№11

$H_0: \xi \sim p_0(x) = 1 \{0,1\}$

$H_1: \xi \sim p_1(x) = \frac{e}{e-1} e^{-x} \{0,1\}$

а) $n=1$ $\qquad \alpha < 1$

$$\frac{L_1}{L_0} = \frac{\frac{e}{e-1} e^{-x}}{1} \geq c - const$$

$e^{-x} \geq B$

$x \leq A$ //

$G: x \leq \alpha$

$$P(x \leq A \mid H_0) = \alpha = \int_0^A dx = A \Rightarrow \alpha_1 = \alpha$$ //

$$W = P(x \leq A \mid H_1) = \int_0^{\alpha} \frac{e}{e-1} e^{-x} dx = \frac{e}{e-1}(1 - e^{-\alpha})$$ //

$$\alpha_2 = 1 - \frac{e}{e-1}(1 - e^{-\alpha})$$ //

б) $n=2$

$$\frac{L_1}{L_0}=\frac{\left(\frac{e}{e-1}\right)^2 e^{-x_1} e^{-x_2}}{1\cdot 1} \geq c$$

$$x_1+x_2 \leq A$$

$$P(x_1+x_2 \leq A \mid H_0)=\iint\limits_{x_1+x_2\leq A} dx_1\,dx_2=\frac{A^2}{2}=\alpha$$

$$A=\sqrt{2\alpha},\quad G: x_1+x_2\leq\sqrt{2\alpha}$$

$$\alpha_1=\alpha$$

$$W=P(x_1+x_2\leq A \mid H_1)=\iint\limits_{x_1+x_2\leq A}\left(\frac{e}{e-1}\right)^2 e^{-x_1-x_2}dx_1\,dx_2=$$

$$=\ldots=\left(\frac{e}{e-1}\right)^2\left(1-e^{-A}-Ae^{-A}\right)$$

$$\alpha_2=1-W$$

c):

$$l = \frac{L_1}{L_0} = \prod_i \frac{P_1(x_i)}{P_0(x_i)} \geq c$$

$$G: \ln l = \sum \underbrace{\ln \frac{P_1(x_i)}{P_0(x_i)}}_{\eta_i} \geq \ln c$$

$$\frac{\sum \eta_i - n M_{\eta_i}}{\sqrt{n D_\eta}} \rightsquigarrow N(0,1)$$

(для один. распр)

$$P(\ln l \geq \ln c \mid H_0) = \alpha$$

$$\eta = \ln \frac{e}{e-1} - x$$

$$\ln l = \sum \ln \left(\frac{e}{e-1} - x_i\right) \geq \ln c$$

$$G: \sum x_i \leq A$$

$$P\left(\frac{\sum x_i - n M[x]}{\sqrt{n D x}} \leq \frac{A - n M[x]}{\sqrt{n D x}} \mid H_0\right) = \alpha$$

(норм. распред)

$$\frac{A - n\cdot\frac{1}{2}}{\sqrt{n\cdot\frac{1}{12}}} = u_\alpha \qquad G: \sum x_i \le \frac{n}{2} + u_\alpha\sqrt{\frac{n}{12}}$$

$$A = \frac{n}{2} + u_\alpha\sqrt{\frac{n}{12}}$$

$$W = P\left(\sum x_i \le A \mid H_1\right) = \cancel{P(\ldots)}$$

$$= P\left(\frac{\sum x_i - nM[x]}{\sqrt{nDx}} \le \frac{A - nM[x]}{\sqrt{nDx}} \,\Big|\, H_1\right) =$$

$$M[x] = \int_0^1 x\,\frac{e}{e-1}e^{-x}\,dx = \frac{e-2}{e-1}$$

$$M[x^2] = \cancel{\int} \cdot \frac{2e-5}{e-1}$$

$$D_B[x] = \frac{e^2-3e+1}{(e-1)^2}$$

$$= \int_{-\infty}^{B} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\,dx, \quad \alpha_2 = 1-W$$

$$B = \frac{\frac{n}{2} + u_\alpha\sqrt{\frac{n}{12}} - n\frac{e-2}{e-1}}{\sqrt{n\frac{e^2-3e+1}{(e-1)^2}}} \xrightarrow[n\to\infty]{} \infty$$

