

The complex two dimensional field on the sensor is given by  $\mathbf{E}(\mathbf{r}) = \mathbf{E}(x, y)$ . There are six fundamental parameters describing a vortex, which in turn completely define the underlying field. In two dimensions, these are  $\mathbf{r}$  and the partial derivative  $\nabla \mathbf{E}(\mathbf{r})$ . These in turn can be transformed into six “morphological parameters”: topological charge  $q$ , rotation  $\rho$ , skewness  $\sigma$ , amplitude  $a$ , and anisotropy  $\alpha$ .

$$\rho = \arctan\left(\frac{\text{Re}(\nabla \mathbf{E})_y}{\text{Re}(\nabla \mathbf{E})_x}\right) \quad (\text{rotation}) \quad (1)$$

$$\sigma = -\arctan\left(\frac{\text{Im}(\nabla \mathbf{E})_x}{\text{Im}(\nabla \mathbf{E})_y}\right) - \rho \quad (\text{skewness}) \quad (2)$$

$$a = \frac{\text{Re}(\nabla \mathbf{E})_x}{\cos \rho} = \frac{\text{Re}(\nabla \mathbf{E})_y}{\sin \rho} \quad (\text{amplitude}) \quad (3)$$

$$\alpha = \frac{-\text{Im}(\nabla \mathbf{E})_x}{(\alpha \sin(\rho + \sigma))} = \frac{-\text{Im}(\nabla \mathbf{E})_y}{(\alpha \cos(\rho + \sigma))} \quad (\text{anisotropy}) \quad (4)$$

$$q = \text{sgn}\left(\left|\begin{pmatrix} \text{Re}(\nabla \mathbf{E})_x & \text{Re}(\nabla \mathbf{E})_y \\ \text{Im}(\nabla \mathbf{E})_x & \text{Im}(\nabla \mathbf{E})_y \end{pmatrix}\right|\right) \quad (\text{charge}) \quad (5)$$

$$\mathbf{E}(x, y) = a\left(x \cos \rho + y \sin \rho + i\alpha(y \cos(\rho + \sigma) + x \sin(\rho + \sigma))\right) \quad (6)$$