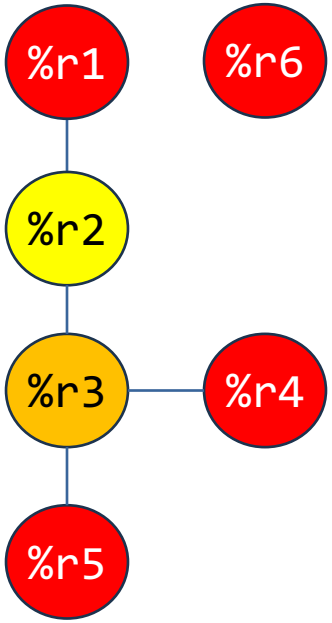


```
main:
        mov     w8, #1
        str     w8, [sp, #28]
        str     w8, [sp, #24]
        ldr     w8, [sp, #28]
        ldr     w9, [sp, #24]
        cbnz    w8, .LBB0_2
.LBB0_1:
        add     w8, w8, w9
        str     w8, [sp, #28]
        b       .LBB0_1
.LBB0_2:
        ldr     w8, [sp, #28]
        add     w8, w9, w8
        str     w8, [sp, #20]
        add     sp, sp, #32
        ret
```

# 寄存器分配=>着色问题（Graph Coloring）

- 使用不超过K种(X9-X15)颜色，要求相邻节点颜色均不同
- 当K≥3时，该问题是NP完全问题（Chaitin的证明）




G.J. Chaitin, *et al*, Register allocation via coloring. *Computer languages*, 1981.

# 基于SAT问题证明

- k-SAT：CNF的每个Clause有不超过k个literals
  - 3SAT是NP-Complete问题
  - 2SAT是多项式复杂度可解
- 如果所有SAT问题可以多项式时间reduce到目标问题，则说明目标问题的难度至少与SAT相当

Literal: $x_1, \overline{x_1}, x_2, \overline{x_2}, x_3, \ldots$

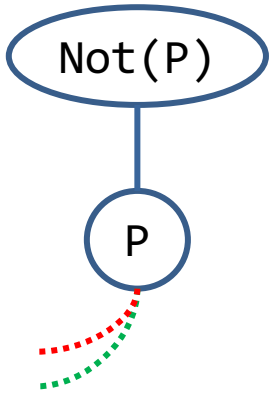
Clause: $l_1 \vee l_2 \vee l_3$

Conjunctive Normal Form: $C_1 \wedge C_2 \wedge \cdots$

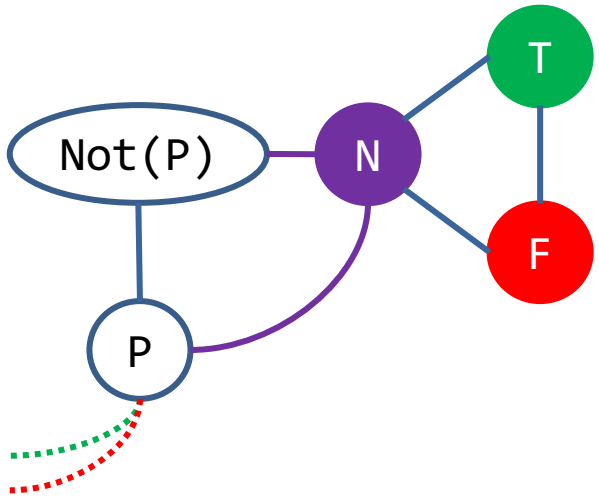
举例：$(x_1 \vee \overline{x_2} \vee x_3) \wedge (x_2 \vee \overline{x_3} \vee x_4) \wedge \cdots$

# 3SAT可以reduce到着色问题

- 构造not和or
- and可以用not和or表示：
  - $C_1 \wedge C_2 = \neg(\neg C_1 \vee \neg C_2)$ …



