

• Use SNOW-PLow to sort

$S = [1, 3, 9, 6, 2, 10, 4, 8, 5, 7]$

• $M = 2$

• Generate only the 1st run

H : sorted

M : unsorted

1) $H = [1, 3]$ // fetch 2 items in M from S

$\min = 1$; $\text{next} = 3$ $\geq \min \Rightarrow H = [3, 9]$, $M = []$

$\min = 3$; $\text{next} = 6$ $\geq \min \Rightarrow H = [6, 9]$, $M = []$

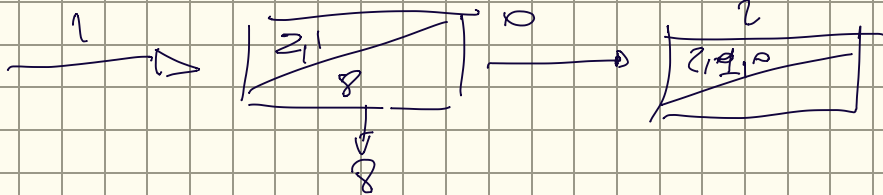
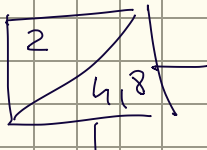
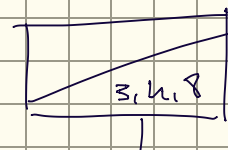
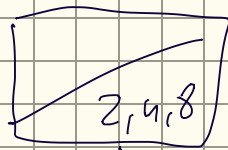
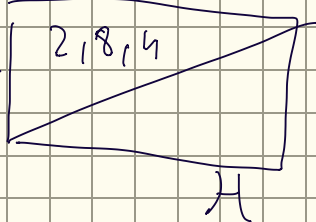
$\min = 6$; $\text{next} = 2$ $< \min \Rightarrow H = [9]$, $M = [2]$

$\min = 9$; $\text{next} = 10$ $\geq \min \Rightarrow H = [10]$, $M = [2]$

$\min = 10$; $\text{next} = 4$ $< \min \Rightarrow H = []$, $M = [2, 4]$

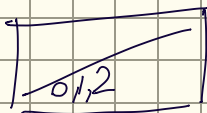
2) Snow Plow with $M = 3$; $S = 2, 8, 4, 3, 2, 1, 0$

1st run



H is empty \Rightarrow end of 1st run

2nd run



end of 2nd run!

PROVE LOWER BOUND FOR SORTING (I/O) when n MULTIPLE
 MS with optimizations

$$\begin{array}{l} \text{new} \\ \text{old} \end{array} \left. \begin{array}{l} \left(\frac{n}{B} \right) B! \\ \left(\frac{n}{B} \right) \end{array} \right\} \begin{array}{l} \frac{n}{B} \\ t - \frac{n}{B} \end{array} \left. \begin{array}{l} \text{to reads} \\ \text{the leaves (permutation)} \\ \Rightarrow \text{a path leaf - root at} \\ \text{the comparisons performed} \end{array} \right\}$$

$n! = \text{permutations}$

$$\Rightarrow \left[\left(\frac{n}{B} \right) B! \right]^{\frac{n}{B}} \cdot \left(\frac{n}{B} \right)^{t - \frac{n}{B}} \geq n!$$

$$\cancel{\left(\frac{n}{B} \right)^{\frac{n}{B}}} \cdot (B!)^{\frac{n}{B}} \cdot \frac{\left(\frac{n}{B} \right)^t}{\cancel{\left(\frac{n}{B} \right)^{\frac{n}{B}}}} \geq n!$$

$$\log_2 (B!)^{\frac{n}{B}} + \log_2 \left(\frac{n}{B} \right)^t \geq n \log_2 n$$

