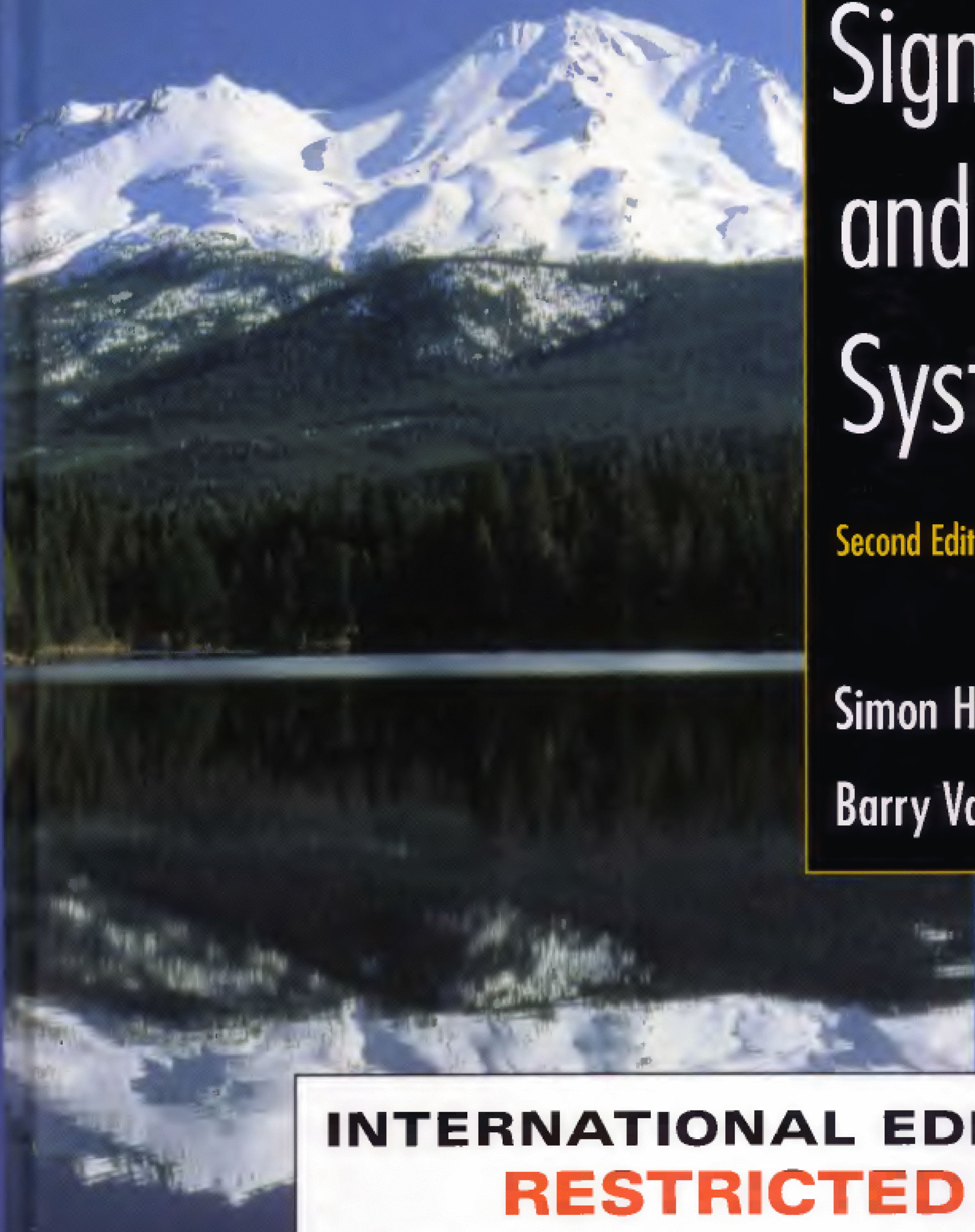

Signals and Systems

Second Edition

Simon Haykin

Barry Van Veen

EXECUTIVE EDITOR *Bill Zobrist*
SENIOR MARKETING MANAGER *Katherine Hepburn*
SENIOR PRODUCTION EDITOR *Caroline Sieg*
SENIOR DESIGNER *Maddy Lesure*
ILLUSTRATION COORDINATOR *Gene Aiello*
COVER PHOTO *Erich Ziller/Eastman's West*

This book was set in Sabon Roman by Preparé Inc. and printed and bound by Hamilton Printing Company. The cover was printed by Brady Palmer.

