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Vehicle Steering Control with MPC for Target Trajectory Tracking of Autonomous Reverse Parking

Tsutomu Tashiro

***Abstract*— In this paper, vehicle steering control following target trajectory by MPC (Model Predictive Control) is proposed. First, vehicle turning characteristic is modeled. Here, the model is mainly composed of vehicle rotation model and steering model. Next, MPC controller is derived from quadratic cost function. This controller instructs vehicle rotation radius directly translated into steering angle. It minimizes difference of the position between target trajectory and the predicted when their moving directions are equal. And finally the controller is verified with simulation.**

I. Introduction

Parking assist system operating steering angle has been developed and put on the market[1]-[5]. Main technology to achieve it is divided into two, environmental recognition like detection of obstacle, parking space, vehicle position and so on, and vehicle motion control. The motion control for parking assist is furthermore divided into two, target trajectory generation and tracking control. With regard to target trajectory generation, there are researches designing trajectory easy to follow by steering operation[5],[6] or generating the shortest route to parking position[7] based on Ackerman model. And as another approach, the methods to generate trajectory as circle arc [8],[9] or clothoid curve[10] are also presented. These have common features that their algorithm and calculation are quite complex because they undertake a part of tracking control tasks. In other words, tracking controls combined with them are generally unsophisticated and not based on control engineering principle. For example, in many cases, it is performed by open-loop control to generate steering angle by reverse calculation from target trajectory. And in order to compensate difference between target and actual trajectory, there are cases to update target trajectory every timing[10]. Namely the difference is practically compensated not by tracking control but by target trajectory modification. It is performed in target trajectory generation function and such operation makes its responsibility and load more.

Based on this issue, new control design method of target trajectory tracking by MPC is proposed in this paper. Disturbance and modeling error, that are main factors to deviate actual motion from the target, are tried to compensate by the controller minimizing cost function. It also expects to bring advantages to reduce responsibility of target trajectory generation function and enable to systematic control design of tracking control as well as improvement of tracking performance. In this paper, vehicle model and its assumption are explained first, and cost function is designed next. The most remarkable feature is to exclude the concept to correspond timing of target and actual trajectory. Finally controller derived from the function is verified with simulation.

T. Tashiro is with Department of Mechanical Engineering for Transportation, Osaka Sangyo University, Nakagaito 3-3-3, Daito-shi, Osaka 574-8530, Japan. (phone: +81-72-875-3001; fax: +81-72-871-1289; e-mail: tashiro@tm.osaka-sandai.ac.jp).

II. Modeling

A. Structure of Control System

Figure 1 shows functional structure of trajectory tracking control system in this paper. It is divided into tracking control, vehicle rotation model, steering model and longitudinal model. Tracking control is designed in this paper, and its reference input is target trajectory and manipulation output is request rotation radius. Sensing data for the control is provided by sensing block that is not shown in Fig.1.

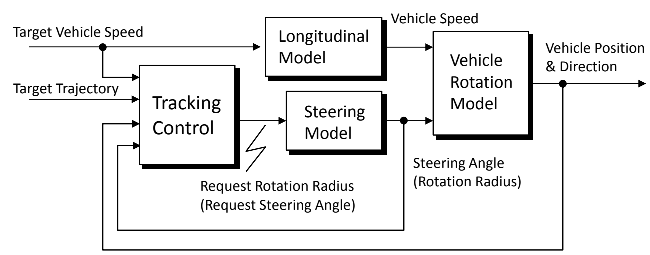

Figure 1. Functional Structure of Control System

A kind of parking managed in this paper is limited for the trajectories by one direction steering like reverse parking as Fig. 2 and the trajectories including switch-back of steering direction like parallel parking are not assumed. The design method is also possible to be applied to parallel parking. In order to realize switch-back operation, the system should be defined as hybrid dynamical system. However it is not referred in this paper.

