GP: $1+r+r^2+r^3+\dots = \frac{1}{1-r}$ $(r<1)$

11/07/18

Z-TRANSFORM

$$F(z) = \sum_{k=-\infty}^{\infty} f(k)\, z^{-k}$$

Q: (1) If $f(k) = \{\underset{\uparrow\, k=0}{3}, 2, 5, \underset{k=3}{4}\}$ find z transform of $f(k)$.

(2) $f(k) = \{\underset{k=-2}{3}, -5, \underset{\uparrow\, k=0}{2}, 0, 4, \underset{k=3}{3}\}$

Ans: (2)

$$F(z) = \sum_{k=-\infty}^{\infty} f(k)\, z^{-k}$$

$$F(z) = \sum_{k=-2}^{3} f(k)\, z^{-k}$$

$$= f(-2)z^2 + f(-1)z^1 + f(0)z^0 + f(1)z^{-1} + f(2)z^{-2} + f(3)z^{-3}$$

$$F(z) = 3z^2 - 5z + 2 + 0 + \frac{4}{z^2} + \frac{3}{z^3}$$

(1) $$F(z) = \sum_{k=-\infty}^{\infty} f(k)\, z^{-k}$$

$$F(z) = \sum_{k=0}^{3} f(k)\, z^{-k}$$

$$= f(0)z^0 + f(1)z^{-1} + f(2)z^{-2} + f(3)z^{-3}$$

$$F(z) = 3 + \frac{2}{z} + \frac{5}{z^2} + \frac{4}{z^3}$$

find z-transform:

Q:1 If $f(k) = 5^{k}$; $k \geq 0$

Q:2 If $f(k) = \begin{cases} a^{k} & k < 0 \\ b^{k} & k \geq 0 \end{cases}$ $\quad F(z) = \sum_{k=-\infty}^{\infty} f(k) z^{-k}$

$$F(z) = \sum_{k=-\infty}^{k=-1} f(k) \quad z^{-k} \qquad + \sum_{k=0}^{\infty} f(k) z^{-k}$$

$$= \sum_{k=-\infty}^{-1} a^{k} z^{-k} + \sum_{k=0}^{\infty} b^{k} z^{-k}$$

$$= \left[a^{-1} z + a^{-2} z^{2} + a^{-3} z^{3} + \dots\right] + \left[1 + b z^{-1} + b^{2} z^{-2} + \dots\right]$$

$$= \left[\frac{z}{a} + \left(\frac{z}{a}\right)^{2} + \left(\frac{z}{a}\right)^{3} + \dots\right] + \left[1 + \frac{b}{z} + \left(\frac{b}{z}\right)^{2} + \dots\right]$$

$$= \frac{z}{a}\left[1 + \frac{z}{a} + \left(\frac{z}{a}\right)^{2} + \dots\right] + \left[1 + \frac{b}{z} + \left(\frac{b}{z}\right)^{2} + \dots\right]$$

$$= \frac{z}{a}\left\{\frac{1}{1-(z/a)}\right\} + \frac{1}{1-b/z}$$

$$= \frac{z}{\cancel{a}}\left\{\frac{\cancel{a}}{a-z}\right\} + \frac{z}{z-b}$$

$$= \frac{-z}{z-a} + \frac{z}{z-b}$$

Q:1 $f(k) = 5^{-k}$; $k \geq 0$

$$F(z) = \sum_{k=0}^{\infty} f(k) z^{-k}$$

$$= \sum_{k=0}^{\infty} 5^{+k} z^{-k}$$

$$F(z) = 1 + 5^{1} z^{-1} + 5^{2} z^{-2} + \cdots$$

$$= 1 + \frac{5}{z} + \left(\frac{5}{z}\right)^2 + \left(\frac{5}{z}\right)^3 + \cdots$$

$$= \frac{1}{1 - 5/z} = \frac{z}{z-5}$$

Q: 3 $\quad f(k) = 2^{|k|}$

$$\text{or} \quad f(k) = \begin{cases} 2^{-k} & k \leq 0 \\ 2^{k} & k \geq 0 \end{cases}$$

$$F(z) = \sum_{k=-\infty}^{\infty} f(k) z^{-k}$$

$$= \sum_{k=-\infty}^{-1} f(k) z^{-k} + \sum_{k=0}^{\infty} f(k) z^{-k}$$

$$= \sum_{k=-\infty}^{-1} 2^{-k} z^{-k} + \sum_{k=0}^{\infty} 2^{k} z^{-k}$$

$$= \left[2^1 z^1 + 2^2 z^2 + 2^3 z^3 + \cdots\right] + \left[1 + 2^1 z^{-1} + 2^2 z^{-2} + \cdots\right]$$