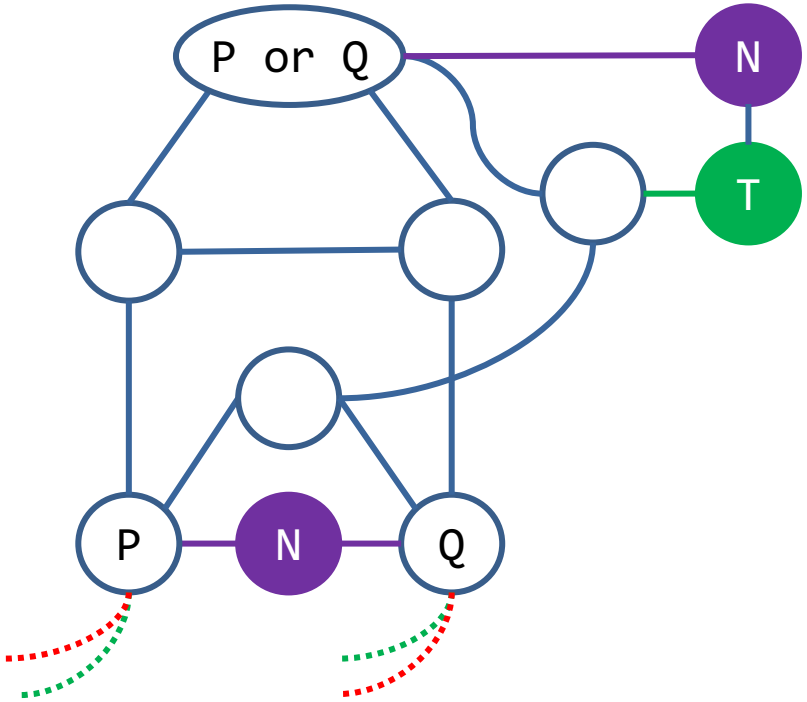

构造not




构造not

Albert R. Meyer和他的学生，MIT

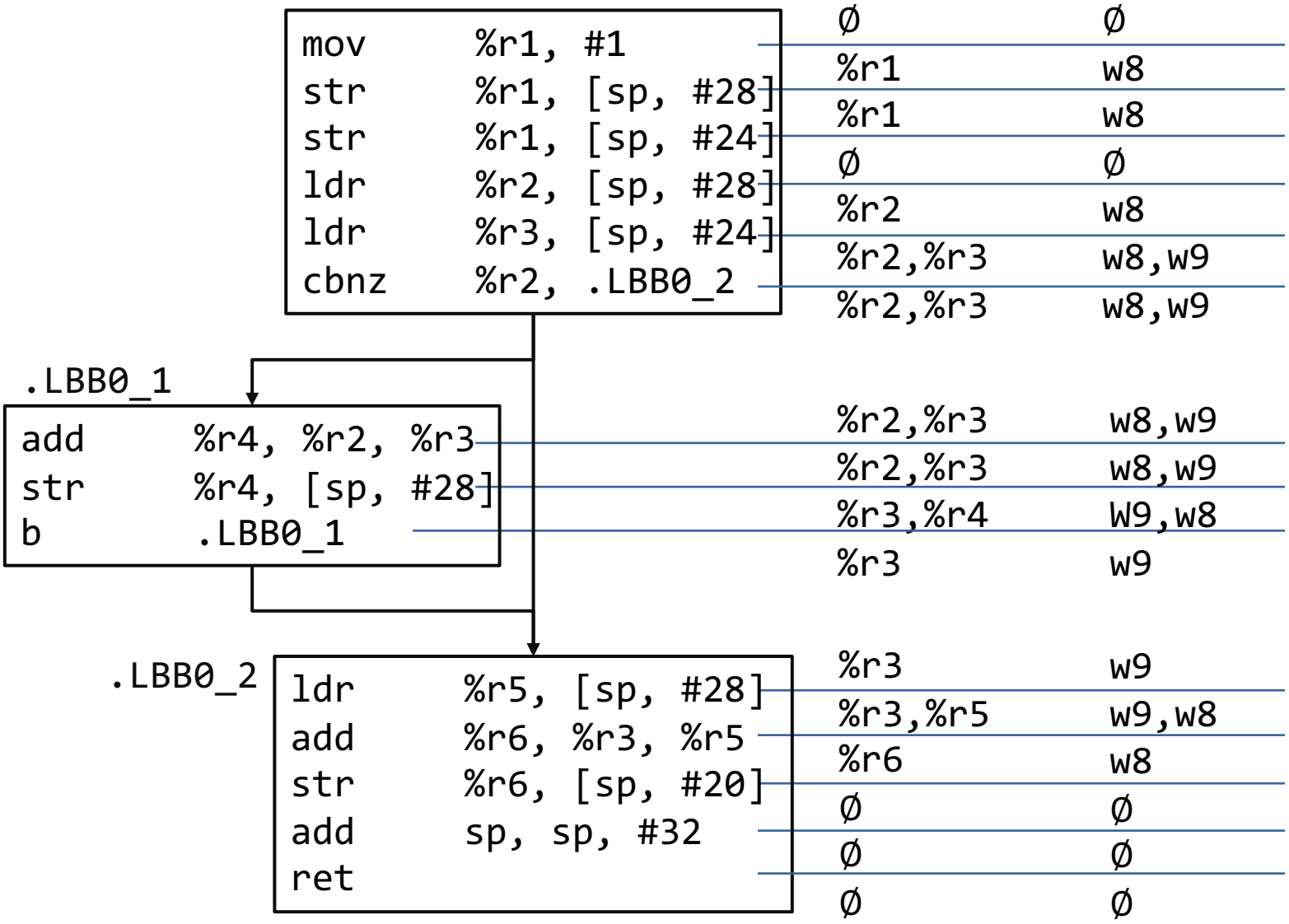
# 3SAT可以reduce到着色问题




构造or

# 二、着色算法

# 线性扫描算法：先到先得

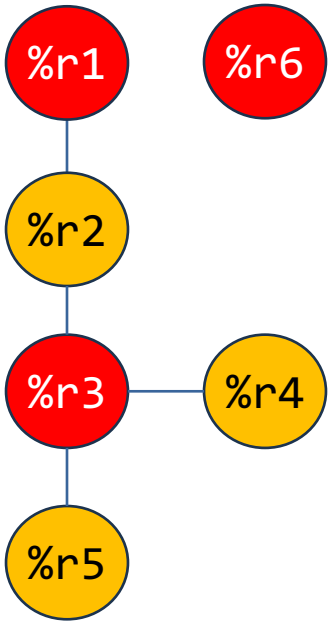

mov %r1, #1
str %r1, [sp, #28]
str %r1, [sp, #24]
ldr %r2, [sp, #28]
ldr %r3, [sp, #24]
cbnz %r2, .LBB0_2
.LBB0_1
add %r4, %r2, %r3
str %r4, [sp, #28]
b .LBB0_1
.LBB0_2
ldr %r5, [sp, #28]
add %r6, %r3, %r5
str %r6, [sp, #20]
add sp, sp, #32
ret
Ø Ø
%r1 w8
%r1 w8
Ø Ø
%r2 w8
%r2,%r3 w8,w9
%r2,%r3 w8,w9
%r2,%r3 w8,w9
%r2,%r3 w8,w9
