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MPC-Based Path Tracking Controller Design for Autonomous Ground Vehicles

Chen Shen[1,2],Hongyan Guo[1,2⋆], Feng Liu[2], Hong Chen[1,2]

1. State Key Laboratory of Automotive Simulation and Control, Jilin University (Campus NanLing), Changchun 130025, PR China
E-mail: chenh@jlu.edu.cn

2. College of Communication Engineering, Jilin University (Campus NanLing), Changchun 130025, PR China

Abstract: It is an important and essential aspect for autonomous ground vehicles to follow the desired path. In this manuscript, a path tracking controller using model predictive control (MPC) method is proposed. In order to describe the vehicle motion and its dynamics, kinematics and dynamics integrated model is first presented. Then, the control strategy is proposed and MPC is employed to design the path tracking controller. In the following, the stability analysis is carried out and it shows that the system is proven to be asymptotically stable and theoretically it has no static error. In order to validate the effectiveness of the proposed MPC-based path tracking controller, veDYNA-Simulink joint simulations under different velocities and road friction coefficient are carried out and the results illustrate that the path tracking algorithm obtains good tracking performance.

Key Words: Path tracking, MPC, Stability

1 Introduction

With the increasingly serious problem of traffic congestion and road safety, autonomous vehicle has become an increasingly important topic. An essential part of vehicle autonomy consists of path planning and tracking control that enables it to safely maneuver under different conditions[1]. Based on the road information, path planning aims to obtain an anti-collision path according to environmental factors. Then, in path tracking, the key is to design the velocity and steering controller to follow the established reference path through velocity and steering control. Due to the strong coupling, steering and velocity control are discussed in respective, which have been described in great detail by many authors.

The early discussed methods directly connect the vehicle steering to the lateral position error. Then, W. Junmin presented a steering control system in which vehicle yaw rate is actively controlled to achieve trajectory tracking. [2]. Moreover, a nested PID steering control is proposed which consists of two algorithms including inter-loop and outer loop [3]. The outer loop is used to calculate the ideal yaw rate depending on lateral position and the velocity and lateral acceleration of autonomous vehicle are taken into consideration in the inter-loop.

All the methods mentioned above simply depend on the accuracy of the current environmental parameters and is tough to take dynamics constraints into consideration. With the rapid development of the computing power of hardware, researchers proposed the model predictive control which has significant advantages in dealing with the constraints in path tracking. The real-time path planning and tracking method based on predictive error feedback control as well as fuzzy logic control are proposed, which is adaptive to the uncertainty of road condition[4]. A receding horizon optimization framework is proposed which used the vector of thrust forces and moments as control input and by the nonlinear model predictive method, the depth and orientation of the AUV can also be taken into consideration.[5]. In addition, in order to obtain better control accuracy and flexibility, the robust model predictive control has been proposed to eliminate the position error and heading error. Thaker proposed a multi-switch prediction model control scheme based on dynamic compensation for trailer according to different velocity and variable slip rate [6].

However, almost all of these control algorithms are applied in practice without in-depth theoretical analysis. In conclusion, this paper presents an MPC based tracking controller and theoretical analysis about stability and static error. Assuming that the longitudinal velocity is constant, an MPC based rolling horizon scheme is first proposed focusing on lateral control, which establishes the kinematic and dynamic state-space models of the vehicle. The front steering angle is used as the input, and the lateral displacement is established as the output. In the formulation of the control strategy, the primary task is to reduce the error between an ideal path and real path for path tracking. Then, the fuel consumption is considered transformed to shortest path, besides, the actuator's action should be decreased within a diminutive range. Moreover, three weight coefficients proposed aims to balance the importance of the three parts respectively. The stability analysis proves that the system is asymptotically stable and the model error is zero without interference. To prove the effectiveness of the controller, veDYNA-Simulink Joint simulations are conducted at last.

The remainder of this paper is organized as follows. In section II, the processes of establishing kinematic and dynamic model are described. Section III presents the MPC based path-tracking controller for the autonomous vehicle. In order to verify the performance of the controller, simulations are carried out in section IV. The conclusions are given in section V.

