(1)

$\lim_{t\to t^*} x(t) = -\infty$

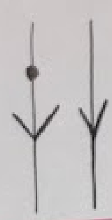
Case: $t^* < \infty$

$\lim_{t\to t^*} x(t) = \pm\infty$

If $+\infty$ $\Rightarrow\Leftarrow$

Because

(1) We have a $f(x)<0$ But If ∞, (positive) then $\Rightarrow\Leftarrow$



$f(x)<0$
It's decreasing
$\forall x \in (-\infty, x]$

If $-\infty$
Then It's ok.

Because

(2) we have a $f(x)<0$. we do have negative Case.

Case: $t^* = \infty$

solution lives on $[0,\infty]$

$\lim_{t\to\infty} x(t) = -\infty$ (3)

Because this Case we have negative.

$|x(t)| \leq M$

Since decreasing and Bound

$\Downarrow$

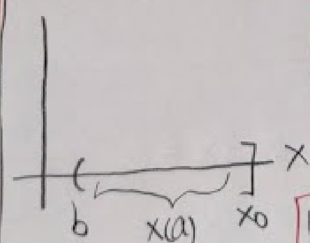
(4)

It's decreasing Because $\dot{x} = f(x) < 0$

$\lim_{t\to\infty} x(t) = b$, b is $b < x(0)$

like professor say.

$\boxed{\exists \delta < 0,\ f(x(a)) < \delta < 0\ \forall a > 0}$.



Case (1).

If $\lim_{t\to\infty} f(x(a)) = 0$

But in our Case. $f(x(a))$ should less than 0. $\Rightarrow$ B equcliBrium.

$\Rightarrow\Leftarrow$ So $f(x(a)) \neq 0$.

Next page proof Continoue

$X(t) = \int_0^t x' \, da$ using fundamental thm of calculus

$X(t) = \int_0^t f(x(a)) \, da + C$

But we know that

$$\int_0^t f(x(a)) \, da + C \leq \int_0^t S \, da + C$$

which $\int_0^t S \, da + C = S(t) + C$

This apply

$$\Rightarrow \lim_{t\to\infty} X(t) \leq \lim_{t\to\infty} S(t) + C$$

Remember $X' = f(x)$ and $f(x) < 0$. It's negative.

$$\Rightarrow \lim_{t\to\infty} X(t) \leq \lim_{t\to\infty} S(t) + C = -\infty$$ S is negative which $-\infty$.

$$\Rightarrow \lim_{t\to\infty} X(t) \leq -\infty$$ ▨

2. $\dot{x} = 2-3x$.

equivaliBria.

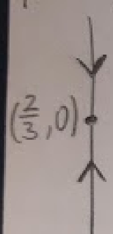
$0 = 2-3x$

$\boxed{x = \frac{2}{3} \text{ is equivliBria}}$

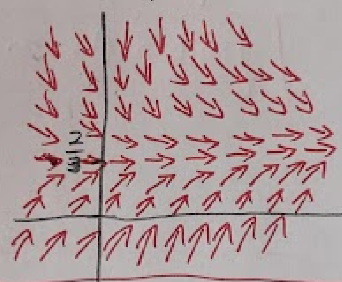
$3x = 2$

$x = \frac{2}{3}$

Phase portrait.



Slope field. $= (1, 2-3x)$



$x > \frac{2}{3} \Rightarrow x(0) > \frac{2}{3}$

$\lim_{t\to t^*} x(t) = \frac{2}{3}$

$x = \frac{2}{3} \Rightarrow x(0) = \frac{2}{3}$

$\lim_{t\to t^*} x(t) = \frac{2}{3}$

$x < \frac{2}{3} \Rightarrow x(0) < \frac{2}{3}$

$\lim_{t\to t^*} x(t) = \frac{2}{3}$

Explain

$x > \frac{2}{3} \Rightarrow x(0) > \frac{2}{3} \quad \lim_{t\to t^*} x(t) = \frac{2}{3}$

$x = \frac{2}{3} \Rightarrow x(0) = \frac{2}{3} \quad \lim_{t\to t^*} x(t) = \frac{2}{3}$

$x < \frac{2}{3} \Rightarrow x(0) < \frac{2}{3} \quad \lim_{t\to t^*} x(t) = \frac{2}{3}$

EquivaliBra. is $\boxed{\text{Sinks}}$

Solve the ODE expelicitly.

