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14th Triennial World Congress, Beijing, P.R. China

E-2c-21-3

A NONLINEAR PID CONTROLLER WITH APPLICATIONS

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Abstract: PID controllers have been the most commonly used industrial controllers. A linear PID controller is usually adequate for a nominal process. However, it often behaves poorly for a process with changes or uncertainties in operating conditions or environmental parameters. A nonlinear PID controller is introduced in this paper to alleviate this limitation. The controller gain and integral time are automatically adjusted according to the control error. Two simple nonlinear functions are proposed for the controller gain and integral time respectively. The proposed nonlinear controller is applied to two processes, a linear robotic process and a nonlinear thermoplastic injection molding process. Application examples show that the proposed nonlinear PID controller outperforms the linear PID controller in set-point tracking and load disturbance rejection. *Copyright © 1999 IFAC*

Keywords: PID controllers, control design, nonlinear control, control algorithms, control applications

1. INTRODUCTION

During the 1930s proportional-integral-derivative (PID) controllers became commercially available. Although there are numerous control algorithms in control theory and engineering, PID controllers have been the most popular and the most commonly used industrial controllers. This is primarily due to the simplicity and good performance of PID controllers. The controller principle, dynamics, design and tuning are discussed in numerous references.

A linear PID controller is described by the following relation

$$u(t) = K_p\left[e(t) + \frac{1}{T_i}\int_0^t e(t)dt + T_d\frac{de(t)}{dt}\right], \qquad (1)$$

where u represents control action; K_p, T_i, and T_d are controller gain, integral time, and derivative time, respectively; e denotes the error between the process set-point and the actual output.

A PID controller consists of three terms representing three different control modes, i.e., proportional, integral, and derivative control actions. The proportional (P) control provides a simple correction proportional to control error to the manipulated variable. The integral (I) control integrates the control error over time to eliminate possible offset. The derivative (D) control considers the rate of

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changes in the control error and takes an anticipatory action to correct the manipulated variable. For PID control, only three parameters are needed to be tuned. The three control modes of P, I, and D behave with some features of intelligence.

Compared with many model-based control algorithms, PID control is model-free and process-based. No process model is required to implement a PID control system. The controller parameters can be tuned by trial and error. This is an important advantage for process control where accurate process models are usually unavailable especially for complex processes such as reactor and distillation column processes.

Although linear PID control is usually adequate for a nominal process, it behaves poorly for a process with changes in operating conditions or environmental parameters. For example, a positive or negative deviation of process gain from its nominal value at which the PID controller is tuned results in oscillatory or sluggish process response. Requirements of fast process response and small overshoot are usually conflicting. In addition, a large control error leads to an aggressive integral control action that may result in undesirable reset windup.

Efforts have been made to overcome the above drawbacks. Cheung and Luyben (1980) investigated the performance of PI control with nonlinear gain. Nonlinear integral control action was considered by Ghreichi and Farison (1986). Recently, Seraji (1998) reported a class of nonlinear PID controllers, which are comprised of a sector-bounded nonlinear gain in cascade with a linear PID controller. An interesting nonlinear PI compensator was proposed by Shahruz and Schwartz (1997).

This paper introduces a new nonlinear PID (NPID) control algorithm to enhance linear PID control. The structure of the linear PID controller is retained in the NPID controller, but the controller gain and integral time are designed to be nonlinear functions of the control error. The proposed NPID controller is applied to two processes, a robotic process and a thermoplastic injection molding process.

2. A NONLINEAR PID CONTROLLER

The proposed NPID controller has the structure of the standard linear PID controller of equation (1). The controller gain and integral time are, however, not fixed. They are designed to be nonlinear functions of the control error e. Similar to equation (1), the NPID control algorithm is described by

$$u(t) = K_p(e)\left[e(t) + \frac{1}{T_i(e)}\int_0^t e(t)dt + T_d \frac{de(t)}{dt}\right]. \quad (2)$$

The nonlinear controller gain, $K_p(e)$, and integral time, $T_i(e)$, are respectively discussed below.