2 Vehicle Model

The proposed control algorithm for path tracking relies on the MPC, thus the 2-DOF bicycle linear vehicle models are required to generate the desired four parameters includ-

⋆ corresponding author

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ing lateral position, yaw rate, yaw angle and sideslip angle. And the bicycle model consists of two components (dynamic model and kinematic model) which describe the vehicle kinematics as shown in Figures 1 and the lateral dynamics as shown in Figures 2.

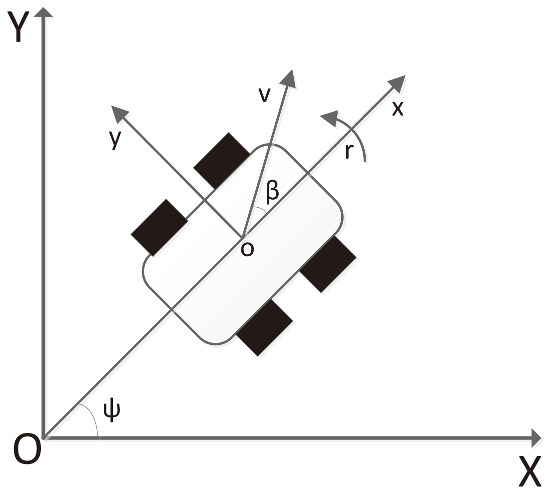


Fig. 1: Vehicle kinematic relation

Assuming vehicle as a rigid body with non-deformable wheels, according to geometry relationship as shown in Figure 1, the kinematic equations are described as follows:

$$\begin{cases} \dot{x} = v\cos(\psi+\beta) \\ \dot{y} = v\sin(\psi+\beta) \\ \dot{\psi} = r \end{cases} \tag{1}$$

Where x is the longitudinal position of the vehicle centroid; v is the velocity of the vehicle centroid; y is the lateral position of the vehicle mass center; β is the sideslip angle; r is the yaw rate of the vehicle. Through the formula of small angle assumption[9], vehicle kinematics model simplification shows:

$$\begin{cases} \dot{x} = v \\ \dot{y} = v(\psi+\beta) \\ \dot{\psi} = r \end{cases} \tag{2}$$

The path tracking system involving tight coupling of a number of parameters,which is difficult to be defined[10]. In this paper, only the steering control is studied, so for improving the flexibility of the system, the longitudinal dynamics is neglected.The equation for the lateral dynamics is established as (3).

$$\begin{cases} mv_x(\dot{\beta}+r) = F_{yf} + F_{yr} \\ I_z\dot{r} = aF_{yf} - bF_{yr} \end{cases} \tag{3}$$

The F_{yf} and F_{rf} are front and rear wheel lateral force of the vehicle in respective; m is the mass of the vehicle; I_z is the vehicle yaw moment of inertia; δ_f is steering angle.Here the v_x is longitudinal velocity which is set to be a constant for a short period of time; The linear tire model is employed to describe the front and rear lateral forces:

$$F_{yi} = 2C_i\alpha_i, i = f, r \tag{4}$$

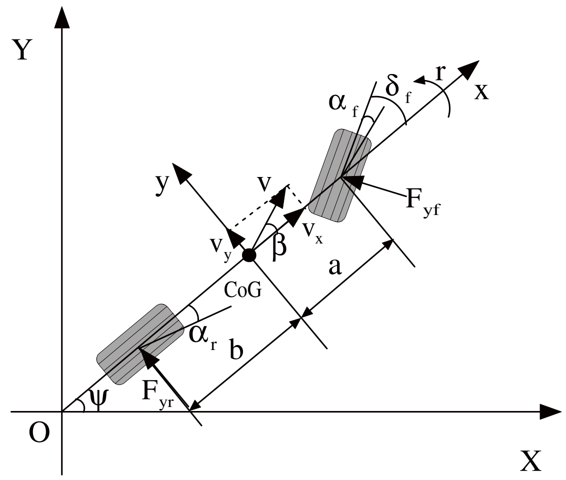


Fig. 2: 2-DOF linear bicycle model

According to the vehicle coordinate system, front and rear wheel sideslip angle can be approximated as:

$$\begin{cases} \alpha_f = \beta + \frac{ar}{v_x} - \delta_f \\ \alpha_r = \beta - \frac{br}{v_x} \end{cases} \tag{5}$$

