Inverse Kinematics

From Position to Angles

Inverse Kinematics Today!

- Two-link Robot Algebraic
- Three Link Robot – Any way
- RRP Robot

Two-link Manipulator IK (Algebraic)

Two-link Manipulator IK (Algebraic)

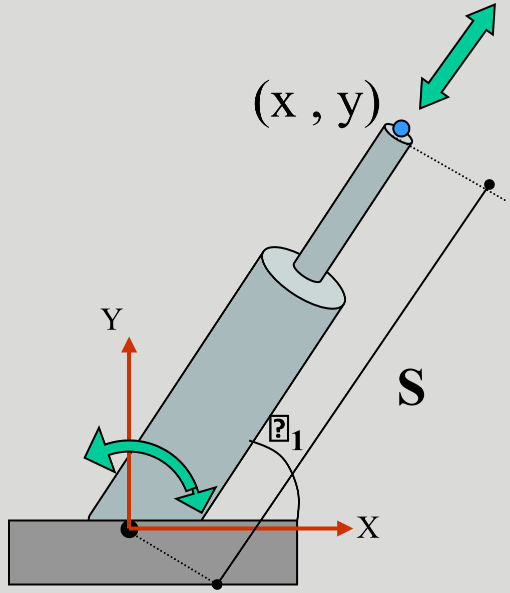
Two-link Manipulator IK (Algebraic)

Three-link Manipulator IK

RRP Arm

A Simple Example

Revolute and Prismatic Joints Combined



Finding ?:

A Simple Example

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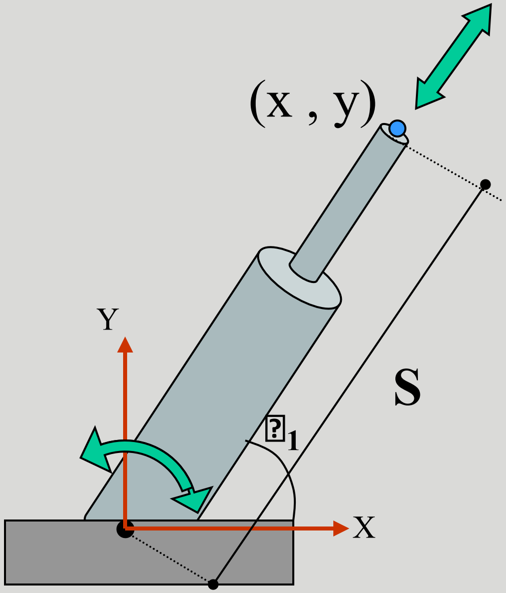

Finding ?:

$$\theta = \arctan(y, x)$$

More Specifically:

$$\theta = \arctan 2(y, x)$$

arctan2() specifies that it's in the first quadrant

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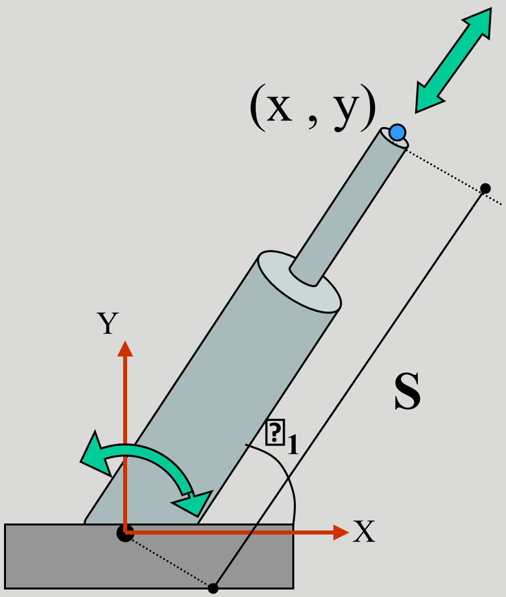

Finding θ:

$$\theta = \arctan(y, x)$$

More Specifically:

$$\theta = \arctan 2(y, x)$$

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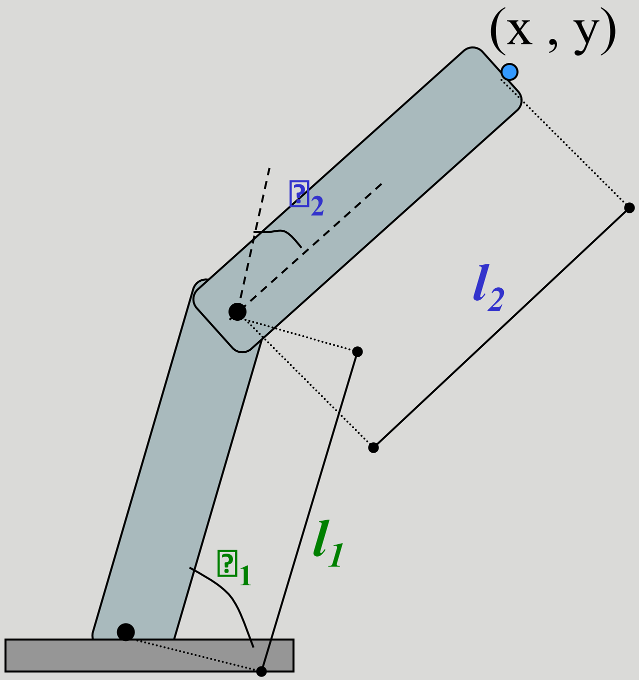
Finding **S**:

$$S = \sqrt{(x^2 + y^2)}$$

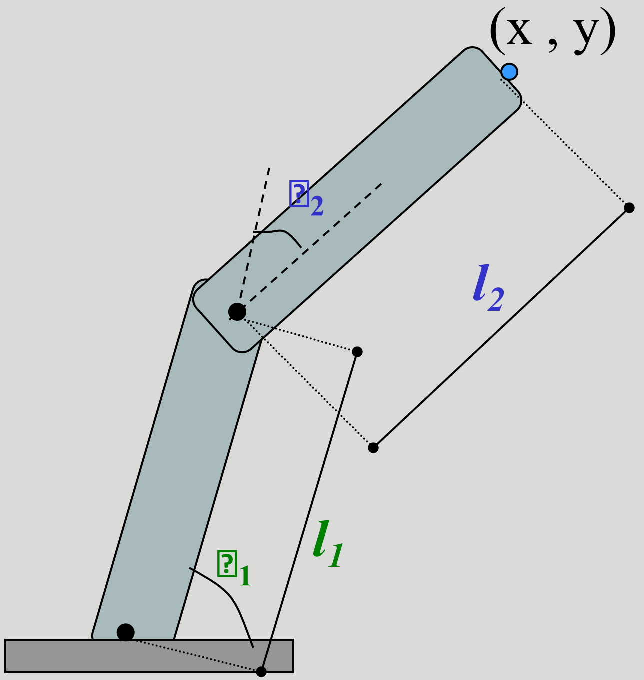
Inverse Kinematics of a Two Link Manipulator



Given: l_1, l_2, x , y

Find: θ_1, θ_2

Inverse Kinematics of a Two Link Manipulator

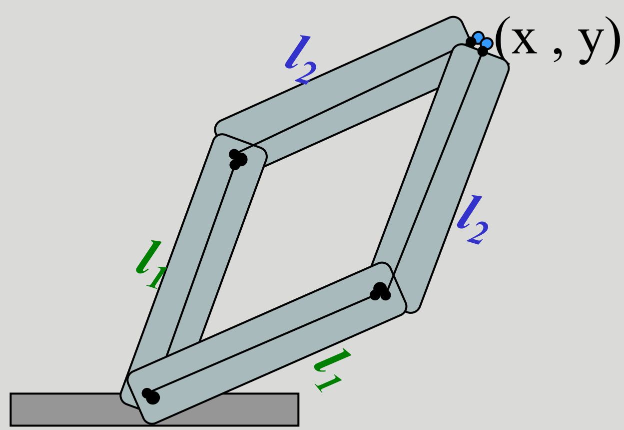


Given: l_1, l_2, x , y

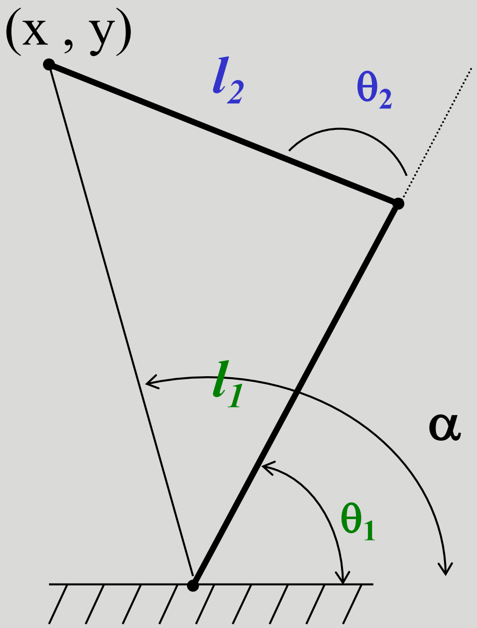
Find: θ_1, θ_2

Redundancy:

A unique solution to this problem does not exist. Notice, that using the "givens" two solutions are possible. Sometimes no solution is possible.



