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Article *in* Forschung im Ingenieurwesen · May 2001

DOI: 10.1007/s100100100060

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Forschung im Ingenieurwesen 66 (2001) 211–216 © Springer-Verlag 2001

Kinematics of four-wheel-steering vehicles

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Abstract In this paper, we review some aspects of the kinematical theory of four-wheel-steering (4WS) vehicles and present some new results and conclusions that we came across during our research on the subject. In a first paragraph, we compare the turning radius of two-wheel-steering (2WS) and 4WS vehicles and draw some interesting conclusions on the manoeuvring ability of either of them. In a second paragraph, we present a general kinematical analysis of 4WS vehicles by considering either the simplified two-wheel (bicycle) model or the more complete four-wheel model. In this analysis, we assume the sideslip angles of the wheels as non-negligible and we derive the general formulae relating the steering angles of the wheels to the geometrical data of the vehicle. By taking as zero the steering angles of the rear wheels, we derive from the above relations the well-known formulae of Ackermann–Jeantaux that are valid for 2WS vehicles.

Kinematik der vierradgelenkten Fahrzeuge

Zusammenfassung In diesem Beitrag werden einige Gesichtspunkte der Kinematik-Theorie von vierradgelenkten Fahrzeugen angegeben und neue Ergebnisse und Schlußfolgerungen präsentiert. An erster Stelle wird der Drehradius von vorderradgelenkten und vierradgelenkten Fahrzeugen verglichen und Folgerungen über deren Maneuvrierbarkeit gezogen. Anschließend wird eine allgemeingültige, kinematische Analyse von vierradgelenkten Fahrzeugen präsentiert, unter Berücksichtigung entweder des vereinfachten Zweierrad Modells (Fahrrad) oder des vollständigeren Vierrad Modells. In dieser Analyse wird angenommen, das der Schräglaufwinkel des jeweiligen Rades nicht vernachlässigbar ist und die allgemeine Formel zwischen des Einschlagwinkels der Räder und der geometrischen Daten des Fahrzeuges hergeleitet wird. Mit der Annahme, dass die Hinterradeinschlagwinkel null sind, wird aus den oberen Beziehungen die bekannte Ackermann–Jeantaux Formel hergeleitet, die für die vorderradgelenkten Fahrzeuge gültig ist.

Received: 9. January 2001

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List of symbols

a	absolute value of the distance of front axle to the centre of gravity of the sprung mass
b	absolute value of the distance of rear axle to the centre of gravity of the sprung mass
k	ratio δ_r/δ_f
ℓ	wheelbase
R	distance of centre of mass of a vehicle from the instantaneous centre of rotation
R_2	distance of the instantaneous centre of rotation to the longitudinal axis of the vehicle
R_{2WS}	turning radius of a 2WS vehicle
R_{4WS}	turning radius of a 4WS vehicle
$2t$	track of a vehicle
α_j	sideslip angle of the jth wheel (j = f for front wheels, j = r for rear wheels)
β	sideslip angle of the vehicle
δ_j	steer angle of the jth wheel (j = f for front wheels, j = r for rear wheels)

Subscripts

f	front wheel
G	centre of mass
i	inner wheel to the negotiated curve
j	jth wheel of the vehicle (j = f for front wheels, j = r for rear wheels)
o	outer wheel to the negotiated curve
r	rear wheel
2WS	two-wheel-steering
4WS	four-wheel-steering

1 Introduction

The last decade considerable efforts and money have been invested in the development of four-wheel-steering (4WS) vehicles, aiming to improve the safety of operation of vehicles by improving their stability behaviour and their steering and manoeuvring ability.

The scope of this paper is to review some aspects of the kinematical theory of 4WS vehicles and present some results and conclusions that we came across during our research on the subject. With the exception of [3], we have no knowledge of a publication on the kinematics of 4WS vehicles, most of the effort of the researchers having been concentrated on the dynamics of 4WS vehicles.

In a first paragraph, we compute the turning radius of two-wheel-steering (2WS) and of four-wheel-steering (4WS) vehicles and by comparison, we draw some interesting conclusions on the manoeuvring ability of each one of them.

