PHYSICS 258 Lab Manual[1]

Contents

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Lab 6

The Counting Experiment

Learning objectives

We are observing random radioactive decays to explore how various probability distributions describe discrete events. We will investigate Poisson probability distributions and how they changes to Gaussian probability distributions when we shift the mean to large values.

Introduction

Radioactive decay is a random process in which a nucleus decays to its daughter nucleus which has to be at a lower energy level. The most commonly known radioactive decays are α and β decays which are often accompanied by the emission of a γ ray. Physicists have developed various types of detectors to measure the particles emitted in radioactive decays. One of the most common detectors are Geiger counters which are typically used to detect beta particles and gamma rays. Radioactivity can be measured in order to discover the amount of radiation a material emits or the amount of radiation absorbed by a human or mammal. The unit for measuring radioactive emissions is the becquerel (Bq). The Bq indicates the number of decays per second.

In this experiment, we will use ^{137}Cs as the radiation source. Cs-137 randomly emits γ radiation, where each event is independent from each other. The rate at which a sample undergoes radioactive decays is called activity which depends on the number of radioactive nuclei in a sample and the half-life of the source. The isotope ^{137}Cs which we use in this experiment has a half life of 30 years. All radioactive decays are independent of each other, and the half life of the source can be considered constant with respect to the measurement period. Therefore, it is reasonable to model the emission of our γ-ray source with a Poisson distribution.

Theory

The Poisson distribution is a special case of the binomial distribution and appropriate to model discrete events [1]. A Poisson distribution can be assumed when

- when each event is independent from the other,
- and when the average rate of the events does not change significantly during the observation time.

Being that the length scale of this experiment, which is on the order of an hour, is much less than the length scale of the half-life of ^{137}Cs. The Poisson distribution defines the probability

$$P(x;\mu) = \frac{\mu^x e^{-\mu}}{x!} \tag{13}$$

of observing exactly x events in a trial of average rate μ. Unlike the Gaussian distributions, which is defined by two parameters, the Poisson distribution is defined by only one, μ. Conveniently, it can be shown that the mean count of the distribution is defined by μ and the standard deviation is

$$\sigma = \sqrt{\mu}\ . \tag{14}$$

The Poisson distribution is derived from the binomial distribution function, which models the success of an event x with probability p over n measurements [2]. The Binomial distribution is given by

$$P(x) = \frac{n!}{x!\,(n-x)!}\ p^x\ \ (1-p)^{n-x}\ . \tag{15}$$

If the average rate of events is taken as pn, then this function becomes

$$P(x) = \frac{n!}{x!\,(n-x)!}\ \frac{\mu^x}{n^x}\ \left(1-\frac{\mu}{n}\right)^{n-x}\ . \tag{16}$$

Taking the limit of the binomial distribution as n approaches infinity it can be shown that

$$\lim_{n\to\infty} P(x) \approx \frac{\mu^x e^{-\mu}}{x!}\ . \tag{17}$$

Placing a detector close to a γ source, or using a very strong source one can observe the central limit theorem. In other words, for large μ the Poisson distribution approximates to a Gaussian distribution by

$$\lim_{\mu\to\infty} \frac{\mu^x e^{-\mu}}{x!} \approx \frac{1}{\sqrt{2\pi\mu}} e^{\frac{-(x-\mu)^2}{2\mu}}\ , \tag{18}$$

where $\sigma = \sqrt{\mu}$ as with the Poisson distribution.

Experiment

You are referred to [3] to learn about the working principles of a Geiger detector. When a γ ray hits the Geiger tube, the counter responds with a clicking sound on the internal speaker and an output pulse is generated. If several pulses are output the rate of these pulses directly corresponds to the rate of photons hitting the Geiger tube. In this experiment, we will be recording data at different sampling rate, i.e., adjusting the bin width during which we count decays, with various runtime intervals as shown in Table 1.

After each repetition, you should save the PASCO file (ideally you save it under a new name) so you don't accidentally lose the data. You want to record the data in a spreadsheet

Experiment	Runtime	Repetitions (Measurements)	Sampling Rate	Source Position
E1	5 min	3	5 Hz	Position 1
E2	5 min	3	2 s	Position 1
E3	1 min	20	10 Hz	Position 1
E4	5 min	5	2 s	Position 2
Tech.	5400 s	1	5 Hz	≈ 4 cm

Table 1: Run list for counting experiment. Do not change the source position between experiments E1-E3. Increase the distance for experiment E4 according to the instructions provided in the text.

with columns, each column represent one repetition. An example of how a data file should be compiled is shown in Table 2. Once you are done with the repetitions of an experiment, save the PASCO file, export the data to a spreadsheet, and take a look at the spreadsheet to verify that it was saved correctly. After each experiment, delete all the data and start with an empty spreadsheet for the next experiment.