The Geometric Solution



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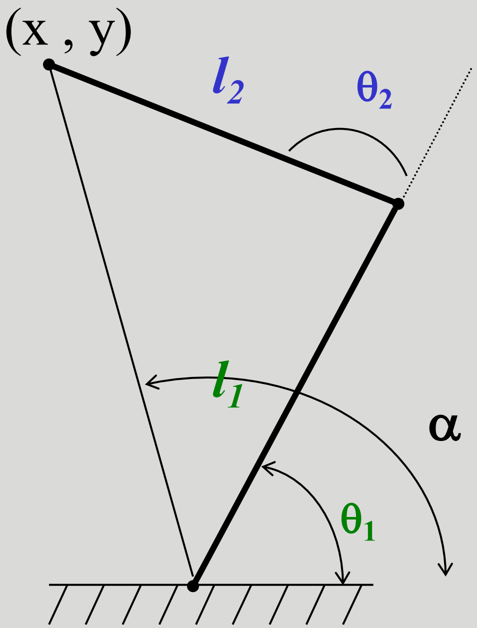
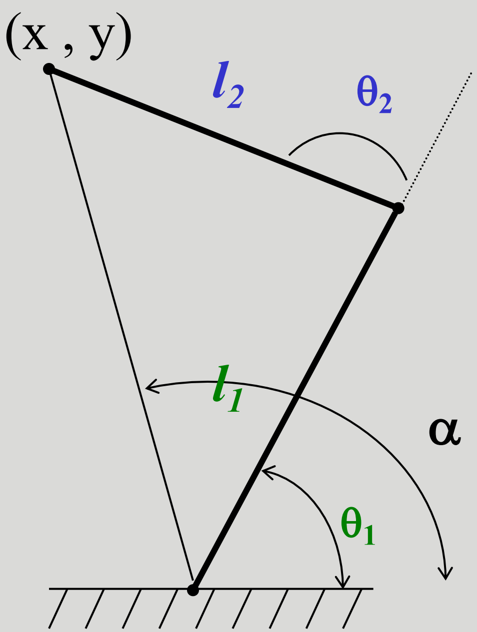


Using the Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

The Geometric Solution



Using the Law of Cosines:

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$$(x^2 + y^2) = l_1^{\,2} + l_2^{\,2} - 2 l_1 l_2 \cos(180 - \theta_2)$$

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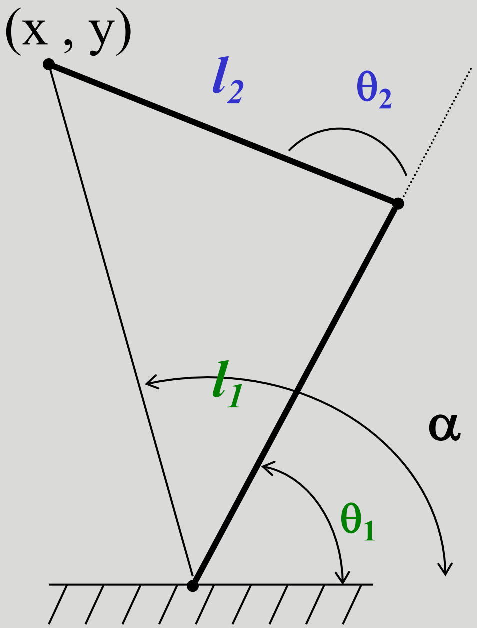

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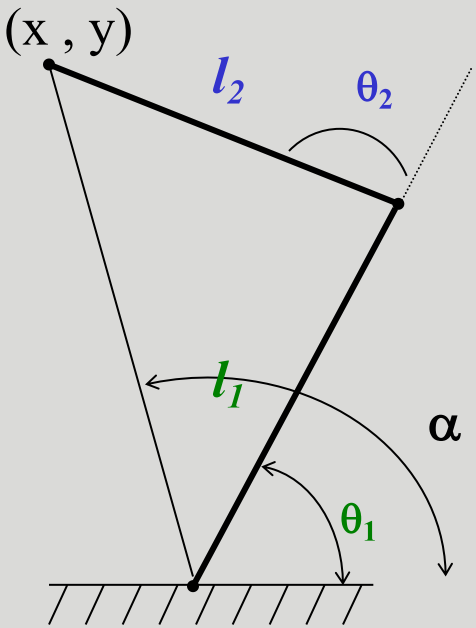

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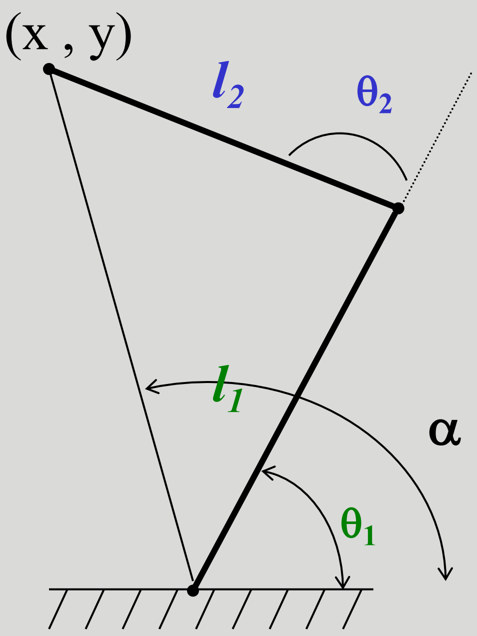

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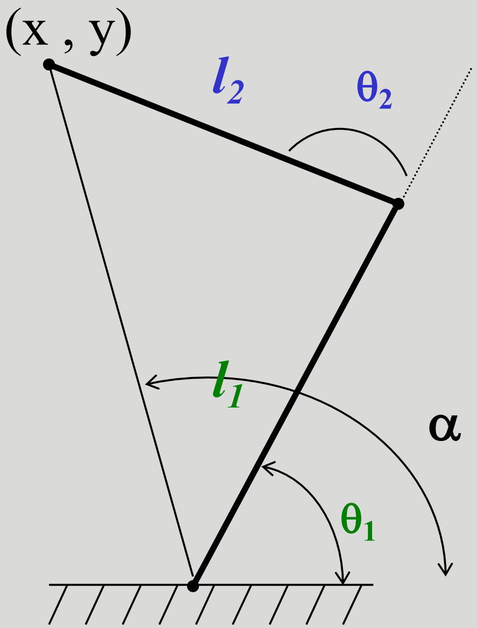
$$\theta_2 = \arccos\left(\frac{x^2 + y^2 - l_1^{\,2} - l_2^{\,2}}{2l_1l_2}\right)$$

Redundant since θ_2 could be in the first or fourth quadrant.

The Geometric Solution



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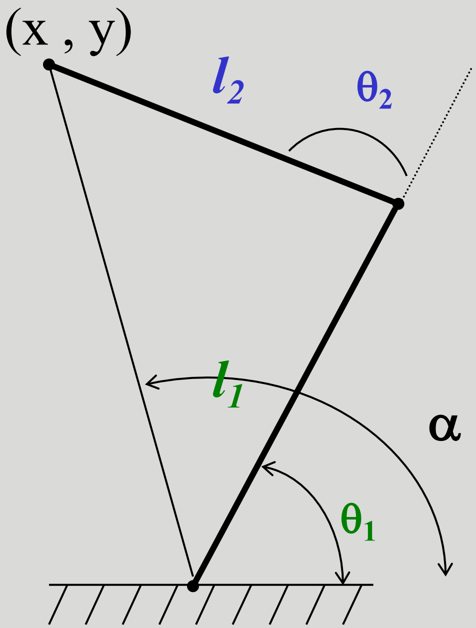

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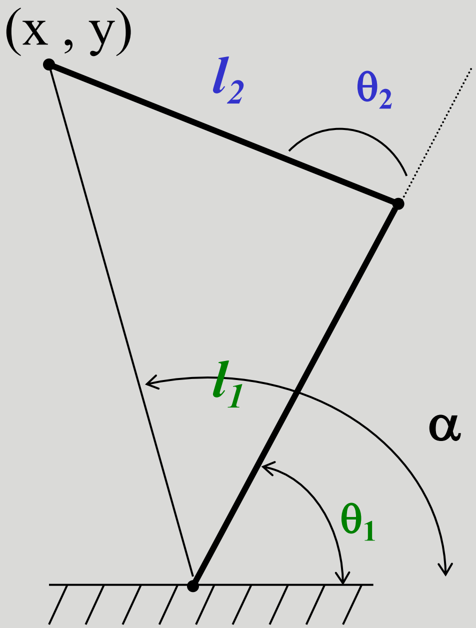

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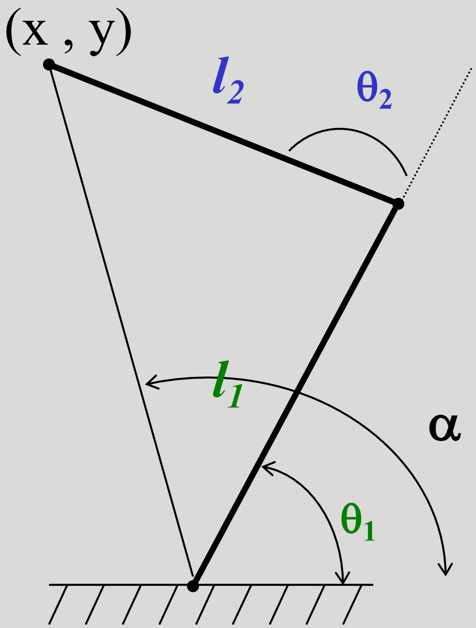