In a second paragraph, we present a general kinematical analysis of 4WS vehicles by considering either the simplified two-wheel (bicycle) model or the more complete four-wheel model. In this analysis, we assume the sideslip angles of the wheels as non-negligible and we derive the general formulae relating the steering angles of the wheels to the geometrical data of the vehicle. By zeroing the value of sideslip angles of the wheels in these formulae, we obtain the simplified formulae that are valid for small speeds of motion of the vehicle. In addition, by taking as zero the steering angles of the rear wheels, we derive the well-known formulae of Ackermann–Jeantaux that are valid for 2WS vehicles.

2
Turning radius of 2WS and of 4WS vehicles

Referring to Fig. 1 and considering the triangle $(I_{2WS}\,A_r\,A_f)$, we can easily verify that the turning radius of a 2WS vehicle is

$$R_{2WS} = (A_r A_f)/\sin\delta_f = \ell/\sin\delta_f \quad . \tag{1}$$

Referring to Figs. 1 and 2 and considering the triangle $(I_{4WS}\,I_m\,A_f)$, we can also obtain the tuning radius of a 4WS vehicle as follows

$$R_{4WS} = (I_m A_f)/\sin\delta_f \quad . \tag{2}$$

Relation (2) is valid when the rear wheels of the 4WS vehicle turn either in the same direction as the front ones or in the opposite direction. However, the length $(I_m A_f)$ appearing in it does not have the same value in these two cases.

Comparison of the turning radii given by relations (1) and (2) gives us the following results:

$R_{4WS} < R_{2WS}$	when the rear wheels turn in opposite direction than the front ones
$R_{4WS} > R_{2WS}$	when the rear wheels turn in the same direction than the front ones

Consequently, a 4WS vehicle presents a manoeuvring advantage over a 2WS vehicle only if its rear wheels can turn in the opposite direction than its front wheels. In that case, we have a relative reduction of the turning radius that is equal to

$$\begin{aligned}
|R_{4WS} - R_{2WS}|/R_{4WS} &= |(I_m A_f)/\sin\delta_f - (A_r A_f)/\sin\delta_f|/R_{4WS}\\
&= |(I_m A_f)/\sin\delta_f - \ell/\sin\delta_f|/R_{4WS}\\
&= [(A_r I_m)/\sin\delta_f]/R_{4WS}\\
&= [(I_m I_{4WS})\tan\delta_r/\sin\delta_f]/R_{4WS}\\
&= \{[(I_m A_f)\cos\delta_f]\tan\delta_r\}/R_{4WS}\\
&= [R_{4WS}\tan\delta_r/\tan\delta_f]/R_{4WS}
\end{aligned} \tag{3}$$

and consequently

$$|R_{4WS} - R_{2WS}|/R_{4WS} = \tan\delta_r/\tan\delta_f \quad . \tag{4}$$

Manoeuvring ability is important in urban areas, where usually a speed limit of 50 km/h or lower applies and the sideslip angle of the wheels is negligible. Consequently, the steering system of a 4WS vehicle must have the