%r3,%r4 W9,w8
%r3 w9
%r3 w9
%r3,%r5 w9,w8
%r6 w8
Ø Ø
Ø Ø
Ø Ø

# 贪心法着色

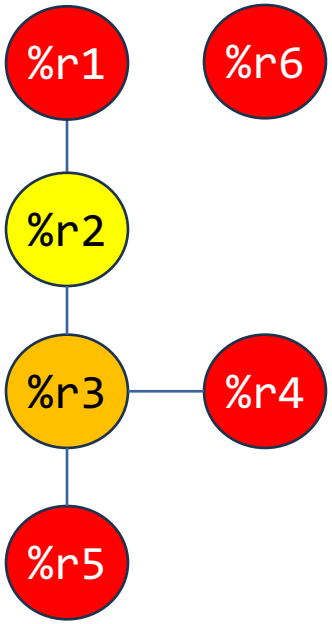
颜色顺序：


- 策略：根据邻居节点颜色，为当前节点选取可用的颜色；

```
贪心算法着色
Input: G=(V,E)
Output: Assignment of colors
For i = 1..n do
  Let c be the lowest color not used in Neighbor(vi)
  Set Col(vi) = c
```



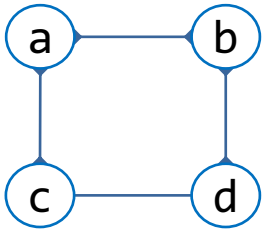

着色顺序是：1-2-3-4-5-6



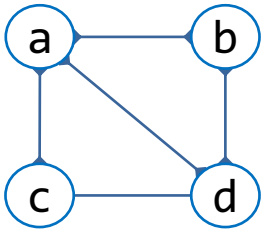

着色顺序是：1-6-4-3-2-5

# 一类特殊的着色问题：弦图（Chordal Graph）

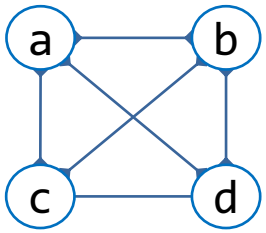
- 任意长度大于3的环都有弦（chord）
- 多项式时间可解
- 静态单赋值形式的干扰图都是弦图