This book is printed on acid free paper. ♾

Library of Congress Cataloging-in-Publication Data
Haykin, Simon S., 1931-
Signals and systems / Simon Haykin, Barry Van Veen. -2nd ed.
p. cm.
Includes index.
ISBN 0-471-16474-7 (cloth : alk. paper)
1. Signal processing. 2. System analysis. 3. Linear time invariant systems.
4. Telecommunication systems. I. Van Veen, Barry. II. Title.
TK5102.5.H37 2002
621.382′2—dc21 2002027040
CIP

ISBN(Domestic) 0471-16474-7
ISBN(WIE) 0471-37851-8

Printed in the United States of America

10 9 8 7 6 5 4

Preface

The "Signals and Systems" Course in the Electrical Engineering Undergraduate Curriculum

A course on "signals and systems" is fundamental to the study of the many fields that constitute the ever-expanding discipline of electrical engineering. Signals and systems serves as the prerequisite for additional coursework in the study of communications, signal processing, and control. Given the pervasive nature of computing, concepts from signals and systems, such as sampling, are an important component of almost every electrical engineering field. Although the signals and systems that arise across these diverse fields are naturally different in their physical make-up and application, the principles and tools of signals and systems are applicable to all of them. An introductory course on "signals and systems", commonly takes one of two forms:

- A one-semester course that focuses on the analysis of deterministic signals and an important class of systems known as linear time-invariant (LTI) systems, with practical examples drawn from communication and control systems.
- A two-semester course that expands on the one-semester course by including more detailed treatment of signal processing, communication and control systems.

This course is usually offered at the sophomore or junior level and assumes the student has a background in calculus and introductory physics.

How this Book Satisfies the Essential Needs of this Course

Given the introductory nature of the signals and systems course and diversity of applications for the topic, the textbook must be easy to read, accurate, and contain an abundance of insightful examples, problems, and computer experiments to expedite learning the fundamentals of signals and systems in an effective manner. This book has been written with all of these objectives in mind.

The second edition builds on the first edition's success at providing a balanced and integrated treatment of continuous- and discrete-time forms of signals and systems. This approach has the pedagogical advantage of helping the student see the fundamental similarities and differences between continuous- and discrete-time representations and reflects the integrated nature of continuous- and discrete-time concepts in modern engineering practice. One consistent comment from users of the first edition and reviewers of the second is that the compelling nature of our approach becomes very apparent in Chapter 4 with the coverage of sampling continuous-time signals, reconstruction of continuous-time signals from samples, and other applications involving mixtures of different signal classes. The integrated approach is also very efficient in covering the large range of topics that are typically required in a signals and systems course. For example, the properties of all four Fourier representations are covered side-by-side in Chapter 3. Great care has been taken in the presentation of the integrated approach to enhance understanding and avoid confusion. As an example of this, the four Fourier representations are treated in Chapter 3 as similar, yet distinct representations that apply to distinct signal classes. Only after the student has mastered them individually is the possibility of using Fourier representations to cross the boundaries between signal classes introduced in Chapter 4.

Given the mathematical nature of signal representation and system analysis, it is rather easy for the reader to lose sight of their practical application. Chapters 5, 8, and 9 deal with applications drawn from the fields of communication systems, design of filters, and control systems in order to provide motivation for the reader. In addition, considerable effort has been expended in the second edition to provide an application focus throughout the tool-oriented chapters by including an abundance of application-oriented examples. A set of six theme examples, introduced in Chapter 1 and revisited throughout the remaining chapters, is used to show how different signal representation and system analysis tools provide different perspectives on the same underlying problem. The theme examples have been selected to sample the broad range of applications for signals and systems concepts.