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$$\theta_1 = \alpha - \bar{\theta}_1$$

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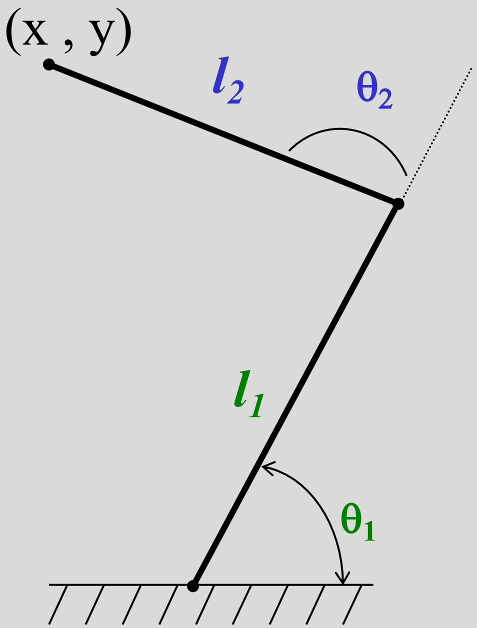
Redundancy caused since θ_2 has two possible values

$$\theta_1 = \arctan 2(y, x) - \arcsin\left(\frac{l_2\sin(\theta_2)}{\sqrt{x^2 + y^2}}\right)$$

The Algebraic Solution



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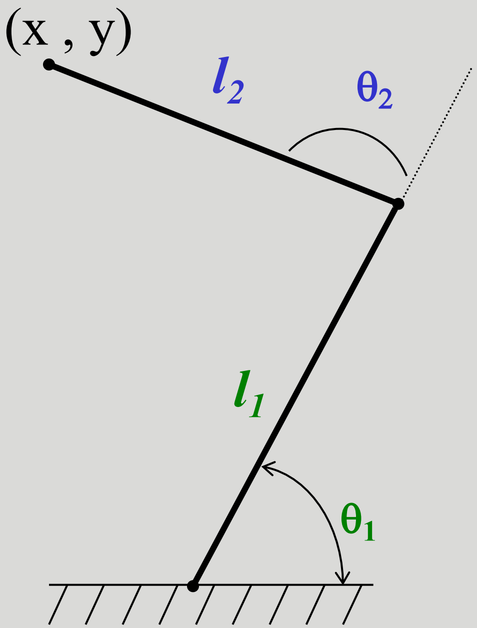

$$c_1 = \cos\theta_1$$

$$c_{1+2} = \cos(\theta_2 + \theta_1)$$

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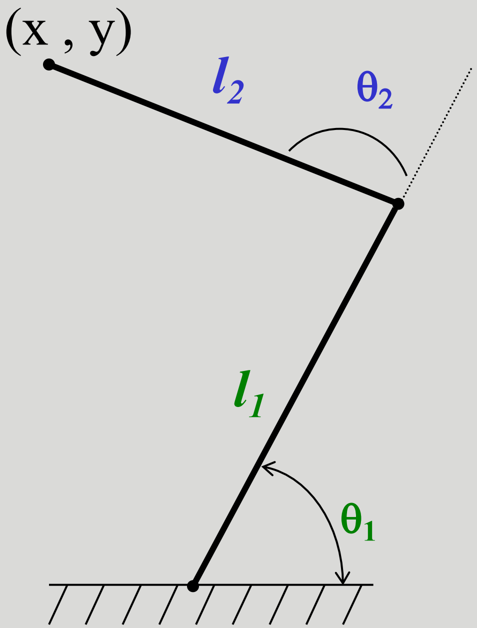

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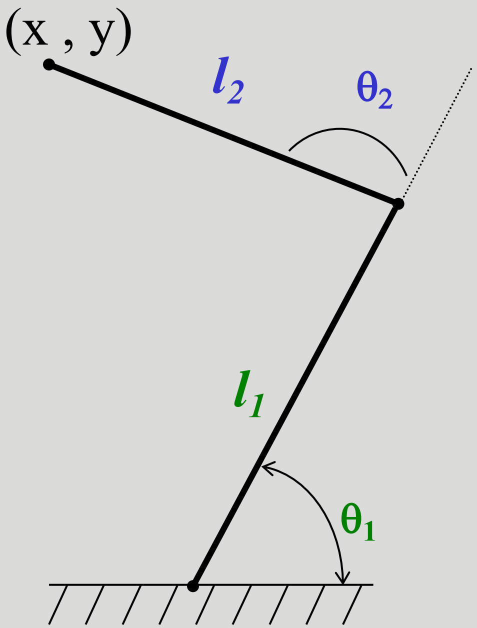

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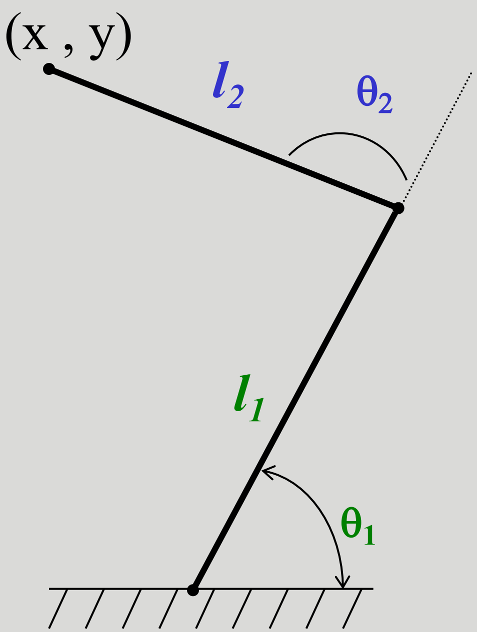

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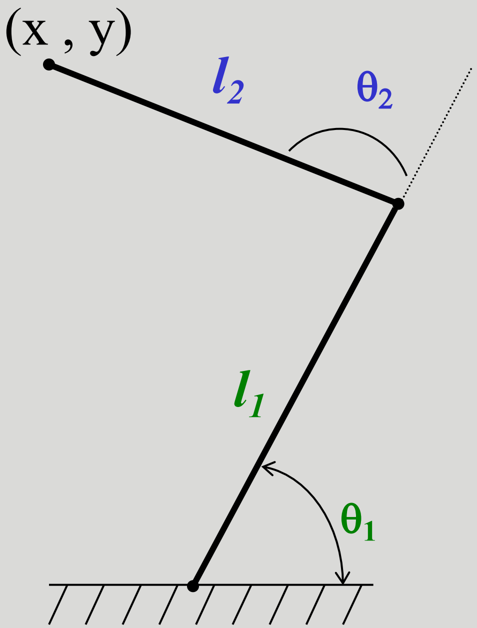

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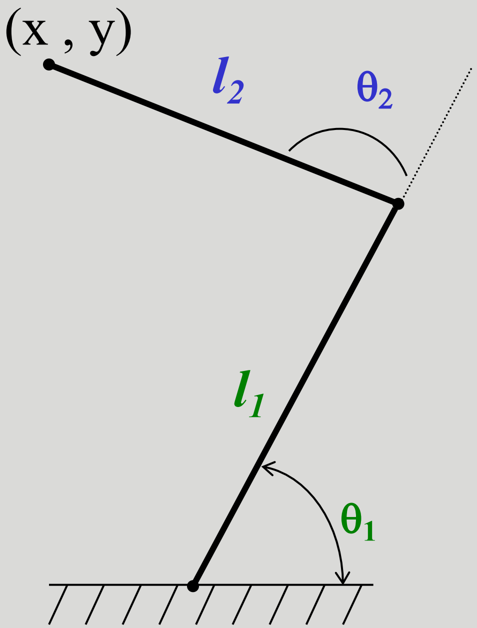

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$$\cos(a \pm b) = (\cos a)(\cos b) \mp (\sin a)(\sin b)$$

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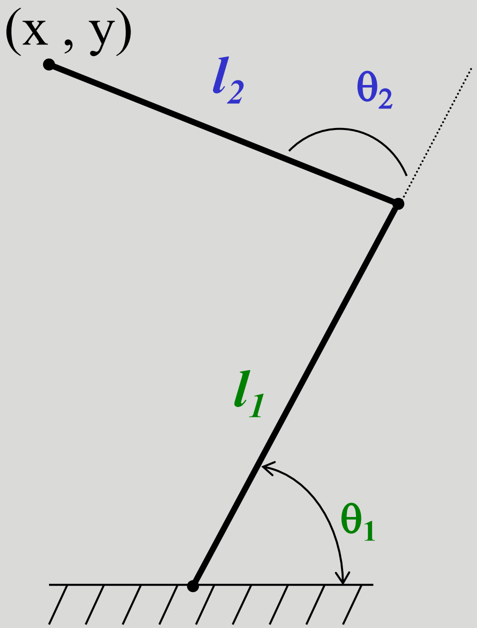

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$$= l_1^{\,2} + l_2^{\,2} + 2 l_1 l_2\, \boxed{c_2}$$

