This document has some practice questions to help you prepare for the midterm.

1 Derivatives

1.1 Differentiation

1. Use the limit definition to compute the derivative of $f(x) = \frac{x+1}{2-x}$.

2. Use the limit definition to compute the derivative of $f(x) = \cos(3x)$.
   Try to simplify your answer as much as possible, you may need this trigonometric identity $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$.

3. Use the limit definition to compute the derivative of $f(x) = \frac{a}{x}$.

4. Which of the following functions is **not continuous**?
   a) $x^2 - 2x + 1$, b) $\cos(x^2)$, c) $|x - 1|$

1.2 Total derivative

1. Compute the total derivative of the Beattie-Bridgeman equation of state for the pressure,

$$p(T, \nu) = \frac{RT}{\nu^2}\left(1 - \frac{c}{\nu T^3}\right)(\nu + B) - \frac{A}{\nu^2}, \tag{1}$$

   where A and B are some functions of the molar volume ν, here we will consider them as constants.

2. Derive is the total derivative of a linear model of the form,

$$f(\mathbf{x}) = \sum_{i=1}^{m} w_i x_i, \quad \text{where} \quad \mathbf{x} = [x_1, x_2, \cdots, x_m]. \tag{2}$$

3. Derive is the total derivative of a polynomial expansion of the form,

$$f(x) = \sum_{i=1}^{m} w_i x^i \quad \text{where} \quad x^i \quad \text{is x to the power of } i. \tag{3}$$

1.3 Partial derivatives

1. The virial expansion is a model of thermodynamic equations of state, and it is given by,

$$P(V_m, T) = \frac{R}{T}\left(1 + \frac{B(T)}{V_m} + \frac{B(T)}{V_m^2} + \cdots\right), \tag{4}$$

where V_m is the molar volume.
Compute the partial derivate of $P(V_m, T)$ with respect to the molar volume.

2. Compute the partial derivative of $f(x, y) = (2x + 6y)^4$ with respect to the variable y

$$\frac{\partial f(x, y)}{\partial y} = \frac{\partial}{\partial y}(2x + 6y)^4 \tag{5}$$

3. Compute the partial derivative of $f(x_1, x_2) = \frac{e^{x_1}}{e^{x_1}+e^{x_2}}$ with respect to the variable x_1

$$\frac{\partial f(x_1, x_2)}{\partial x_1} = \frac{\partial}{\partial x_1}\left(\frac{e^{x_1}}{e^{x_1} + e^{x_2}}\right) \tag{6}$$

2 Linear Algebra

1. What is the result of $\mathbf{u}\mathbf{v}^\top$, given

$$\mathbf{u} = \begin{pmatrix}1\\2\\3\end{pmatrix} \quad \mathbf{v} = \begin{pmatrix}4\\5\\6\end{pmatrix} \quad . \tag{7}$$

Remember that $\mathbf{v}^\top$ is a row vector, meaning the transpose of a column vector.

2. Compute the Manhattan distance of the vector

$$\mathbf{u} = \begin{pmatrix}1\\2\\3\end{pmatrix} \tag{8}$$

The Manhattan distance is defined as, $d_M(\mathbf{x}) = \sum_i |x_i|$.

3. Given the following two matrices $\mathbf{A}$ and $\mathbf{B}$, where $\mathbf{A}$ has 10 rows and 100 columns, and $\mathbf{B}$ has 200 rows and 10 columns. Which of the following operations is doable,
a) $\mathbf{A}\,\mathbf{B}$, b) $\mathbf{B}\,\mathbf{A}$
For the selected option, what is the size of the output tensor.

4. What is $B^\top$ ($\top$: transpose),

$$B = \begin{pmatrix}5 & 7 & 1\\2 & 0 & 1\end{pmatrix} \tag{9}$$

5. What is the output of $\mathbf{A}\,\mathbf{A}^\top$?

$$\mathbf{A} = \begin{pmatrix}-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\\ 0 & 0 & 1\end{pmatrix} \tag{10}$$

The End.