MINIMAL ORDERED PERFECT HASHING (graph)

- given a set of strings: $\{\underset{0}{aa}, \underset{1}{ba}, \underset{2}{bb}, \underset{3}{bc}, \underset{4}{ca}\}$

=> assign codes that are ranking of the strings starting from 0

- given $h(x)_s$: 2 chars of the strings

$$h_1(c'c'') = 2 \cdot \text{rank}(c') \cdot \text{rank}(c'') \bmod 11$$

$$h_2(c'c'') = 5 \cdot \text{rank}(c') + \text{rank}(c''+1) \bmod 11$$

	h	h_1	h_2
aa	0	8	2
ba	1	1	7
bb	2	7	8
bc	3	2	9
ca	4	5	1

rank	a	b	c
	2	3	4

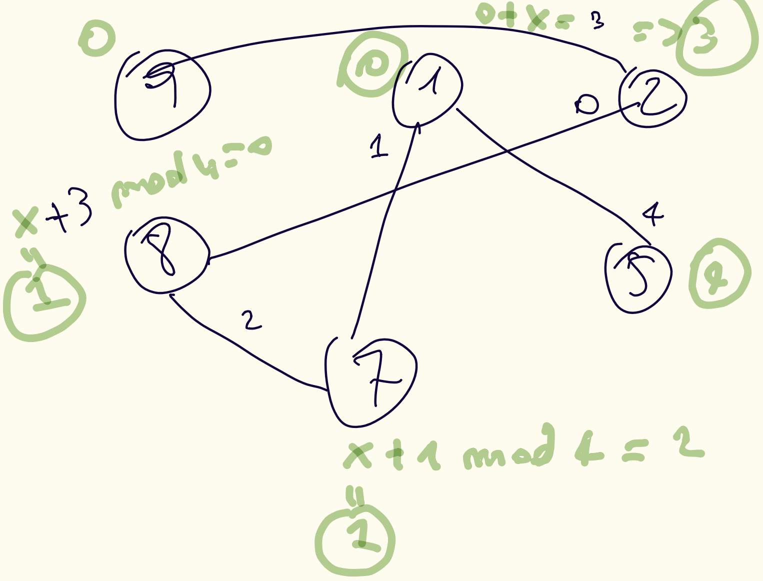
$h_1(cc) = 2 \cdot 4 \cdot 4 = 32$ mod $11 = 10$

$h_2(cc) = 5 \cdot 4 + 5 = 25$ mod $11 = 3$

compute function g (represented by an array) → domain from max values => [0, 10] cuz mod 11

g

=> build the graph with 11 nodes [0, 10]



=> ACYCLIC => solvable
(if cyclic => no solvable

=> START from the last node
& propagate (5) 0

final result

$h(t) = g(h_1(t)) + g(h_2(t)) \underbrace{\mod 5}_{\#\text{ of strings}}$

g	0	1	2	3	4	5	6	7	8	9	10
	0	0	3	0	0	4	0	1	1	0	0

any values. as 0

•) SHOW $h(BA) = g(\underbrace{h_1(ba)}_{1}) + g(\underbrace{h_2(ba)}_{7}) \mod 5 := 1$

$g(1) = 0$, $g(7) = 1$

and

$h(cc) = g(\underbrace{h_1(cc)}_{10}) + g(\underbrace{h_2(cc)}_{3}) \mod 4 = 0$

$g(10) = 0$, $g(3) = 0$

Cukoo Hashing

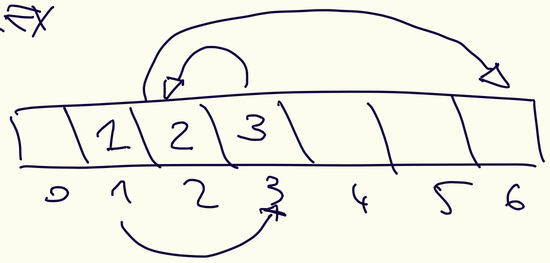
: table of $m = 7$ $7 \Rightarrow$ positions