The text has been written with the aim of offering maximum teaching flexibility in both coverage and order of presentation, subject to our philosophy of truly integrating continuous- and discrete-time concepts. When continuous- and discrete-time concepts are introduced sequentially, such as with convolution in Chapter 2 and Fourier representations in Chapter 3, the corresponding sections have been written so that the instructor may present either the continuous- or discrete-time viewpoint first. Similarly, the order of Chapters 6 and 7 may be reversed. A two-semester course sequence would likely cover most, if not all, of the topics in the book. A one-semester course can be taught in a variety of ways, depending on the preference of the instructor, by selecting different topics.

Structure Designed to Facilitate and Reinforce Learning

A variety of features have been incorporated into the second edition to facilitate and reinforce the learning process. We have endeavored to write in a clear, easy to follow, yet precise manner. The layout and format has been chosen to emphasize important concepts. For example, key equations and procedures are enclosed in boxes and each example is titled. The choice and layout of figures has been designed to present key signals and systems concepts graphically, reinforcing the words and equations in the text.

A large number of examples are included in each chapter to illustrate application of the corresponding theory. Each concept in the text is demonstrated by examples that emphasize the sequence of mathematical steps needed to correctly apply the theory and by examples that illustrate application of the concepts to real-world problems.

An abundance of practice is required to master the tools of signals and systems. To this end, we have provided a large number of problems with answers immediately following introduction of significant concepts, and a large number of problems without answers at the end of each chapter. The problems within the chapters provide the student with immediate practice and allow them to verify their mastery of the concept. The end of the chapter problems offer additional practice and span a wide range of difficulty and nature, from drilling basic concepts to extending the theory in the text to new applications of the material presented. Each chapter also contains a section illustrating how MATLAB, acronym for MATrix LABoratory and product of The Math Works, Inc., may be used to explore concepts and test system designs within the context of a "Software Laboratory". A complementary set of computer-oriented end of chapter problems is also provided.

New to the Second Edition of the Book

In general terms, this new edition of the book follows the organization and philosophy of the first edition. Nevertheless, over and above new examples and additional problems, some important changes have been made to the book. In addition to the layout and format improvements noted above, long sections in the first edition have been broken up into smaller units. The significant changes to each chapter are summarized as follows:

- Chapter 1: Two new sections, one on Theme Examples and the other on electrical noise, have been added. The Theme Examples, six in number, illustrate the broad range of problems to which signals and systems concepts apply and provide a sense of continuity in subsequent chapters of the book by showing different perspectives on the same problem. Two new subsections, one on MicroElectroMechanical Systems (MEMS) and the other on derivatives of the unit-impulse function, have also been added.
- Chapter 2: The treatment of discrete- and continuous-time convolution has been reorganized into separate, yet parallel sections. The material introducing the frequency response of LTI systems has been removed and incorporated into Chapter 3. The treatment of differential and difference equations has been expanded to clarify several subtle issues.
- Chapter 3: The chapter has been written with increased emphasis on applications of Fourier representations for signals through the introduction of new examples, incorporation of filtering concepts contained in Chapter 4 of the first edition, and reordering the presentation of properties. For example, the convolution property is presented much earlier in the second edition because of its practical importance. Derivations of the discrete-time Fourier series, Fourier series, and discrete-time Fourier transform have been removed and incorporated as advanced problems.
- Chapter 4: The focus has been tightened as reflected by the new title. Material on frequency response of LTI systems has been moved to Chapter 3 and advanced material on interpolation, decimation, and fast convolution has been removed and incorporated as advanced problems.
- Chapter 5: A new section on the Costas receiver for demodulation of double sideband-suppressed carrier modulated signals has been added.

- Chapter 6: The definition of the unilateral Laplace transform has been modified to include impulses and discontinuities at $t = 0$ and the material on Bode diagrams in Chapter 9 of the first edition is now incorporated in the discussion of graphical evaluation of frequency response.
- Chapter 9: A new section on the fundamental notion of feedback and "why feedback?" has been introduced. Moreover, the treatment of feedback control systems has been shortened, focusing on the fundamental issue of stability and its different facets.
- Chapter 10: The epilogue has been completely rewritten. In particular, more detailed treatments of wavelets and the stability of nonlinear feedback systems have been introduced.
- Appendix F: This new appendix presents a tutorial introduction to MATLAB.