Only Unknown

Note:

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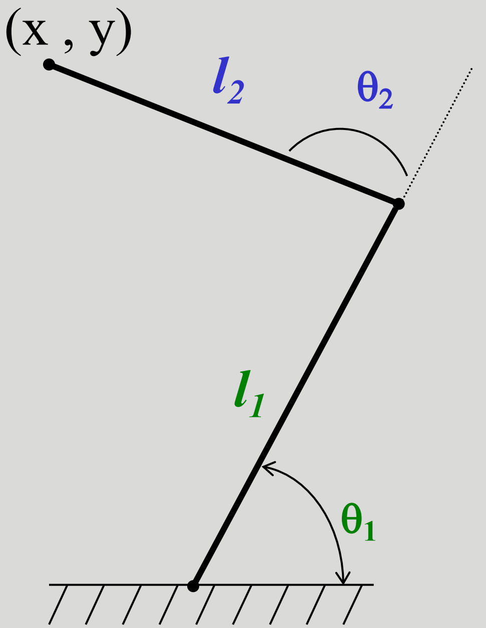
The Algebraic Solution



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$$= l_1^{\,2} + l_2^{\,2} + 2l_1 l_2 \left(c_1(c_{1+2}) + s_1(\sin_{1+2})\right)$$
$$= l_1^{\,2} + l_2^{\,2} + 2l_1 l_2\, \boxed{c_2} \leftarrow \text{Only Unknown}$$
$$\therefore \theta_2 = \arccos\left(\frac{x^2 + y^2 - l_1^{\,2} - l_2^{\,2}}{2l_1 l_2}\right)$$

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We know what θ_2 is from the previous slide. We need to solve for θ_1 . Now we have two equations and two unknowns (sin θ_1 and cos θ_1)

$$x = l_1 c_1 + l_2 c_{1+2}$$

$$y = l_1 s_1 + l_2 \sin_{1+2}$$

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\begin{aligned}
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&= l_1 c_1 + l_2 c_1 c_2 - l_2 s_1 s_2 \\
&= c_1 (l_1 + l_2 c_2) - s_1 (l_2 s_2)
\end{aligned}
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$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} (l_1 + l_2 c_2) & (-l_2 s_2) \\ (l_2 s_2) & (l_1 + l_2 c_2) \end{bmatrix} \begin{bmatrix} c_1 \\ s_1 \end{bmatrix}$$

$$\theta_1 = \arctan 2(s_1, c_1)$$

$$
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Alternative approach

$$\theta_1 = \arctan 2(s_1, c_1)$$

$$x = l_1 c_1 + l_2 c_{1+2}$$
$$= l_1 c_1 + l_2 c_1 c_2 - l_2 s_1 s_2$$
$$= c_1 (l_1 + l_2 c_2) - s_1 (l_2 s_2)$$

Note:

$$\cos(a \pm b) = (\cos a)(\cos b) \mp (\sin a)(\sin b)$$
$$\sin(a \pm b) = (\cos a)(\sin b) \pm (\cos b)(\sin a)$$

$$y = l_1 s_1 + l_2 \sin_{1+2}$$
$$= l_1 s_1 + l_2 s_1 c_2 + l_2 s_2 c_1$$
$$= c_1 (l_2 s_2) + s_1 (l_1 + l_2 c_2)$$

We know what θ_2 is from the previous slide. We need to solve for θ_1 . Now we have two equations and two unknowns (sin θ_1 and cos θ_1)

$$c_1 = \frac{x + s_1 (l_2 s_2)}{(l_1 + l_2 c_2)}$$

$$y = \frac{x + s_1 (l_2 s_2)}{(l_1 + l_2 c_2)} (l_2 s_2) + s_1 (l_1 + l_2 c_2)$$

Substituting for c_1 and simplifying many times

$$= \frac{1}{(l_1 + l_2 c_2)} \left(x\, l_2 s_2 + s_1 (l_1^2 + l_2^2 + 2 l_1 l_2 c_2) \right)$$

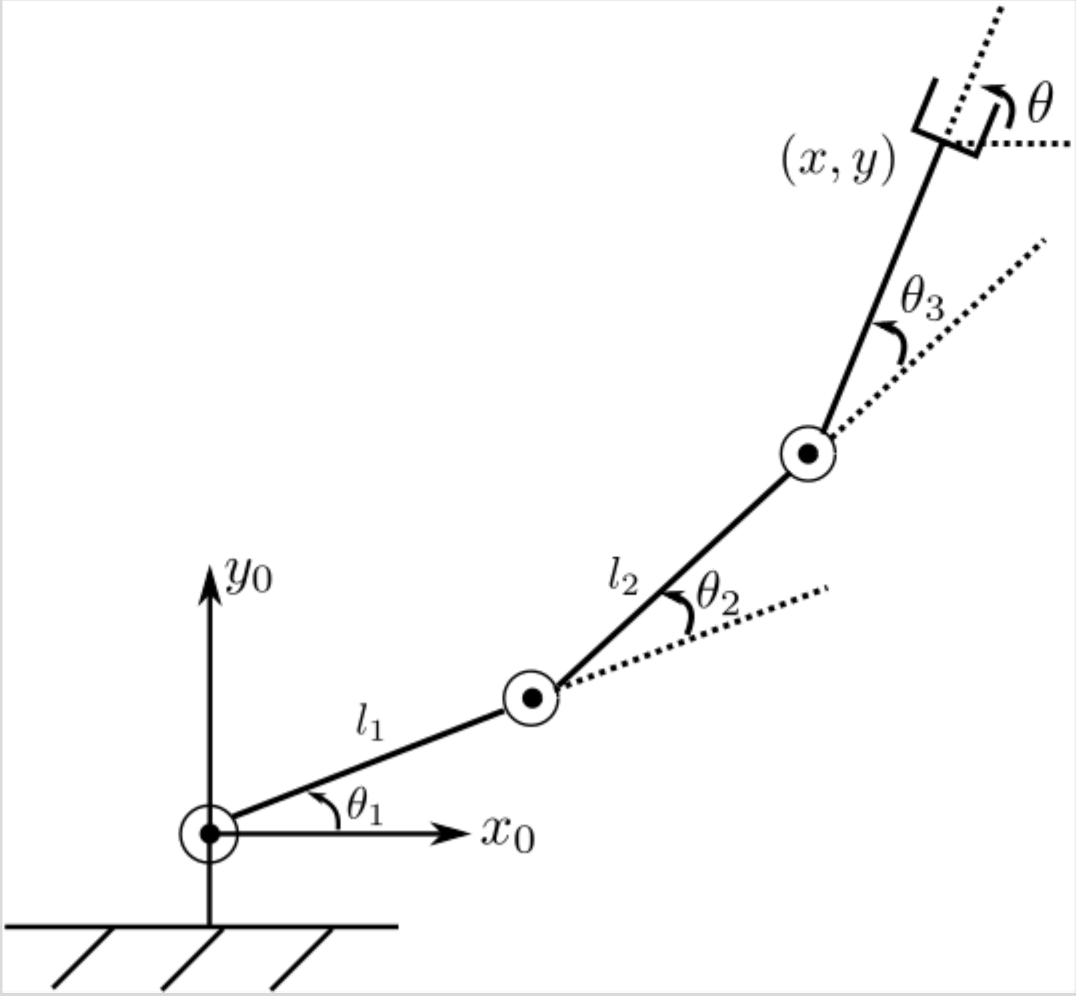
Notice this is the law of cosines and can be replaced by $x^2 + y^2$

