

Tutorial-1 DAA

AMAN PANDEY
Sec-H
Rollno-25
Univ. Rollno-2016615

Ans-1: Asymptotic Notation: are the mathematical notations used to describe the running time of an algorithm.

Types:

1) Big-O Notation(O): It represent upper Bound of algorithm.

$$f(n) = O(g(n)) \text{ if } f(n) \leq c * g(n)$$

2) Omega Notation(Ω): It represents lower bound of Algorithm.

$$f(n) = \Omega(g(n)) \text{ if } f(n) \geq c * g(n).$$

3) Theta Notation(Θ): It represent upper and lower bound of algorithm.

$$f(n) = \Theta(g(n)) \text{ if } c_1 g(n) \leq f(n) \leq c_2 g(n).$$

Ans-2) for ($i=1$ to n)
{
 $i = i * 2$
}

$i=1$
 $i=2$
 $i=4$
 $i=8$
 \vdots
 $i=n$

It is forming a GP:

$$a_n = a * r^{n-1}$$

$$a_n = n$$

$$r = 2$$

$$a = 1$$

$$n = 1 \times (2)^{k-1}$$

$$\log n = (k-1) \log 2$$

$$k = \log n + 1$$

$$O(\log n)$$

Ans-3:

$$T(n) = 3T(n-1) \text{ if } n > 0, \text{ otherwise } 1$$

$$T(1) = 3T(0)$$

$$T(1) = 3 \times 1$$

$$T(2) = 3T(1) = 3 \times 3 \times 1$$

⋮

$$T(n) = 3 \times 3 \times 3 \dots$$

$$= 3^n = O(3^n)$$

Ans-4: $T(n) = 2T(n-1) - 1$ if $n > 0$, otherwise 1

$$T(0) = 1$$

$$T(1) = 2T(0) - 1$$

$$= 2 - 1 = 1$$

$$T(2) = 2(T(1)) - 1$$

$$= 1$$

⋮

$$T(n) = 1$$

$$O(1)$$

Ans-5

int $i=1, j=1$

while($j \leq n$)

{

$i++$;

$s = s + i$;

printf(" #");

}

~~$i=1$~~

$s=1$

~~$i=2$~~

$s = 1+2$

$i=3$

$s = 1+2+3$

$i=4$

$s = 1+2+3+4$

⋮

loop ends when $s > n$

$$1+2+3+4 \dots k > n$$

$$\frac{k(k+1)}{2} > n$$

$$k^2 > n$$

$$k > \sqrt{n}$$

$$T(c) = O(\sqrt{n})$$

Ans-6 void func(int n)

```
{
    int i, count=0;
    for(int i=1; i*i<=n; i++)
        count++;
}
```

Loop ends when $i*i > n$
 $k \times k > n$
 $k^2 > n$
 $k > \sqrt{n}$
 $O(n) = \sqrt{n}$

Ans-7.

1st Loop $i = n/2$ to n , $i++$
 $O(n/2) = O(n)$

2nd nested Loop: $j = 1$ to n , $j = j \times 2$
 $j=1$
 $j=2$
 $j=3$
 $j=4$
 $j=n$
 $\Rightarrow O(\log n)$

3rd nested loop $k = 1$ to n
 $k=1$
 $k=2$
 $k=4$
 $O(\log n)$

Time Complexity $T(c) = O(n \log^2 n)$

Ans-8

$$T(n) = T(n-3) + n^2$$

$$T(1) = 1$$

$$T(4) = T(4-3) + 4^2$$

$$= T(1) + 4^2 = 1^2 + 4^2$$

$$T(7) = T(7-3) + 7^2 = 1^2 + 4^2 + 7^2$$

$$T(10) = T(10-3) + 10^2$$

$$= 1^2 + 4^2 + 7^2 + 10^2$$

$$\text{So, } T(n) = 1^2 + 4^2 + 7^2 + 10^2 \dots n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$= O(n^3)$$

$$T(n) = O(n^3).$$

Ans-9 void function(int n)

{ for (int i=1 to n) — n

{ for (j=1; j<=n; j=j+1) — n

{ printf("*");

j j j

i=1 — j=1 to n

i=2 — j=1 to n

i=3 — j=1 to n

so, for i upto n it will take n^2

$$\text{so, } T(n) = O(n^2).$$

Ans-10)

$$f_1(n) = n^k, \quad f_2(n) = c^n$$

Asymptotic relationship between f_1 and f_2
is Big O. i.e.

$$f_1(n) = O(f_2(n)) = O(c^n)$$

is

$$n^k \leq G * c^n \quad [G \text{ is some constant}]$$