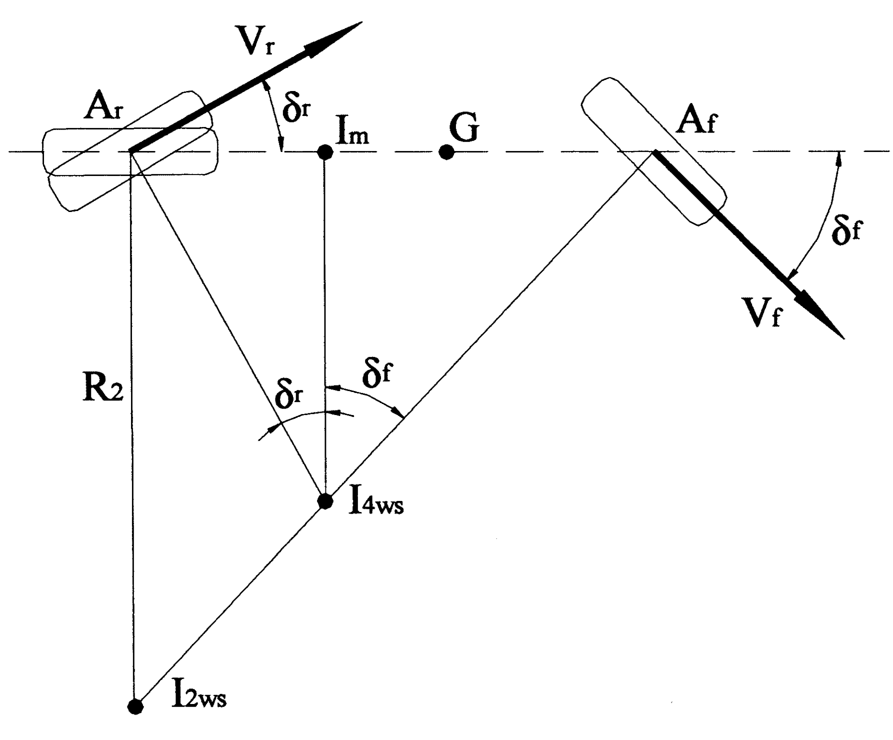

Fig. 1. Two-wheel (bicycle) model of a 4WS vehicle; the rear wheel turns in the opposite direction to the front one

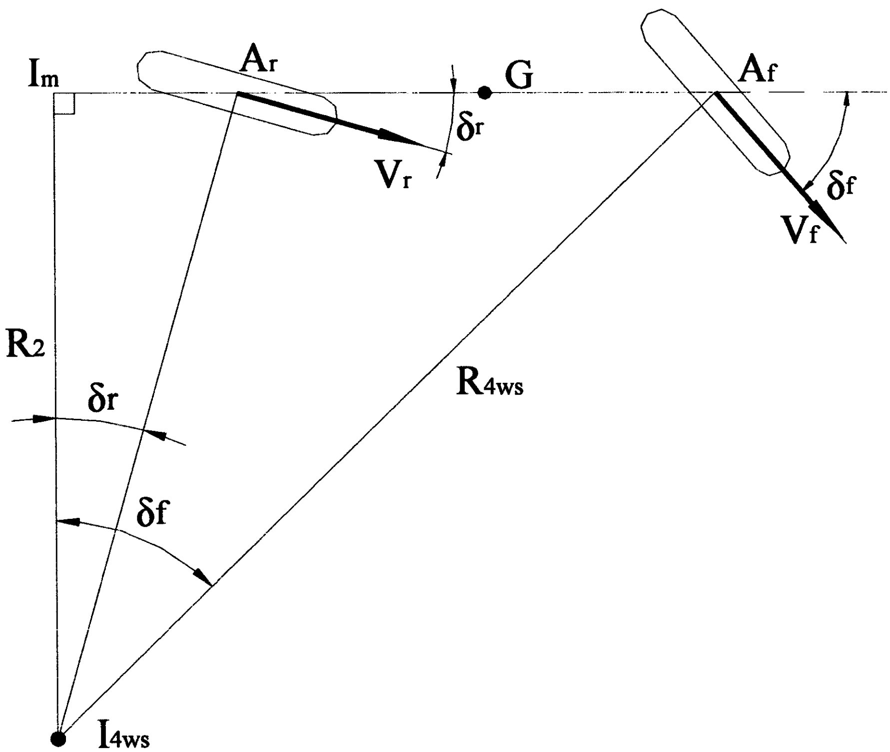

Fig. 2. Two-wheel (bicycle) model of 4WS vehicle; the rear wheel turns in the same direction as the front one

ability, at low speeds, to turn the rear steering wheels in the opposite direction than the front steering wheels. As the vehicle's speeds increases above that limit, we must provide for a progressive evolution of the rear wheels steering angle till it gets the same signum as the front wheels steering angle.