$$s_1 = \frac{y(l_1 + l_2 c_2) - x\, l_2 s_2}{x^2 + y^2}$$

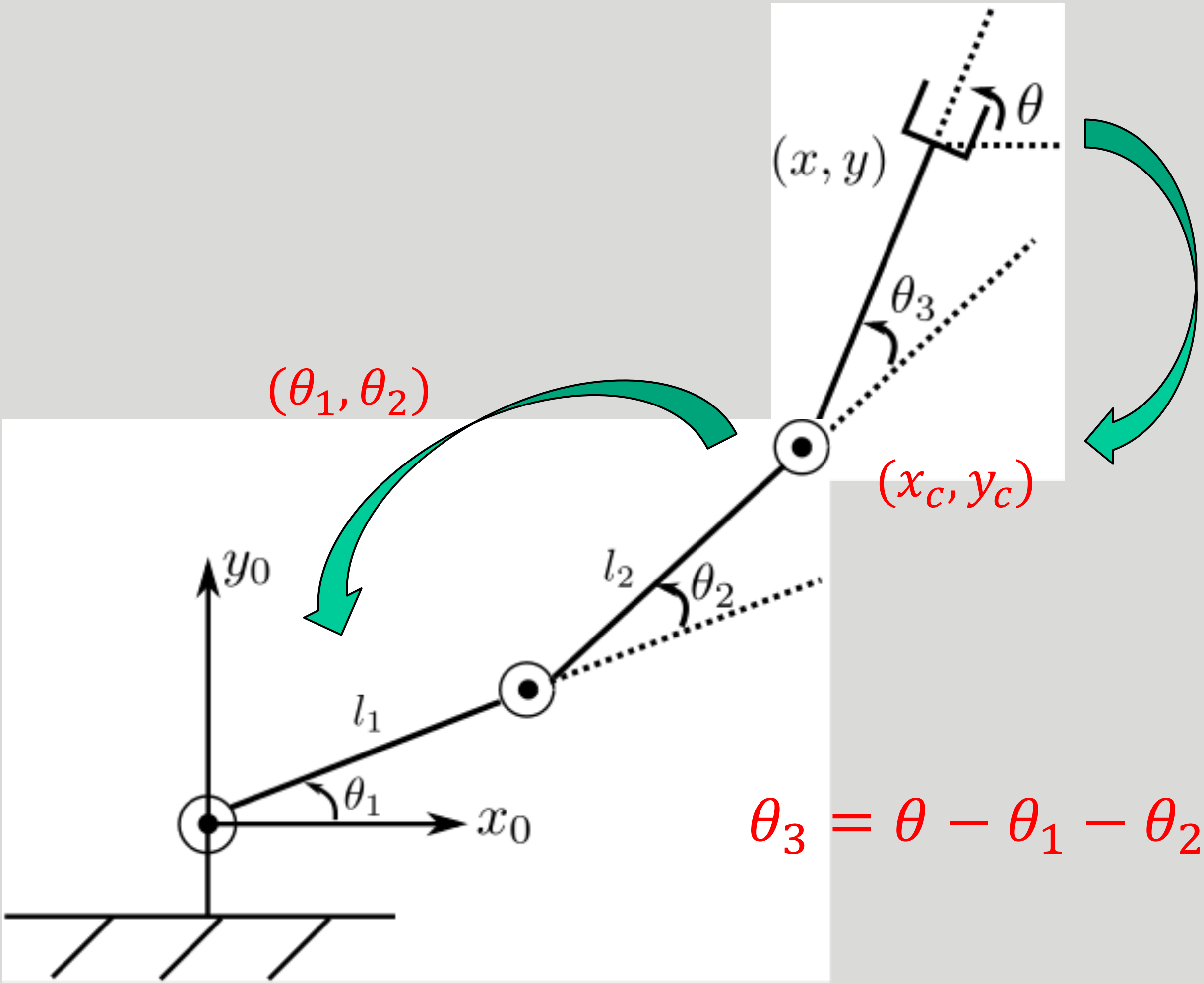
$$\theta_1 = \arctan 2(s_1, c_1)$$

Three-link Manipulator IK

Three-link Robot



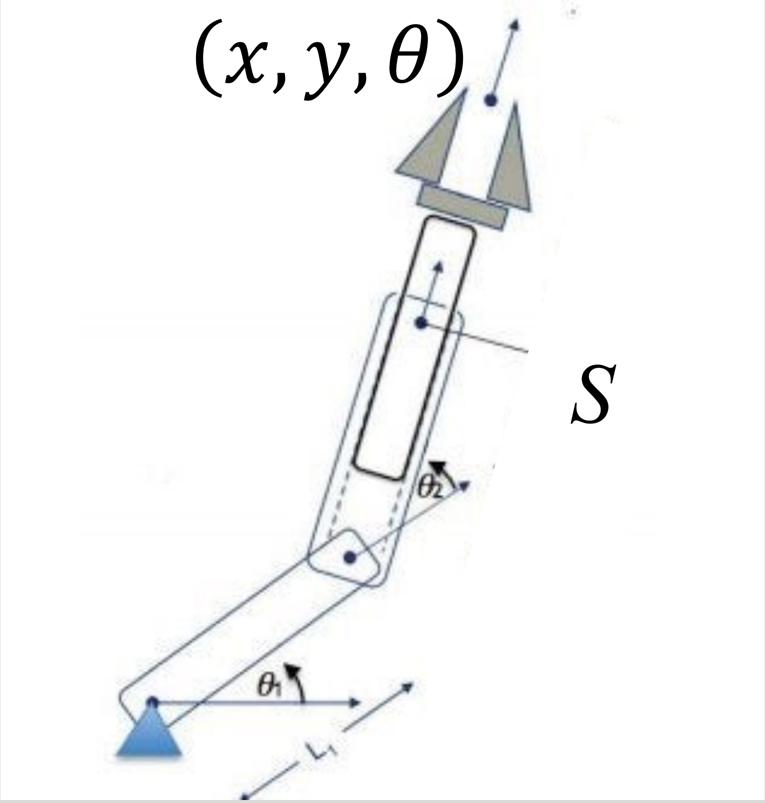
Three-link to Two-link Robot



$$\theta_3 = \theta - \theta_1 - \theta_2$$

RRP Arm

$$x = l_1 c_1 + S c_{12}$$
$$y = l_1 s_1 + S s_{12}$$
$$\theta = \theta_1 + \theta_2$$



RRP Arm

$$x = l_1 c_1 + S c_{12} \longrightarrow x - S c_{12} = l_1 c_1$$

$$y = l_1 s_1 + S s_{12} \longrightarrow y - S s_{12} = l_1 s_1$$

$$\theta = \theta_1 + \theta_2$$

RRP Arm

$$x = l_1 c_1 + S c_{12} \longrightarrow x - S c_\theta = l_1 c_1$$

$$y = l_1 s_1 + S s_{12} \longrightarrow y - S s_\theta = l_1 s_1$$

$$\theta = \theta_1 + \theta_2$$

RRP Arm

$$x = l_1 c_1 + S c_{12} \longrightarrow (x - S c_\theta = l_1 c_1)^2$$

$$y = l_1 s_1 + S s_{12} \longrightarrow (y - S s_\theta = l_1 s_1)^2$$

$$\theta = \theta_1 + \theta_2$$

$$x^2 + S^2 c_\theta^2 - 2xS c_\theta + y^2 + S^2 s_\theta^2 - 2yS s_\theta = l_1^2 c_1^2 + l_1^2 s_1^2$$

RRP Arm

$$x = l_1 c_1 + S c_{12} \longrightarrow (x - S c_\theta = l_1 c_1)^2$$

$$y = l_1 s_1 + S s_{12} \longrightarrow (y - S s_\theta = l_1 s_1)^2$$

$$\theta = \theta_1 + \theta_2$$

$$\underline{x^2} + \underline{S^2 c_\theta^2} - 2xS c_\theta + \underline{y^2} + \underline{S^2 s_\theta^2} - 2yS s_\theta$$
$$= \underline{l_1^2 c_1^2} + \underline{l_1^2 s_1^2}$$

RRP Arm

$$x = l_1 c_1 + S c_{12} \longrightarrow (x - S c_\theta = l_1 c_1)^2$$

$$y = l_1 s_1 + S s_{12} \longrightarrow (y - S s_\theta = l_1 s_1)^2$$

$$\theta = \theta_1 + \theta_2$$

$$x^2 + S^2 c_\theta^2 - 2xSc_\theta + y^2 + S^2 s_\theta^2 - 2ySs_\theta$$
$$= l_1^2 c_1^2 + l_1^2 s_1^2$$

$$x^2 + y^2 + S^2 - (2x + 2y) c_\theta S + S^2 = l_1^2$$

RRP Arm

$$x = l_1 c_1 + S c_{12} \longrightarrow (x - S c_\theta = l_1 c_1)^2$$

$$y = l_1 s_1 + S s_{12} \longrightarrow (y - S s_\theta = l_1 s_1)^2$$

$$\theta = \theta_1 + \theta_2$$

$$x^2 + S^2 c_\theta^2 - 2xSc_\theta + y^2 + S^2 s_\theta^2 - 2ySs_\theta = l_1^2 c_1^2 + l_1^2 s_1^2$$

$$x^2 + y^2 + S^2 - (2x + 2y) c_\theta S = l_1^2$$

$$S^2 - (2x + 2y) c_\theta S + x^2 + y^2 - l_1^2 = 0$$

RRP Arm

$$S = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = l_1 c_1 + S c_{12} \longrightarrow (x - S c_\theta = l_1 c_1)^2$$