$\dot{x} = 2-3x$

$\frac{dx}{dt} = 2-3x$

$\frac{1}{2-3x} dx = dt$

$\int \frac{1}{2-3x} dx = \int dt$

$\ln(2-3x) \cdot \frac{-1}{3} = t + C_1$

$\ln(2-3x) = -3t + C_2$

$2-3x = e^{-3t+C_2}$

$\to 2-3x = e^{C_2} e^{-3t}$

$2-3x = C^3 e^{-3t}$

$-3x = C^3 e^{-3t} - 2$

$x = C_4 e^{-3t} + \frac{2}{3}$

$x(0) \Rightarrow C_4 + \frac{2}{3} = x_0$

$C_4 = x_0 - \frac{2}{3}$

$x(t) = (x_0 - \frac{2}{3}) e^{-3t} + \frac{2}{3}$

Plot solution for $x(0) \in \{-4, -3, -1.5, 3, 4\}$.

$$x(t) = (x_0 - \frac{2}{3})e^{-3t} + \frac{2}{3}$$

$$x(0) = (x_0 - \frac{2}{3})e^{-3(0)} + \frac{2}{3}$$

$$x(0) = x_0 - \frac{2}{3} + \frac{2}{3} = x_0 \text{ , so } x(0) = x_0$$

Now. Case $x(0) = -4$.

$$\boxed{\lim_{t\to\infty} (-4 - \frac{2}{3})e^{-3t} + \frac{2}{3}}$$

This equal to $\frac{2}{3}$

When $\lim_{t\to\infty} e^{-3t} = e^{-\infty} = 0$

so $\lim_{t\to\infty} e^{-3t} = 0$

Case $x(0) = -3$

$$\boxed{\lim_{t\to\infty}(-3 - \frac{2}{3})e^{-3t} + \frac{2}{3} = \frac{2}{3}}$$

Case $x(0) = 3$

$$\boxed{\lim_{t\to\infty}(3 - \frac{2}{3})e^{-3t} + \frac{2}{3} = \frac{2}{3}}$$

Case $x(0) = -1.5$

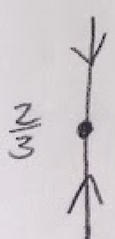
$$\boxed{\lim_{t\to\infty}(-1.5 - \frac{2}{3})e^{-3t} + \frac{2}{3} = \frac{2}{3}}$$

Case $x(0) = 4$

$$\boxed{\lim_{t\to\infty}(4 - \frac{2}{3})e^{-3t} + \frac{2}{3} = \frac{2}{3}}$$

Solution for $x(0) \in \{-4, -3, -1.5, 3, 4\}$ is all $\frac{2}{3}$.

All point are attracting to $\frac{2}{3}$.



for number greater than $\frac{2}{3}$, It's attract to $\frac{2}{3}$ such as (3, 4)

for number smaller than $\frac{2}{3}$, It's attract to $\frac{2}{3}$ such as (-4, -3, -1.5)

3). $\dot{x} = ax(1-x)$ where $a<0$.

equilibria.

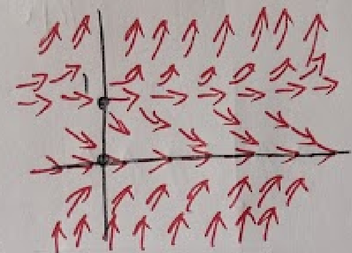
$0 = ax(1-x)$

$0 = ax - ax^2 \Rightarrow a(x-x^2) \Rightarrow (a)(x)(1-x)$.

$\boxed{x=0 \text{ or } x=1}$ equiVliBria.

Slopefield. $= (1, ax(1-x)) \to (1,0)$ or $\to (1,1)$

phase portrait



If $x(0)>1$
$\lim_{t \to t^*} x(t) = \infty$
If $x(0)<0$
$\lim_{t \to t^*} x(t) = 0$
If $0<x(0)<1$
$\lim_{t \to t^*} x(t) = 0$

Exlplain where the solution $x(t)$ when $t \to \infty$.

If $x(0)>1$ $\lim_{t \to t^*} x(t) = \infty$.
If $x(0)<0$ $\lim_{t \to t^*} x(t) = 0$.
If $0<x(0)<1$ $\lim_{t \to t^*} x(t) = 0$.
If $x(0)=0$. $\lim_{t \to t^*} x(t) = 0$.
If $x(0)=1$ $\lim_{t \to t^*} x(t) = 1$

Determine if equilibra are sources or sinks.

If $x = 0$, then x is sink.
If $x = 1$, then x is sources.