3 Kinematical analysis using a two-wheel (bicycle) model of the vehicle

3.1 4WS vehicle, sideslip angles not neglected

Considering the triangles $(I\,I_m\,A_f)$ and $(I\,I_m\,A_r)$ in Fig. 3 and using the following abbreviations

$$a = (GA_f) \tag{5}$$

$$b = (GA_r) \tag{6}$$

$$e = (GI_m) \tag{7}$$

we obtain

$$(I_m A_f) = |a| + |e| = R_2 \tan(\delta_f - \alpha_f) \tag{8}$$

$$(A_r I_m) = |b| - |e| = R_2 \tan(\alpha_r - \delta_r) \ . \tag{9}$$

By adding the above two relations and taking into consideration the obvious relation

$$\ell = (A_r I_m) + (I_m A_f) = |a| + |b| \tag{10}$$

we obtain

$$\tan(\delta_f - \alpha_f) + \tan(\alpha_r - \delta_r) = \ell / R \cos\beta \ . \tag{11}$$

Relation (11) can also be written as

$$\tan(\delta_f - \alpha_f) - \tan(\delta_r - \alpha_r) = \ell / R \cos\beta \ . \tag{12}$$

Also, by considering the relations

$$R_2 = R \cos\beta \tag{13}$$

$$e = R \sin\beta \ , \tag{14}$$

we can write

$$|a| + |e| = |a| + R \sin\beta \tag{15}$$

$$|b| - |e| = |b| - R \sin\beta \ . \tag{16}$$

Consequently we obtain

$$\begin{aligned}\tan(\delta_f - \alpha_f) &= (|a| + R\sin\beta)/R\cos\beta \\ &= |a|/(R\cos\beta) + \tan\beta\end{aligned} \tag{17}$$

$$\begin{aligned}\tan(\delta_r - \alpha_r) &= -[(|b| - R\sin\beta)/R\cos\beta] \\ &= -[|b|/(R\cos\beta) - \tan\beta]\end{aligned} \tag{18}$$

and finally we have

$$\delta_f = \arctan[|a|/(R\cos\beta) + \tan\beta] + \alpha_f \tag{19}$$

$$\delta_r = -\arctan[|b|/(R\cos\beta) - \tan\beta] + \alpha_r \tag{20}$$

$$\begin{aligned}\delta_f + \delta_r = {} & \arctan[|a|/(R\cos\beta) + \tan\beta] \\ & - \arctan[|b|/(R\cos\beta) - \tan\beta] + \alpha_f + \alpha_r \ .\end{aligned} \tag{21}$$

By introducing the ratio

$$k = \delta_r / \delta_f \ , \tag{22}$$

we can also write

$$\begin{aligned}\delta_f = {} & [1/(1+k)]\{\arctan[|a|/(R\cos\beta) + \tan\beta] \\ & - \arctan[|b|/(R\cos\beta) - \tan\beta]\} + \alpha_f + \alpha_r\end{aligned} \tag{23}$$

$$\begin{aligned}\delta_r = {} & [k/(1+k)]\{\arctan[|a|/(R\cos\beta) + \tan\beta] \\ & - \arctan[|b|/(R\cos\beta) - \tan\beta]\} + \alpha_f + \alpha_r \ .\end{aligned} \tag{24}$$

