1. Consider $G(x) = 4x(1-x)$ and its schematic itineraries. Find a point that lies in the subinterval LLR and LLL.

1) The point for LLR: $\frac{1}{8}$

$4(x)(1-x)$, plug in $\frac{1}{8}$ ($\frac{1}{8}$ in L)

$= 4(\frac{1}{8})(1-\frac{1}{8}) = (\frac{1}{2})(\frac{7}{8}) = \frac{7}{16}$ ($\frac{7}{16}$ is left)

$\Rightarrow 4(\frac{7}{16})(1-\frac{7}{16}) = (\frac{7}{4})(\frac{9}{16}) = \frac{63}{64}$ ($\frac{63}{64}$ is a right)

Therefore; $\frac{1}{8}$ is a point for the subinterval LLR.

2) The point for LLL: $\frac{1}{30}$

$4x(1-x)$ plug in $\frac{1}{30}$ ($\frac{1}{30}$ in L)

$= 4(\frac{1}{30})(1-\frac{1}{30}) = (\frac{2}{15})(\frac{29}{30}) = \frac{58}{450}$ ($\frac{58}{450}$ is left)

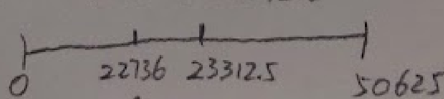
plug in $\frac{58}{450}$

$\Rightarrow 4(\frac{58}{450})(1-\frac{58}{450}) = (\frac{232}{450})(\frac{392}{450}) = (\frac{90944}{202500})$

It's a L because $= (\frac{45472}{101250})$

$50625 \div 2 = 25312.5 \rightarrow = (\frac{22736}{50625})$

$22736 < 25312.5$



Left side.

Therefore; $\frac{1}{30}$ is a point for subinterval LLL.

2) Let x_0 be a point in the Subinterval RLLRRRLRLR. Answer and Explain the following.

a) Is x_0 less than, equal to or greater than $1/2$.

$x_0 \in R$.

Since x_0 is belong R. we need to check

x_0

$\swarrow \quad \downarrow \quad \searrow$

L $\quad$ = $\quad$ R

Less than $\frac{1}{2}$ $\quad$ equal to $\frac{1}{2}$ $\quad$ greater than $\frac{1}{2}$

Because $x_0 \in R$, which is not less than $\frac{1}{2}$.

Only need to check x_0 is

① equal to $\frac{1}{2}$. (It could be)

② greater than $\frac{1}{2}$.

Check equal to $\frac{1}{2}$ (The reason is $\frac{1}{2}$ is also R).

$f(x) = 4x(1-x)$

$f(\frac{1}{2}) = 4(\frac{1}{2})(1-\frac{1}{2})$

$= 2 \cdot (\frac{1}{2}) = 1$

But 1 is on the Right side.

our Subinterval for $f(x_0) =$ RL.

It's belong to Left side,

Which equal to $\frac{1}{2}$ it also not correct.

Therefore; x_0 is greater than $\frac{1}{2}$.

b) $f^6(x_0)$ is less than, equal or greater than $\frac{1}{2}$.

$f^6(x_0) =$ LRLR

$f^6(x_0)$

$\swarrow \quad \downarrow \quad \searrow$

L $\quad$ = $\quad$ R

Because the first character is L, which R is not the case.

proof on next page.

now we test the equal to $\frac{1}{2}$. Because it could be the case.

$4x(1-x)$, plug in $\frac{1}{2}$. and $\frac{1}{2}$ is a L and R, continoue.

$\Rightarrow 4(\frac{1}{2})(1-\frac{1}{2}) = (\frac{4}{2})(\frac{1}{2}) = 1$. It's a R, Continoue.

$\Rightarrow 4(1)(1-1) = 4(0) = 0$. It's a L, Continoue

$\Rightarrow 4(0)(1-0) = 0$, but $f^6(x_0) = LRLR$, which last one is R.
0 should be L. therefore; equal is not the case

$\boxed{f^6(x_0) \text{ is Less than } \frac{1}{2}}$

Extra challenge. $f^5(x_0)$:

$f^5(x_0) = RLRLR$.