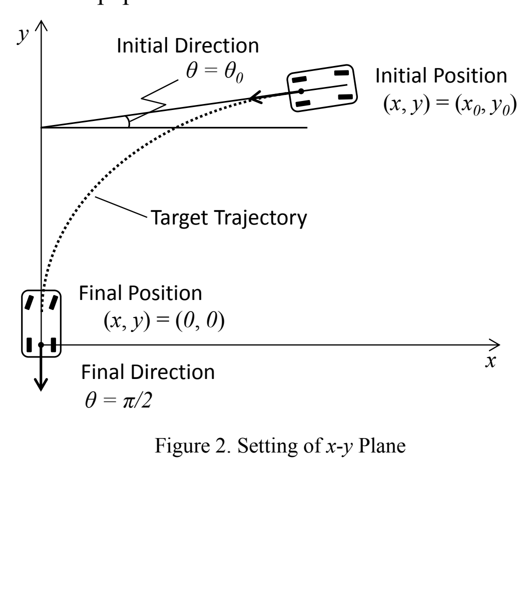

Figure 2. Setting of x-y Plane

Vehicle speed is controlled by longitudinal controller included in longitudinal model and target vehicle speed is given to tracking control as information to calculate request rotation radius. Namely the tracking control is designed based on vehicle rotation model and steering model with the assumption that vehicle speed corresponds to target speed by longitudinal model. These models are described as discrete time system in order to apply to MPC.

B. Vehicle Rotation Model

Vehicle rotation model expresses characteristic from tire angle to vehicle motion. In this paper, vehicle motion is defined by x-y coordinates and moving angle θ that shows vehicle moving direction. The axis of coordinates is defined as Fig. 2 that the final target position is located at $(x, y) = (0, 0)$ and $\theta = \pi/2$. Vehicle is assumed to move based on the geometry shown in Fig. 3.

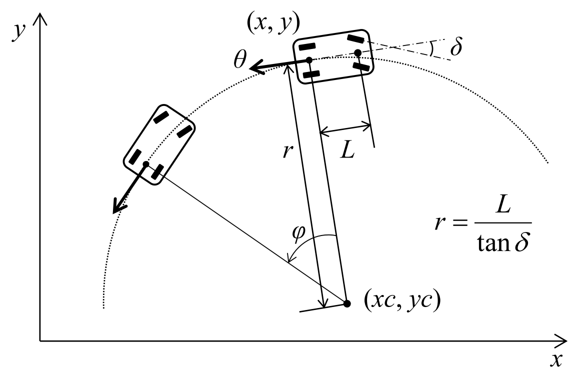

Figure 3. Geometry of Vehicle Rotation

In this case, tire angle and vehicle rotation radius is related as (1). Where r is vehicle rotation radius, δ is tire angle and L is wheel base.

$$r = L / \tan\delta \tag{1}$$

It means when tire angle is decided, vehicle rotation center (x_c, y_c) and rotation angle φ are also decided as (2).

$$\begin{aligned} xc &= x + r\sin\theta \\ yc &= y - r\cos\theta \\ \varphi &= v\,T / r \end{aligned} \tag{2}$$

Where v is vehicle speed and T is sampling period. And when present control timing is t, vehicle position and direction at next timing $t+T$ are decided as Fig. 4. As a result, they are described by (3). Where subscript t and $t+T$ mean their control timing.

$$\begin{aligned} x_{t+T} &= (x_t - xc_t)\cos\varphi_t - (y_t - yc_t)\sin\varphi_t + xc_t \\ y_{t+T} &= (x_t - xc_t)\sin\varphi_t + (y_t - yc_t)\cos\varphi_t + yc_t \\ \theta_{t+T} &= \varphi_t + \theta_t \end{aligned} \tag{3}$$