NOTATION:
 \Rightarrow from now
 \log will be
 in base 2

$$\frac{n}{B} (B \log B) + t (B \log \frac{n}{B}) \geq n \log n$$

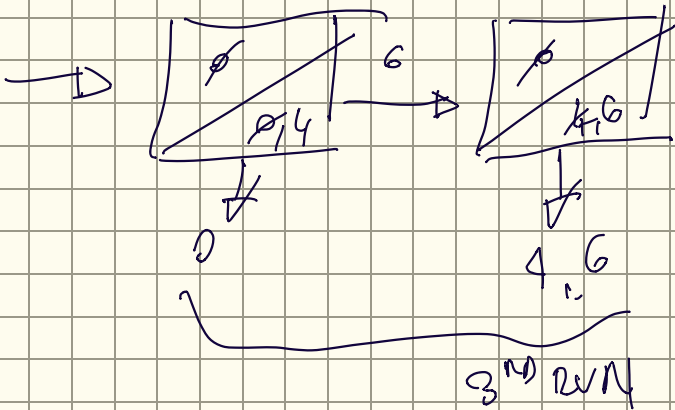
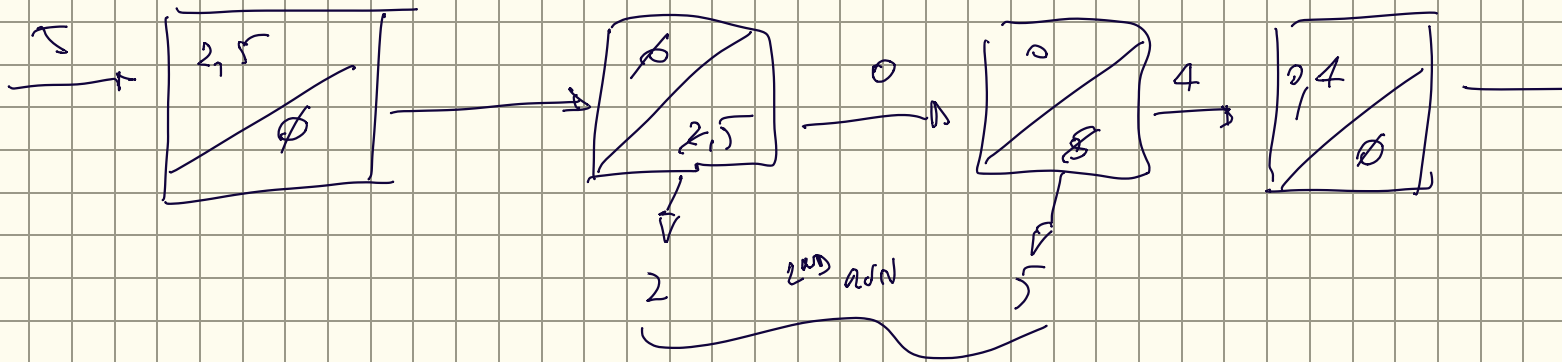
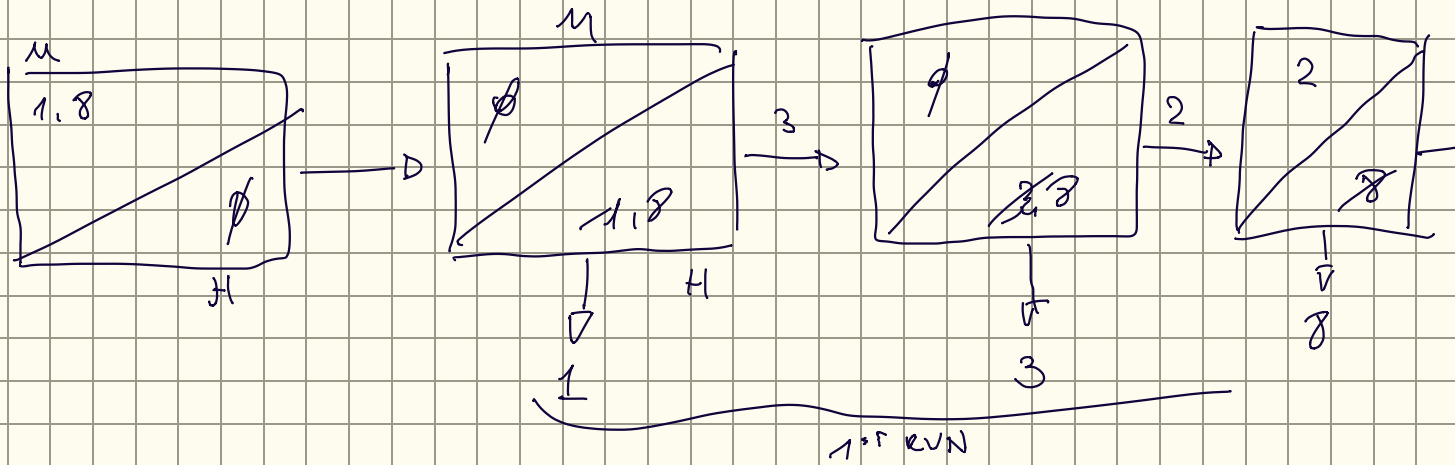
$$n \log B + t (B \log \frac{n}{B}) \geq n \log n$$

$$t (B \log \frac{n}{B}) \geq n \log n - n \log B$$

$$\frac{t (B \log \frac{n}{B})}{\cancel{B \log \frac{n}{B}}} \geq \frac{n \log \frac{n}{B}}{\cancel{B \log \frac{n}{B}}}$$

$$t \geq \frac{n}{B} \log_{\frac{n}{B}} \frac{n}{B} \quad \text{that asymptotically is } \frac{n}{B}$$