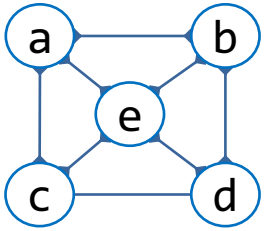
非弦图



弦图

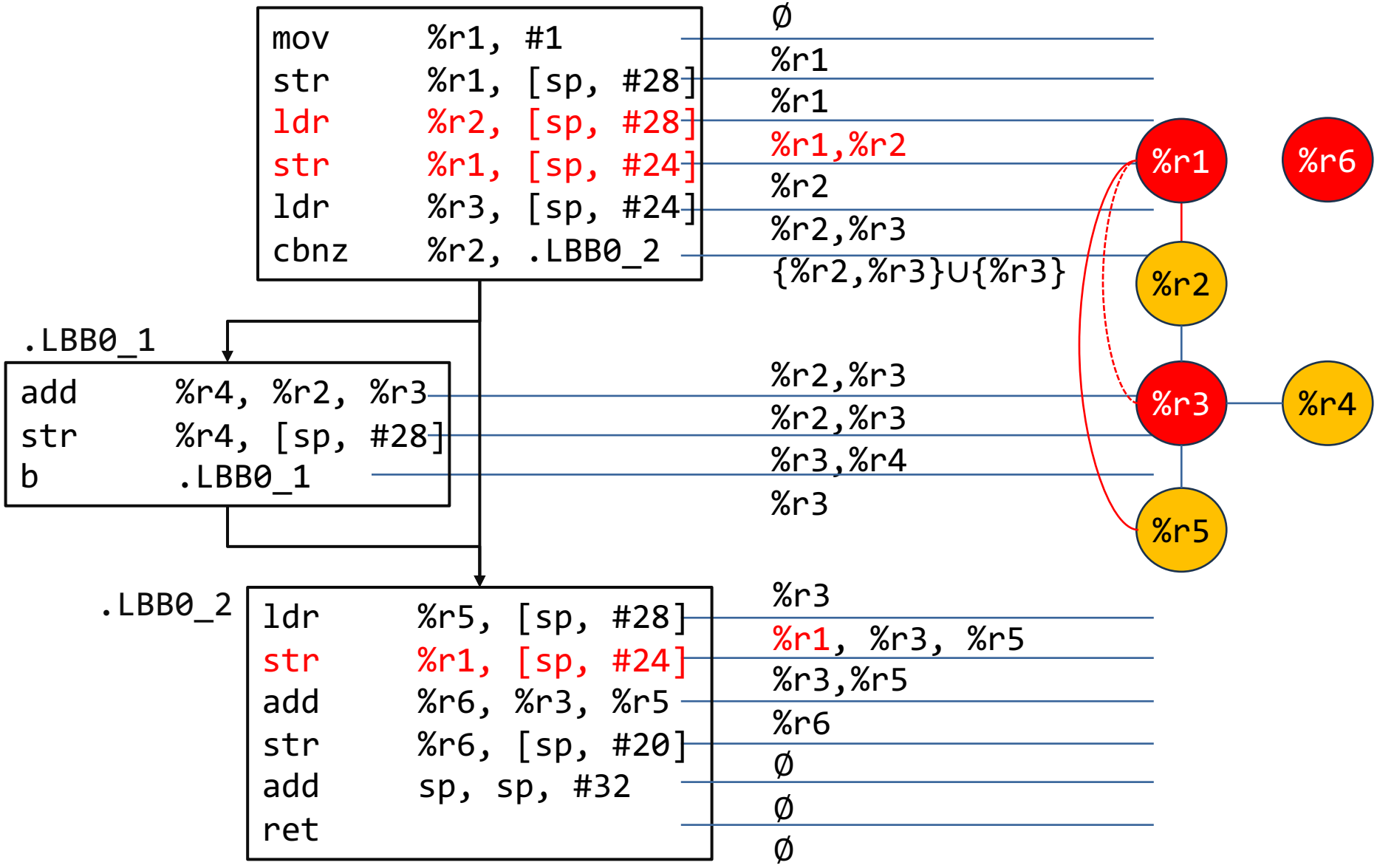


弦图



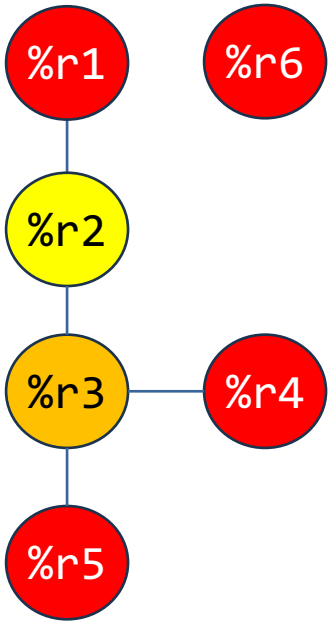
非弦图

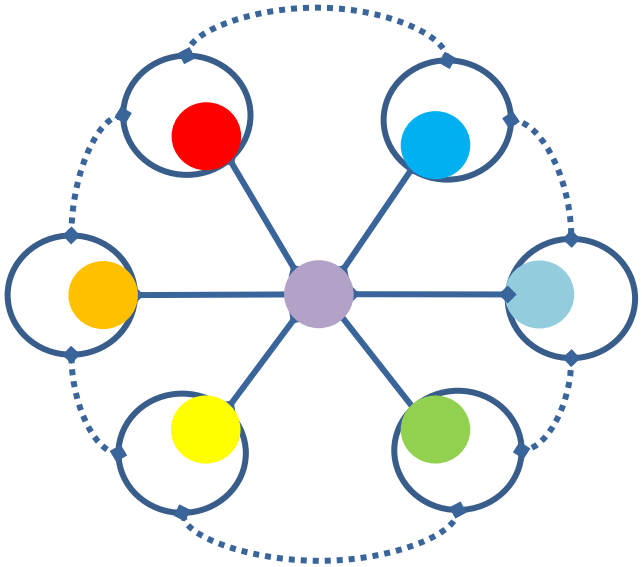
# 尝试为SSA构造非弦图？



# 着色思路

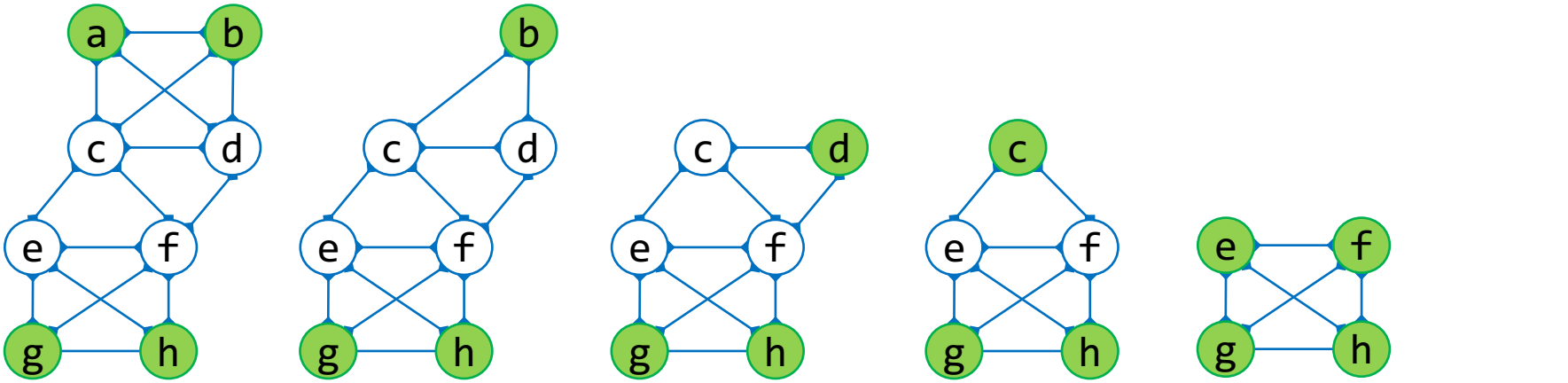