$\Rightarrow$ SHOW CONTENT OF THE TABLE INSERTING THE KEY

$h_1(x) = x \bmod 7$

$h_2(x) = 3x \bmod 7$

m =



buf := x

• keys to insert:

$\{3, 2, 1, 8, 6\}, 15$

• When both cells after h_1 & h_2 are occupied:

$\Rightarrow$ PREFER ALWAYS h_1 TO INSERT

$h_1(3) = 3 \bmod 7 = 3$

$h_2(3) = 3 \cdot 3 \bmod 7 = 2$

$h_1(2) = 2 \bmod 7 = 2$

$h_2(2) = 6 \bmod 7 = 6$

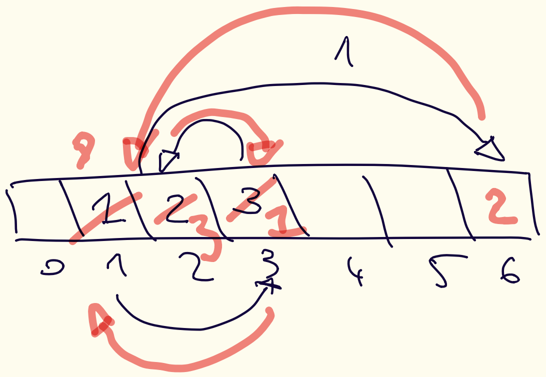
$h_1(1) = 1 \bmod 7 = 1$

$h_2(1) = 3 \cdot 1 \bmod 7 = 3$

$h_1(8) = 1$ 1st kick out

$h_2(8) = 24 \bmod 7 = 3$

m =

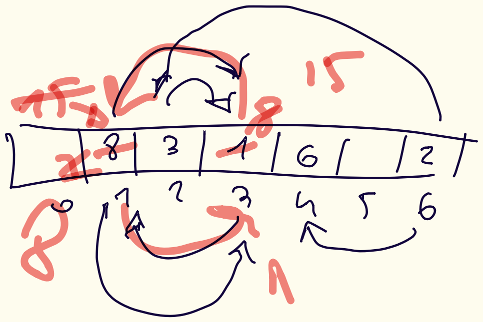


- kick out
- insert the arrow

$h_1(6) = 6$ X

$h_2(6) = 18 \bmod 7 = 4$ ✓

m =



$\Rightarrow$ cycle

$h_1(15) = 1$ kick

$h_2(15) = 15 \cdot 3 \bmod 7$

$= 45 \bmod 7 =$

$= 3$

$\Rightarrow$ loop with 15 involving 1, 8, 15 TRY h_2

$\Rightarrow$ 15 CANNOT BE INSERTED

=> Should take other 2 h(x)

STRINGS: $S = \{AA, AC, BB, CC\}$ compute MOPE

given $h_1(xy) = x + y \mod 7$

$h_2(xy) = x + 2 \cdot y \mod 7$

CODES are

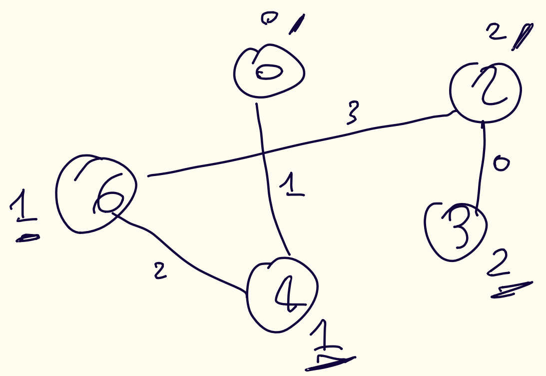
$A = 1$
$B = 2$
$C = 3$

key	h (Edge label)	h_1 (NODES LABEL)	h_2 (NODES LABEL)
AA	0	2	$1+2\cdot1 \mod 7 = 3$
AC	1	4	$1+2\cdot3 \mod 7 = 0$
BB	2	4	$2+2\cdot2 \mod 7 = 6$
CC	3	6	$3+2\cdot3 \mod 7 = 2$