$$= \left[(2z) + (2z)^2 + (2z)^3 + \cdots\right] + \left[1 + \left(\frac{2}{z}\right) + \left(\frac{2}{z}\right)^2 + \cdots\right]$$

$$= (2z)\left[1 + (2z) + (2z)^2 + \cdots\right] + \left[1 + \left(\frac{2}{z}\right) + \left(\frac{2}{z}\right)^2 + \cdots\right]$$

$$= (2z)\left\{\frac{1}{1-(2z)}\right\} + \frac{1}{1 - (2/z)}$$

$$= \frac{(2z)}{1-2z} + \frac{z}{(z-2)} \Rightarrow \boxed{F(z) = \frac{z}{z-2} - \frac{2z}{2z-1}}$$

Use: $\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \dots$

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Q: $f(k) = \frac{1}{k} \quad (k \geq 1)$

$$F(z) = \sum_{k=-\infty}^{\infty} f(k) z^{-k}$$

Q: $f(k) = \frac{3^k}{k} \quad (k \geq 1)$

Ans: $f(k) = \frac{3^k}{k} \quad k \geq 1$

$$F(z) = \sum_{k=1}^{\infty} f(k) z^{-k}$$

$$F(z) = \sum_{k=1}^{\infty} \frac{3^k}{k} z^{-k}$$

$$= \frac{3}{1} z^{-1} + \frac{3^2}{2} z^{-2} + \frac{3^3}{3} z^{-3} + \frac{3^4}{4} z^{-4} + \dots$$

$$= \frac{\frac{3}{z}}{1} + \frac{\left(\frac{3}{z}\right)^2}{2} + \frac{\left(\frac{3}{z}\right)^3}{3} + \frac{\left(\frac{3}{z}\right)^4}{4} + \dots$$

$$= -\left\{ \frac{-(3/z)}{1} - \frac{(3/z)^2}{2} - \frac{(3/z)^3}{3} - \frac{(3/z)^4}{4} + \dots \right\}$$

$$\boxed{F(z) = -\log\left(1 - \frac{3}{z}\right)}$$

Q: Find Z-transform of (i) $f(k) = c^k \cos\alpha k$ ~~k≥0~~ $(k \geq 0)$

and (ii) $f(k) = c^k \sin\alpha k$

$$F(z) = \sum_{k=-\infty}^{\infty} f(k) z^{-k}$$

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(i) $f(k) = c^k \cos\alpha k$

$$= c^k \left[\frac{e^{i\alpha k} + e^{-i\alpha k}}{2}\right]$$

$$f(k) = \frac{1}{2}\left[c^k e^{i\alpha k} + c^k e^{-i\alpha k}\right] \qquad k \geq 0$$

$$F(z) = \sum_{k=0}^{\infty} f(k) z^{-k}$$

$$= \sum_{k=0}^{\infty} \frac{1}{2}\left[c^k e^{i\alpha k} z^{-k} + c^k e^{-i\alpha k} z^{-k}\right]$$

$$= \frac{1}{2} \sum_{k=0}^{\infty}\left[(c e^{i\alpha} z^{-1})^k + (c e^{-i\alpha} z^{-1})^k\right]$$

$$= \frac{1}{2}\Big[\left[1 + (c e^{i\alpha} z^{-1})^1 + (c e^{i\alpha} z^{-1})^2 + \cdots\right] +$$

$$\left[1 + (c e^{-i\alpha} z^{-1}) + (c e^{-i\alpha} z^{-1})^2 + \cdots\right]\Big]$$

<u>Using Σ GP:</u>

$$= \frac{1}{2}\left[\frac{1}{1 - \left(\frac{c e^{i\alpha}}{z}\right)} + \frac{1}{1 - \frac{c e^{-i\alpha}}{z}}\right]$$

$$= \frac{1}{2}\left[\frac{z}{z - c e^{i\alpha}} + \frac{z}{z - c e^{-i\alpha}}\right]$$

$$= \frac{z}{2}\left[\frac{z - c e^{-i\alpha} + z - c e^{i\alpha}}{(z - c e^{i\alpha})(z - c e^{-i\alpha})}\right]$$

$$= \frac{z}{2}\left[\frac{2z - c\,(e^{i\alpha} + e^{-i\alpha})}{(z - c e^{i\alpha})(z - c e^{-i\alpha})}\right]$$

$$= \frac{z}{2}\left[\frac{2z - 2c\left(\frac{e^{i\alpha}+e^{-i\alpha}}{2}\right)}{z^2 - zce^{-i\alpha} - zce^{i\alpha} + c^2}\right]$$