For experiment E4, place the ^{137}Cs source in the clamp and adjust the height until an average of 5-7 clicks per interval is observed. Do not move the source or detector once you are happy with the count rate.

For experiment Tech, the lab technician will set up one measurement with the run conditions outlined in Table 1. Data will be recorded during class and made available on myCourses at the end of the lab session.

Inverse Square Law of Radiation

Use the provided setup to demonstrate the inverse square law of radiation. What are the limitations?

Procedure

- Make sure to record your source number.
- Connect the BNC output of the Geiger counter to the digital input 1 of the PASCO interface. See figure 10 for set-up reference.
- Turn on the counter (switch on the side) and the PASCO interface, then open the appropriate file for this experiment. **Note:** The counter should be clicking at a very low rate when no source is nearby as there is ambient radiation from the environment. Don't forget to measure the background activity and include it in your analysis.
- Before starting the experiment, try to locate the Geiger-Müller tube by placing the source at various locations and observe how the counting rate changes accordingly.

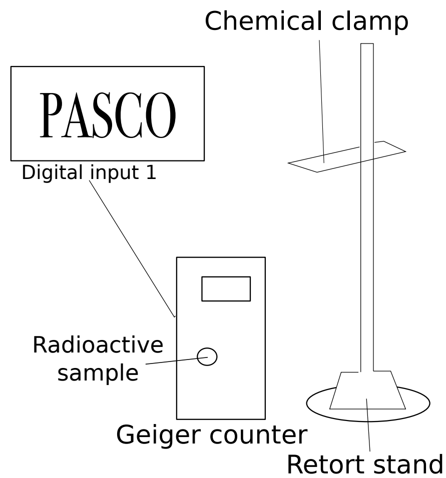

Figure 10: The set up of the experiment. You can put the radioactive source on the chemical clamp to adjust the distance between the source and the detector.

- Start recording data. A histogram will fill and the average rate should be around 2-5 counts/bin . If the rate is larger either flip the source or use the retort stand to increase the distance between source and counter. **Note:** Every time you change the distance you have to stop the data recording, delete the data, and start a new data acquisition process.
- Once you have placed the ^{137}Cs source such that the average count rate is around 2-5 counts per bin you should **not change** the position of neither Geiger counter nor the source for measurements E1 to E3 listed in Table 1. Once you have determined the source position you should delete all data to start with a fresh data file.

Analysis

Don't panic. You have two weeks to complete your lab reports. Before you start, calculate today's activity of your radioactive source. Look up the necessary data in the sky database.

For data analysis we will convert our data tables, which consist of counts per time bin, into histograms. We will then fit Gaussian and Poissonian distributions to the histogram and compare χ^2 and residual plots with each other. Determine the count rate (in Hz or Bq) for each analysis. It is recommended to write python scripts so that you can run different data files with the same script to facilitate data analysis and graph plotting.

Here is how your data should look like in a table, each column represents one repetition:

Data number	Counts/bin R1	Counts/bin R2	Counts/bin R3	Counts/bin $\cdots$	Counts/bin R_i
1	$c_{1,1}$	$c_{2,1}$	$c_{3,1}$	$\cdots$	$c_{i,1}$
2	$c_{1,2}$	$c_{2,2}$	$c_{3,2}$	$\cdots$	$c_{i,2}$
3	$c_{1,3}$	$c_{2,3}$	$c_{3,3}$	$\cdots$	$c_{i,3}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
j	$c_{1,j}$	$c_{2,j}$	$c_{3,j}$	$\cdots$	$c_{i,j}$

Table 2: Sample data file. $c_{i,j}$, where i refers to the column (i.e. repetition 1), j represent the order of data (i.e. the jth data). For example, $c_{2,9}$ refers to the 9^{th} data of the second repetition.

Analysis Experiment E1

- Create a histogram which has "counts per time bin" as your x-axis and the number of occurrence as your y-axis for each repetition. The width of the bin should be 1 unit (i.e. a count-per-bin histogram). Perform a least squares fitting of the Gaussian and Poissonian distributions. Determine individual and the average count rates.
- Add the three data sets, i.e. add $c_{1,1}$, $c_{2,1}$, and $c_{3,1}$, and so on. Create a histogram of the summed data and fit a Gaussian and a Poissonian distributions on the summed data. Determine the count rate.

Analysis Experiment E2

- Create a count-per-bin histogram. Perform a least squares fitting of the Gaussian and Poissonian distributions. Determine the count rate.
- Add the three data sets, i.e. add $c_{1,1}$, $c_{2,1}$, and $c_{3,1}$, and so on. Create a histogram of the summed data and fit a Gaussian and a Poissonian distributions on the averaged data. Determine the count rate.

