Consider the logistic map $G(x) = 4x(1-x)$. Give an itinerary of period 6 point where the initial condition is the Right intervel $R = [\frac{1}{2}, 1]$. Prove that the Given itinerary indeed belongs to period 6 points

Fixed point $4x(1-x) = x$ $\Rightarrow x = 0$, $\Rightarrow x = \frac{3}{4}$

$4(1-x) = 1$

$4 - 4x = 1$

$4x = 3$

Since $R \subseteq f^6(R)$ apply theorem f^6 Has a fixed point.

WTS: periodic 6 points

RLLLRLL An Example from class, but

$k=1$, $G(x) = x$; $x \in L$ $\quad G(x) = \frac{1}{2}$

$G(x) \in L$, $G(x) \in R$ $\Rightarrow x = \frac{1}{2} \Rightarrow G(\frac{1}{2}) = 1 \Rightarrow\Leftarrow$

RLLLRL

$k=2$ $G^2(x) \in L$ and R $\quad G^2(x) = \frac{1}{2} \Rightarrow x = \frac{1}{2}$ $\quad G^2(\frac{1}{2}) = G(1) =$

Since $R \subseteq f^6(R)$ apply theorem. f^6 has a fixed point.

It's a period 6 orbits

2. Let $f: \mathbb{R} \to \mathbb{R}$ be continous map with a fixed point b. prove that if b

$b < f(x) < x$ for all $x \in (b, c]$, then:

$$\lim_{k\to\infty} f^k(c) = b. \qquad \text{WTS.}$$

proof: $f(b) = b$, $\quad b < f(x) < x, \ \forall x \in (b, c]$

$$\begin{aligned} x_k &= c \\ x_{k-1} &= f^k(c) \\ x_{k-2} &= f^{k-1}(c) = f(x_{k-1}) \\ &\vdots \\ x_0 &= f(c) \Rightarrow f(x_1) \end{aligned}$$

(a) Limit exists

$c = b < f(x_k) < x_k$

$b < x_{k-1} < x_k \Longrightarrow x_{k-1} \in (b, c]$

Apply Aagin: x_{k-1} is $b < f(x_{k-1}) < x_{k-1}$

$b < x_{k-2} < x_{k-1} \Longrightarrow x_{k-2} \in (b,$

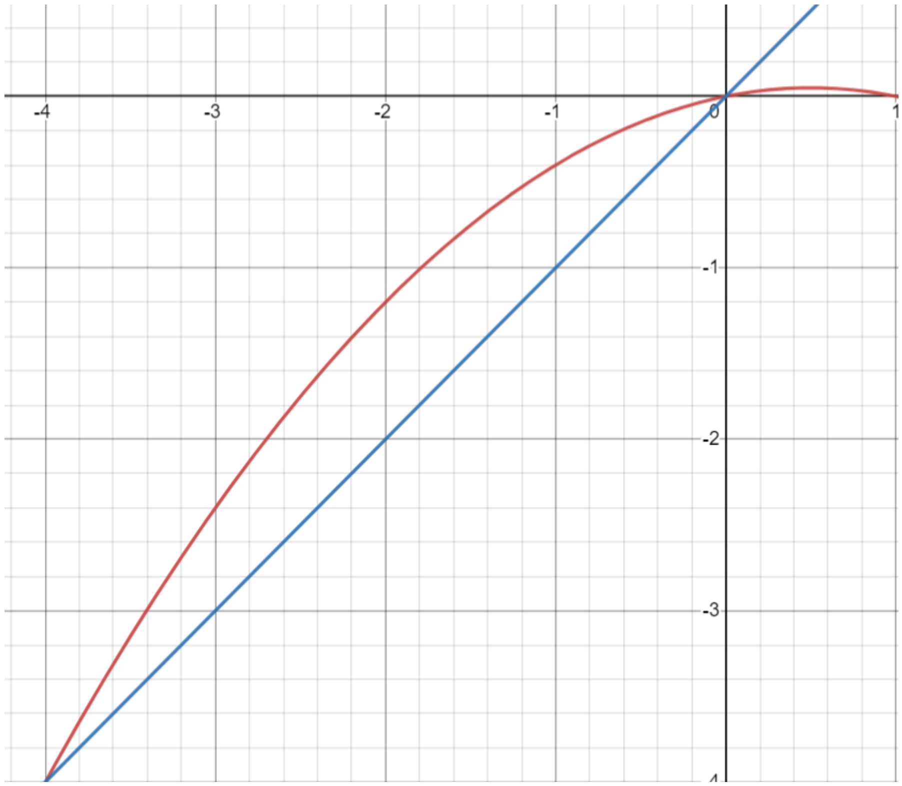
$c = b < x_0 < x_1 \ldots < x_{k-2} < x_{k-1} < x_k$

x_k is decreasing sequence, It's diverge.