- 在图上搜索团(clique)
  - 团：所有节点两两连接
  - 着色所需颜色数与团的大小一致
- 找最大团也是np-hard问题
- 着色顺序不引入非团节点带来的颜色限制即可
  - 单纯消除序列可达到上述目的



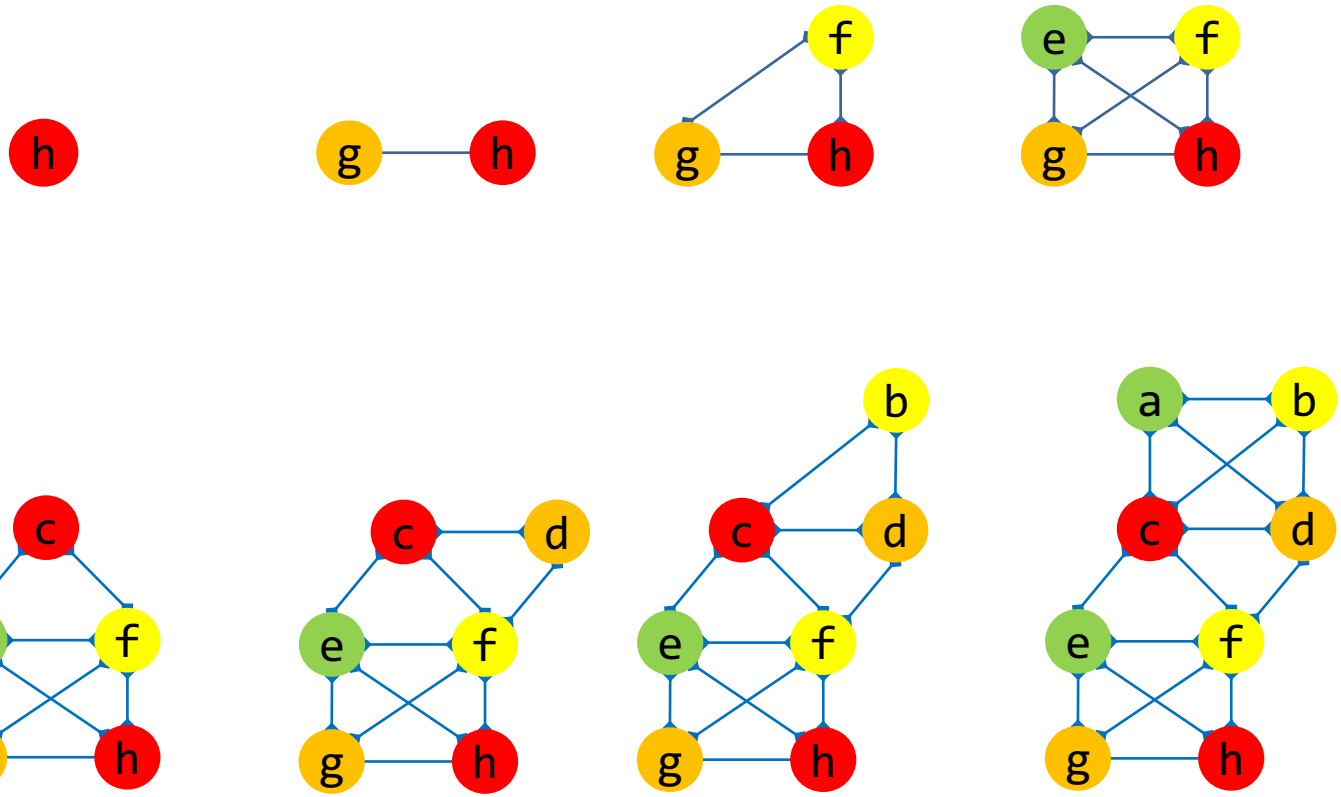
# 单纯消除序列（Simplicial Elimination Ordering）