$AA = 1+1 = 2$
$AC = 1+3 = 4$
$BB = 2+2 = 4$
$CC = 3+3 = 6$

$m = 7$

$[0, 6]$



mod 4

$h(xy) = g(h_1(xy)) + g(h_2(xy)) \mod 4$

	0	1	2	3	4	5	6
g	0	0	2	2	1	0	1

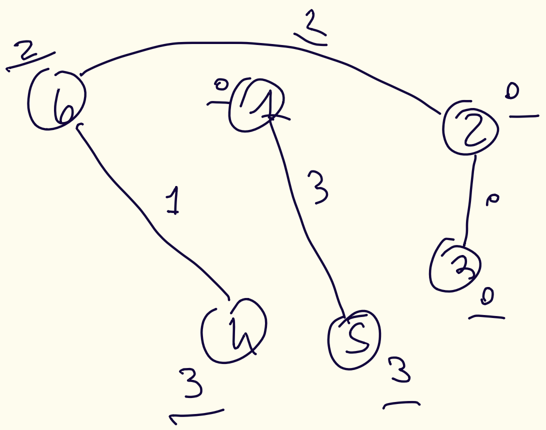
MOPT

$S = \{ \underset{0}{11}, \underset{1}{22}, \underset{2}{33}, \underset{3}{44} \}$

$h_1(xy) = x + y \mod 7$

$h_2(xy) = x + 2y \mod 7$

keys	h	h_1	h_2
11	0	2	$1+2\cdot1 \text{ m}7 = 3$
22	1	4	$2+2\cdot2 \text{ m}7 = 6$
33	2	6	$3+2\cdot3 \text{ m}7 = 2$
44	3	1	$4+2\cdot4 \text{ m}7 = 5$



remember mod 4

$h(t) = g(t(h_1)) + g(t(h_2)) \mod x$

0	1	2	3	4	5	6
0	0	0	0	3	3	2

MOPF

$$S = \{ \underset{0}{AA}, \underset{1}{AC}, \underset{2}{BC}, \underset{3}{CC} \}$$

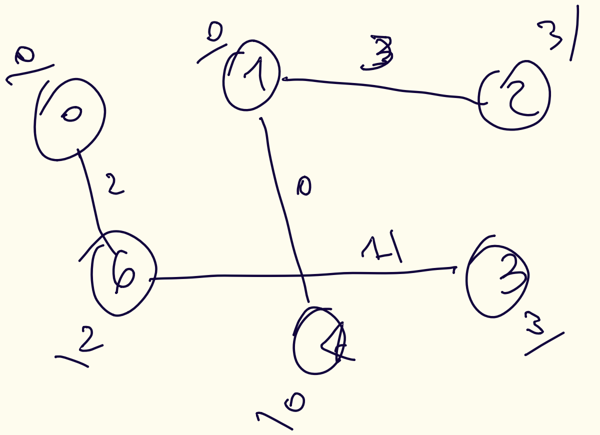
$$h_1(xy) = 3\,\mathrm{rank}(x) + \mathrm{rank}(y) \bmod 7$$

$$h_2(xy) = \mathrm{rank}(x) + \mathrm{rank}(y) \bmod 7$$

$$\mathrm{rank}(x) = \overset{a}{2}; \overset{b}{3}; \overset{c}{4}$$

$x = a, b, c$

keys	h	h_1	h_2
AA	0	$3 \cdot 2 + 2 \text{ m}7 = 1$	4
AC	1	$3 \cdot 2 + 4 \text{ m}7 = 3$	$2 + 4 = 6$
BC	2	$3 \cdot 3 + 4 \text{ m}7 = 6$	$3 + 4 = 7 \text{ m}7 = 0$
CC	3	$3 \cdot 4 + 4 \text{ m}7 = 2$	$4 + 4 \text{ m}7 = 1$



mod 4

$$h(t) = g(h_1(t)) + g(h_2(t)) \bmod \cancel{4}$$

$x = 4$

0	0	3	3	0	0	2
0	1	2	3	4	5	6