where C_f and C_r are front and rear wheel lateral stiffness; α_f and α_r are front and rear sideslip angle. Combining (3), (4) and (5), the formulas of 2-DOF vehicle are described as follows

$$\begin{cases} \dot{y} = v(\psi+\beta) \\ \dot{\psi} = r \\ \dot{\beta} = \frac{2(C_f+C_r)}{mv_x}\beta + (\frac{2(aC_f-bC_r)}{mv_x^2} - 1)r - \frac{2C_f}{mv_x}\delta_f \\ \dot{r} = \frac{2(aC_f-bC_r)}{I_z}\beta + \frac{2(a^2C_f+b^2C_r)}{I_zv_x}r - \frac{2aC_f}{I_z}\delta_f \end{cases} \tag{6}$$

The control input is δ_f and the y is viewed as the control output.Then, selecting the $[y\ \psi\ \beta\ r]$ as state variable, the state space model could be obtained as follows

$$\begin{cases} \dot{X} = AX + BU \\ Y = CX + DU \end{cases} \tag{7}$$

$$A = \begin{bmatrix} 0 & v & v & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{2(C_f+C_r)}{mv_x} & \frac{2(aC_f-bC_r)}{mv_x^2} - 1 \\ 0 & 0 & \frac{2(aC_f-bC_r)}{I_z} & \frac{2(a^2C_f+b^2C_r)}{I_zv_x} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ -\frac{2C_f}{mv_x} \\ -\frac{2aC_f}{I_z} \end{bmatrix}, C = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T, D = 0$$

Then,by discrediting at sample time T_s with Euler method, the discrete-time model could consequently obtained as follows

$$\begin{cases} X(k+1) = A_cX(k) + B_c\delta_f(k) \\ Y(k) = C_cX(k) \end{cases} \tag{8}$$

3 Path tracking controller design

In this section, the control strategy is described firstly based on the path tracking scheme in [7, 8] and then the MPC based controller for path tracking is designed. At last, stability analysis and static difference analysis prove that the system is reliable theoretically.

3.1 control strategy

For dealing with complex traffic conditions, the state space model of the vehicle is combined with dynamics and kinematics In addition,ignoring the longitudinal dynamics, a lateral control block is designed to control the vehicle following the reference path according to the lateral displacement. The real position of the vehicle are obtained through vehicle system and the local reference that can be recognized by the controller is calculated by two points search and first order Lagrange interpolation.

The structure of the MPC based controller is described in Figure 3.

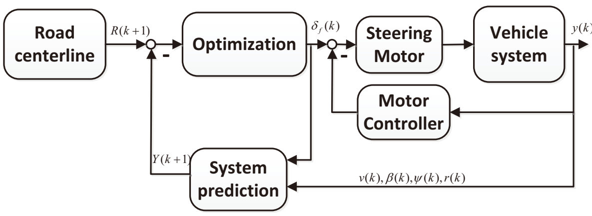


Fig. 3: control strategy

3.2 MPC based controller design

The control input of previous methods can only be calculated according to the current state, which is a process of constant error and constant correction. The MPC [11]used in this paper can predict the state in multiple sampling periods to improve the control accuracy for path tracking. P is viewed as the prediction horizon and M is the control horizon. Then,according to MPC control strategy, the first element is implemented and calculation is repeated at each time instant. And the predictive output can be obtained at sample time k as follows,

$$\begin{aligned} y(k+1) &= C_cA_cx(k) + C_cB_c\delta_f(k) \\ y(k+2) &= C_cA_cx(k+1) \\ &\quad + C_cB_c\delta_f(k+1) \\ &\vdots \\ y(k+M) &= C_cA_c^Mx(k) + \cdots \\ &\quad + C_cB_c\delta_f(k+M-1) \\ &\vdots \\ y(k+P) &= C_cA_c^Px(k) + \cdots \\ &\quad + C_c\sum_{i=1}^{P-M+1} A_c^{i-1}B_c\delta_f(k+M-1) \end{aligned} \tag{9}$$

Define the output sequence and the input sequence as:

$$Y(k+1) = \begin{bmatrix} y(k+1) \\ y(k+2) \\ \vdots \\ y(k+P) \end{bmatrix} \tag{10}$$

$$\delta_f(k) = \begin{bmatrix} \delta_f(k) \\ \delta_f(k+1) \\ \vdots \\ \delta_f(k+M-1) \end{bmatrix} \tag{11}$$