- **单纯点**（simplicial）：所有邻居组成一个团
- **完美消除序列**：按照该序列消除的每一个点都是单纯点
- **单纯消除序列**：完美消除序列的逆序
- 如果一个图是弦图，则该图存在完美消除序列



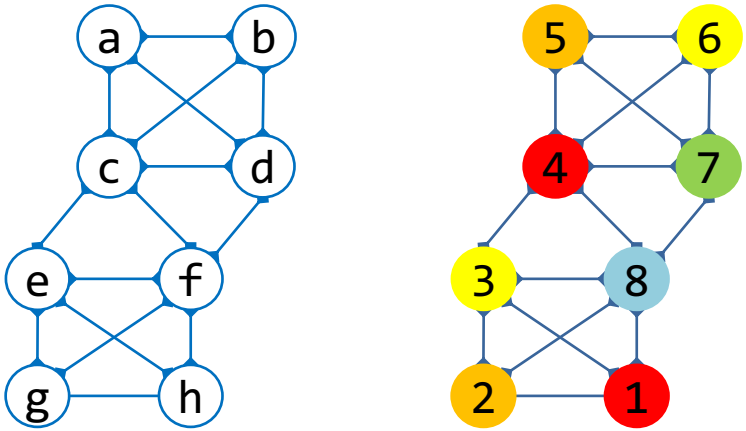
# 基于单纯消除序列着色

- 每次在已着色团的基础上新增一个点，连接该团的所有点



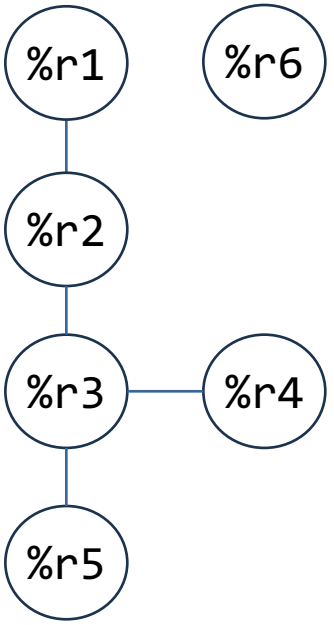
# 如果不遵循非单纯消除序列着色

- 所需颜色数可能需要超过最大团大小



# 最大势算法求单纯消除序列

- Maximum Cardinality Search
- 思路：搜索与已着色节点邻居最多的点
  - 维护一个所有点的向量，每次选取值最大的点；
  - 选取一个点后，则其邻居计数加1。




| 步骤 | 选取 | %r1 | %r2 | %r3 | %r4 | %r5 | %r6 |
|---|---|---|---|---|---|---|---|
| 1 | %r1 | | 1 | 0 | 0 | 0 | 0 |
| 2 | %r2 | | | 1 | 0 | 0 | 0 |
| 3 | %r3 | | | | 1 | 1 | 0 |
| 4 | %r4 | | | | | 1 | 0 |
| 5 | %r5 | | | | | | 0 |
| 6 | %r6 | | | | | | |

# 算法参考

**Maximum Cardinality Search**

Input: $G = (V, E)$

Output: Simplicial elimination ordering $v_1, \ldots, v_n$

For all $v_i \in V$

  $w(v_i) = 0$

Let $W = V$

For i = 1,..,n do

  Let $v$ be a node with max weight in $W$

  Set $v_i = v$

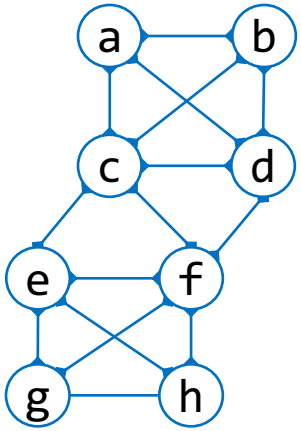
  For all $u \in W \cap N(v)$

    $w(u) = w(u) + 1$

  $W = W \backslash \{v\}$

# 练习

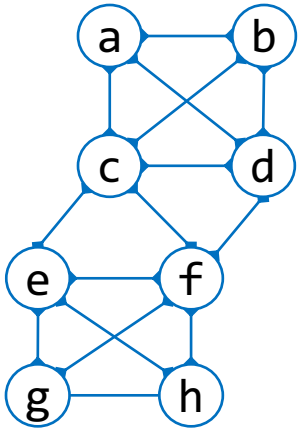
- 求下列冲突图的单纯消除序列




| 步骤 | 选取 | a | b | c | d | e | f | g | h |
|---|---|---|---|---|---|---|---|---|---|
| | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | a | | | | | | | | |
| 2 | b | | | | | | | | |
| 3 | c | | | | | | | | |
| 4 | d | | | | | | | | |
| 5 | f | | | | | | | | |
| 6 | e | | | | | | | | |
| 7 | g | | | | | | | | |
| 8 | h | | | | | | | | |