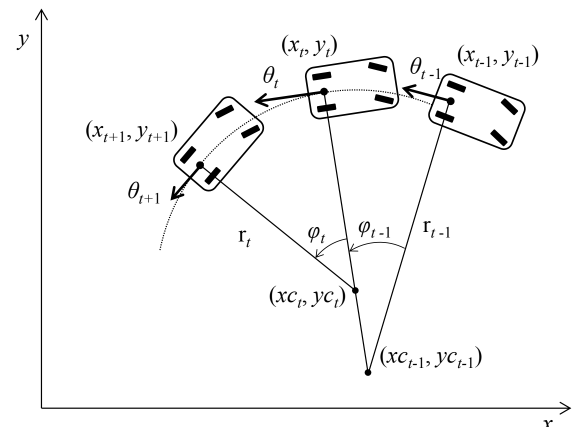

Figure 4. Position and Direction at Next Timing

C. Steering Model

Steering model expresses characteristic from request tire angle to actual tire angle. Tire angle is assumed to be operated by EPS (Electric Power Steering). EPS is composed of various parts - electric motor, torsion bar, gear, rack, pinion and so on. And its structure and characteristic is complex. EPS model needs to describe by four order state space function even though it is simplified one[11]. However, in this paper, EPS controller has been already designed and implemented in steering model and assumed to achieve first order lag characteristic from target tire angle to actual tire angle. In addition, as shown in Fig. 3 and (1), tire angle is directly translated into vehicle rotation radius. Therefore, request rotation radius is equivalent to request tire angle practically. So input of steering model is regarded as request rotation radius hereafter. In this case, steering model is expressed by (4). Here, u is request rotation radius and τ is a constant concerned with time constant.

$$r_{t+1} = (1 - \tau)\, r_t + \tau\, u_t \tag{4}$$

D. Longitudinal Model

Tracking controller designed in this paper does not control vehicle speed as mentioned at the beginning of this chapter. Longitudinal controller, achieving target vehicle speed, has been already designed and implemented in longitudinal model. And tracking controller receives vehicle target speed based on the assumption that vehicle speed corresponds to target speed.

E. Vehicle Motion Model

By combination of these models, vehicle motion is described by following nonlinear state space equation. Input of the model u, request rotation radius, is manipulation output of tracking control, and v, vehicle speed, is external input.

$$\begin{bmatrix} x_{t+T} \\ y_{t+T} \\ \theta_{t+T} \\ r_{t+T} \end{bmatrix} = \begin{bmatrix} -r_t \sin(\theta_t + v_t T / r_t) + r_t \sin\theta_t + x_t \\ r_t \cos(\theta_t + v_t T / r_t) - r_t \cos\theta_t + y_t \\ v_t T / r_t + \theta_t \\ (1 - \tau) r_t + \tau\, u_t \end{bmatrix} \tag{5}$$

III. Control Design

A. Design Principle

Based on features of tracking control for autonomous parking, following items are taken into account as core issues of control design.

- Effective utilization of future information about position and direction on target trajectory
- Simplification of controller and calculation for its derivation
- Control not based on time axis but *x-y* coordinates
- Avoidance of performance deterioration by vehicle speed difference between target and actual motion

In case of tracking control for autonomous parking, target trajectory is defined between initial position and final target position. It means that future target position is defined at the initial timing of control. And this is quite an important and useful feature to achieve better performance. Based on the fact, MPC is applied because it can utilize future motion data predicted by model. For example, it is one of the effective ideas to use cost function including future data as (6) that minimizes differences between future target and predicted variables.

$$V = \sum_{j=1}^{n} \Big\{ \left(\hat{x}_{t+jT} - x_{t+jT}\right)^2 + \left(\hat{y}_{t+jT} - y_{t+jT}\right)^2 + \alpha\left(\hat{\theta}_{t+jT} - \theta_{t+jT}\right)^2 + \beta\left(r_{t+jT} - r_{t+(j-1)T}\right)^2 \Big\} \tag{6}$$

Here, $\hat{x}, \hat{y}$ and $\hat{\theta}$ are target value of x, y and θ. α and β are relative weights. Equation (6) includes rotation radius r. It is employed in order to avoid sudden motion of vehicle and steering wheel. In particular, the issue regarding steering wheel is quite important. In case of EPS, tire and steering wheel are mechanically linked, so when tire angle that is directly calculated from vehicle rotation radius changes suddenly, steering wheel angle also changes. There is an assumption that driver does not touch steering wheel during autonomous parking control, however its sudden motion might be possible to catch in and injure driver's hand. Therefore this term should be also included in cost function.

With regard to simplification of control calculation, it is practically impossible to derive controller minimizing the cost function analytically without any simplification. In particular, the feature of autonomous parking must be utilized well for simplification.