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$$= \frac{z}{2}\left[\frac{2z - 2c\cos\alpha}{z^2 - zc \cdot 2\left(\frac{e^{i\alpha}+e^{-i\alpha}}{2}\right) + c^2}\right]$$

$$= \frac{z}{2} \times 2\left[\frac{z - c\cos\alpha}{z^2 - 2zc\cos\alpha + c^2}\right]$$

$$\boxed{f(z) = \frac{z^2 - zc\cos\alpha}{z^2 - 2zc\cos\alpha + c^2}}$$ [NOTE: LEARN]

(ii) $f(k) = c^k \sin\alpha k \qquad (k \geq 0)$

$$= c^k\left[\frac{e^{i\alpha k} - e^{-i\alpha k}}{2i}\right]$$

$$= \frac{1}{2i}\left[c^k e^{i\alpha k} - c^k e^{-i\alpha k}\right]$$

$$F(z) = \sum_{k=0}^{\infty} f(k) z^{-k}$$

$$= \sum_{k=0}^{\infty} \frac{1}{2i}\left[c^k e^{i\alpha k} z^{-k} - c^k e^{-i\alpha k} z^{-k}\right]$$

$$= \frac{1}{2i}\sum_{k=0}^{\infty}\left[(ce^{i\alpha}z^{-1})^k - (ce^{-i\alpha}z^{-1})^k\right]$$

$$= \frac{1}{2i}\left[\left[1 + (ce^{i\alpha}z^{-1}) + (ce^{i\alpha}z^{-1})^2 + \dots\right] - \left[1 + (ce^{-i\alpha}z^{-1}) + (ce^{-i\alpha}z^{-1})^2 + \dots\right]\right]$$

$$= \frac{1}{2i}\left[\frac{1}{1 + \left(\frac{e^{i\alpha}c}{z}\right)} - \frac{1}{1 - \frac{ce^{i\alpha}}{z}}\right]$$

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$$F(z) = \frac{1}{2i}\left[\frac{z}{z - ce^{i\alpha}} - \frac{z}{z - ce^{-i\alpha}}\right]$$

$$= \frac{z}{2i}\left[\frac{1}{z - ce^{i\alpha}} - \frac{1}{z - ce^{-i\alpha}}\right]$$

$$= \frac{z}{2i}\left[\frac{\cancel{z} - ce^{-i\alpha} - \cancel{z} + ce^{i\alpha}}{(z - ce^{i\alpha})(z - ce^{-i\alpha})}\right]$$

$$= \frac{z}{2i}\left[\frac{c(e^{i\alpha} - e^{-i\alpha})}{z^2 - zce^{-i\alpha} - zce^{i\alpha} + c^2}\right]$$

$$= \frac{z}{2i}\left[\frac{2ci\,\frac{(e^{i\alpha} - e^{-i\alpha})}{2i}}{z^2 - zc \times 2\left(\frac{e^{i\alpha} + e^{-i\alpha}}{2}\right) + c^2}\right]$$

$$= \frac{z}{2i}\left[\frac{2ci\sin\alpha}{z^2 - 2zc\cos\alpha + c^2}\right]$$

$$= \frac{z\,\cancel{2}\cancel{i}c\sin\alpha}{\cancel{2}\cancel{i}\,(z^2 - 2zc\cos\alpha + c^2)}$$

$$\boxed{F(z) = \frac{zc\sin\alpha}{z^2 - 2zc\cos\alpha + c^2}}$$

(Learn)

Q: Find z-transform of :

(i) $f(k) = \sin\left(\frac{\pi}{2}k + \beta\right)$

(ii) $f(k) = \cos(ak + b)$

Ans: (i) $f(k) = \sin\left(\frac{\pi}{2}k + 3\right)$

$f(k) = \sin\left(\frac{\pi}{2}k\right)\cos 3 + \cos\left(\frac{\pi}{2}k\right)\sin 3$

$z[f(k)] = \cos 3 \; z\left[\sin\left(\frac{\pi}{2}k\right)\right] + \sin 3 \; z\left[\cos\left(\frac{\pi}{2}k\right)\right] \longrightarrow (A)$

Using:

$z[c^k \cos\alpha k] = \dfrac{z^2 - zc\cos\alpha}{z^2 - 2zc\cos\alpha + c^2}$

put $\alpha = \pi/2$, $c = 1$

$z\left[\cos\frac{\pi}{2}k\right] = \dfrac{z^2}{z^2+1}$

$\longrightarrow (1)$

$z[c^k \sin\alpha] = \dfrac{zc\sin\alpha}{z^2 - 2zc\cos\alpha + c^2}$

put $\alpha = \pi/2$, $c = 1$

$z\left[\sin\left(\frac{\pi}{2}k\right)\right] = \dfrac{z}{z^2+1}$

$\longrightarrow (2)$

Substitute (1) & (2) in (A)