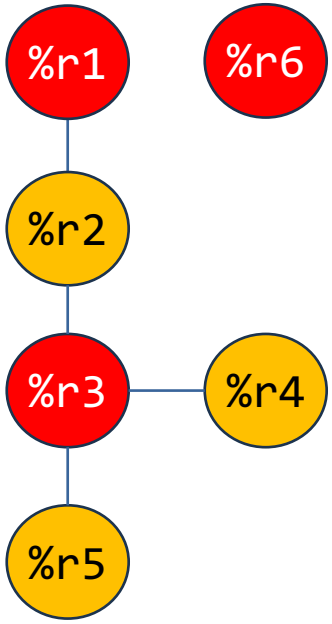
# 练习

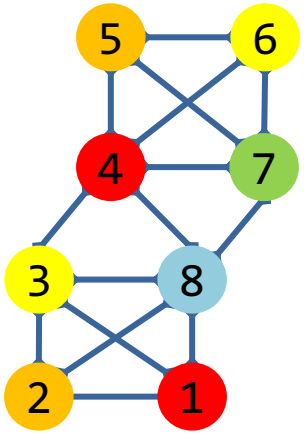
- 求下列冲突图的单纯消除序列

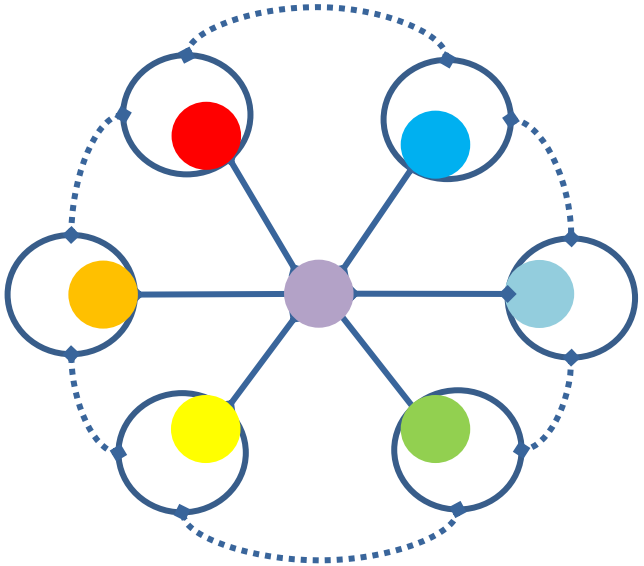



| 步骤 | 选取 | a | b | c | d | e | f | g | h |
|---|---|---|---|---|---|---|---|---|---|
| | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | a | | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 2 | b | | | 2 | 2 | 0 | 0 | 0 | 0 |
| 3 | c | | | | 3 | 1 | 1 | 0 | 0 |
| 4 | d | | | | | 1 | 2 | 0 | 0 |
| 5 | f | | | | | 2 | | 1 | 1 |
| 6 | e | | | | | | | 2 | 2 |
| 7 | g | | | | | | | | 3 |
| 8 | h | | | | | | | | |

