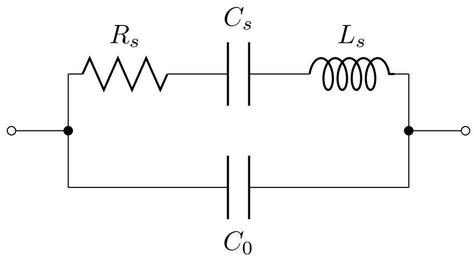
What We Know About QCM Centrifuge Measurements

Last Update: May 29, 2013

Contents

1 Equivalent Circuit

The typical circuit used to analyze QCM behavior is called the *Butterworth van Dyke* (BvD) circuit. It consists of a capacitor C_s, an inductor L_s, and a resistor R_s in series with a parallel capacitance C_0.



The top branch is the *motional branch*, and relates to the crysal and its interaction with the environment. The bottom is the *static branch*, representing the paracitic capitances of the quartz and its driver. In the SRS QCM200[**?**], and probably any other similar compensated phase locked oscillator circuit, C_0 is nulled with additional circuitry. This is absolutely crutial, as the parallel C_0 pertdubes the resonance frequency of the circuit by about $0.825\,\mathrm{Hz\,pF^{-1}}$. The SRS manual gives a higher value of $2\,\mathrm{Hz\,pF^{-1}}$.

Typical values for the 1 in 5 MHz AT cut quartz crystal used in with the QCM200[**?**] are

R_s	$400\,\Omega$ (water), $10\,\Omega$ (air)
C_s	33 fF (SRS manual)
L_s	30 mH (SRS manual), 40 mH (my prediction)
C_0	20 pF (SRS manual)

The circuit may be also be solved using the following second order linear differential equation for charge

$$L\ddot{q} + R\dot{q} + q/C = V(t) \tag{1}$$

The natural frequency is

$$f_0 = \frac{1}{2\pi LC} \tag{2}$$

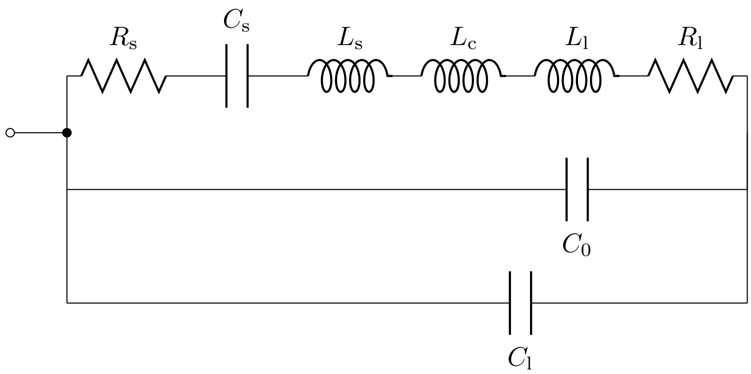
Where C in the above equation takes into account both C_0 and C_s

$$C = \frac{C_s C_0}{C_s + C_0} \tag{3}$$

This, as well as more complicated equivalent circuit models can also be computed directly using SPICE, as per the following code snippet

```
* Butterworth van Dyke equivalent circuit
V1 0 1 ac 1 dc 0
Rs 1 2 100
R1 2 3 375
C0 2 0 20e-12
C1 3 4 33.0e-15
L1 4 0 30e-3
.control
ac lin 100000 5.052e6 5.063e6
write bvd.raw all
```

Liquid sensing under a rigid coating is modeled with a modified Butterworth van Dyke circuit as follows



1.1 Effect of Resonance on Changing the Calues of the Elements

From Equation **??** the following relationships between f and the individual circuit elements are clear

- As R_s goes up, f is unchanged, but the linewidth broadens.

- As C_s goes up, f *decreases* and vice versa.
- As L_s goes up, f *decreases* and vice versa.
- As C_0 goes up, f *increases.*

2 Measurements of Frequency and Resistance

The QCM200 measures both absolute and relative frequency, f, and resistance, R. Unfortunately, we only have absolute resistance measurements so the frequencies are relative through each run.