$z[f(k)] = \cos 3\left(\dfrac{z}{z^2+1}\right) + \sin 3\left[\dfrac{z^2}{z^2+1}\right]$

(ii) $f(k) = \cos(ak + b)$

$= \cos ak \cos b - \sin ak \sin b$

$= \cos b \cos ak - \sin b \sin ak$

$z[f(k)] = \cos b \; z[\cos ak] - \sin b \; z[\sin ak] \longrightarrow (A)$

Use: $z[c^k \cos\alpha k] = \dfrac{z^2 - zc\cos\alpha}{z^2 - 2zc\cos\alpha + c^2}$

put $\alpha = a$, $c = 1$

$z[\cos ak] = \dfrac{z^2 - z\cos a}{z^2 - 2z\cos a + 1}$

$\longrightarrow (1)$

$z[c^k \sin\alpha k] = \dfrac{zc\sin\alpha}{z^2 - 2zc\cos\alpha + c^2}$

put $\alpha = a$, $c = 1$

$z(\sin ak) = \dfrac{z\cancel{c}\sin a}{z^2 - 2zc\cos a + 1}$

$\longrightarrow (2)$

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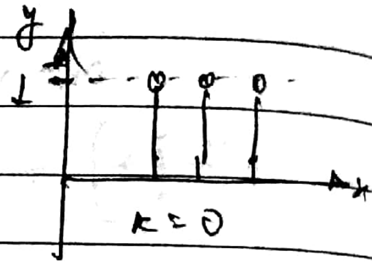
Subs 1 & 2 in A

$$z[f(k)] = \cos b \left[\frac{z^2 - z\cos a}{z^2 - 2z\cos a + 1}\right] - \sin b \left[\frac{zc \sin a}{z^2 - 2zc\cos a + 1}\right]$$

Q: Find z transform of $u(k)$ [unit step signal]

$$u(k) = \begin{cases} 1 & (k \geq 0) \\ 0 & \text{otherwise.} \end{cases}$$



$$\boxed{f(k) = u(k)}$$

$$z[u(k)] = \sum_{k=0}^{\infty} u(k)\, z^{-k}$$

$$= \sum_{k=0}^{\infty} 1\, z^{-k}$$

$$= 1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \dots$$

$$= \frac{1}{1 - 1/z} = \frac{z}{z-1}$$

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* PROPERTIES of z transform:

(1) CHANGE of SCALE PROPERTY:

If $z[f(k)] = F(z)$

then $z[a^k f(k)] = F\left(\frac{z}{a}\right)$

(2) Multiplication by k:

If $z[f(k)] = F(z)$

then $z[k\, f(k)] = -z \frac{d}{dz}[F(z)]$

(3) SHIFTING Property:

If $z[f(k)] = F(z)$

then $z[f(k-n)] = z^{-n} F(z)$

$z[f(k+n)] = z^{n} F(z)$

(4) CONVOLUTION THEOREM: (*)

(दोनों का z-transform निकालके Multiply करो)

If two sequence $f(k)$ and $g(k)$ is defined as

$h(k) = f(k) * g(k)$

then $z[h(k)] = z[f(k) * g(k)]$

$\boxed{H(k) = F(k) \cdot G(k)}$

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Q: Find z transform of : (1) $z[a^k u(k)]$