(b) $x^* = b$?

$$\begin{aligned} x^* = \lim_{k\to\infty} (x_{k-1}) &= \lim_{k\to\infty} f(x_{k-2}) \\ &= f(\lim_{k\to\infty} x_{k-2}) \\ &= f(x^*) \end{aligned}$$

x^* is a fixed point $= [b, c)$.



-4
-3
-2
-1
0
1
-1
-2
-3

0.5
-0.5
0
0.5
-0.5

Let $f(x) = 0.2x(1-x)$ be a map $\mathbb{R}$. Using Theorem 3.23 of the textbook and a Cobweb analysis, identify the Basin of the sink 0. Explain it and print screen capture of your analysis.

Recall them 3.23.

① If $f(b)=b$ and $x<f(x)<b$ for $\forall\, x$ in $[a,b)$, then $f^k(a) \to b$

② If $f(b)=b$ and $b<f(x)<x$ for $\forall\, x$ in $(b,c]$, then $f^k(c) \to b$

$f(x)=0.2x(1-x)$

Fixed point

$0.2x(1-x)=x$

$0.2(1-x)=1$

$0.2-0.2x=1$

$-0.2x=0.8$

$x=-4$ or $x=0$

$f'(x)=0.2-0.4x$

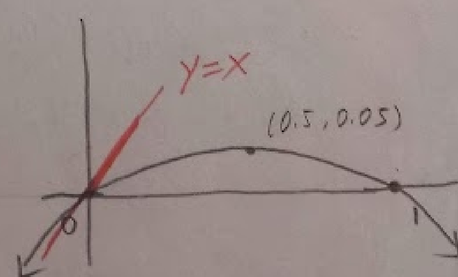
$f'(0)=0.2<1$ $\boxed{0 \text{ is Sink. attracting}}$

only focus on 0 Because is attracting

$f'(-4)=0.2-0.4(-4)$

$=0.2+1.6$

$=1.8>1$ -4 is source



analyze (0) $(-4,0] \subseteq$ Basin(0)

$[0,1) \subseteq$ Basin(0)

From theorem, basin(0) = $(-4,0] \cup [0,1)$.

Show that if f and g have negative Schwarzian, then $f \circ g$ has negative Schwarzian.

$$S(f)(x) = \frac{f'''(x)}{f'(x)} - \frac{3}{2}\left(\frac{f''(x)}{f'(x)}\right)^2$$

Given f and g have negative schwarzian.

Let use an example in class. Assume $f = ax(1-x)$

$$\left.\begin{aligned} f(x) &= ax - ax^2 \\ f'(x) &= a - 2ax \\ f''(x) &= -2a \\ f'''(x) &= 0 \end{aligned}\right\} \text{ so } \frac{f'''(x)}{f'(x)} - \frac{3}{2}\left(\frac{f''(x)}{f'(x)}\right)^2 \Rightarrow -\frac{3}{2}\left(\frac{-2a}{a-2ax}\right)^2$$

No matter what we use for a. It's alway negative, so f is negative Schwarzian

Same idea as g is negative schwarzian

Example

$$f \circ g \Rightarrow a(ax(1-x))(1-(ax(1-x)))$$

$$\Rightarrow a(ax-ax^2)(1-(ax-ax^2))$$

$$\Rightarrow (a^2x - a^2x^2)(ax^2 - ax + 1)$$

$$\Rightarrow 2a^3x^3 - a^3x^2 + a^2x - a^3x^4 - a^2x^2$$

$$\Rightarrow f'(x) = 6a^3x^2 - 2a^3x + a^2 - 4a^3x^3 - 2xa^2$$

$$\Rightarrow f''(x) = 12a^3x - 2a - 12a^3x^2 - 2a^2$$

$$\Rightarrow f'''(x) = 12a^3 - 24a^3x$$

$$a(6ax - 6ax^2 - 1) - 1$$

$$= \frac{12a(1-2x)}{6ax^2 - 2ax + 1 - ax^4 - x^2} - \frac{3}{2}\left(\frac{6a^2x - 1 - 6a^2x^2 - a}{a(6ax^2 - 2ax + 1 - ax^4 - x^2)}\right)$$

$$a(6x^2 - 2x + 1 - x^4) - x^2 \qquad a(a(6x^2 - 2x + 1 - x^4) - x^2))$$

$$\frac{12(1-2x)}{(6x^2-2x+1-x^4)-x^2} - \frac{3}{2}\left(\frac{a(6ax-6ax^2)-1}{a(a(6x^2-2x+1-x^4))-x^2}\right)$$

$$\Rightarrow -\frac{3}{2}\left(\frac{(6ax-6ax^2)-1}{a(6x^2-2x+1-x^4)-x^2}\right) \Rightarrow -\frac{3}{2}\left(\frac{(6x-6x^2)-1}{(6x^2-2x+1-x^4)-x^2}\right)$$

It's also always negative. which fog is negative.