According to Eq. (9), (10) and (11) the predictive formula is concluded as follows:

$$Y(k+1) = S_xx(k) + S_u\delta_f(k) \tag{12}$$

where

$$S_x = \begin{bmatrix} C_cA \\ C_cA^2 \\ \vdots \\ C_cA^P \end{bmatrix} \tag{13}$$

$$S_u = \begin{bmatrix} C_cB_c & 0 & \cdots & & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ C_cA_c^{M-1}B_c & \cdots & & \cdots & C_cB_c \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ C_cA_c^{P-1}B_c \cdots & & \cdots & & \sum_{i=1}^{P-M+1} C_cA_c^iB_c \end{bmatrix} \tag{14}$$

When designing the controller, the tracking accuracy is the primary goal. In addition, the various range of front steering angle is limited. Then, considering the energy consumption by minimizing the path, a cost function is determined and weighting factors are introduced due to their contradiction.

$$\begin{aligned} J = \Gamma_y &\sum_{i=1}^{P} \|y(k+i) - r(k+i)\|^2 \\ &+ \Gamma_u \sum_{i=1}^{N} \|u(k+i)\|^2 \\ &+ \sum_{i=1}^{P} \Gamma_e(\|\Delta y(k+i)^2 + \Delta x(k+i)^2\|) \end{aligned} \tag{15}$$

where

$$\begin{aligned} \Gamma_{\mathrm{y}} &= \mathrm{diag}(\Gamma_{y,1}, \Gamma_{y,2}, \cdots, \Gamma_{y,P}) \\ \Gamma_{\mathrm{u}} &= \mathrm{diag}(\Gamma_{u,1}, \Gamma_{u,2}, \cdots, \Gamma_{u,M}) \\ \Gamma_{\mathrm{e}} &= \mathrm{diag}(\Gamma_{e,1}, \Gamma_{e,2}, \cdots, \Gamma_{e,P}) \end{aligned} \tag{16}$$

This unconstrained optimization problem for path tracking can be transformed into calculating the minimum of the cost function. Because there is no limit to the control input, the exact solutions of the control input at every time can be obtained through deriving the output equation.

$$\begin{aligned} \delta_f = &(S_{\mathrm{u}}^T\Gamma_{\mathrm{y}}^T\Gamma_{\mathrm{y}}S_{\mathrm{u}} + \Gamma_{\mathrm{u}}^T\Gamma_{\mathrm{u}} + S_u^TS_e^T\Gamma_e^T\Gamma_eS_eS_u)^{-1} \\ &\times (S_u^T\Gamma_y^T\Gamma_yE_p - S_u^TS_e^T\Gamma_e^T\Gamma_eS_eS_xX) \end{aligned} \tag{17}$$

where

$$E_p = R(k+1) - S_xX(k) \tag{18}$$

$$S_e = \begin{bmatrix} 1, -1, 0, \cdots, 0 \\ 0, 1, -1, \cdots, 0 \\ \vdots \quad \ddots \quad \vdots \\ 0, 0, \cdots, \quad 1 \end{bmatrix} \tag{19}$$

3.3 stability analysis

The path tracking control of the vehicle is complex including myriad parameters, so simplification of the vehicle has been made before controller design which could cause the uncertainty of the control system, mainly the stability and the accuracy. The stability of the MPC based controller is necessary to be proved in various outdoor conditions.

$$x(k+1) = A_c x(k) + B_c \delta_f(k) \tag{20}$$

define $M = S_u^T \Gamma_y^T \Gamma_y S_u + \Gamma_u^T \Gamma_u + S_u^T S_e^T \Gamma_e^T \Gamma_e S_e S_u$, $N = S_u^T \Gamma_y^T \Gamma_y$, $O = S_u^T S_e^T \Gamma_e^T \Gamma_e S_e S_x$ Substitute (17) into (20) the closed-loop system is defined as follows

$$\begin{aligned} x(k+1) = (A_c - B_c S_x MN - B_c MO)x(k) \\ + B_c MNR(k+1) \end{aligned} \tag{21}$$

From Figure 4, the eigenvalues of [$A_c - B_c S_x MN - B_c MO$] are all in the unit circle.In other words, the system roots are all on the left of the real axis, which prove that the system is asymptotically stable.