$f^5(x_0)$

L e R

Because It's ER, which L is not the case

$\Rightarrow$ check equal case

$\Rightarrow 4(\frac{1}{2})(1-\frac{1}{2}) = 1$. It's a R, but second character is L.

which only R, the $f^5(x_0)$ is greater than $\frac{1}{2}$.

```
#function for question number 3
F(x, y) = [x^2 - 5x + y, x^2]
γ2(t) = [0.2*cos(t), 0.2*sin(t)]
parametric_2d(F, 3, γ2, 0:0.02:(2*π))

F(x, y) = [x^2 - 5x + y, x^2]
γ4(t) = [0.2*cos(t), 0.2*sin(t)] + [3,9]
parametric_2d(F, 3, γ4, 0:0.02:(2*π))

#function for question number 4
F(x, y) = [-x^2 - 0.5 *y - 0.56, x] | F (generic function with 1 method)
γ(t) = [0.001*cos(t), 0.001*sin(t)] - [0.7, 0.7] | γ (generic function with 1 method)
parametric_2d(F, 10, γ, 0:0.02:(2*π)) | Figure()


F(x, y) = [-x^2 - 0.5 *y - 0.56, x] | F (generic function with 1 method)
γ1(t) = [0.001*cos(t), 0.001*sin(t)] - [0.8, 0.8] | γ1 (generic function with 1 method)
parametric_2d(F, 10, γ1, 0:0.02:(2*π)) | Figure()
```

The code for question number 3 and 4 in HW3

Find the Fixed point of $g(x,y) = (x^2-5x+y, x^2)$

$y = x^2$, $x = x^2-5x+y$

$\Rightarrow x = x^2-5x+x^2$

$x = 2x^2-5x$

$0 = 2x^2-6x$

$0 = (2x)(x-3)$

$x=0$ or $x=3$

Fixed point

when $x=0$, $y=0$ or $x=3$, $y=9$

For Fixed point $(0,0)$

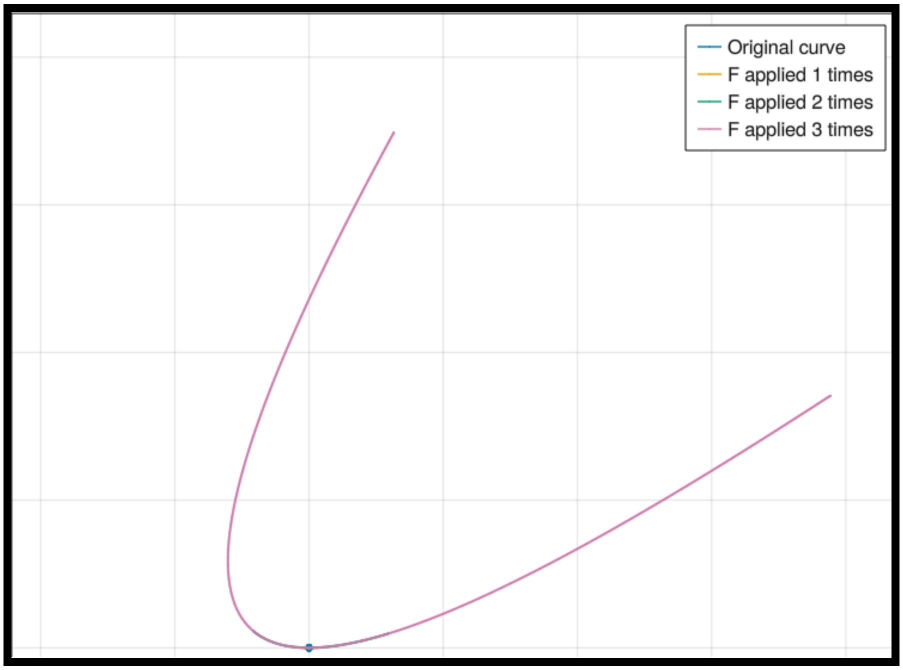
The stablility is saddle.