Regarding control not based on time axis but *x-y* coordinates, if target position and direction are achieved, practically its timing does not become any matters in autonomous parking. In case of (6), target and predicted variables at the same timing are compared. It means, even though actual motion is on the target trajectory, if vehicle speed used in target calculation and the actual are not corresponded, their position and direction are regarded as different one. In order to eliminate this issue, they should compare not based on time axis but *x-y* coordinates.

And tracking control cannot manage the performance of vehicle speed control because it is performed by longitudinal control. Therefore tracking control should be designed with principle to be robust for vehicle speed fluctuation.

B. Modification of Cost Function

In this section, cost function V is improved so as to achieve the items listed in the previous section. Specifically target and predicted motion are compared not at the same timing but at the timing that their directions are corresponded. Here, when θ corresponds to $\hat{\theta}$, x, y and θ are described as $\tilde{x}, \tilde{y}$ and $\tilde{\theta}$ respectively. This assumption is expressed as following equation.

$$\tilde{\theta}_{t+T} = v_t T / r_t + \theta_t = \hat{\theta}_{t+T} \tag{7}$$

By means of (7), $\tilde{x}(t+T)$ and $\tilde{y}(t+T)$ are written as (8) by present state variables and the target.

$$\begin{aligned} \tilde{x}_{t+T} &= -r_t \sin\hat{\theta}_{t+T} + r_t \sin\hat{\theta}_t + x_t \\ \tilde{y}_{t+T} &= r_t \cos\hat{\theta}_{t+T} - r_t \cos\hat{\theta}_t + y_t \end{aligned} \tag{8}$$

And when x, y and θ of (6) are replaced by $\tilde{x}, \tilde{y}$ and $\tilde{\theta}$, cost function V is transformed as V_1.

$$V_1 = \sum_{j=1}^{n} \Big\{ \left(\hat{x}_{t+jT} - \tilde{x}_{t+jT}\right)^2 + \left(\hat{y}_{t+jT} - \tilde{y}_{t+jT}\right)^2 + \beta\left(r_{t+jT} - r_{t+(j-1)T}\right)^2 \Big\} \tag{9}$$

Because $\tilde{\theta}$ is equal with $\hat{\theta}$, its term becomes zero and diminished. And for control design in next chapter, this V_1 is used as cost function to derive tracking controller. The aims to modify the function are mainly two. First one is not to compare target and predicted variables at same timing. They are compared at the timing that their directions are same as mentioned. Therefore, timing error between target and predicted caused by vehicle speed error does not give influence to control performance. Second one is to simplify the calculation by replacing argument of trigonometric function with known value, target direction. However the simplification causes difference between request rotation radius u from V and V_1. It is verified as a tracking performance with simulation in next chapter.

C. Derivation of Controller

Shown as Fig. 5, the number of control horizon, n of (9), is 4 in this control design. Target positions are calculated from present position, prediction time, horizon number and target vehicle speed. The target direction of the first period is calculated as the direction from the present position to fourth target position in order to reduce the difference between target and actual trajectory. The target direction of the second, third and fourth period are same with the direction at each target position in order to follow target trajectory.

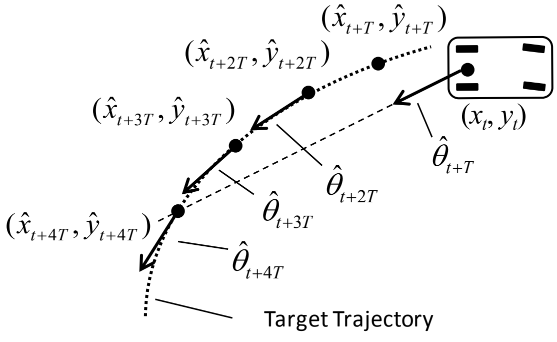

Figure 5. Target Direction for Control

Predicted positions $(\tilde{x}_{t+T}, \tilde{y}_{t+T})$, $(\tilde{x}_{t+2T}, \tilde{y}_{t+2T})$, $(\tilde{x}_{t+3T}, \tilde{y}_{t+3T})$ and $(\tilde{x}_{t+4T}, \tilde{y}_{t+4T})$ are calculated from recurrence formula (8). In addition to the condition written previously, upper limit of tire angle δ_{max} is also taken into account as (10).

$$\delta < \delta_{max} \tag{10}$$

These equations are applied to cost function (9). Consequently, u_t, u_{t+T}, u_{t+2T} and u_{t+3T} minimizing V_1 are derived. Among them, u_t is used as manipulation output of tracking control.