3 Cause For Different Types of Signals

3.1 Electric Fields

> For a resonator to exhibit a sensitivity, the dc electric field must have a component along the axis of the crystal. True AT-cuts with regular electrodes therefore exhibit little, if any, sensitivity to electric fields. [?]

3.2 Magnetic Fields

> Quartz resonators inherent magnetic field sensitivity is probably smaller than $10\,\mathrm{T}^{-1}$ for fields smaller than $10\,\mathrm{T}$. [?]

3.3 Temperature

I did a quick qualitiative test to assertain the sign of the change in both f and R by placing the crystal next to either *a*) a hot source, consisting of a beaker filled with hot water and *b*) a cold source, consisting of a beaker filled with ice

Heat made f go up and R go down. Cold made f go down and R go up. R goes back to its original value much more slowly than f.

The SRS manual quotes a first order temperature dependence of $8\,\mathrm{Hz}\,{}^{\circ -1}\,{}^{\circ}\mathrm{C}$ and $4\,\Omega\,{}^{\circ -1}\,{}^{\circ}\mathrm{C}$ in water based on its temperature coefficent of viscosity.

Temperature dependence is a second major issue. It is small for AT-cut crystals; however, temperature fluctuations cause fluctuations in Rq inversely proportional to Q [7]. This effect is small compared to temperature ef- fects having their origin in properties of the measurand. In liquid appli- cations, the most temperature-sensitive value is the liquid viscosity. Here, temperature-induced variations in frequency increase with , whereas mass sensitivity increases with . Therefore, an elevated resonance frequency is helpful.

3.4 Pressure

3.5 Mechanical Stress

When I push on the holder from the top, f goes up and R goes down, but not by much. The same is true for flipping the holder and pressing from the other side.

3.5.1 Orientation and Gravity

3.6 Sedimentation Potential

3.7 Viscous Penetration Depth

3.8 Hydrostatic Pressure

3.9 Sauerbrey Mass Loading

The Sauerbrey equation has been modified to predict the frequency and resistance changes.

$$\Delta f = -f_u^{3/2} \left(\frac{\rho_l \eta_l}{\pi \rho_q \mu_q} \right)^{1/2} \tag{4}$$

where

f_u	frequency of unloaded crystal
μ_q	shear modulus of quartz
ρ_q	density of quartz
ρ_l	density of contact liquid
η_l	viscosity of contact liquid

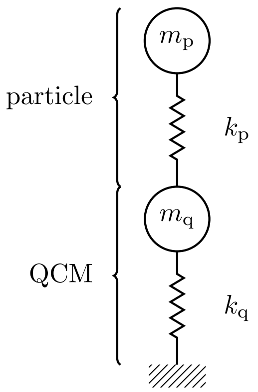
$$\Delta R = 2 n f_s L_u \left(\frac{4\pi f_s \rho_l \eta_l}{\rho_q \mu_q} \right)^{1/2} \tag{5}$$

where

ΔR	change in series resistance
n	number of sides in contact with liquid
f_s	oscillation frequency
L_u	intrinsic inductance of unloaded crystal

From these equations, classic liquid loading in this thin film Sauerbrey approximation causes an decrease in Δf and an increase in δR.

3.10 Dybwad Model



4 Buffer Shifts

5 Pressing Down

6 Oligos

7 Lambda DNA

8 Mechanical Assembly

8.1 PDMS Cell

9 Shift Directions for Unloading

sample	Δf	ΔR
Sauerbrey	$\uparrow$	$\downarrow$
air	$\uparrow$	$\uparrow$
water	$\uparrow$	$\downarrow$
HBFR	$\downarrow$	$\uparrow$

Plot Shapes?

10 Important Numerical Quantities

temperature dependent viscosity of water	$\mu(T) = 2.414 \times 10^{-5} \times 10^{247.8/(T-140)}$