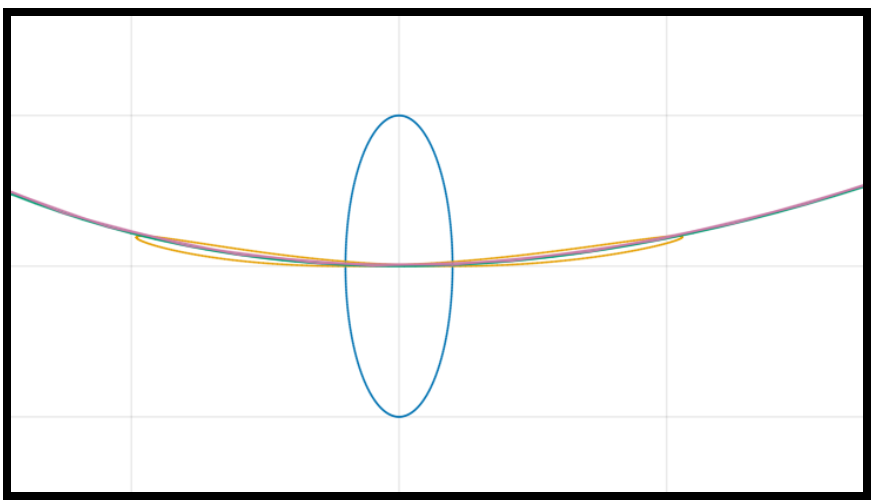
Because the left side and right side direction are repelling and the up (down) side are getting closer. when F applied 3 times. this case more show. has atleast one attracting and one repelling.

For Fixed point $(3,9)$

The stablilty is repelling

Because (After F applied 3 times) we can see the side direction are trying to leave away, which it's a repelling.





The left side and right side are repelling, which also shows the left side's direction and right side's direction are leaving far away. It's also interesting that the up and down sides are attracting because they get closer. To sum up! The graph shows (When F applied 3 times) has at least one attracting direction and at least one repelling direction.Therefore, stability is saddle.

Find the Fixed poine of $g(x,y) = (x^2-5x+y,\ x^2)$

$y = x^2$, $x = x^2-5x+y$

$\Rightarrow x = x^2-5x+x^2$

$x = 2x^2-5x$

$0 = 2x^2-6x$

$0 = (2x)(x-3)$

$x=0$ or $x=3$

Fixed point

when $x=0$, $y=0$ or $x=3$, $y=9$

For Fixed point $(0,0)$

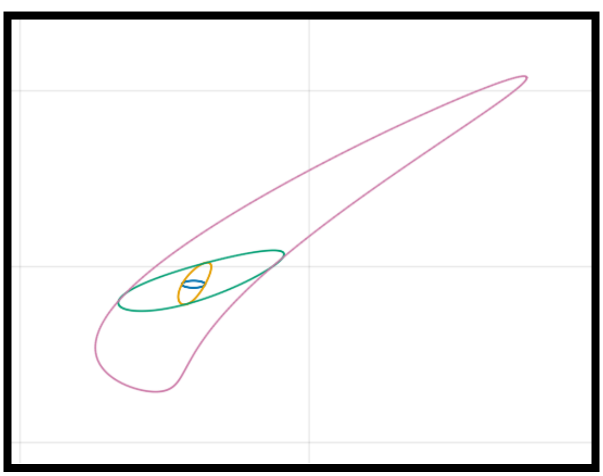
The stablitity is saddle.

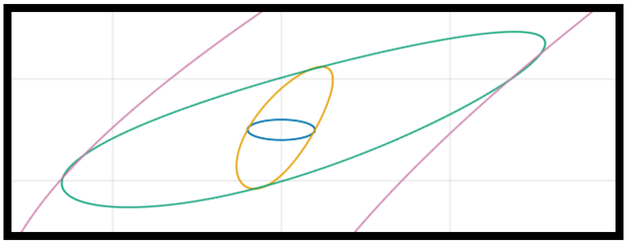
Because the left side and right side direction are repelling and the up (down) side are getting closer. when F applied 3 times, this case more show. has atleast one attracting and one repelling.