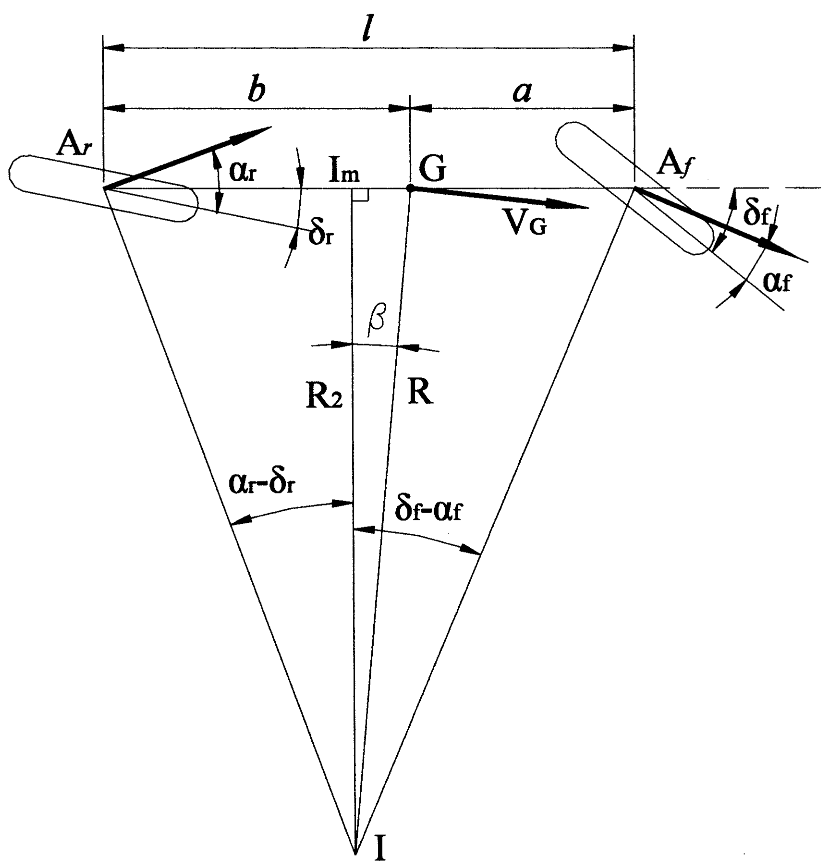

Fig. 3. Two-wheel (bicycle) model of a 4WS vehicle (the side-slip angles of the tires are not neglected)

3.2 4WS vehicle, sideslip angles neglected

When the vehicle is moving at low speed, the impact of the sideslip angles can be neglected. Consequently, we can make the assumption

$$\alpha_f = \alpha_r = 0 \ . \tag{25}$$

By introducing these values in the formulae (12), (17) to (21), (23) and (24) we obtain

$$(\tan\delta_f - \tan\delta_r) = \ell/R\cos\beta \tag{26}$$

$$\begin{aligned}\tan\delta_f &= (|a| + R\sin\beta)/R\cos\beta \\ &= |a|/(R\cos\beta) + \tan\beta\end{aligned} \tag{27}$$

$$\begin{aligned}\tan\delta_r &= -\ [(|b| - R\sin\beta)/R\cos\beta] \\ &= -\ [|b|/(R\cos\beta) - \tan\beta]\end{aligned} \tag{28}$$

$$\delta_f = \arctan[|a|/(R\cos\beta) + \tan\beta] \tag{29}$$

$$\delta_r = -\arctan[|b|/(R\cos\beta) - \tan\beta] \tag{30}$$

$$\begin{aligned}\delta_f + \delta_r = {} & \arctan[|a|/(R\cos\beta) + \tan\beta] \\ & - \arctan[|b|/(R\cos\beta) - \tan\beta]\end{aligned} \tag{31}$$

$$\begin{aligned}\delta_f = [1/(1+k)]\{ & \arctan[|a|/(R\cos\beta) + \tan\beta] \\ & - \arctan[|b|/(R\cos\beta) - \tan\beta]\}\end{aligned} \tag{32}$$

$$\begin{aligned}\delta_r = [k/(1+k)]\{ & \arctan[|a|/(R\cos\beta) + \tan\beta] \\ & - \arctan[|b|/(R\cos\beta) - \tan\beta]\} \ .\end{aligned} \tag{33}$$

3.3 2WS vehicle, side-slip angles not neglected

By assuming that the rear wheel steering angle is zero

$$\delta_r = 0 \ , \tag{34}$$

we derive from the relation (12) the formulae that are valid for a 2WS vehicle with non-negligible sideslip angles