Supplements

The following supplements are available from the publishers website:

www.wiley.com/college/haykin

PowerPoint Slides: Every illustration from the text is available in PowerPoint format enabling instructors to easily prepare lesson plans.

Solutions Manual: An electronic Solutions Manual is available for download from the website. If a print version is required, it may be obtained by contacting your local Wiley representative. Your representative may be determined by finding your school on Wiley's CONTACT/Find a Rep webpages.

MATLAB resources: M-files for the computer-based examples and experiments are available.

About the Cover of the Book

The cover of the book is an actual photograph of Mount Shasta in California. This picture was chosen for the cover to imprint in the mind of the reader a sense of challenge, exemplified by the effort needed to reach the peak of the Mount, and a sense of the new vistas that result from climbing to the peak. We thus challenge the reader to master the fundamental concepts in the study of signals and systems presented in the book and promise that an unparalleled viewpoint of much of electrical engineering will be obtained by rising to the challenge.

In Chapter 1 we have included an image of Mount Shasta obtained using a synthetic aperture radar (SAR) system. A SAR image is produced using many concepts from the study of signals and systems. Although the SAR image corresponds to a different view of Mount Shasta, it embodies the power of signals and systems concepts for obtaining different perspectives of the same problem. We trust that motivation for the study of signals and systems begins with the cover.

Acknowledgments

In writing the second edition, we have benefited enormously from insightful suggestions and constructive input received many instructors and students that used the first edition, anonymous reviewers, and colleagues. We are deeply grateful to Professor Aziz Inan of University of Portland for carefully reading the entire manuscript for both accuracy and readability and making innumerable suggestions to improve the presentation. In addition, the following colleagues have generously offered detailed input on the second edition:

- Professor Yogesh Gianchandani, *University of Michigan*
- Professor Dan Cobb, *University of Wisconsin*
- Professor John Gubner, *University of Wisconsin*
- Professor Chris Demarco, *University of Wisconsin*
- Professor Leon Shohet, *University of Wisconsin*
- Mr. Jacob Eapen, *University of Wisconsin*
- Dr. Daniel Sebald

We are grateful to them all for helping us in their own individual ways shape the second edition into its final form.

Barry Van Veen is indebted to his colleagues at the University of Wisconsin for the opportunity to regularly teach the Signals and Systems class. Simon Haykin thanks his students, past and present, for the pleasure of teaching them and conducting research with them.

We thank the many students at both McMaster and Wisconsin, whose suggestions and questions have helped us over the years to refine and in some cases rethink the presentation of the material in the book. In particular, we thank Chris Swickhamer and Kris Huber for their invaluable help in preparing some of the computer experiments, the Introduction to MATLAB, the solutions manual, and in reviewing page proofs.

Bill Zobrist, Executive Editor of Electrical Engineering texts, has skillfully guided the second edition from conception to completion. We are grateful for his strong support, encouragement, constructive input, and persistence. We thank Caroline Sieg for dexterously managing the production process under a very tight schedule, and Katherine Hepburn (Senior Marketing Manager) for her creative promotion of the book.

We are indebted to Fran Daniele and her staff of Preparé Inc. for their magnificent job in the timely production of the book; it was a pleasure to work with them.

Lastly, Simon Haykin thanks his wife Nancy, and Barry Van Veen thanks his wife Kathy and children Emily, David, and Jonathan for their support and understanding throughout the long hours involved in writing this book.