For Fixed point $(3,9)$

The stablilty is repelling

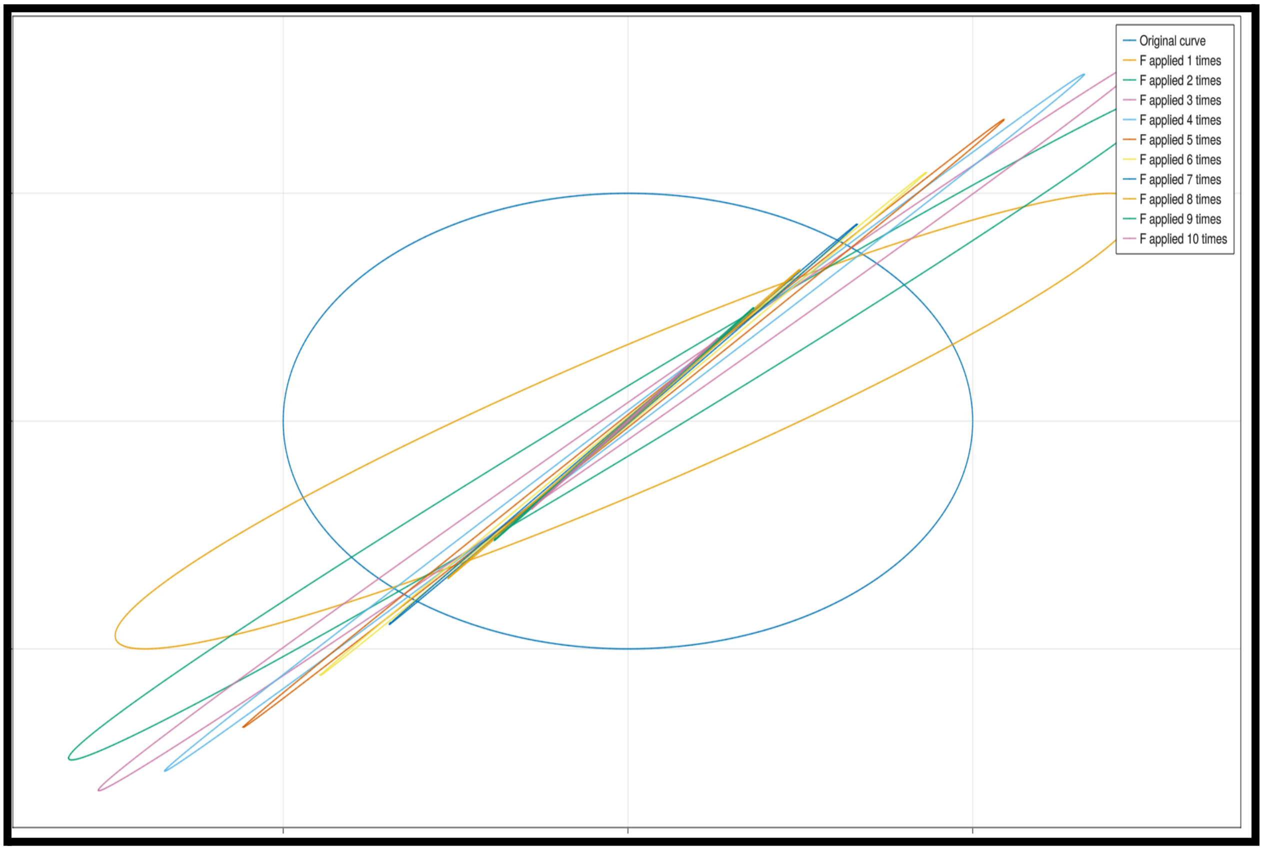
Because (After F applied 3 times) we can see the side direction are trying to leave away, which it's a repelling.





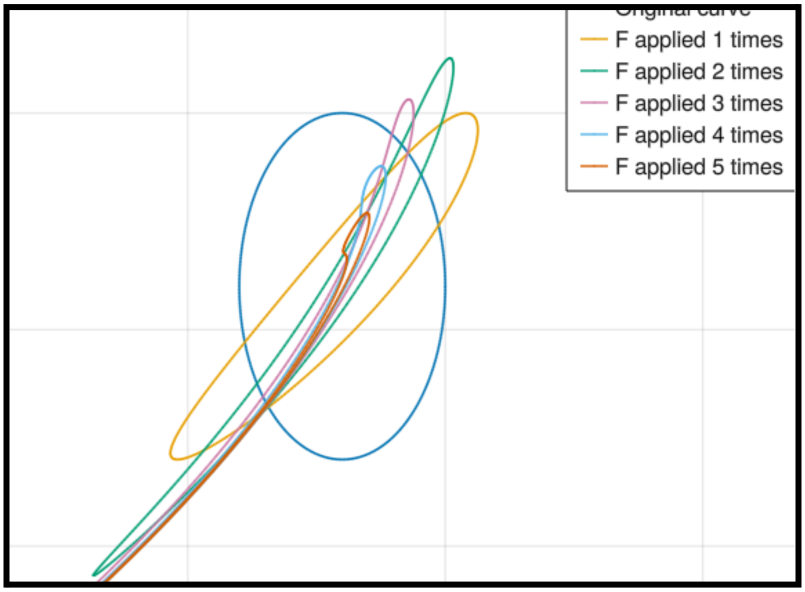
The stability for this graph is repelling because when graph F is applied 3 times, we can see that all the side's direction are trying to leave away. To sum up! The stability has more than one repelling direction, which is a repelling.