Snow plow with $K=2$ & $S = 1, 8, 3, 2, 5, 0, 4, 6$



SLOW POW

Refume: All unsorted array of M 's items

R = build a min-heap over M 's items

Set $U = \emptyset$

While ($H \neq \emptyset$) do

min = extract min from the H

write min to the output

next = read next item from S

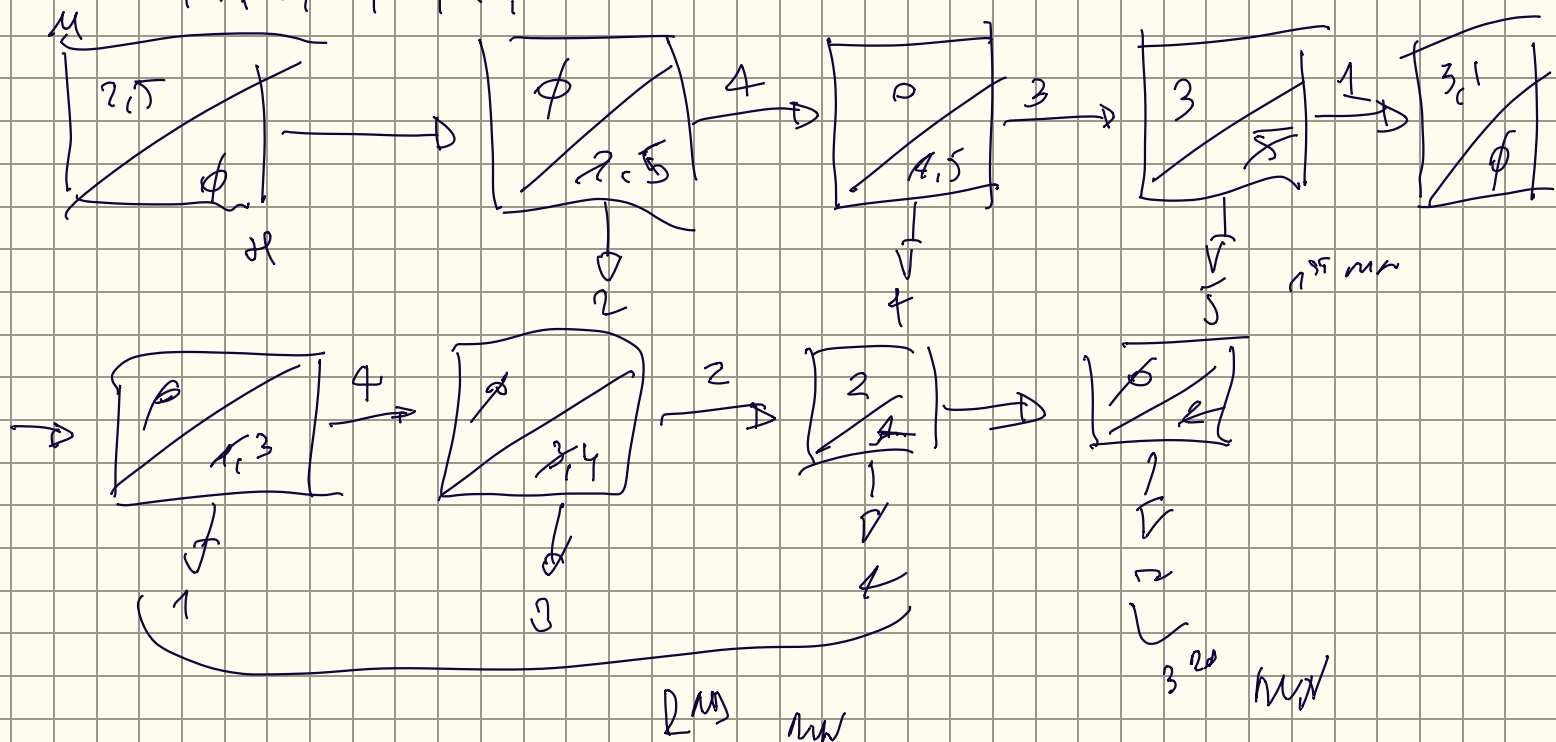
if (next < min)

write next in U

else

write next in H

$S = 2, 5, 4, 3, 1, 4, 2$ $M = 7$



Runs: $2, 4, 5 / 1, 3, 4 / 2$

SNOW PLOW : $n=2$

$S = 1, 3, 9, 10, 7, 6, 5, 4, 3, 8$