IV. Verification

A. Simulation Condition

Simulation condition is explained in this section. Vehicle position and direction on the *x-y* plane are assumed to measure precisely without time delay. As target trajectory case (i) and (ii) are applied. Both are designed based on steering operation by driver. The case (i) is necessary to increase tire angle during second half of control as shown in Fig. 6 and 8, and the other case (ii) to reach upper limit of tire angle as shown in Fig. 7. As an initial simulation setting, position, direction and tire angle of target trajectory correspond to real ones.

Control period is 40ms and prediction period is possible to set as its integral multiplication. In the evaluation of this paper, the cases of 40ms, 120ms and 240ms are tested with same horizon number 4. In general, shorter prediction period improve tracking performance, but tire angle tends to be operated quickly and suddenly. On the other hand, longer prediction period enables to take more future target information and tire angle operation becomes smoother, however deterioration of tracking performance is anticipated. With regard to calculation of simulation, geometric part of the model is same with the model for control design shown in previous chapter. However, its calculation period is 1ms, much shorter than control period, in order to get rid of the influence of discrete time calculation. And dynamic part of the model about steering motion is realized by more detail and higher orders model than that for control. As mechanical and characteristic constants, wheel base L is 2.5m. Nominal parameter τ of steering model is chosen as time constant becomes 0.25s and target vehicle speed is 9km/h. And their influences are evaluated with errors. Design parameter β of V_1 is 10^{-5} and it is decided based on simulation results. Proposed controller in this research is assumed to realize vehicle rotation motion and not to straight driving. So when straight driving is judged from target direction, steering angle is instructed zero to steering model.

B. Simulation Result

In this section, examples of simulation result are shown. Figure 6 shows a result with target trajectory (i), tire angle is necessary to increase during second half of control, by different prediction period. Figure 6(a) and (b) show vehicle trajectory for *x-y* plane and tire angle for time axis respectively. As shown in Fig.6(a), all the controls can guide car to final parking position. However, as the prediction period becomes longer, the small difference with target on the way is observed. While as for tire angle, the prediction period becomes longer, tire angle operation starts earlier and its motion becomes smoother.

Figure 7 shows the results with target trajectory (ii), tire angle reaches its upper limit. Figure7(a) and (b) show vehicle trajectory and tire angle. As well as Fig. 6, the controller with shorter period achieves better tracking. However the controller with longer period makes maximum tire angle small. These results indicate that length of prediction period can use as a parameter to balance tracking performance and tire angle operation.

Figure 8 shows simulation results with target trajectory (i) when vehicle speed has an error. The prediction period is 40ms. Vehicle speed used in the controller is 9km/h but real one is 7km/h or 11km/h. Figure 8(a) and (b) show vehicle trajectory and tire angle respectively. As the vehicle speed becomes faster, the influence of delay in steering system becomes bigger. However as shown in Fig. 8(a), the controller can follow the target in each speed. And as shown in Fig.8(b), tire angle for higher speed becomes slightly bigger than for lower speed in order to compensate the delay. Verification in same vehicle speed conditions with target trajectory (ii) is also tested and similar kind of result with target trajectory (i) is observed.

In addition to them, influence of modeling error about response of steering system, 30% faster and slower than nominal condition, is verified. As a result, the controller can realize good target tracking performance.