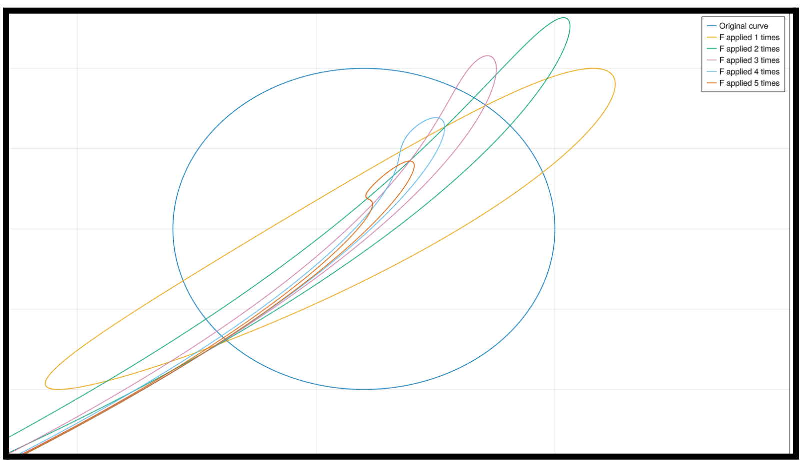
(-0.7,-0.7)

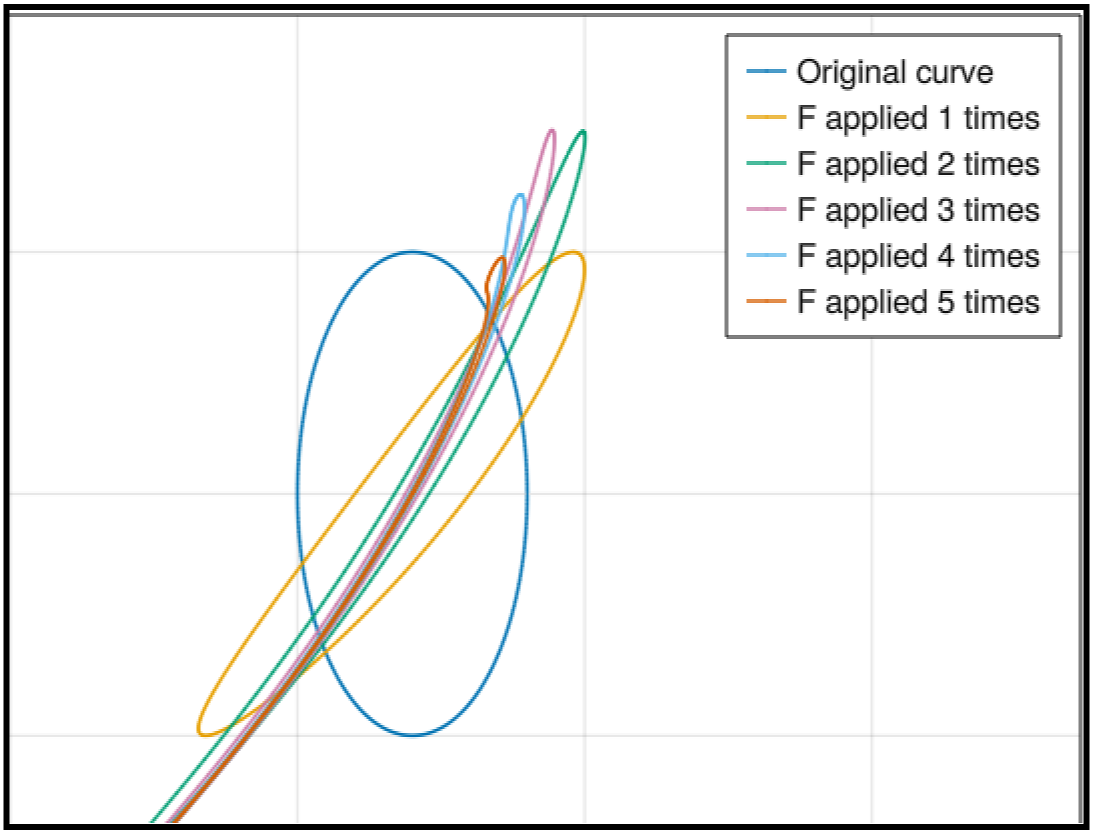


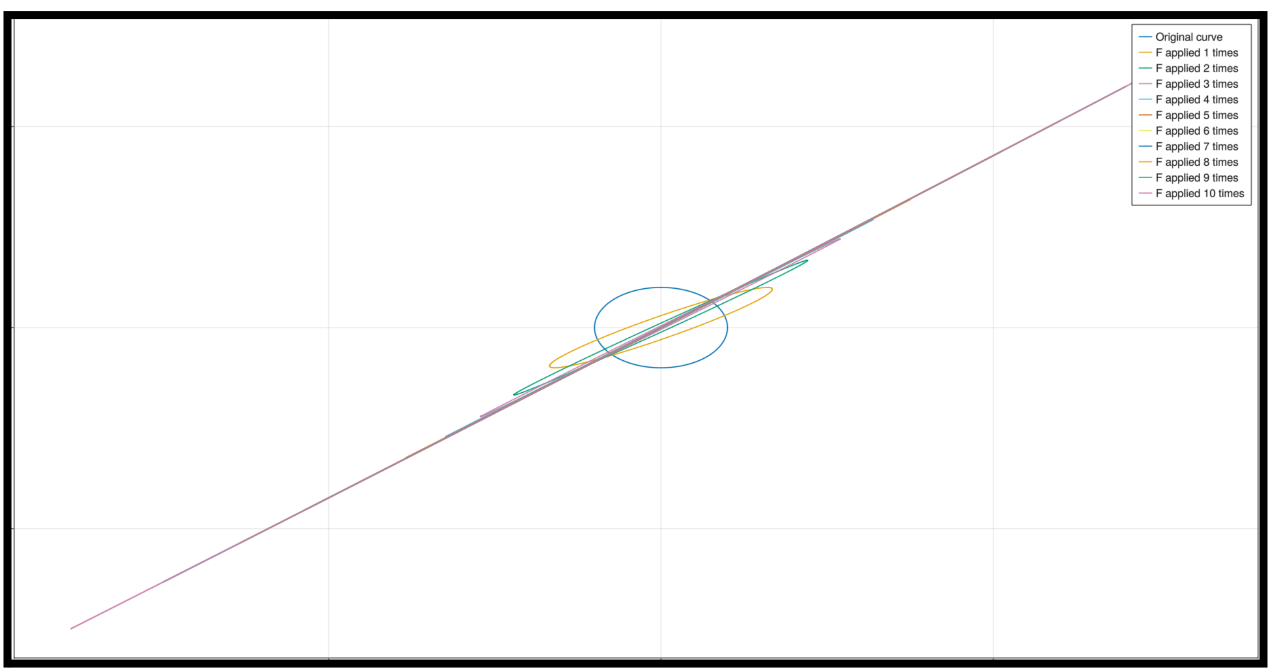
As you can see in the graph, The upper left side and lower right side seem like saddle points at first, but after applying more times, it shows it's attractive later. To sum up! The graph shows (When F applied 10 times) , it shows the direction for the upper left side and lower right side are getting close to each other when we applied more times. Therefore, stability is a attracting and it's a sink. **Proof with Professor in the office hour**

(-0.8,-0.8)


F applied 1 times
F applied 2 times
F applied 3 times
F applied 4 times
F applied 5 times


Original curve
F applied 1 times
F applied 2 times
F applied 3 times
F applied 4 times
F applied 5 times


Original curve
F applied 1 times
F applied 2 times
F applied 3 times
F applied 4 times
F applied 5 times


Original curve
F applied 1 times
F applied 2 times
F applied 3 times
F applied 4 times
F applied 5 times
F applied 6 times
F applied 7 times
F applied 8 times
F applied 9 times
F applied 10 times

For Fixed point (-0.8, -0.8)

Based on my observed, The stability is a saddle.

Reason: As we can see in the image above, It look almost the Same like (-0.7,-0.7). The upperleft and lower Right direction are repelling. But other direction are attacting.

The upper left side and lower right side are repelling, which also shows the upper left side's direction and lower right side's direction of F are leaving away. It's also interesting that the other sides are attracting because they get closer. To sum up! The graph shows (When F applied 5 times) has at least one attracting direction and at least one repelling direction. Therefore, stability is a saddle.