$$\tan(\delta_f - \alpha_f) + \tan\alpha_r = \ell/R\cos\beta \tag{35}$$

$$\tan(\delta_f - \alpha_f) = \ell/R\cos\beta - \tan\alpha_r \tag{36}$$

Relation (36) gives also the value of δ_f

$$\delta_f = \arctan[\ell/R\cos\beta - \tan\alpha_r] + \alpha_f \ . \tag{37}$$

3.4 2WS vehicle, sideslip angles neglected

Assuming, as before, that

$$\alpha_f = \alpha_r = 0 \tag{38}$$

we simplify formulae (36) and (37) as follows

$$\tan\delta_f = \ell/R\cos\beta = \ell/|b| \tag{39}$$

$$\delta_f = \arctan[\ell/R\cos\beta] = \arctan(\ell/|b|) \ . \tag{40}$$

4 Kinematical analysis using a four-wheel model of the vehicle

4.1 4WS vehicle, side-slip angles not neglected

Considering the triangles $(I\,I_{mo}\,A_{fo})$, $(I\,I_{mi}\,A_{fi})$, $(I\,I_{mo}\,A_{ro})$ and $(I\,I_{mi}\,A_{ri})$ in Fig. 4 we obtain

$$|a_2| = (R_2 + t)\tan(\delta_{fo} - \alpha_{fo}) \quad (41)$$

$$|a_2| = (R_2 - t)\tan(\delta_{fi} - \alpha_{fi}) \quad (42)$$

$$|b_2| = (R_2 + t)\tan(\delta_{ro} - \alpha_{ro}) \quad (43)$$

$$|b_2| = (R_2 - t)\tan(\delta_{ri} - \alpha_{ri}) \ , \quad (44)$$

where the symbols a_2 and b_2 are defined as follows

$$a_2 = I_{mo}A_{fo} = I_{mi}A_{fi} \quad (45)$$

$$b_2 = I_{mo}A_{ro} = I_{mi}A_{ri} \ . \quad (46)$$

But we have

$$\ell = |a_2| + |b_2| \ . \quad (47)$$

By adding Eqs. (41) to (43), we obtain

$$\ell = (R_2 + t)[\tan(\delta_{fo} - \alpha_{fo}) + \tan(\delta_{ro} - \alpha_{ro})] \ . \quad (48)$$

Also, by adding Eqs. (42) to (44), we obtain

$$\ell = (R_2 - t)[\tan(\delta_{fi} - \alpha_{fi}) + \tan(\delta_{ri} - \alpha_{ri})] \ . \quad (49)$$

The Eqs. (48) and (49) can also be written as follows

$$R_2 + t = \ell/[\tan(\delta_{fo} - \alpha_{fo}) + \tan(\delta_{ro} - \alpha_{ro})] \quad (50)$$

$$R_2 - t = \ell/[\tan(\delta_{fi} - \alpha_{fi}) + \tan(\delta_{ri} - \alpha_{ri})] \ . \quad (51)$$

By subtracting (51) from (50) and rearranging the terms we can write

$$1/[\tan(\delta_{fo} - \alpha_{fo}) + \tan(\delta_{ro} - \alpha_{ro})] - 1/[\tan(\delta_{fi} - \alpha_{fi}) + \tan(\delta_{ri} - \alpha_{ri})] = 2t/\ell \ . \quad (52)$$

4.2 4WS vehicle, sideslip angles neglected

As stated before, when the vehicle is moving at low speed, the impact of the sideslip angles can be neglected. Consequently, we can use the assumption

$$\alpha_{fo} = \alpha_{ro} = \alpha_{fi} = \alpha_{ri} = 0 \quad (53)$$

to simplify Eq. (52) as follows

$$1/(\tan\delta_{fo} + \tan\delta_{ro}) - 1/(\tan\delta_{fi} + \tan\delta_{ri}) = 2t/\ell \ . \quad (54)$$