(2) $z[2^k u(k)]$

(3) $z[k^4 \cdot 2^k]$; $k \geq 0$

(4) $z[k^2 e^{-ak}]$; $k \geq 0$

Ans: 1 we know $z[u(k)] = \frac{z}{z-1}$

$$z[a^k u(k)] = F\left(\frac{z}{a}\right)$$

$$= \frac{z/a}{\frac{z}{a}-1}$$

$$= \frac{z}{z-a}$$

Ans: 2 we know $z[u(k)] = \frac{z}{z-1}$

$$z[2^k u(k)] = F\left(\frac{z}{2}\right)$$

$$= \frac{z/2}{\frac{z}{2}-1}$$

$$= \frac{z}{z-2}$$

Ans: 4 $z[e^{-ak}] = \sum_{k=0}^{\infty} f(k) z^{-k}$

$$= \sum_{k=0}^{\infty} e^{-ak} z^{-k}$$

$$= \sum_{k=0}^{\infty} (e^{-a} z^{-1})^k$$

$$= 1 + e^{-a}z^{-1} + (e^{-a}z^{-1})^2 + \dots$$

$$= \frac{1}{1 - \frac{e^{-a}}{z}}$$

$$z[e^{-ak}] = \frac{z}{z-e^{-a}}$$

$$z[k^2 e^{-ak}] = z[k \cdot k e^{-ak}] =$$

$$z[k e^{-ak}] = -z \frac{d}{dz} F(z)$$

Ans: 3 $z[k^2 e^{k \log 2}]$

$$z[e^{k \log 2}] = \sum_{k=0}^{\infty} f(k) z^{-k}$$

$$= \sum_{k=0}^{\infty} (e^{\log 2} z^{-1})^k$$

$$= \sum_{k=0}^{\infty} (2z^{-1})^k$$

$$= 1 + \frac{2}{z} + \left(\frac{2}{z}\right)^2 + \dots$$

$$= \frac{1}{1-2/z} = \frac{z}{z-2} = z[e^{k \log 2}]$$

$$z[k e^{k \log}] = (-z) \frac{d}{dz}\left(\frac{z}{z-2}\right)$$

$$= -z \left\{ \frac{(z-2)-z}{(z-2)^2} \right\}$$

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$\Rightarrow \frac{d}{dz}\frac{z}{(z-e^{-a})}$

$=(-z)\frac{\cancel{(z-e^{-a})}-\cancel{z}}{(z-e^{-a})^2}$

$=\frac{ze^{-a}}{(z-e^{-a})^2}$

$z[k\cdot ke^{-ak}] = -z\frac{d}{dz}\frac{ze^{-a}}{(z-e^{-a})^2}$

$=-z\left[\frac{(z-e^{-a})^2 e^{-a} - ze^{-a}\,2(z-e^{-a})}{(z-e^{-a})^4}\right]$

$=-z\left[\frac{(z^2+e^{-2a}-2ze^{-a})e^{-a} - 2ze^{-a}(z-e^{-a})}{(z-e^{-a})^4}\right]$

$=-z\,\frac{e^{-a}(z-e^{-a})\,[z-e^{-a}-2z)}{(z-e^{-a})^4}$

$=\frac{-ze^{-a}(z-e^{-a})(-z-e^{-a})}{(z-e^{-a})^4}$

$=\frac{ze^{-a}(z+e^{-a})}{(z-e^{-a})^3}$

$z[ke^{k\log 2}] = \frac{2z}{(z-2)^2}$

$z[k\cdot k2^k] = (-z)\frac{d}{dz}\frac{2z}{(z-2)^2}$

$=(-z)\left[\frac{(z-2)^2\,2 - 2z\,2(z-2)}{(z-2)^4}\right]$

$=(-z)2(z-2)\frac{[z-2-2z]}{(z-2)^4}$

$=\frac{+2(z-2)z\,(z+2)}{(z-2)^4}$

$=\frac{2z(z+2)}{(z-2)^3}$

Q: Find $z[k^2 a^{(k-1)} u(k-1)]$

$z[u(k)] = \frac{z}{z-1}$

$\cancel{z[u(k-1)] = z^{-1}F(z)}$

$\cancel{= z^{1-1}\frac{z}{z-1} = \frac{z}{z-1}}$

$z[a^k u(k)] = F(z/a)$

$=\frac{z/a}{\frac{z}{a}-1} = \frac{z}{z-a}$

$$Z[a^k u(k)] = \frac{z}{z-a}$$

$$Z[a^{k-1} u(k-1)] = +z^{+n} F(z) \qquad (n=-1)$$

$$= +z^{-1} \frac{z}{z-a}$$

$$= \frac{1}{z-a}$$

$$Z[k a^{k-1} u(k-1)] = -z \frac{d}{dz}\left(\frac{1}{z-a}\right)$$

$$= -z \frac{-1 \cdot 1}{(z-a)^2}$$

$$= \frac{z}{(z-a)^2}$$

$$Z[k^2 a^{k-1} u(k-1)] = -z \frac{d}{dz} \frac{z}{(z-a)^2}$$

$$= -z \left[\frac{(z-a)^2 - z\, 2(z-a)}{(z-a)^4}\right]$$

$$= (-z)(z-a) \frac{[z-a-2z]}{(z-a)^4}$$

$$= \frac{(-z)(-z-a)}{(z-a)^3}$$

$$Z[k^2 a^{k-1} u(k-1)] = \frac{z(z+a)}{(z-a)^3}$$

Q: $Z[k2^k u(k-1)]$

$$Z(u(k)) = \frac{z}{z-1}$$

$$Z(u(k-1)) = +z^{-1} \frac{z}{z-1} = \frac{1}{z-1}$$

$$Z(2^k u(k-1)) = F(z/2)$$

$$= \frac{1}{\frac{z}{2}-1} = \frac{2}{z-2}$$

$$Z[k\ 2^k u(k-1)] = -z \frac{d}{dz} \frac{2}{(z-2)}$$

$$= -z \frac{-2}{(z-2)^2}$$

$$= \frac{2z}{(z-2)^2}$$

NOTE: The Term denominator take common.