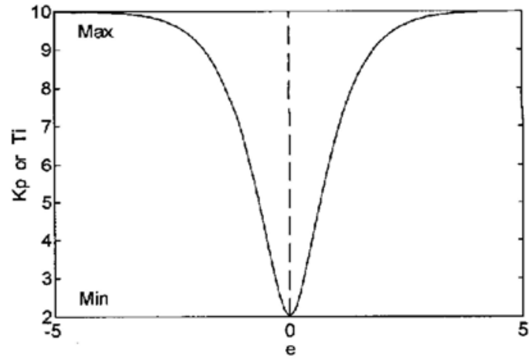

Figure 1: Nonlinear controller gain and integral time.

As discussed previously, requirements of fast process response and small overshoot are usually conflicting. A large controller gain is a benefit for fast reduction of a large control error, while it may lead to a large overshoot or an oscillatory response. A conservative controller gain is favorable for system stability, but it results in a sluggish response. These properties imply that the controller gain is expected to be aggressive for large control error or conservative for small control error. Therefore, we propose a three-parameter nonlinear controller gain as a function of the control error e

$$K_p = K_{\max} - (K_{\max} - K_{\min})(1 + a_p |e|)\exp(-a_p |e|), \quad (3)$$

where K_{max}, K_{min}, and a_p are positive parameters, $K_{max} > K_{min}$. It is clear that the maximum and minimum values of the controller gain are K_{max} and K_{min} respectively, i.e., the controller gain is bounded by $K_{max} \geq K_p \geq K_{min}$. Figure 1 gives a plot of the above nonlinear controller gain, K_p, as a function of e. It is seen that the controller gain, K_p, increases with the increase of $|e|$ and decreases with the decrease of $|e|$. K_p takes its maximum value, K_{max}, when $|e|$ tends to infinity and takes its minimum value, K_{min}, when e vanishes. To obtain an improved performance, K_{max} and K_{min} may be respectively selected to be larger and smaller than the nominal gain value of a linear PID controller. It is noted that the slope of K_p is determined by parameter a_p.

Similarly, the integral control action is expected to be conservative for large control error and aggressive for small control error. Hence, to improve integral control performance, a three-parameter nonlinear integral time is proposed as a function of the control error e

$$T_i = T_{\max} - (T_{\max} - T_{\min})(1 + a_i |e|)\exp(-a_i |e|), \quad (4)$$

where T_{max}, T_{min}, and a_i are positive parameters, $T_{max} > T_{min}$. The maximum and minimum values of the integral time are T_{max} and T_{min} respectively, i.e., the integral time is bounded by $T_{max} \geq T_i \geq T_{min}$. Figure 1 shows a plot of the above nonlinear integral time, T_i,

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as a function of e. It is clearly seen that the integral time, T_i, increases with the increase of $|e|$ and decreases with the decrease of $|e|$. This implies that the integral control becomes conservative or aggressive with the increase or decrease of $|e|$, respectively. T_i takes its maximum value, T_{max}, when $|e|$ tends to infinity and takes its minimum value, T_{min}, when e vanishes. T_{max} and T_{min} may be chosen to be larger and smaller than the nominal integral time of a linear PID controller. The slope of T_i is determined by parameter a_i.

It should be mentioned that the proposed NPID algorithm (2) has a fixed derivative time, T_d, although both controller gain, K_p, and integral time, T_i, are variable. This is based on the consideration that applications of PI control are more common than PID control in industry because process measurements are usually noisy.

To illustrate the superior performance of the proposed NPID controller, two applications are discussed in the next section.

3. APPLICATIONS

3.1 Robotic Process

The first application of the proposed NPID control is for a linear, second-order, robotic process discussed by Seraji (1998). The process is governed by

$$G(s) = \frac{K}{\omega^2 s^2 + 2\xi\omega s + 1}, K = 1, \omega = 0.2, \xi = 2. \quad (5)$$

A unit step change in set-point is introduced at time $t = 0$ as an excitation of the process. Suppose that the process has the same load transfer function as equation (5). To test the load rejection ability of the NPID controller, a load step change of magnitude -0.5 is introduced at $t = 1$.