Simon Haykin
Barry Van Veen

To God
who created the universe
and gives meaning to our lives
through His love

Contents

CHAPTER 3 *Fourier Representations of Signals and Linear Time-Invariant Systems* 195

CHAPTER 4 *Applications of Fourier Representations to Mixed Signal Classes* 341

CHAPTER 5 *Application to Communication Systems* 425

CHAPTER 6 *Representing Signals by Using Continuous-Time Complex Exponentials: the Laplace Transform* 482

CHAPTER 7 *Representing Signals by Using Discrete-Time Complex Exponentials: the z-Transform* 553

Notation

$[\cdot]$ indicates discrete valued independent variable, e.g. $x[n]$
$(\cdot)$ indicates continuous valued independent variable, e.g. $x(t)$

- Complex numbers

$\lvert c \rvert$	magnitude of complex quantity c
$\arg\{c\}$	phase angle of complex quantity c
$\mathrm{Re}\{c\}$	real part of c
$\mathrm{Im}\{c\}$	imaginary part of c
c^*	complex conjugate of c

- Lower case functions denote time-domain quantities, e.g. $x(t)$, $w[n]$
- Upper-case functions denote frequency- or transform-domain quantities

$X[k]$	discrete-time Fourier series coefficients for $x[n]$
$X[k]$	Fourier series coefficients for $x(t)$
$X(e^{j\Omega})$	discrete-time Fourier transform of $x[n]$
$X(j\omega)$	Fourier transform of $x(t)$
$X(s)$	Laplace transform of $x(t)$
$X(z)$	z-transform of $x[n]$

- Boldface lower-case symbols denote vector quantities, e.g., $\mathbf{q}$
- Boldface upper-case symbols denote matrix quantities, e.g., $\mathbf{A}$
- Subscript δ indicates continuous-time representation of a discrete-time signal

$x_\delta(t)$	continuous-time representation for $x[n]$
$X_\delta(j\omega)$	Fourier transform of $x_\delta(t)$

- Sans serif type indicates MATLAB variables or commands, e.g., `X = fft(x,n)`
- 0^0 is defined as 1 for convenience
- arctan refers to the four quadrant inverse tangent function and produces a value between $-\pi$ and π radians

Principal Symbols

j	square root of -1
`i`	square root of -1 used by MATLAB
T_s	sampling interval of T_s in seconds
T	fundamental period for continuous-time signal in seconds

N	fundamental period for discrete-time signal in samples
ω	(angular) frequency for continuous-time signal in radians/second
Ω	(angular) frequency for discrete-time signal in radians
ω_o	fundamental (angular) frequency for continuous-time periodic signal in radians/second
Ω_o	fundamental (angular) frequency for discrete-time periodic signal in radians
$u(t), u[n]$	step function of unit amplitude
$\delta[n], \delta(t)$	unit impulse
$H\{\cdot\}$	representation of a system as an operator H
$S^\tau\{\cdot\}$	time shift of τ units
$H^{\text{inv}}, h^{\text{inv}}$	superscript inv denotes inverse system
$*$	denotes convolution operation
$\circledast$	periodic convolution of two periodic signals
$H(e^{j\Omega})$	discrete-time system frequency response
$H(j\omega)$	continuous-time system frequency response
$h[n]$	discrete-time system impulse response
$h(t)$	continuous-time system impulse response
$y^{(h)}$	superscript (h) denotes homogeneous solution
$y^{(n)}$	superscript (n) denotes natural response
$y^{(f)}$	superscript (f) denotes forced response
$y^{(p)}$	superscript (p) denotes particular solution
$\xleftrightarrow{DTFS;\,\Omega_o}$	discrete-time Fourier series pair with fundamental frequency Ω_o
$\xleftrightarrow{FS;\,\omega_o}$	Fourier series pair with fundamental frequency ω_o
$\xleftrightarrow{DTFT}$	discrete-time Fourier transform pair
$\xleftrightarrow{FT}$	Fourier transform pair
$\xleftrightarrow{\mathcal{L}}$	Laplace transform pair
$\xleftrightarrow{\mathcal{L}_u}$	unilateral Laplace transform pair
$\xleftrightarrow{z}$	z-transform pair
$\xleftrightarrow{z_u}$	unilateral z-transform pair
$\text{sinc}(u)$	$\dfrac{\sin(\pi u)}{\pi u}$
$\cap$	intersection
$T(s)$	closed-loop transfer function
$F(s)$	return difference
$L(s)$	loop transfer function