Solve (4) explicitly for initial contidition $x(0)=x_0$

$\dot{x} = x^2+1$

$\frac{dx}{dt} = x^2+1$

$dt = \frac{dx}{x^2+1} \Rightarrow \int dt = \int \frac{1}{(x^2+1)} dx$

$t + C_1 = \tan^{-1}(x)$

$x = \tan(t+C_1)$

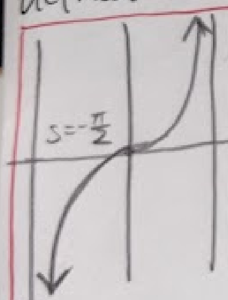
$x(0) \Rightarrow \tan(C_1) = x_0$

$C_1 = \tan^{-1}(x_0)$

$\boxed{x(t) = \tan(t + \tan^{-1}(x_0))}$

Suppose $x(t)$ is a solution of (4) with initial condition $x(0)=x_0$

Find $t*$ such that $[0, t*)$ is the maximal initial where $x(t)$ is defined.



$x(t) = \tan(t+\tan^{-1}(x_0))$

Let $s = t + \tan^{-1}(x_0)$

which $x(t) = \tan(s)$

$S = \frac{\pi}{2}$ and the graph always increasing, and

$\boxed{S = \frac{\pi}{2} \Rightarrow \frac{\pi}{2} = t + \tan^{-1}(x_0) \Rightarrow t* = \frac{\pi}{2} - \tan^{-1}(x_0)}$ step 1

step 2

$\lim_{t \to t^*} x(t) = x(t^*)$

$= \tan(\underbrace{\tfrac{\pi}{2} - \tan^{-1}(x_0)}_{\text{we Assume is } S})$

$= \tan(s)$.

The answer is clear: $[0, \frac{\pi}{2} - \tan^{-1}(x_0))$

Plot the solution.

$x(t) = \tan(t + \tan^{-1}(x_0))$

and $x(0) \Rightarrow \tan(0 + \tan^{-1}(x_0)) \Longrightarrow x(0) = x_0$

Example I use: $x(0) \in \{\frac{\pi}{2}, 2, \pi, 4, \frac{3\pi}{2}\}$

By last Question. I have $[\, 0, \frac{\pi}{2} - \tan^{-1}(x_0))$

but this only if $\boxed{S = \frac{\pi}{2} \Longrightarrow t + \tan^{-1}(x_0)}$

$\boxed{\tan(S) = \tan(t + \tan^{-1}(x_0))}$

$S = \frac{\pi}{2}$

SO S go to $\frac{\pi}{2}$

Because $\lim_{S \to \frac{\pi}{2}} \tan(S) = \infty$

So Example

$$\lim_{t \to \frac{\pi}{2}} \tan(t + \tan^{-1}(\tfrac{\pi}{2})) = \lim_{t \to \frac{\pi}{2}} \tan(t) + \frac{\pi}{2} = \infty$$

$$\lim_{t \to \frac{\pi}{2}} \tan(t + \tan^{-1}(2)) = \lim_{t \to \frac{\pi}{2}} \tan(t) + 2 = \infty$$

$$\lim_{t \to \frac{\pi}{2}} \tan(t + \tan^{-1}(\pi)) = \lim_{t \to \frac{\pi}{2}} \tan(t) + \pi = \infty$$

$$\lim_{t \to \frac{\pi}{2}} \tan(t + \tan^{-1}(4)) = \lim_{t \to \frac{\pi}{2}} \tan(t) + 4 = \infty$$

$$\lim_{t \to \frac{\pi}{2}} \tan(t + \tan^{-1}(\tfrac{3\pi}{2})) = \lim_{t \to \frac{\pi}{2}} \tan(t) + \frac{3\pi}{2} = \infty$$

4. $\dot{x} = x^2+1$

1) equilibria.

$0 = x^2+1$

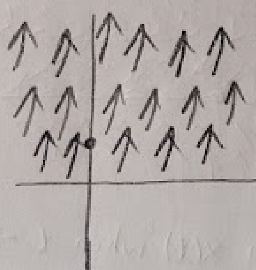
$x^2 = -1$

No equilibria. Because $x^2 \neq -1$.

Phase portrait

alway increasing

Slope field



Explain where the solutions $x(t)$ tend when $t \to t^*$.

$\lim_{t \to t^*} X(t) = \infty$ It's always positive ∞.

The phase portrait and slope field, and the solution I try

It's show all solution are point to ∞.