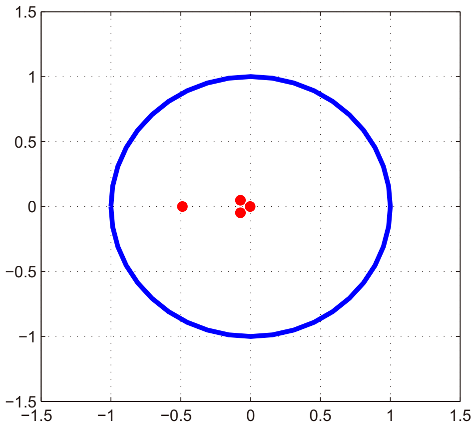


Fig. 4: Eigenvalues

3.4 static difference analysis

In order to analysis the static difference of the proposed MPC control, the analysis is carried out in this subsection. The dimension of the control output in the system is equal to the control input, so $C_c B_c$ is square and reversible. $\Gamma_u = 0_{m \times m}$, $\Gamma_y = I_{p \times p}$, $\Gamma_e = I_{p \times p}$ and $m = p$, the control input is simplified from (17) to (22).

$$\begin{aligned} \delta_f = -[S_u^{-1} S_e^T S_e S_x + S_u^{-1} S_e^{-1} {S_e^T}^{-1} S_e^T S_e S_x] x(k) \\ + [S_u^{-1} + S_u^{-1} S_e^{-1} {S_e^T}^{-1}] E_p \end{aligned} \tag{22}$$

The expressing formula of the control input is unfolded and arranged into a concise formula

$$\delta_f = {C_c B_c}^{-1} (r(k+1) - C_c A_c x(k)) \tag{23}$$

Substitute (23) into (20), the Simplified closed-loop control system is obtained as follow

$$\begin{aligned} x(k+1) = B_c {C_c B_c}^{-1} (r(k+1) - C_c A_c x(k)) \\ + A_c x(k) \end{aligned} \tag{24}$$

Combination with $y(k+1) = C_c x(k+1)$, the output is calculated

$$\begin{aligned} y(k+1) = C_c B_c {C_c B_c}^{-1} (r(k+1) - C_c A_c x(k)) \\ + C_c A_c x(k) = r(k+1) \end{aligned} \tag{25}$$

Further deducing

$$y(k+i) = r(k+i), i = 2, 3, \cdots, p \tag{26}$$

The system has no static error derived from the previous formula. In order to further illustration, simulation is carried out and the result is shown in Figure 5 according to a step curve reference which begin at 2 s.

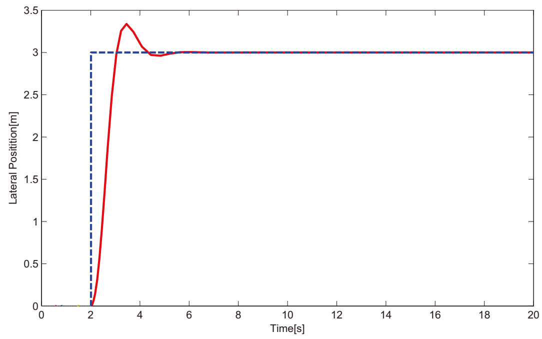


Fig. 5: Step simulation

4 Simulation

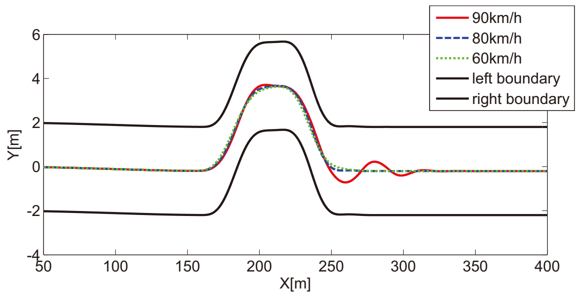
In order to verify the effectiveness of the MPC based controller, veDYNA-Simulink joint simulations are carried out under different conditions based on autonomous vehicle model of Hongqi vehicle. The reference is double line change which is suitable to check the controller performance. Three groups of comparisons experiment under dry asphalt pavement (μ=0.9), wet asphalt pavement (μ=0.6) and ice-covered (μ=0.2) road are carried out.