$$y = l_1 s_1 + S s_{12} \longrightarrow (y - S s_\theta = l_1 s_1)^2$$

$$\theta = \theta_1 + \theta_2$$

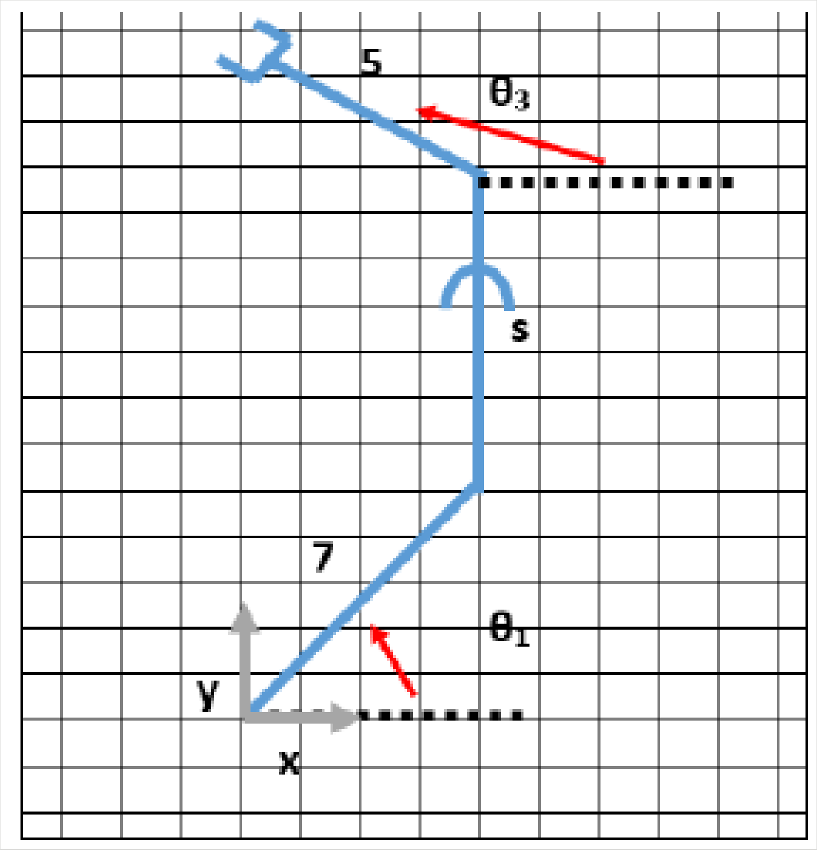
$$x^2 + S^2 c_\theta^2 - 2xS c_\theta + y^2 + S^2 s_\theta^2 - 2yS s_\theta = l_1^2 c_1^2 + l_1^2 s_1^2$$

$$x^2 + y^2 + S^2 - (2x c_\theta + 2y s_\theta) S = l_1^2$$

$$\boxed{S^2 - (2x c_\theta + 2y s_\theta) S + x^2 + y^2 - l_1^2 = 0}$$

RPR

Now we have a RPR arm with angles described from the horizontal. The prismatic link must always be vertical. Find θ_1, s, and θ_3 in terms of (x, y, and θ) of the end effector:



RPR

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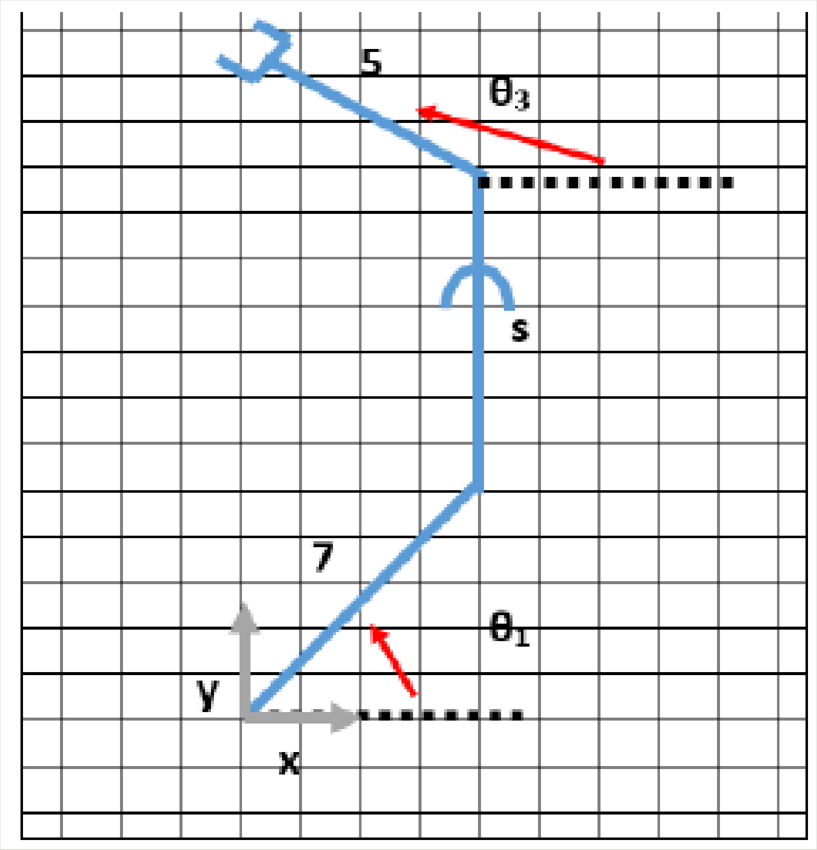
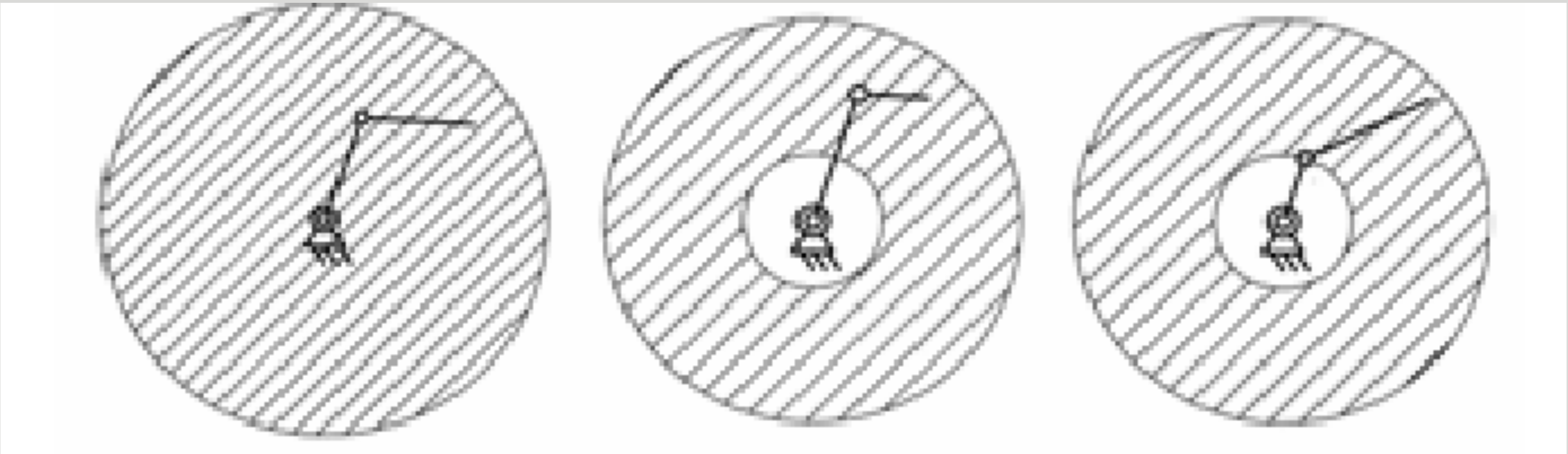
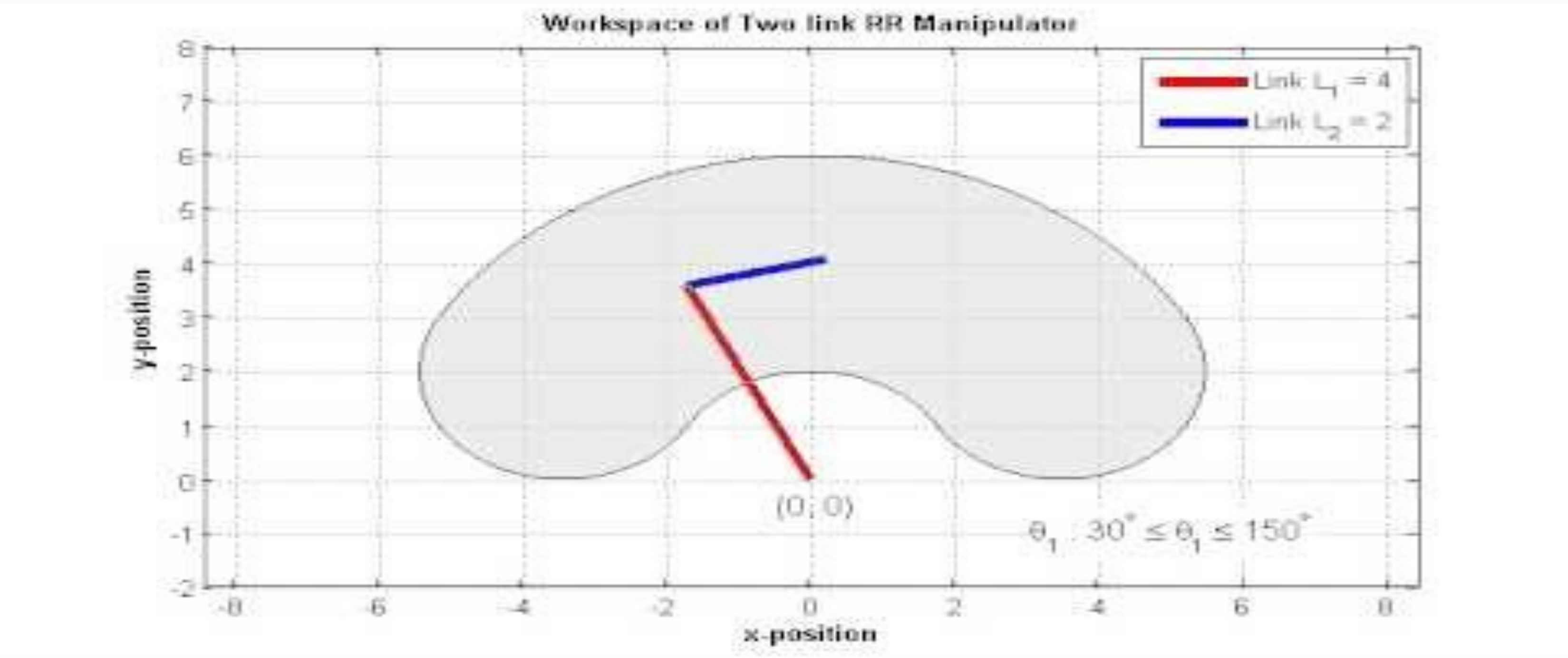


