Tutorial-4 DAA

1) $T(n) = 3T(n/2) + n^2$

Ans: $a = 3, b = 2 \quad f(n) = n^2$

$$n^{\log_b^a} = n^{\log_2^3}$$

$$n^2 > n^{\log_2^3}$$

$$T(n) = \Theta(n^2)$$

2) $T(n) = 4T(n/2) + n^2$

$a = 4, b = 2 \quad f(n) = n^2$

$$n^{\log_b^a} = n^{\log_2^4}$$

$$= n^2 = f(n)$$

$$T(n) = \Theta(n^2 \log n)$$

3) $T(n) = T(n/2) + 2^n$

$a = 1, b = 2$

$$n^{\log_2^1} = n^0 = 1$$

$$1 < 2^n$$

$$T(n) = \Theta(2^n).$$

4) $T(n) = 2^n T(n/2) + n^n$

Not applicable.

5) $T(n) = 16T(n/4) + n$

$$n^{\log_b^a} = n^{\log_4^{16}}$$

$$= n^2$$

$$n^2 > f(n)$$

$$T(n) = \Theta(n^2)$$

6) $T(n) = 2T(n/2) + n\log n$

$a = 2 \quad b = 2$

$n^{\log_2 2} = n$

$f(n) > n \qquad T(n) = \Theta(n \log n)$

7) $T(n) = 2T(n/2) + n/\log n$

$a = 2, \; b = 2$

$n^{\log_b a} = n^{\log_2 2} = n$

$n > f(n)$

$T(n) = \Theta(n)$

8) $T(n) = 2T(n/4) + n^{0.51}$

$a = 2, \; b = 4$

$n^{\log_b a} = n^{\log_4 2} = n^{0.5}$

$f(n) > n^{0.5}$

$T(n) = \Theta(n^{0.51})$

9) $T(n) = 0.5\,T(n/2) + 1/n$

Not applicable as $a < 1$.

10) $T(n) = 16T(n/4) + n!$

$n^{\log_b a} = n^2$

$n^2 < n!$

$T(n) = \Theta(n!)$

11) $T(n) = 4T(n/2) + \log n$

$n^{\log_b a} = n^2$

$n^2 > \log n$

$T(n) = \Theta(n^2)$

12) $T(n) = 3T(\text{sqrt}(n))\,T(n/2) + \log n$

∴ Not applicable as a is not constant.

13) $T(n) = 3T(n/2) + n$

$a = 3,\ b = 2$

$n^{\log_2 3} = n^{1.58}$

$n^{1.58} > n$

$T(n) = O(n^{1.58})$.

14) $T(n) = 3T(n/3) + \sqrt{n}$

$n^{\log_3 3} = n$

$n > \sqrt{n}$

$T(n) = \Theta(n)$

15) $T(n) = 4T(n/2) + c \cdot n$

$n^{\log_2 4} = n^2 > c \cdot n$

$T(n) = \Theta(n^2)$

16) $T(n) = 3T(n/4) + n \log n$

$a = 3,\ b = 4$

$n^{\log_4 3} = n^{0.79}$

$n^{0.79} < n \log n$

$T(n) = \Theta(n \log n)$

17) $T(n) = 3T(n/3) + n/2$

$n^{\log_3 3} = n$

$n > n/2$

$T(n) = \Theta(n)$

18) $T(n) = 6T(n/3) + n^2 \log n$

$n^{\log_3^6} = n^{1.63}$

$n^{1.63} < n^2 \log n$

$T(n) = \Theta(n^2 \log n)$

19) $T(n) = 4T(n/2) + n/\log n$

$n^{\log_5^a} = n^{\log_2^4} = n^2$

$n^2 > n/\log n$

$T(n) = \Theta(n^2)$

20) $T(n) = 64T(n/8) - n^2 \log n$

Not applicable as $f(n)$ is not an increasing function.

21) $T(n) = 7T(n/3) + n^2$

$n^{\log_3^7} = n^{1.7}$

$n^{1.7} < n^2$

$T(n) = \Theta(n^2)$

22) $T(n) = T(n/2) + n(2 - \cos n)$

Not applicable.