When a linear PI controller is applied, the tuned controller settings are $K_p = 5$ and $T_i = 0.2$, as discussed by Seraji (1998). A nonlinear PI controller is employed with a nonlinear gain of equation (3) and nonlinear integral time of equation (4). The controller settings are tuned to be $K_{max} = 15$, $K_{min} = 4$, $a_p = 1/0.6$, $T_{max} = 5$, $T_{min} = 0.4$, and $a_i = 1/0.6$. Figure 2 shows a comparison of the closed-loop responses of the standard linear PID controller (dash-dotted line) and the NPID controller (solid line). The corresponding variations of the nonlinear controller gain, K_p, and integral time, T_i, are plotted in Figure 3. Compared with the fixed parameter settings of the linear PID controller, it is clear from Figure 3 that K_p and T_i of the NPID controller are automatically adjusted based on control error. At $t = 0$, $|e| = 1$ is large, $K_p = 11.84$ and $T_i = 3$ are also large. K_p and T_i then decrease with the decrease of absolute e. At $t = 0.3$, K_p and T_i have been closed to their minimum values of 4 and 0.4 respectively. These automatic adjustments of K_p and T_i result in a fast set-point tracking without oscillations, as shown in Figure 2. When a load appears at $t = 1$ that leads to a control error, K_p and T_i change automatically to reject the load rapidly. It is clearly seen from Figure 2 that the NPID controller outperforms the linear PID controller in both set-point tracking and load disturbance rejection.

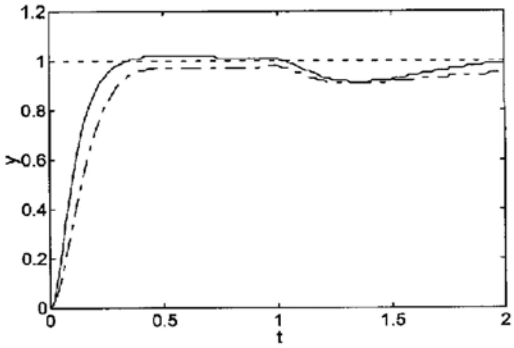

Figure 2: Closed-loop responses of the standard linear PID controller (dash-dotted line) and the NPID controller (solid line).

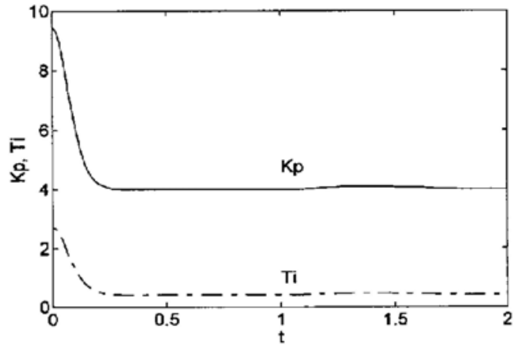

Figure 3: Plots of nonlinear controller gain K_p (solid line) and T_i (dash-dotted line) versus time t.

3.2 Thermoplastic Injection Molding Process

Now the application of the proposed NPID controller to a thermoplastic injection molding process is considered. Injection molding is an indispensable process in polymer processing industry. It produces a wide range of plastics economically and efficiently for industrial, agricultural, electronic, and household usage. A complete description of the injection molding process can be found in a number of books, say Rubin (1972). Injection molding is a typical cyclic process. Each cycle consists of three separate phases: filing, packing and holding, and cooling. The injection velocity control in the filling phase is considered here. The injection begins with a screw axial movement with a velocity profile and a continuous increase of the cavity pressure and ends with the cavity being completely filled.