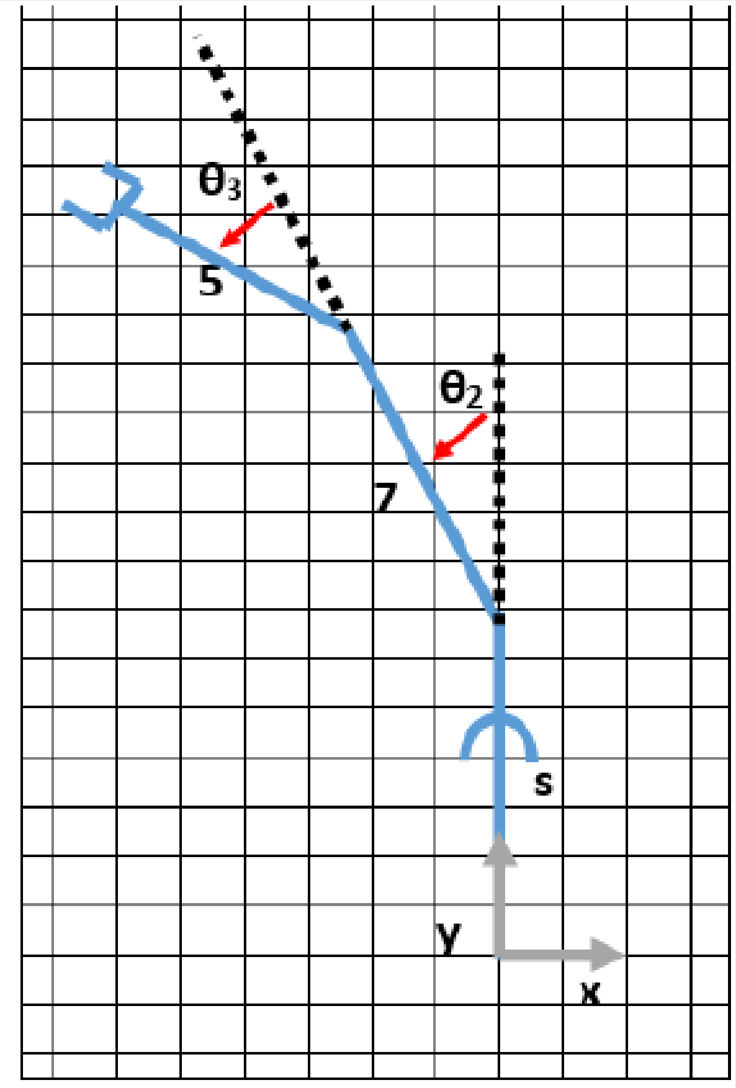
Workspace of Two-Link



Workspace of Two-Link



PRR Arm

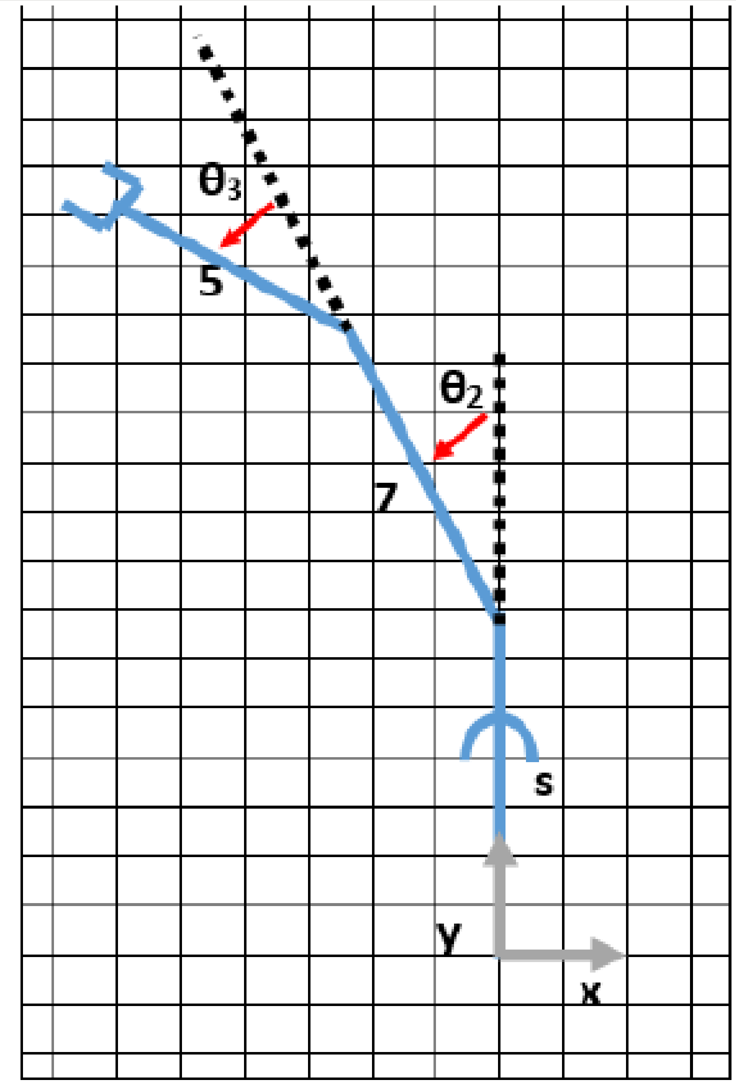


We have a PRR arm as shown below. The first joint is a prismatic joint starting the origin and continuing up along the y axis. It is of length s_1 and it is limited from 5 to 10 cm. The second and third joints are revolute joints with angles defined relative to the previous link where 0 is in line with the previous link and angles increasing counter-clockwise. The second link is of length 7. The third link is of length 5. The revolute joints (θ_2 and θ_3) are constrained between 0 and 180 degrees. The angle of the end effector is with respect to the positive x axis.

How many solutions are there to place the end effector at (0,22)?

How many solutions are there to place the end effector at (7,1)?

PRR Arm



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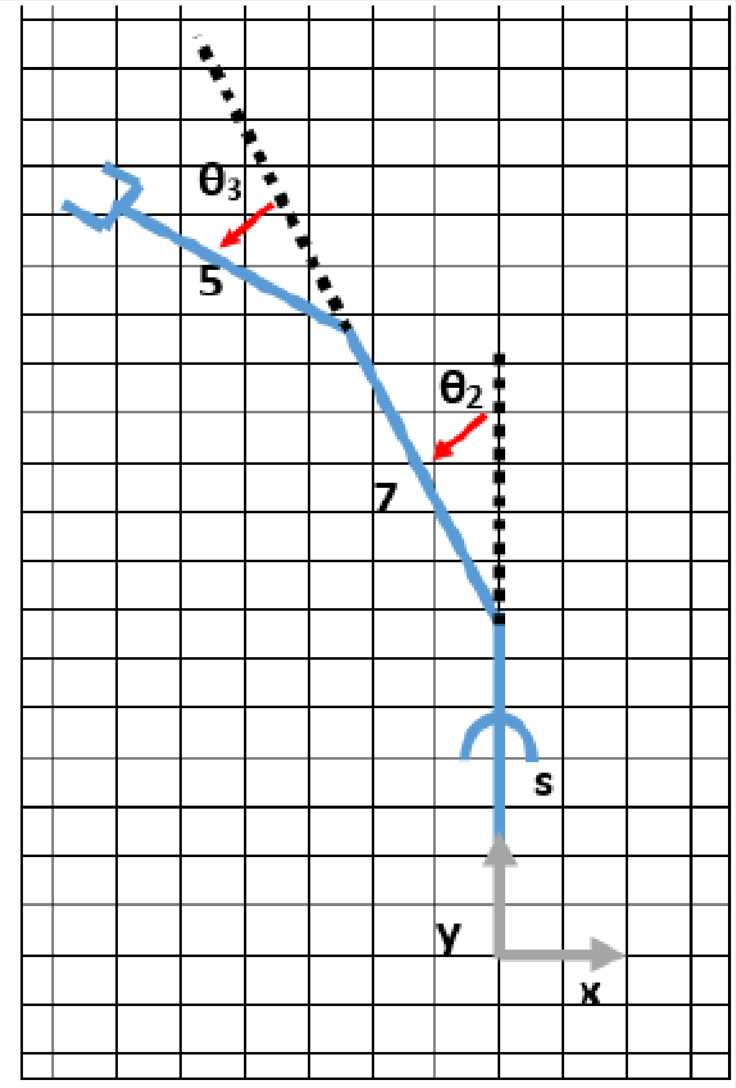
PRR Arm