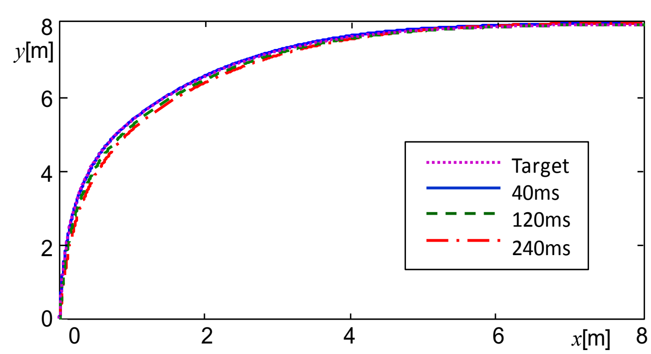

Figure 6(a). Vehicle Trajectory

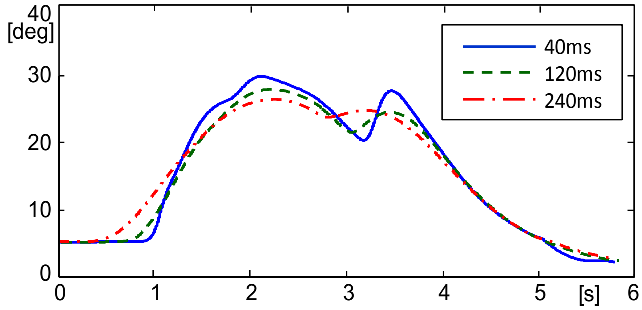

Figure 6(b). Tire Angle

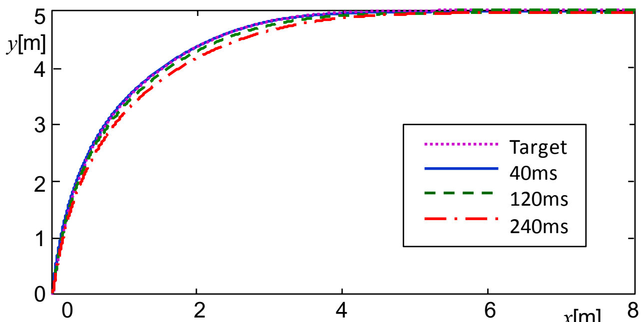

Figure 7(a). Vehicle Trajectory

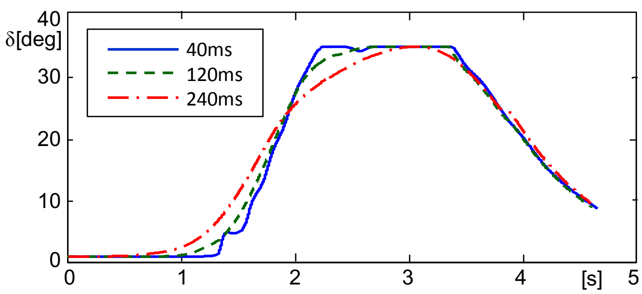

Figure 7(b). Tire Angle

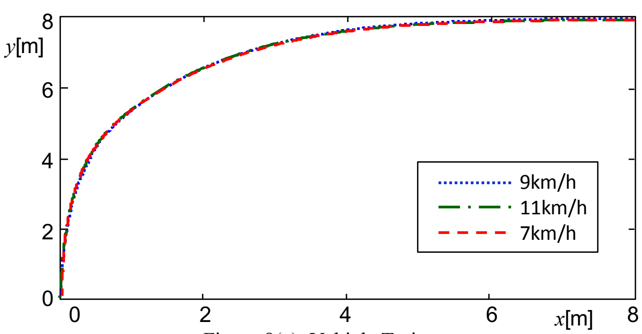

Figure 8(a). Vehicle Trajectory

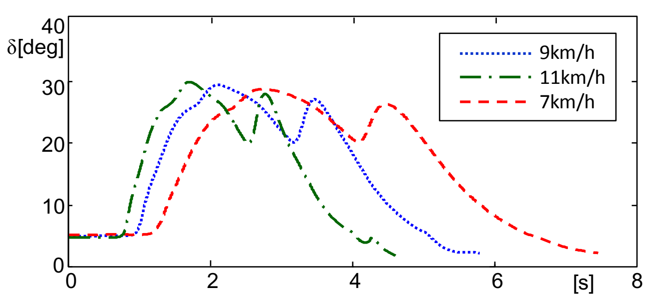

Figure 8(b). Tire Angle

V. CONCLUSION

Vehicle steering control with MPC for target trajectory tracking of autonomous reverse parking is designed. Based on the feature of autonomous parking system, concept of time is excluded from cost function. It brings some effects as keeping performance from changing of vehicle speed, steering system characteristic and so on. This controller is also expected to simplify target trajectory generation algorithm and reduce its responsibility.

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