$\Rightarrow$ критерий сост.//

d) $G: x_{min} < c$

$P(x_{min} < c | H_0) = \alpha \Leftrightarrow$

$\Leftrightarrow 1-(1-F(c))^n = \cancel{1-(1-c)^n}\ 1-(1-c)^n$

$c = 1-(1-\alpha)^{\frac{1}{n}}$

$G: x_{min} < 1-(1-\alpha)^{\frac{1}{n}}$

$\alpha_1 = \alpha$

$W = P(\vec{X}_n \in G | H_1) = P(x_{min} < c | H_1) =$

$= 1-\left(1-\frac{e}{e-1}(1-e^{-c})\right)^n = 1-\left(1-\frac{e}{e-1}\left(1-e^{-1+(1-\alpha)^{\frac{1}{n}}}\right)\right)$

$=1-\left(\frac{e-1-e\left(1-e^{-1+(1-\alpha)^{\frac{1}{n}}}\right)}{e-1}\right)^n =$

$= 1-\left(\frac{-1+e^{(1+\alpha)^{\frac{1}{n}}}}{e-1}\right)^n =$

$= 1\left(\frac{e-1+e\frac{1}{n}\ln(\alpha+1)+o(\frac{1}{n^2})}{e-1}\right) \underset{n\to\infty}{\not\longrightarrow} 1 \Rightarrow$

$NT12$

$n=2$

	P_1	P_2	P_3	P_4
H_0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{3}$
H_2	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$\alpha=0,2$

$H_0: P_i \cdot P_j$

$H_1: P_i \cdot P_j = \frac{1}{16}$

	1	2	3	4
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{24}$	$\frac{1}{12}$
2	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{24}$	$\frac{1}{12}$
3	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{36}$	$\frac{1}{18}$
4	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{9}$

$L: \frac{L_0}{L_1}$

	1	2	3	4
1	1	1	$\frac{3}{2}$	$\frac{3}{4}$
2	1	1	$\frac{3}{2}$	$\frac{3}{4}$
3	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{9}{4}$	$\frac{9}{8}$
4	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{9}{8}$	$\frac{9}{16}$

№13

$n_1 = 139$ $n_2 = 1000$

$\Delta t = 8$мм ← очень важно

$\sigma_{1g} = 5,722$ мм

$\sigma_{1ш} = 4,612$ мм

$\sigma_{2g} = 6,161$ мм

$\sigma_{2ш} = 5,055$ $\alpha = 0,05$

Рассмотрим для длины:

$H_0: \varphi_{1g}^2 = \varphi_{2g}^2$ $H_1: \varphi_{1g}^2 \neq \varphi_{2g}^2$

$S_{n_1}^2 = \frac{n_1}{n_1-1} \sigma_{g1}^2 = \dots$

$S_{n_2}^2 = \dots$

$\tilde{\Delta} = \left(\frac{S_{n_1}}{S_{n_2}}\right)^2 = \left(\frac{n_1}{n_1-1} \cdot \frac{n_2-1}{n_2} \frac{\sigma_{g1}}{\sigma_{g2}}\right)^2 \approx 0,876$

q(H: F(138,999)

$P_{val} = P(\Delta \geq \tilde{\Delta} / H_0) \approx 0,8?5$

нет оснований отвергнуть но//

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$H_0: \varphi_{1g}^2 = \varphi_{2g}^2$ $H_1: \varphi_{1g}^2 \neq \varphi_{2g}^2$

$S_{n_1}^2 = \frac{n_1}{n_1 - 1} \sigma_{g1}^2 \approx 1$

$S_{n_2}^2 = \dots$

$\tilde{\Delta} = \left(\frac{S_{n_1}}{S_{n_2}}\right)^2 = \left(\frac{n_1}{n_1-1} \cdot \frac{n_2-1}{n_2} \frac{\sigma_{g1}}{\sigma_{g2}}\right)^2 \approx 0,876$

q(H: F(138, 999)

$P_{val} = P(\Delta \geq \tilde{\Delta} / H_0) \approx 0,8$~~?~~$5$

нет оснований отвергнуть но

Шарин

аналогично:

$\tilde{\Lambda} \approx 0{,}84$

$q(t) : F(1{,}7;\ 38{,}995)$

$P_{val} \approx 0{,}85$

нет основ. отвергнуть H_0 //