Transfer Function

- Defined as the ratio of the Laplace transform of the output signal to that of the input signal (think of it as a gain factor!)
- Contains information about dynamics of a Linear Time Invariant system
- Time domain

$$u(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = h(t) * u(t)$$

Laplace transform

Inverse Laplace transform

- Frequency domain

$$U(s) \rightarrow \boxed{H(s)} \rightarrow Y(s) = H(s)U(s)$$

Mass-Spring-Damper System

- ODE

$$M\ddot{y}(t) + b\dot{y}(t) + ky(t) = u(t)$$

- Assume all initial conditions are zero. Then take Laplace transform,

$$Ms^2Y(s) + bsY(s) + kY(s) = U(s)$$

Output

$$\boxed{\frac{Y(s)}{U(s)}} = \boxed{\frac{1}{Ms^2 + bs + k}}$$

Transfer function

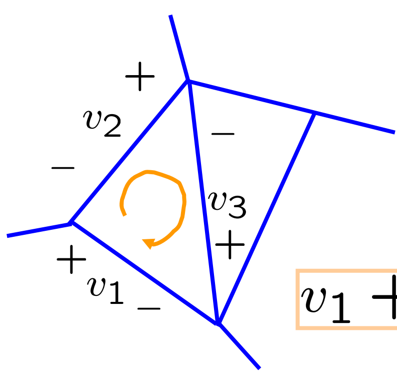
Input

Transfer Function

- Differential equation replaced by algebraic relation *Y(s)=H(s)U(s)*
- If U(s)=1 then Y(s)=H(s) is the impulse response of the system
- If U(s)=1/s, the unit step input function, then Y(s)=H(s)/s is the step response
- The magnitude and phase shift of the response to a sinusoid at frequency w is given by the magnitude and phase of the complex number $H(jw)$
- Impulse: $\mathcal{L}[\delta(t)] = \int_0^\infty \delta(t)e^{-st}dt = 1$
- Unit step: $\mathcal{L}[1(t)] = \int_0^\infty e^{-st}dt = \frac{1}{s}$

Kirchhoff's Voltage Law

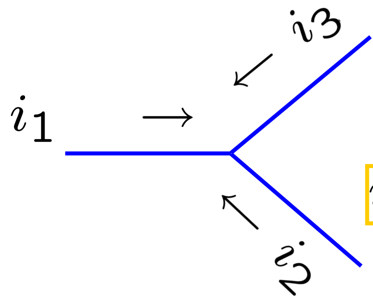
- The algebraic sum of voltages around any closed loop in an electrical circuit is zero.



$$\boxed{v_1 + v_2 + v_3 = 0}$$

Kirchhoff's Current Law

- The algebraic sum of currents into any junction in an electrical circuit is zero.



$$\boxed{i_1 + i_2 + i_3 = 0}$$