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The dynamic behaviors of injection molding are determined by a complex interaction among material properties, mold geometry and a number of process variables such as injection velocity, cavity pressure, nozzle pressure, hydraulic pressure, a group of barrel temperatures, nozzle temperature, etc. Among these variables, the injection velocity plays a critical role in injection molding quality. It indicates the rate of the polymer melt flowing to the mold cavity and significantly affects the cavity pressure and consequently the products quality such as shear rate and stress, shrinkage, impact strength, and surface quality. Productivity is also dependent on the velocity setting.

Efforts have been made for setting a proper velocity profile, say Yao, Bhattacharya, Kosior, *et al* (1994), Chao and Maul (1989). But keeping the velocity tracking a desired profile is not easy due to the complex effects of many factors on the velocity dynamics. Open-loop control is common at present for velocity control. However, it cannot give high control performance due to the complexity of the process dynamics and inevitable disturbances. Furthermore, an open-loop control algorithm developed for one injection molding machine and/or one mold cannot be applied to another machine and/or another mold due to the dynamics difference for different machines and molds. Even for the same machine and mold, the open-loop control performance deteriorates when operation conditions change. Optimization of the velocity profile is, thus, senseless unless the velocity can track the desired curve accurately in different production environments. Closed-loop control of the velocity is, therefore, vital for improving the velocity control performance and the products quality.

There have been a few reports on closed-loop control of the injection velocity. Experimental controls have been demonstrated by Zhang, Leonard and Speight (1996) with adaptive control and Tsoi and Gao (1998) with fuzzy logic. The design of these control schemes are, however, complicated for industrial implementation. The NPID controller proposed in this paper is adopted for closed-loop control of the injection velocity.

The injection velocity process behaves with nonlinearity and pure time delay, and it is time varying. A detailed analysis of injection velocity dynamics can be found in Tian and Gao (1998). A first-order plus delay process model with a nonlinear gain is adopted to approximate the injection velocity process

$$G(s) = \frac{K}{Ts+1}e^{-ds}. \quad (6)$$

The input and output variables of the above process transfer function is servo-valve opening (%) and injection velocity (m/s) respectively. For a cup mold, HDPE material, and the barrel temperature of 200°C, the model parameters are identified to be $K = 0.0006 \sim 0.0011 \mathrm{m{\cdot}s^{-1}/\%}$ depending on excitations of the process, $T \approx 0.007$s, and $d \approx 0.015$s. It is also noted that there is a large pure time delay of about 0.08s at the beginning of injection.

Injection velocity control is a typical tracking problem. The desired injection velocity is determined according to mold, material, barrel temperature, etc. It is defined in injection velocity profile. Figure 4 shows a typical velocity profile, where v denotes injection velocity. Because the velocity profile is available before any control action is imposed on the process, set-point changes can thus be shifted forward by a period of process delay time (0.015s in this example) to compensate this delay and speed up the closed-loop response. On the other hand, the initial step change of the velocity profile is transformed into a smaller step change plus a ramp change to overcome possible initial peak in the closed-loop response. The shifting and transformation of the velocity profile has been discussed in detail by Tian and Gao (1998). Corresponding to the desired velocity curve in Figure 4 in solid line, a transformed and shifted velocity curve is also depicted in Figure 4 in dash-dotted line.

When a linear PI controller is employed, the controller parameters should be tuned in the worst case to ensure the system stability. The derivative control action is not employed here due to intensive noises in the process. The ITAE criterion is adopted to tune the controller parameters, resulting in the PI controller settings of K_p = 265%·s/m and T_i = 0.01035s for set-point tracking. These settings of the PI controller has been employed by Tian and Gao (1998) for the velocity control with a novel double-controller scheme. With the velocity profile of Figure 4, a typical closed-loop system response is shown in Figure 5 in dash-dotted line (Tian and Gao, 1998).