# 思考：为何最大势算法能得到单纯消除序列？



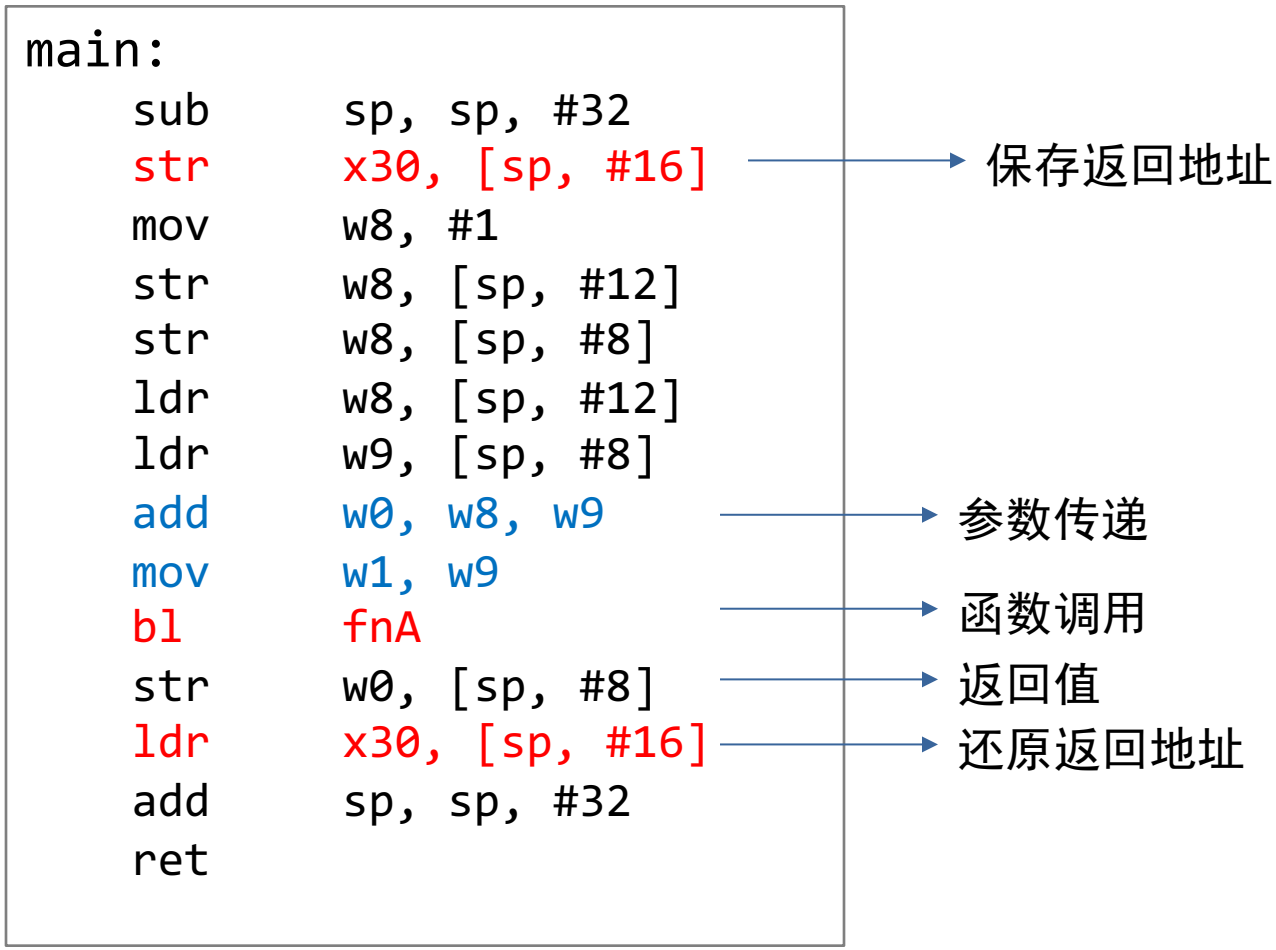



# 三、预分配和溢出

# 函数调用需要遵守的寄存器规则

| aarch64寄存器 | 调用规约 | 注释 |
| --- | --- | --- |
| X0-X7 | 参数1-8 | Caller-saved |
| X0-X1 | 返回值 | Caller-saved |
| X8 | 特殊用途：间接调用返回地址 | Caller-saved |
| X9-X15 | 临时寄存器 | Caller-saved |
| X16-X17 | 特殊用途：Intra-Procedure-Call | Caller-saved |
| X18 | 特殊用途：平台寄存器 | Caller-saved |
| X19-X28 | 普通寄存器 | Callee-saved |
| X29 | 栈帧基指针 | Caller-saved |
| X30 | 返回地址 | Caller-saved |
| SP | 栈顶指针 | Callee-saved |

# 函数调用示例


main:
sub sp, sp, #32
str x30, [sp, #16]
mov w8, #1
str w8, [sp, #12]
str w8, [sp, #8]
ldr w8, [sp, #12]
ldr w9, [sp, #8]
add w0, w8, w9
mov w1, w9
bl fnA
str w0, [sp, #8]
ldr x30, [sp, #16]
add sp, sp, #32
ret
保存返回地址
参数传递
函数调用
返回值
还原返回地址

# 临时寄存器的使用：示例

```
main:
    sub     sp, sp, #32
    str     x30, [sp, #16]
    mov     w8, #1
    str     w8, [sp, #12]
    str     w8, [sp, #8]
    ldr     w8, [sp, #12]
    ldr     w9, [sp, #8]
    add     w0, w8, w9
    mov     w1, w9
    str     w9, [sp, #4]
    str     w8, [sp]
    bl      fnA
    ldr     w8, [sp]
    ldr     w9, [sp, #4]
    add     w10, w8, w9
    str     w10, [sp, #8]
    ldr     x30, [sp, #16]
    add     sp, sp, #32
    ret
```

将临时寄存器入栈

函数调用

将临时寄存器还原

何时用到普通寄存器：X19-X28？

# 参数过多怎么办?

```
%b2 = call i32 @fnB(
     i32 %a2, i32 %b1,
     i32 %a2, i32 %b1,
     i32 %a2, i32 %b1,
     i32 %a2, i32 %b1,
     i32 %a2, i32 %b1 )
```

```
mov     w0, w8
mov     w1, w9
mov     w2, w8
mov     w3, w9
mov     w4, w8
mov     w5, w9
mov     w6, w8
mov     w7, w9
mov     x10, sp
str     w8, [x10]
mov     x10, sp
str     w9, [x10, #8]
bl      fnB
```

# 其它溢出考量因素

- 寄存器不足时应优先溢出哪个虚拟寄存器？
  - 线性统计：代码中出现次数最多的
  - 考虑控制流：代码运行次数最多的
- 目标：最少的溢出次数

# 思考：指令调度可否优化寄存器使用？

- 构造一个程序说明