INVERSE - Z - Transform:

$$(1-x)^{-1} = 1+x+x^2+x^3+\cdots$$

$$(1+x)^{-1} = 1-x+x^2-x^3+\cdots$$

$$(1-x)^{-2} = 1+2x+3x^2+4x^3+\cdots$$

$$(1+x)^{-2} = 1-2x+3x^2-4x^3+\cdots$$

$\Big\}\ |x|<1$

Q: Find inverse z transform of

$$F(z) = \frac{1}{(z-3)(z-2)}$$

for

(1) $|z|<2$

(2) $|z|>3$

(3) $2<|z|<3$

$$\frac{1}{(z-3)(z-2)} = \frac{(z-2)-(z-3)}{(z-3)(z-2)}$$

$$F(z) = \frac{1}{z-3} - \frac{1}{z-2}$$

(1) $|z|<2$ or $|z|<3$

$$\left|\frac{z}{2}\right|<1 \quad \text{or} \quad \left|\frac{z}{3}\right|<1$$

$$F(z) = \frac{1}{3\left(\frac{z}{3}-1\right)} - \frac{1}{2\left(\frac{z}{2}\right)-1}$$

$$= \frac{1}{2}\left(1-\frac{z}{2}\right)^{-1} - \frac{1}{3}\left(1-\frac{z}{3}\right)^{-1}$$

$$= \frac{1}{2}\left[1+\frac{z}{2}+\left(\frac{z}{2}\right)^2+\left(\frac{z}{2}\right)^3\right] - \frac{1}{3}\left[1+\left(\frac{z}{3}\right)^1+\left(\frac{z}{3}\right)^2+\left(\frac{z}{3}\right)^3\right]$$

$$= (-1)\left(\frac{1}{3}+\frac{z}{3^2}+\frac{z^2}{3^3}+\frac{z^3}{3^4}+\cdots\right) + 1\left[\frac{1}{2}+\frac{z}{2^2}+\frac{z^2}{2^3}+\cdots\right]$$

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coefficient of $z^k = \cancel{\sum} (-1) \frac{1}{3^{k+1}} + \frac{1}{2^{k+1}}$ $k \geq 0$

Coefficient of $z^{-k} = (-1) \frac{1}{3^{-k+1}} + \frac{1}{2^{-k+1}}$ $k \leq 0$

(2) $|z| > 3$ or $|z| > 2$

[Greater hai tho inverse.]

$$\left|\frac{1}{z}\right| < \frac{1}{3}$$

$$\left|\frac{3}{z}\right| < 1 \quad \text{or} \quad \left|\frac{2}{z}\right| < 1$$

$$F(z) = \frac{1}{z-3} - \frac{1}{z-2}$$

$$= \frac{1}{z\left(1-\frac{3}{z}\right)} - \frac{1}{z\left(1-\frac{2}{z}\right)}$$

$$= z^{-1}\left[1-\frac{3}{z}\right)^{-1} - \frac{1}{z}\left(1-\frac{2}{z}\right)^{-1}$$

$$= \frac{1}{z}\left[1+\frac{3}{z}+\left(\frac{3}{z}\right)^2+\cdots\right] - \frac{1}{z}\left(1+\frac{2}{z}+\left(\frac{2}{z}\right)^2+\cdots\right)$$

$$= \left(\frac{1}{z}+\frac{3}{z^2}+\frac{3^2}{z^3}+\cdots\right) - \left(\frac{1}{z}+\frac{2}{z^2}+\frac{2^2}{z^3}+\cdots\right.$$

$$z[f(k)] = \sum_{k=-\infty}^{\infty} f(k)\, z^{-k}$$

coeff. of $z^{-k} = \cancel{\sum_{k=1}^{\infty}} 3^{k-1} - 2^{k-1}$ $(k \geq 1)$

Case (3) $2 < |z| < 3$

$2 < |z|$ $\quad$ $|z| < 3$

$\left|\frac{2}{z}\right| < 1$ $\quad$ $\left|\frac{z}{3}\right| < 3$

$$F(z) = \frac{1}{z-3} - \frac{1}{z-2}$$

$$= \frac{1}{3\left(\frac{z}{3}-1\right)} - \frac{1}{z\left(1-\frac{2}{z}\right)}$$

$$= \frac{-1}{3}\left[1-\frac{z}{3}\right]^{-1} - \frac{1}{z}\left(1-\frac{2}{z}\right)^{-1}$$