General Case.

$$S(f\circ g)(x) = \frac{(f\circ g)'''(x)}{(f\circ g)'(x)} - \frac{3}{2}\left(\frac{(f\circ g)''(x)}{(f\circ g)'(x)}\right)$$

$(f\circ g)' = f'(g(x))g'(x)$

$f\circ g)'' = (f'(g(x))g'(x))' = f''(g(x))[g'(x)]^2 + f'(g(x))g''(x)$

$\Rightarrow [f'(g(x))]'g'(x) + f'(g(x))g''(x)$

$\Rightarrow f''(g(x))g'(x)g'(x) + f'(g(x))g''(x)$

$\Rightarrow f''(g(x))[g'(x)]^2 + f'(g(x))g''(x)$

$([g'(x)]^2)'$
$= 2g'(x)g'(x)$

$(f\circ g)''' = (f''(g(x))(g'(x))^2 + f'(g(x))g''(x))'$

$\Rightarrow (f''(g(x)))'([g'(x)]^2)' + (f'g(x))'g''(x) + f'(g(x))g'''(x)$

$\Rightarrow f'''(g(x))[g'(x)]^3 + 2f''(g(x))g'(x)g''(x) + f''(g(x))g'(x)g''(x)$
$+ f'(g(x))g'''(x)$

$$S(f\circ g)(x) = \frac{(f\circ g)'''(x)}{(f\circ g)'(x)} - \frac{3}{2}\left(\frac{(f\circ g)''(x)}{(f\circ g)'(x)}\right)$$

$$\Rightarrow \frac{f'''(g(x))(g'(x))^3 + 3f''(g(x))g'(x)g''(x) + f'(g(x))g'''(x)}{f'(g(x))g'(x)}$$

$$\Rightarrow -\frac{3}{2}\left(\frac{f''(g(x))[g'(x)]^2 + f'(g(x))g''(x)}{f'(g(x))g'(x)}\right.$$

$$\frac{3}{2}\left(\frac{f''(g(x))g'(x)}{f'(g(x))} + \frac{g''(x)}{g'(x)}\right)$$

$$\frac{f'''(g(x)(g'(x))^2}{f'(g(x))} + \frac{3f''(g(x))g''(x)}{f'g(x)} + \frac{g'''(x)}{g'(x)}$$

$$\Rightarrow \frac{f'''(g(x))(g'(x))^2 + 3f''(g(x))g''(x}{f'(g(x)} + \frac{g'''(x)}{g'(x)}$$

$$\frac{f'''g(x)(g'(x)) + \frac{3}{2} f''\left(\frac{g''(x)g(x)}{g'(x)} - g(x)\right) -}{f'(g(x))}$$

$$\frac{f'''(g(x))(g'(x))^2}{f'(g(x))} + \frac{3f''(g(x))g''(x)}{f'(g(x))} - \frac{3}{2}\left(\frac{f''(g(x))g'(x)}{f'(g(x))}\right)$$

$$+ \boxed{\left(-\frac{3}{2}\left(\frac{g''(x)}{g'(x)}\right)\right) + \frac{g'''(x)}{g'(x)}} \Rightarrow$$ become $S(g)(x)$

$f \circ g$ is negative schwarizan

the answer $g^2(x)\, S(f)(x) \cdot S(g)(x)$

therefore. when negative.

$$\Rightarrow \frac{f'''(g(x))(g'(x))^2 +}{f'(g(x))} \square + S(g)(X)$$

$$\frac{6f''(g(x))g''(x) - 3(f''(g(x))g'(x)}{2(f'(g(x))}$$

$$\Rightarrow \frac{(3)(f''(g(x))(2g''(x) - g'(x))}{2(f'(g(x)))}$$

$(2(g''(x)) - g'(x))$

$$\Rightarrow \left(-\frac{3}{2}\right)\left(\frac{f''(g(x))(2(g'(x)))}{f'(g(x))}\right)$$

$$\Rightarrow \left(-\frac{3}{2}\right)\left(\frac{f''(g(x))(-2g''(x) + g'(x))}{f'(g(x))}\right)$$

Let A be $f'(g(x))$

$$\frac{f'''(g(x))(g'(x))^2 + 3f''(g(x))g''(x) - \frac{3}{2}(f''(g(x))g'(x)}{A}$$

$$\Rightarrow \frac{f'''(g(x))(g'(x))^2 + f''(g(x))(3g''(x) - \frac{3}{2}g'(x))}{A}$$

$$\Rightarrow \frac{f'''(g(x))(g'(x))^2 + f''(g(x))(-\frac{3}{2})(-2g''(x) + g'(x))}{A}$$

$$\Rightarrow \boxed{\frac{f'''(g(x))}{A} + \frac{f''(g(x))}{A}\left(-\frac{3}{2}\right)\left(\frac{-2g''(x)}{(g'(x))^2} + \frac{g'(x)}{g'(x))^2}\right)} \Rightarrow \frac{-2g''(x) + g'(x)}{(g'(x))^2}$$

$$\boxed{S(f)(X) \cdot g^2(X)}$$