The proposed NPID control of equations (2) through (4) is now applied to the injection velocity control. As discussed above, the derivative control action is not employed, implying that T_d = 0. The controller parameters are tuned to be K_{max} = 320, K_{min} = 255, a_p = 250, T_{max} = 0.0115, T_{min} = 0.0086, and a_i = 250. It

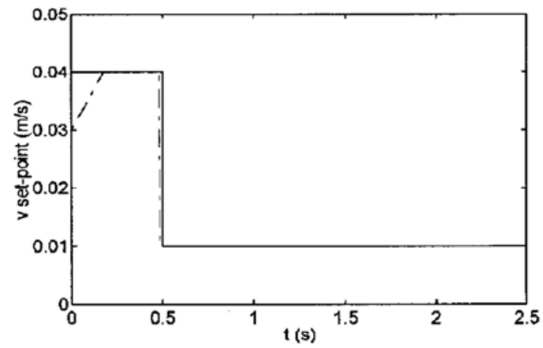

Figure 4: A typical injection velocity profile.

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is clear that both K_p and T_i increase with the increase of control error e. Conversely, both K_p and T_i decrease as e is reduced. Thus, a large e leads to an aggressive proportional control and a conservative integral control. A small e results in a conservative proportional control and an aggressive integral control. A typical closed-loop response of the nonlinear PI controller is shown in Figure 5 in solid line. It is clear from this figure that the nonlinear PI control behaves with a faster set-point tracking than the linear PI control. Figures 6 and 7 depict the corresponding variations of K_p and T_i versus t.

Extensive simulations and experiments of injection velocity control have shown that the proposed NPID controller outperforms the linear PID controller.

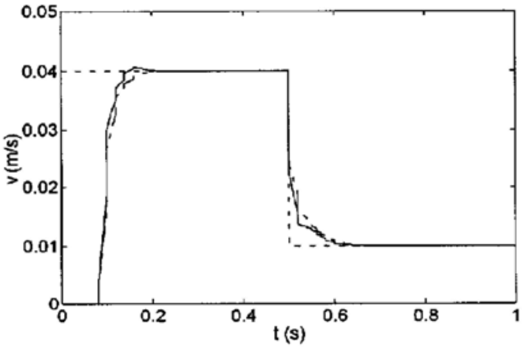

Figure 5: Closed-loop responses of the NPID controller (solid line) and the linear PID controller (dash-dotted line).

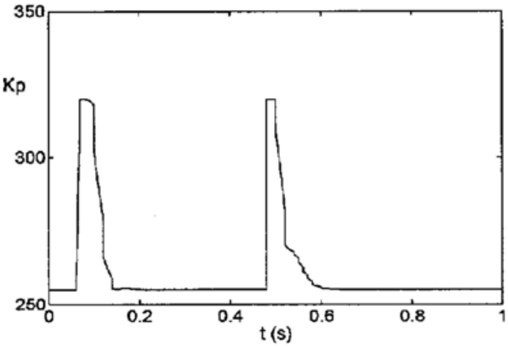

Figure 6: Nonlinear controller gain K_p versus t.

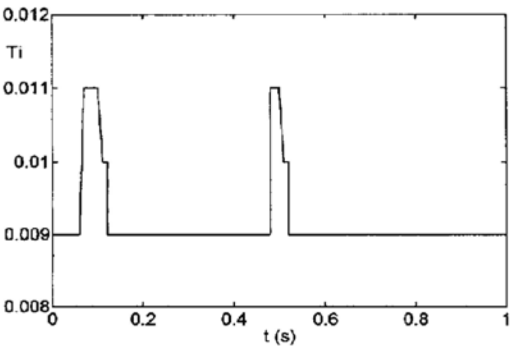

Figure 7: Nonlinear controller integral time T_i.

4. CONCLUSIONS

A NPID controller has been introduced with high performance. The controller has the structure of the standard linear PID controller, while its gain and integral time are designed to be nonlinear functions of control error. Two simple nonlinear functions have been proposed for the nonlinear controller gain and integral time, which increase with the increase of absolute control error and decrease with the decrease of absolute control error. The proposed NPID controller have been applied to two processes, a second-order, linear, robotic process and a first-order plus delay, nonlinear, thermoplastic injection molding process. The applications have shown that the proposed NPID controller is superior to the standard linear PID controller. The proposed NPID controller is, therefore, a promising tool for industrial process control.

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