$$= \frac{-1}{3}\left[1+\frac{z}{3}+\left(\frac{z}{3}\right)^2+\left(\frac{z}{3}\right)^3+\dots\right] - \frac{1}{z}\left[1+\frac{2}{z}+\left(\frac{2}{z}\right)^2+\left(\frac{2}{z}\right)^3+\dots\right]$$

$$= (-1)\left[\frac{1}{3}+\frac{z}{3^2}+\frac{z^2}{3^3}+\frac{z^3}{3^4}+\dots\right] - 1\left[\frac{1}{z}+\frac{2}{z^2}+\frac{2^2}{z^3}+\dots\right]$$

Coeff $z^{k} = (-1)(2)^{k-1}$ $\quad (k \geq 1)$

Coeff $z^{k} = (-1)/3^{k+1}$ $\quad (k \geq 0)$

Coeff $z^{-k} = (-1)/3^{-k+1}$ $\quad (k < 0)$

Find inverse z-transform

Q: $F(z) = \dfrac{2z^2-10z+13}{(z-3)^2(z-2)}$ $\quad 2$

$$\frac{2z^2-10z+13}{(z-2)(z-3)^2} = \frac{A}{z-2} + \frac{B}{z-3} + \frac{C}{(z-3)^2}$$

$$2z^2-10z+13 = A(z-3)^2 + B(z-2)(z-3) + C(z-2)$$

$z=3$	$1 = C(1)$ $\boxed{C=1}$	$z=2$ $\boxed{1=A}$	$z=0$ $13 = 9A+6B-2C$

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$13 = 9 - 2 + 6B$

$13 = 7 + 6B$

$\boxed{B = 1}$

$$F(z) = \frac{1}{z-2} + \frac{1}{z-3} + \frac{1}{(z-3)^2}$$

$2 < |z|$ | $|z| < 3$

$\frac{2}{|z|} < 1$ | $\left|\frac{z}{3}\right| < 1$

$\left|\frac{2}{z}\right| < 1$

$$F(z) = \frac{1}{z\left(1-\frac{2}{z}\right)} + \frac{1}{3\left(\frac{z}{3}-1\right)} + \frac{1}{9\left(\frac{z}{3}-1\right)^2}$$

$$= \frac{1}{z}\left[1-\frac{2}{z}\right)^{-1} - \frac{1}{3}\left(1-\frac{z}{3}\right)^{-1} + \frac{1}{9}\left(1-\frac{z}{3}\right)^{-2}$$

$$= \frac{1}{z}\left[1+\left(\frac{2}{z}\right)+\left(\frac{2}{z}\right)^2+\left(\frac{2}{z}\right)^3+\dots\right) - \frac{1}{3}\left(1+\frac{z}{3}+\left(\frac{z}{3}\right)^2+\dots\right) + \frac{1}{9}\left(1+\frac{2z}{3}+3\left(\frac{z}{3}\right)^2+4\left(\frac{z}{3}\right)^3\dots\right)$$

$$= \left[\frac{1}{z}+\frac{2}{z^2}+\frac{2^2}{z^3}+\dots\right] - 1\left[\frac{1}{3}+\frac{z}{3^2}+\frac{z^2}{3^3}+\dots\right) + \left[\frac{1}{3^2}+\frac{2z}{(3)^3}+\frac{3z^2}{3^4}+\frac{4z^3}{3^5}+\dots\right)$$

coeff of $z^{-k} = 2^{k-1}$ $\quad k \geq 1$

coeff of $z^k = (-1)\frac{1}{3^{k+1}}$ + $\frac{(k+1)}{3^{k+2}}$ $\quad (k \geq 0) \longrightarrow 2$

coeff of $z^{-k} = \frac{(-1)}{3^{-k+1}} + \frac{(-k+1)}{3^{-k+2}}$ $\quad (k < 0)$

from 2 $= \frac{-1}{3^{k+1}} + \frac{k+1}{3(3^{k+1})} = \frac{1}{3^{k+1}}\left[-1+\frac{k+1}{3}\right]$

Coeff $z^k = \frac{1}{3^{k+1}}\left[\frac{k-2}{3}\right] = \frac{k-2}{3(3^{k+1})}$ $\quad k \geq 0$

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coeff $z^{-k} = \frac{-k+2}{3^{-k+2}}$; $(k<0)$

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Q: $F(z) = \frac{z+2}{z^2-2z+1}$ $\quad |z|>1$