4.1 Dry asphalt pavement

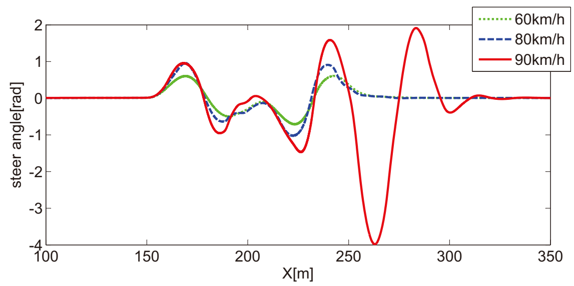
It could be seen from Figure 6 that the tracking error is smaller with the velocity decreasing with tire-road friction coefficient is μ =0.9. In addition, the sideslip angle as well as the yaw rate is much smaller clearly. Besides, the steering angle has the same tendency . From Figure 6 the maximum allowable velocity is approximately 80km/h. When the velocity less than 80km/h, the MPC based controller for path tracking performs well in the condition of dry asphalt pavement and the yaw rate and the sideslip angle, the two key factors of lateral stability are in the allowable range, which ensures the safety of the controlled vehicle system.

4.2 Wet asphalt pavement

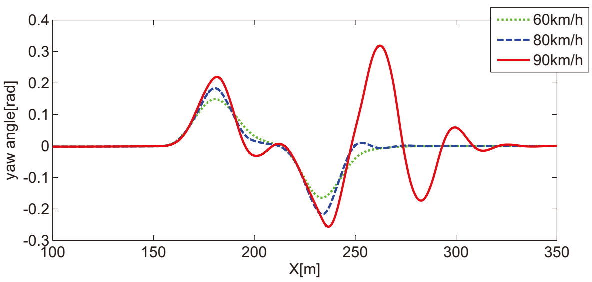
In Figure 7, the friction coefficient of the road is set to 0.6. Compared with the performance of dry asphalt pavement, the maximum allowable velocity under wet asphalt condition is smaller, 40 km/h. It can be seen from Figure 7 that the controlled vehicle has good performance and the range of yaw rate and sideslip angle is within the allowable limits under 40km/h.



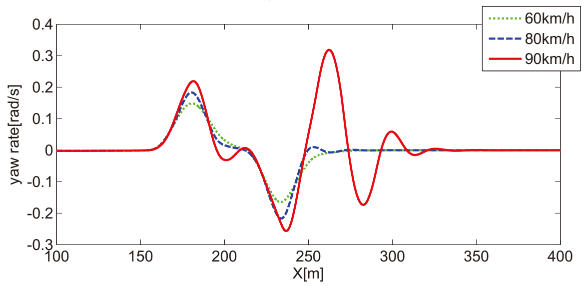
(a) The vehicle's trajectory



(b) sideslip angle

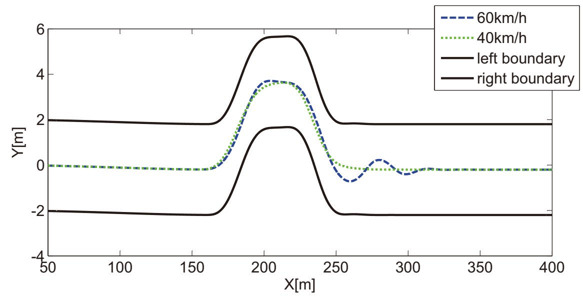


(c) yaw rate

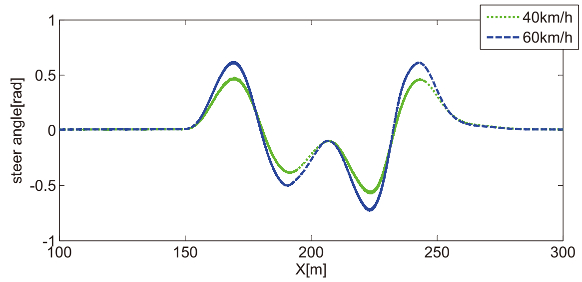


(d) steering angle

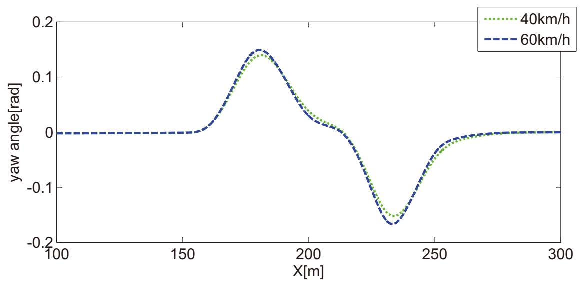
Fig. 6: Simulation results on $\mu = 0.9$



