Sum of n-integers

$A_{s,b} \Rightarrow$ s = jumps b = # items per block B = page

$\Rightarrow A_j = A[\, j \cdot b + 1, (j+1) \cdot b \,]$

- Sum all items in each block before moving to the next one -

$s = b = 1 \Rightarrow$ normal sequential access

b large $\Rightarrow$ block-wise access

s large $\Rightarrow$ random-wise access

- not all values of s takes account all blocks
- If s is coprime with $\frac{n}{b} \Rightarrow$ then $\left\{ s \cdot i \bmod \left(\frac{n}{b}\right) \right\}$ generates a permutation

$\Rightarrow$ thus $A_{s,b}$ touches all block

$\Rightarrow$ sum of n integers

COST

- cpu time $= n$
- space occupancy $= \frac{n}{B}$ pages

1) Case $(s = 1) \Rightarrow A_{1,b}$ $\Rightarrow \forall\, b \in \mathbb{N}$ it scans all block rightward

$\Rightarrow$ takes $O\left(\frac{n}{B}\right)$ I/Os

2) Case $(s = 2,\ b < B)$

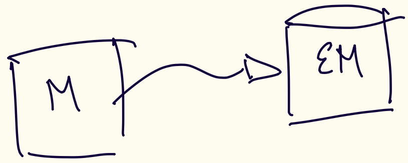
$\Rightarrow$ touches the half of the B pages logically divided small blocks b

$\Rightarrow$ jumping by 2

$$\Rightarrow \left(\frac{2n}{B}\right) \text{ I/Os}$$

$\Rightarrow$ total cost: $\min\left\{ s, \frac{B}{b} \right\} \cdot \frac{n}{b}$ I/Os

Memory: assume input size is $n = (1+\varepsilon)M$ $\Rightarrow$



$n = M + \varepsilon M$

want evaluate $P(\varepsilon)$ = "probability I'm going to access the disk"

$$\frac{\varepsilon M}{n} = \frac{\varepsilon M}{(1+\varepsilon) M} = \frac{\varepsilon}{1+\varepsilon}$$

$\varepsilon M \to$ # of elements on the disk

$n \to$ # total elements

if $P(\varepsilon) = 1 \Rightarrow$ surely go to disk

$P(\varepsilon) = 0 \Rightarrow$ stay in memory

$P(\varepsilon) = \frac{\varepsilon}{1+\varepsilon}$ random behaviour

EVALUATE AVG TIME FOR A STEP

let be: $-(a)$: frequent of steps (load / write) $\to$ M, $\to$ D

$-(1-a)$: " " " (computation) $\to$ CPU

$$(1-a)\cdot\overset{\text{computation}}{\alpha} + a\left[\overset{\text{M cost}}{\beta}\,(1-p(\varepsilon)) + c\cdot\left(p(\varepsilon)\right)\right] =$$

$$\alpha = \beta = 1$$

$$= \underbrace{(1-a)\cdot 1}_{\leq 1} + a\left[\underbrace{1\cdot(1-p(\varepsilon))}_{\leq 1} + c\cdot\left(p(\varepsilon)\right)\right] \approx$$

$$\approx a\cdot c\cdot p(\varepsilon) \qquad a \approx 30\%$$

$$10^5 \leq c \leq 10^6$$

$$\Downarrow \quad 300'000\cdot p(\varepsilon)$$