$|z|>1$

$1<|z|$

$\boxed{\left|\frac{1}{z}\right|<1}$

$= \frac{z+2}{(z-1)^2} = \frac{(z-1)+3}{(z-1)^2}$

$= \frac{1}{z-1} + \frac{3}{(z-1)^2}$

$F(z) = \frac{1}{z\left(1-\frac{1}{z}\right)} + \frac{3}{z^2\left(1-\frac{1}{z}\right)^2}$

$F(z) = \frac{1}{z}\left(1-\frac{1}{z}\right)^{-1} + \frac{3}{z^2}\left(1-\frac{1}{z}\right)^{-2}$

$= \frac{1}{z}\left[1+\frac{1}{z}+\left(\frac{1}{z}\right)^2+\left(\frac{1}{z}\right)^3+\cdots\right] + \frac{3}{z^2}\left[1+\frac{2}{z}+\frac{3}{z^2}+\frac{4}{z^3}+\right]$

$= \left[\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\frac{1}{z^4}+\cdots\right] + 3\left[\frac{1}{z^2}+\frac{2}{z^3}+\frac{3}{z^4}+\frac{4}{z^5}+\cdots\right]$

~~Coeff of z^{-k} = (1) + 3~~

$= \cancel{\frac{1}{z}+\frac{4}{z^2}+\frac{7}{z^3}+\frac{10}{z^4}+\frac{13}{z^5}+\cdots}$

~~Coeff z^{-k} = $(k+3)$ $\quad k\geq 1$~~

Coeff $z^{-k} = (1)^{k-1} + 3(k-1) \quad k\geq 1$

$= 1+3k-3$

$= 3k-2 \quad k\geq 1$

Q:

$\frac{z^2}{\left(z-\frac{1}{4}\right.}$

Case 1:

Q: find inv. z-transform: NOTE:

$$F(z) = \frac{z^2}{\left(z - \frac{1}{4}\right)\left(z - \frac{1}{5}\right)}$$ for ~~roc~~

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① $|z| < \frac{1}{4}$ & $|z| > \frac{1}{5}$

② $|z| < \frac{1}{5}$

$$\frac{z^2}{\left(z - \frac{1}{4}\right)\left(z - \frac{1}{5}\right)} = \frac{A}{\left(z - 1/4\right)} + \frac{B}{z - 1/5}$$

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NOTE: Power of numerator & denominator is same do Partial fraction of $\frac{f(z)}{z}$.

$\left|\frac{1}{5z}\right| < 1$ or $|4z| < 1$

$$\frac{f(z)}{z} = \frac{5}{z-1/4} - \frac{4}{(z-1/5)}$$

$$= \frac{20}{4z-1} - \frac{4 \times 5}{(5z-1)}$$

$$\frac{f(z)}{z} = 20\left[\frac{1}{4z-1} - \frac{4}{5z-1}\right]$$

$$= -20\,(1-4z)^{-1} - \frac{4}{5z}\left(1-\frac{1}{5z}\right)^{-1}$$

$$= -20\left[1+4z+(4z)^2+\dots\right] - \frac{4}{5z}\left[1+\frac{1}{5z}+\frac{1}{(5z)^2}+\dots\right]$$

$$\frac{f(z)}{z} = -20\left[1+4z+(4z)^2\dots\right] - 4\left[\frac{1}{5z}+\frac{1}{(5z)^2}+\frac{1}{(5z)^3}+\dots\right]$$

$$f(z) = -20\left[z+4z^2+4^2z^3+\dots\right] - 4\left[\frac{1}{5}+\frac{1}{5^2 z}+\frac{1}{5^3 z^2}+\dots\right]$$

coeff $z^{+k} = (-20)\,(4)^{k-1}$ $\quad k \geq 1$

coeff $z^{-k} = (-20)\,(4)^{-k-1}$ $\quad k < 1$

coeff $z^{-k} = (-4)\,\frac{1}{5^{k+1}}$ $\quad k \geq 0$

Case 2: $|z| < \frac{1}{5}$ or $|z| < \frac{1}{4}$

$|5z| < 1$ or $|4z| < 1$

$$\frac{f(z)}{z} = 20\left(\frac{1}{4z-1} - \frac{1}{5z-1}\right)$$

$$\frac{f(z)}{z} = 20\left[\frac{-1}{1-4z} + \frac{1}{1-5z}\right].$$

$$= -20(1-4z)^{-1} + 20(1-5z)^{-1}$$

$$= -20\left[1 + 4z + (4z)^2 + \dots\right] + 20\left[1 + 5z + (5z)^2 + \dots\right]$$

coeff $z^k = (-20)\,4^k + 20\,5^k \quad (k \geq 0)$

coeff $z^{-k} = (-20)\,4^{-k} + (20)\cdot 5^{-k} \quad (k < 0)$