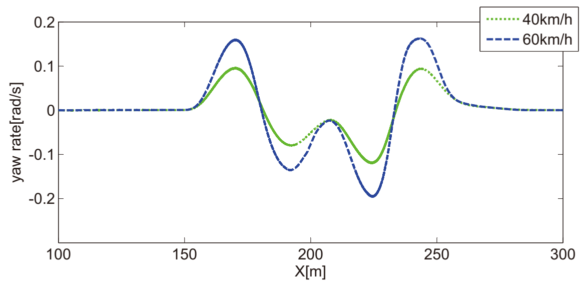
(a) The vehicle's trajectory



(b) sideslip angle



(c) yaw rate



(d) steering angle

Fig. 7: Simulation results on $\mu = 0.6$

4.3 Ice-covered road

It can been seen from Figure 8 that under the ice-covered road condition the vehicle can only keep good performance in low velocity which conforms to the actual situation.

5 Conclusion

An MPC based path tracking scheme for the autonomous vehicle has been designed in this paper, which takes the kinematic and dynamic model into consideration. The system which has no static error without interference is proved asymptotically stable through stability analysis. Then, the veDYNA-Simulink joint simulations are carried out under the double lane change which indicates that the MPC based path-tracking controller for autonomous vehicle is efficient. Due to the fact that some parameters used can not be measured directly, estimation of these parameters will be investigated in a future study.

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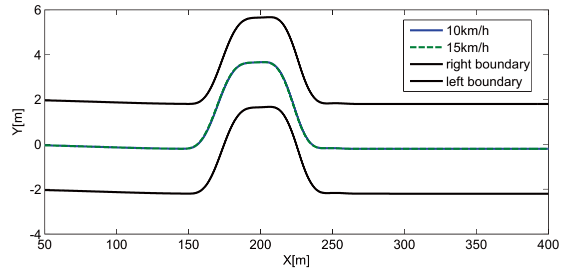
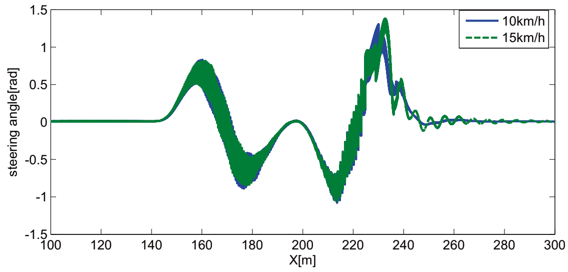
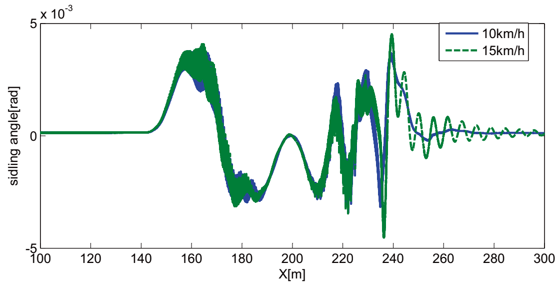


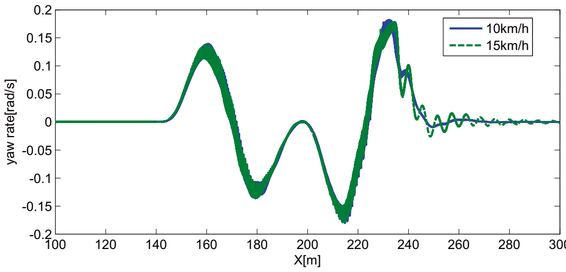
(a) The vehicle's trajectory



(b) sideslip angle



(c) yaw rate



(d) steering angle

Fig. 8: Simulation results on $\mu = 0.2$

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