Abbreviations

A	amperes (units for electric current)
A/D	analog-to-digital (converter)
AM	amplitude modulation
BIBO	bounded input-bounded output
BPSK	binary phase-shift keying
CD	compact disc
CW	continuous wave
D/A	digital-to-analog (converter)
dB	decibel
DSB-SC	double-sideband suppressed carrier
DTFS	discrete-time Fourier series
DTFT	discrete-time Fourier transform
ECG	electrocardiogram
F	Farads (units for capacitance)
FDM	frequency-division multiplexing
FFT	fast Fourier transform
FIR	finite-duration impulse response
FM	frequency modulation
FS	Fourier series
FT	Fourier transform
H	Henries (units for inductance)
Hz	Hertz
IIR	infinite-duration impulse response
LTI	linear time-invariant (system)
MEMS	microelectricalmechanical system
MSE	mean squared error
PAM	pulse-amplitude modulation
PCM	pulse-code modulation
PM	phase modulation
QAM	quadrature-amplitude modulation
RF	radio frequency
ROC	region of convergence
rad	radian(s)
s	second(s)
SSB	single sideband modulation
STFT	short-time Fourier transform
TDM	time-division multiplexing
V	volts (units for electric potential)
VLSI	very large scale integration
VSB	vestigial sideband modulation
WT	wavelet transform

1 Introduction

1.1 *What Is a Signal?*

Signals, in one form or another, constitute a basic ingredient of our daily lives. For example, a common form of human communication takes place through the use of speech signals, in a face-to-face conversation or over a telephone channel. Another common form of human communication is visual in nature, with the signals taking the form of images of people or objects around us.

Yet another form of human communication is electronic mail over the *Internet*. In addition to providing mail, the Internet serves as a powerful medium for searching for information of general interest, for advertising, for telecommuting, for education, and for playing games. All of these forms of communication over the Internet involve the use of information-bearing signals of one kind or another. Other real-life examples in which signals of interest arise are discussed subsequently.

By listening to the heartbeat of a patient and monitoring his or her blood pressure and temperature, a doctor is able to diagnose the presence or absence of an illness or disease. The patient's heartbeat and blood pressure represent signals that convey information to the doctor about the state of health of the patient.

In listening to a weather forecast over the radio, we hear references made to daily variations in temperature, humidity, and the speed and direction of prevailing winds. The signals represented by these quantities help us, for example, to form an opinion about whether to stay indoors or go out for a walk.

The daily fluctuations in the prices of stocks and commodities on world markets, in their own ways, represent signals that convey information on how the shares in a particular company or corporation are doing. On the basis of this information, decisions are made regarding whether to venture into new investments or sell off old ones.

A probe exploring outer space sends valuable information about a faraway planet back to a station on Earth. The information may take the form of radar images representing surface profiles of the planet, infrared images conveying information on how hot the planet is, or optical images revealing the presence of clouds around the planet. By studying these images, our knowledge of the unique characteristics of the planet in question is enhanced significantly.

Indeed, the list of what constitutes a signal is almost endless.

A signal is formally defined as a function of one or more variables that conveys information on the nature of a physical phenomenon. When the function depends on a single variable, the signal is said to be *one dimensional*. A speech signal is an example of a one-dimensional signal whose amplitude varies with time, depending on the spoken word and who speaks it. When the function depends on two or more variables, the signal is said to be *multidimensional*. An image is an example of a two-dimensional signal, with the horizontal and vertical coordinates of the image representing the two dimensions.

1.2 *What Is a System?*

In the examples of signals mentioned in the preceding section, there is always a system associated with the generation of each signal and another system associated with the extraction of information from the signal. For example, in speech communication, a sound or signal excites the vocal tract, which represents a system. The processing of speech signals usually relies on the use of our ears and auditory pathways in the brain. In this case, the systems responsible for the production and reception of signals are biological in nature. These systems could also be implemented using electronic systems that try to emulate or mimic their biological counterparts. For example, the processing of a speech signal may be performed by an automatic speech recognition system in the form of a computer program that recognizes words or phrases.

A system does not have a unique purpose. Rather, the purpose depends on the application of interest. In an automatic speaker recognition system, the function of the system is to extract information from an incoming speech signal for the purpose of *recognizing* or *identifying* the speaker. In a communication system, the function of the system is to *transport* the information contained in a message over a communication channel and deliver that information to a destination in a reliable fashion. In an *aircraft landing system*, the requirement is to keep the aircraft on the extended centerline of a runway.