Analysis Experiment E3

- Create a count-per-bin histogram. Perform a least squares fitting of the Gaussian and Poissonian distributions. Determine the count rate. **These two histograms have to be shown in one or several plots in the main body of the lab report!**
- Add the twenty data sets horizontally, i.e. do $c_{1,1} + c_{2,1} + \ldots + c_{20,1}$. Create a histogram of the summed data and fit a Gaussian and a Poissonian distributions. Determine the count rate. **These two histograms have to be shown in one or several plots in the main body of the lab report!** Compare them to the histograms of the previous paragraph.
- Add the twenty data sets vertically, i.e. do $\sum_j c_{1,j}$, $\sum_j c_{2,j}, \ldots, \sum_j c_{20,j}$. This will give you 20 points. Create a histogram with these 20 values and fit a Gaussian and a Poissonian distribution. Determine the count rate.

Analysis Experiment E4

- Create a count-per-bin histogram. Perform a least squares fitting of the Gaussian and Poissonian distributions. Determine the count rate.
- Add the five data sets, i.e. add $c_{1,1}$, $c_{2,1}$, $c_{3,1}$, $c_{4,1}$, and $c_{5,1}$. Create a histogram of the summed data and fit a Gaussian and a Poissonian distribution. Determine the count rate.

Analysis Experiment Tech.

- Create a count per bin histogram. Perform a least squares fitting of the Gaussian and Poissonian distribution. Determine the count rate.
- Re-bin the data by 2, i.e. combine the consecutive data like $(c_1 + c_2)$, $(c_3 + c_4)$, $(c_5 + c_6)$, $\ldots$. Create a count per bin histogram. Perform a least squares fitting of the Gaussian and Poissonian distributions. Determine the count rate.
- Re-bin the data by 5, i.e. always add five bins such as $(c_1 + \ldots + c_5)$, $(c_6 + \ldots + c_{10})$, $(c_{11} + \ldots + c_{15})$, $\ldots$. Create a count per bin histogram. Perform a least squares fitting of the Gaussian and Poissonian distributions on the binned data. Determine the count rate.
- Re-bin the data by twenty (20), i.e. always add twenty bins such as $(c_1 + \ldots + c_{20})$, $(c_{21} + \ldots + c_{40})$, $(c_{41} + \ldots + c_{60})$, $\ldots$ sum up to a new bin. Create a count per bin histogram. Perform a least squares fitting of the Gaussian and Poissonian distributions on the re-binned data. Determine the count rate.

Here are some examples of how the plots should look like:

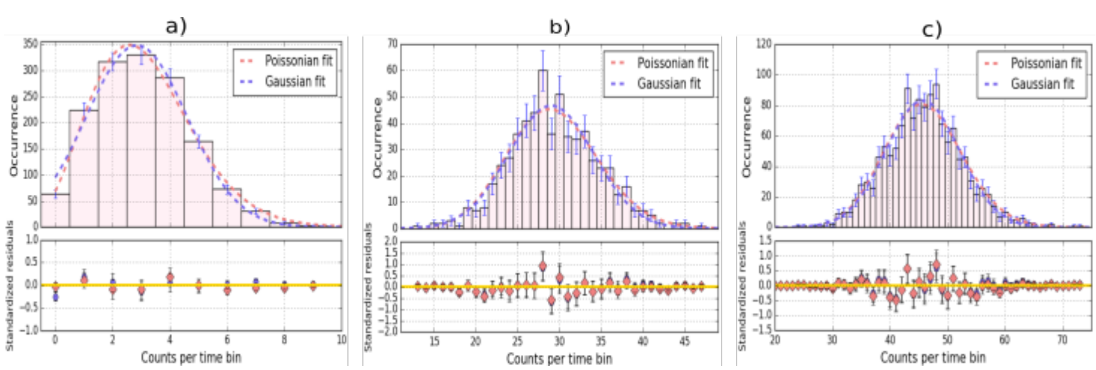

Figure 11: Graph a), b) and c) represents respectively data of E2/R2, E3/horizontal sum and Tech/re-bin by 20. The x-axis is counts per time bin, which depends on the sampling rate of the experiment. The y axis is the frequency of occurrence and the residual plot is standardized.

Required Information on Your Report

Five plots are required to be placed within the body of the lab report:

- Add the plots that are requested in **Analysis Experiment E3**. These should be four plots in total.
- Create a plot showing the count rates that were extracted in experiments E1-E3. How do the uncertainties compare?

References

[1] Hughes I, Hase T. (2010). *Measurements and their Uncertainties*

`Oxford University Press, Chapter 3`

[2] https://stuff.mit.edu/afs/sipb/user/biyeun/Public/8.13/poisson/poisson_statistics_biyeun.pdf, Aug 2016

[3] Knoll, G.F. (2010). *Radiation Detection and Measurement*

`Wiley, 4th edition`