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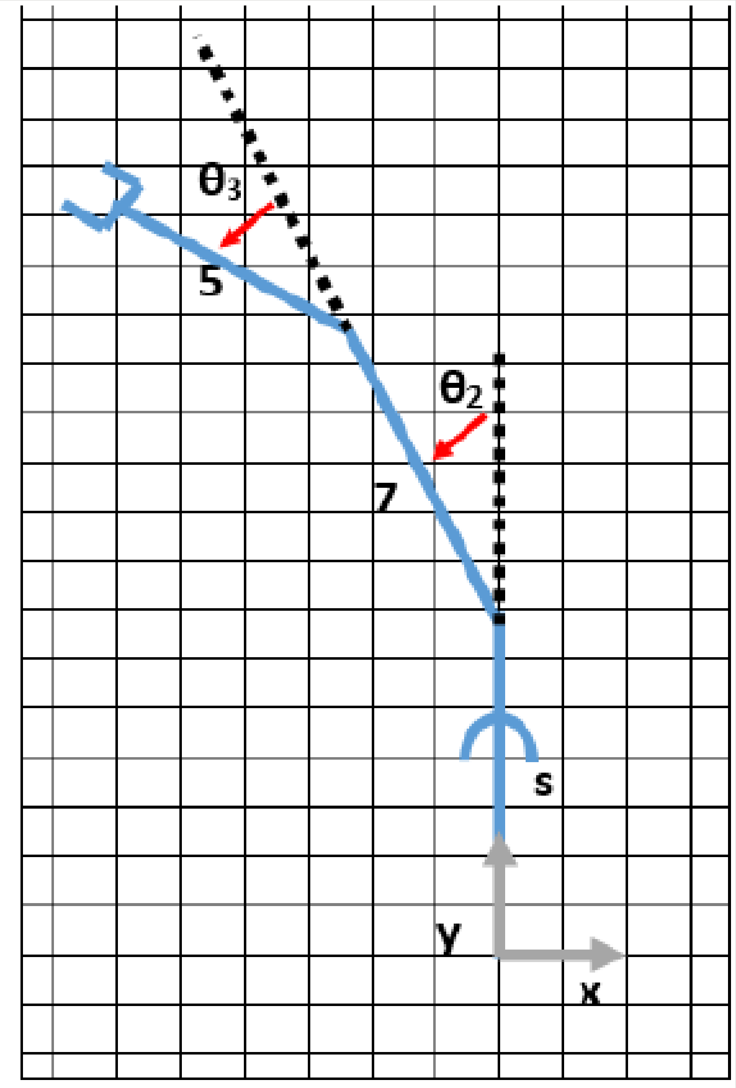
PRR Arm



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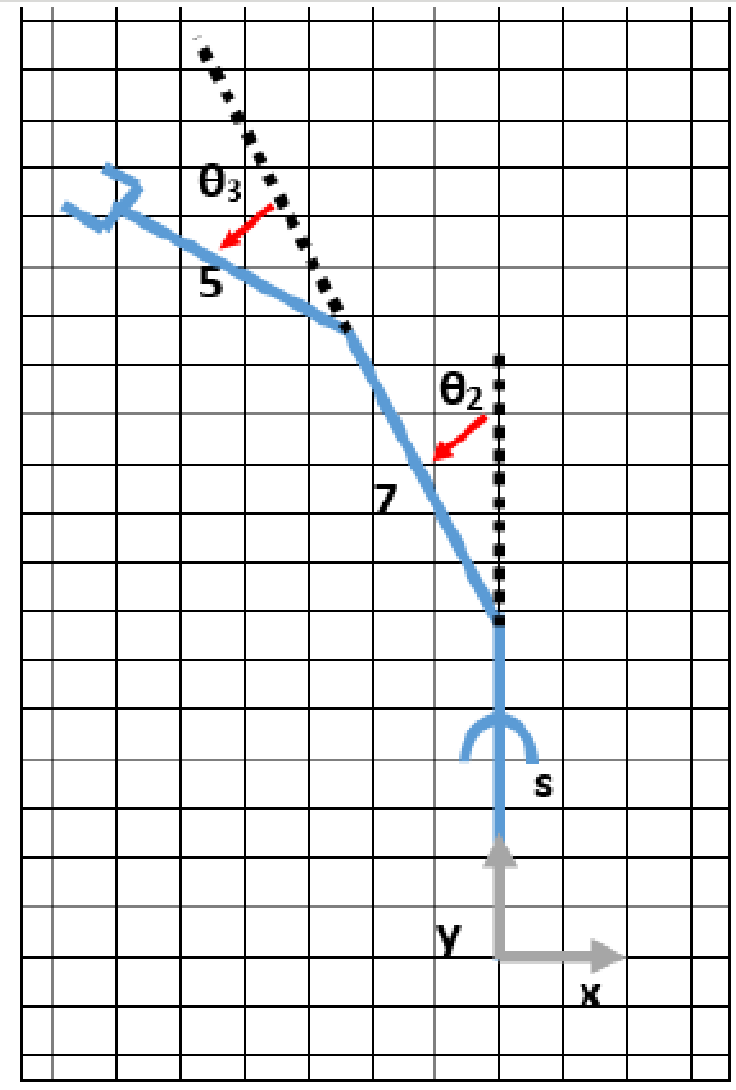
PRR Arm



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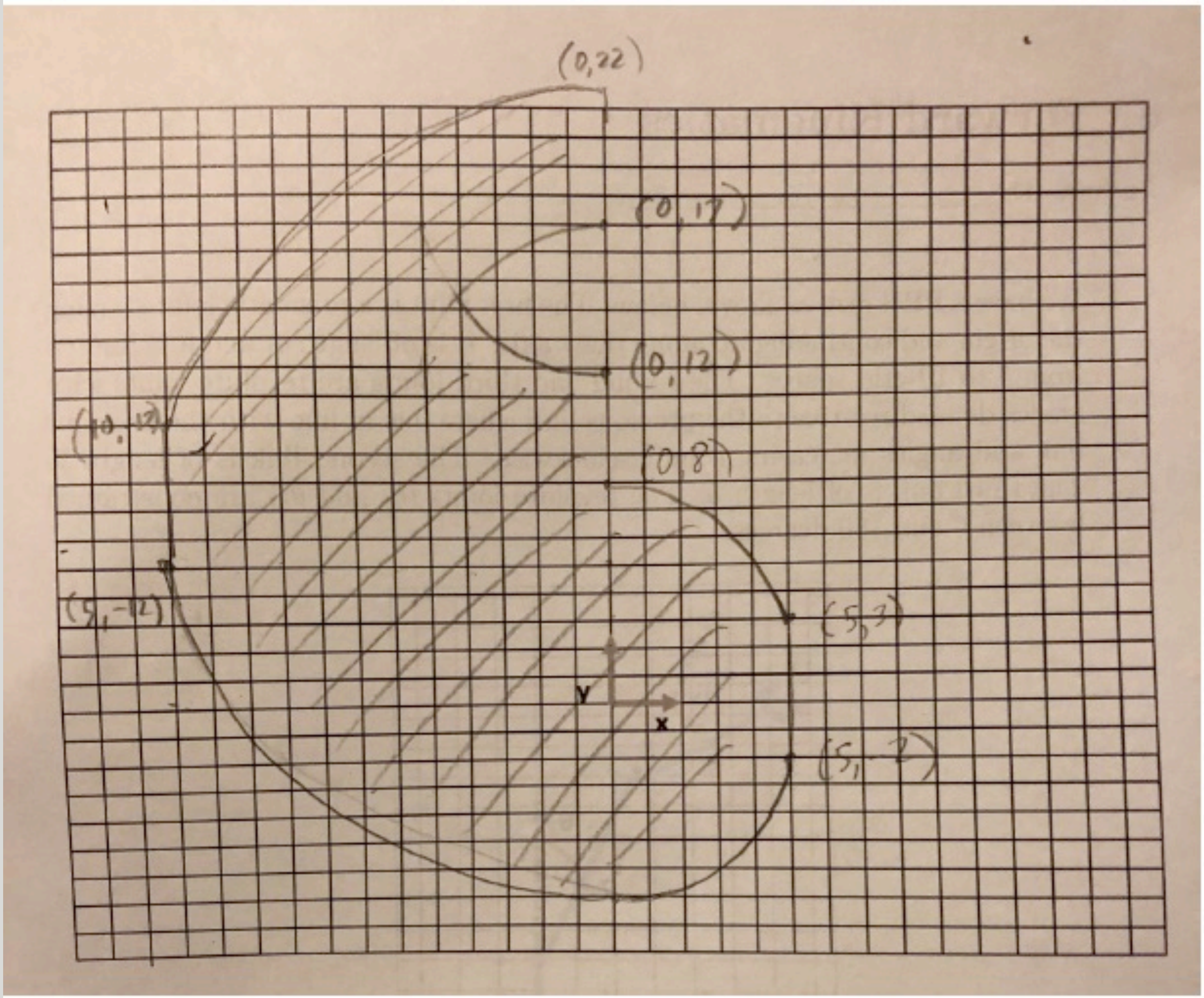
(0,22)
(0,17)
(0,12)
(10,17)
(0,8)
(5,-12)
(5,3)
y
x
(5,-7)