A system is formally defined as an entity that manipulates one or more signals to accomplish a function, thereby yielding new signals. The interaction between a system and its associated signals is illustrated schematically in Fig. 1.1. Naturally, the descriptions of the input and output signals depend on the intended application of the system:

- In an automatic speaker recognition system, the input signal is a speech (voice) signal, the system is a computer, and the output signal is the identity of the speaker.
- In a communication system, the input signal could be a speech signal or computer data, the system itself is made up of the combination of a transmitter, channel, and receiver, and the output signal is an estimate of the information contained in the original message.
- In an aircraft landing system, the input signal is the desired position of the aircraft relative to the runway, the system is the aircraft, and the output signal is a correction to the lateral position of the aircraft.

1.3 *Overview of Specific Systems*

In describing what we mean by signals and systems in the previous two sections, we mentioned several applications. In this section, we will expand on six of those applications,

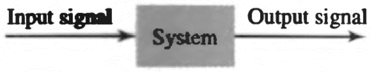

FIGURE 1.1 Block diagram representation of a system.

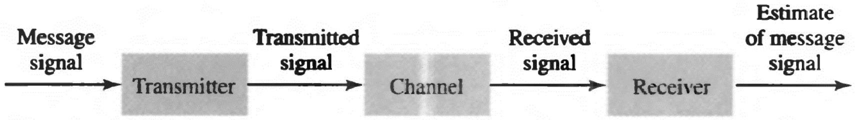

FIGURE 1.2 Elements of a communication system. The transmitter changes the message signal into a form suitable for transmission over the channel. The receiver processes the channel output (i.e., the received signal) to produce an estimate of the message signal.

namely, communication systems, control systems, microelectromechanical systems, remote sensing, biomedical signal processing, and auditory systems.

1.3.1 Communication Systems

As depicted in Fig. 1.2, there are three basic elements to every communication system: the *transmitter*, the *channel*, and the *receiver*. The transmitter is located at one point in space, the receiver is located at some other point separate from the transmitter, and the channel is the physical medium that connects the two together. Each of these three elements may be viewed as a system with associated signals of its own. The purpose of the transmitter is to convert the message signal produced by a source of information into a form suitable for transmission over the channel. The message signal could be a speech signal, a television (video) signal, or computer data. The channel may be an optical fiber, a coaxial cable, a satellite channel, or a mobile radio channel; each of these channels has its specific area of application.

As the transmitted signal propagates over the channel, it is distorted due to the physical characteristics of the channel. Moreover, noise and interfering signals (originating from other sources) contaminate the channel output, with the result that the received signal is a corrupted version of the transmitted signal. The function of the receiver is to operate on the received signal so as to reconstruct a recognizable form (i.e., produce an estimate) of the original message signal and deliver it to its destination. The signal-processing role of the receiver is thus the reverse of that of the transmitter; in addition, the receiver reverses the effects of the channel.

Details of the operations performed in the transmitter and receiver depend on the type of communication system being considered. The communication system can be of an analog or digital type. In signal-processing terms, the design of an *analog communication system* is relatively simple. Specifically, the transmitter consists of a *modulator* and the receiver consists of a *demodulator. Modulation* is the process of converting the message signal into a form that is compatible with the transmission characteristics of the channel. Ordinarily, the transmitted signal is represented as amplitude, phase, or frequency variations of a sinusoidal carrier wave. We thus speak of amplitude modulation, phase modulation, or frequency modulation, respectively. Correspondingly, through the use of amplitude demodulation, phase demodulation, or frequency demodulation, an estimate of the original message signal is produced at the receiver output. Each one of these analog modulation–demodulation techniques has its own advantages and disadvantages.

In contrast, a *digital communication system*, as described below, is considerably more complex. If the message signal is of analog form, as in speech and video signals, the transmitter performs the following operations to convert it into digital form:

- *Sampling*, which converts the message signal into a sequence of numbers, with each number representing the amplitude of the message signal at a particular instant of time.