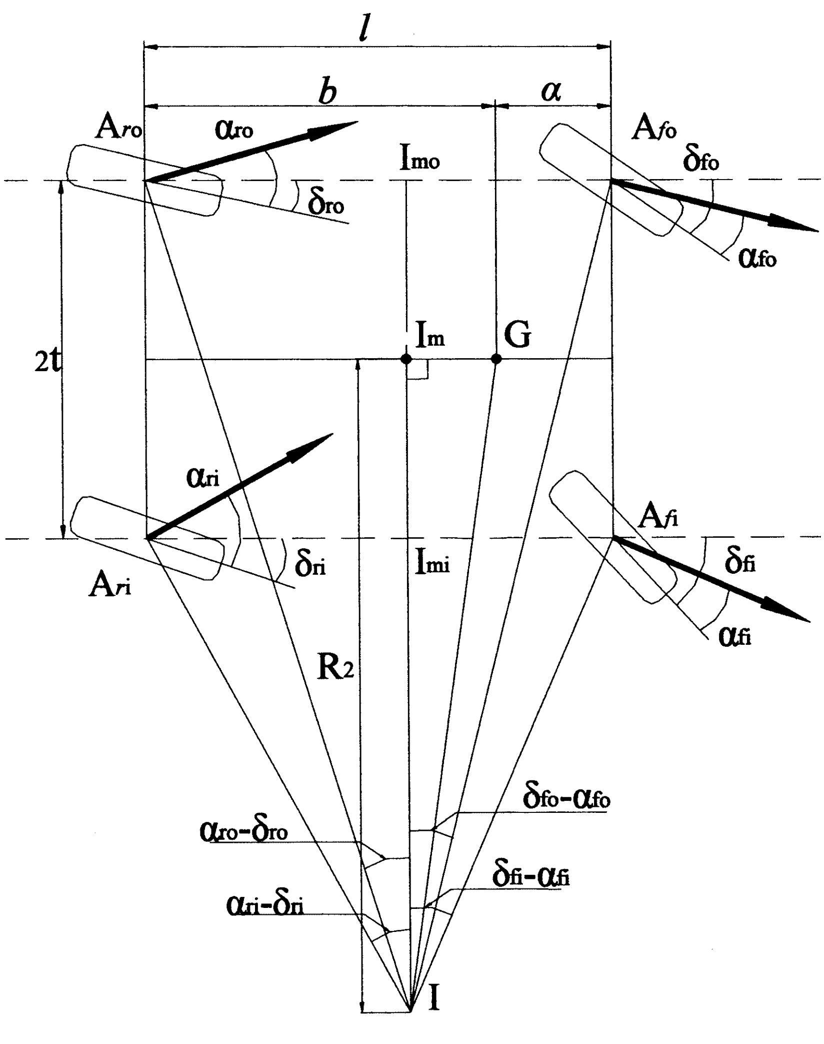

Fig. 4. Four-wheel model of a 4WS vehicle (the side-slip angles of the tires are not neglected)

4.3
2WS vehicle, sideslip angles not neglected

By assuming that the rear wheel steering angles are zero

$$\delta_{ro} = \delta_{ri} = 0 \tag{55}$$

we derive from the relation (52) the formula that is valid for a 2WS vehicle with non-negligible sideslip angles

$$1/[\tan(\delta_{fo} - \alpha_{fo}) + \tan(-\alpha_{ro})] - 1/[\tan(\delta_{fi} - \alpha_{fi}) + \tan(-\alpha_{ri})] = 2t/\ell \ . \tag{56}$$

4.4
2WS vehicle, sideslip angles neglected

Assuming that

$$\alpha_{fo} = \alpha_{ro} = \alpha_{fi} = \alpha_{ri} = 0 \tag{57}$$

and

$$\delta_{ro} = \delta_{ri} = 0 \ , \tag{58}$$

we simplify (52) as follows

$$1/\tan\delta_{fo} - 1/\tan\delta_{fi} = 2t/\ell \ . \tag{59}$$

This relation can also be written in the form known as relation of Ackermann and Jeantaux

$$\operatorname{cotan}\delta_{fo} - \operatorname{cotan}\delta_{fi} = 2t/\ell \ . \tag{60}$$

5
Conclusions

In this paper, we discussed the kinematics of four-wheel-steering vehicles. The following conclusions can be drawn from the analysis presented here:

1. A 4WS vehicle has a manoeuvring advantage over a 2WS vehicle only if its rear wheels can turn in the opposite direction to its front wheels, because only in that case we have a relative reduction of the turning radius.
2. The kinematical analysis of 4WS road vehicles presented above, permits to generalise the well-known theory that Ackermann–Jeantaux developed for the 2WS vehicles. As expected, the kinematical theory of 2WS vehicles is a sub-case of the generalised kinematical theory of 4WS vehicles.

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