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Question Paper Set

(December 2013 - May 2018)

CONTENTS

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APPLIED MATHEMATICS - III

DECEMBER 2013

Con. 7854-13 (REVISED COURSE) QP Code : GX-12040

(3 Hours) [Total Marks : 80]

N.B.: (1) Question No. 1 is compulsory.
(2) Answer any three questions from Q. 2 to Q. 6.
(3) Each question carry equal marks.
(4) Non-programmable calculator is allowed.

1. (a) Find $L^{-1}\left\{\dfrac{e^{4-3s}}{(s+4)^{5/2}}\right\}$ 5

(b) Find the constant a, b, c, d and e If . 5
$f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i\,(4x^3y - exy^3 + 4xy)$ is analytic.

(c) Obtain half range Fourier cosine series for $f(x) = \sin x,\ x \in (0, \pi)$. 5

(d) If r and $\bar{r}$ have their usual meaning and a is constant vector, prove that $\nabla\times\left[\dfrac{a\times\bar{r}}{r^n}\right] = \dfrac{(2-n)}{r^n}a + \dfrac{n(a.\bar{r})\bar{r}}{r^{n+2}}$ 5

2. (a) Find the analytic function $f(c) = u + iv$, If $3u + 2v = y^2 - x^2 + 16\,xy$. 6

(b) Find the z - transform of $\left\{a^{|k|}\right\}$ and hence find the z-transform of $\left\{\left(\dfrac{1}{2}\right)^{|k|}\right\}$ 6

(c) Obtain Fourier series expansion for $f(x) = \sqrt{1-\cos x}$, $x \in (0, 2\pi)$ and hence deduce that 8
$\displaystyle\sum_{n=1}^{\infty}\frac{1}{4n^2-1} = \frac{1}{2}$

3. (a) Find :- (i) $L^{-1}\left\{\dfrac{s}{(2s+1)^2}\right\}$ (ii) $L^{-1}\left\{\log\dfrac{s^2+a^2}{\sqrt{s+b}}\right\}$ 6

(b) Find the orthogonal trajectories of the family of curves $e^{-x}\cos y + xy = \propto$ where $\propto$ is the real constant in xy - plane. 6

(c) Show that $\bar{F} = \left(y\,e^{xy}\cos z\right)i + (x\,e^{xy}\cos z)\,j - (e^{xy}\sin z)\,k$ is irrotational and find the scalar potential for $\bar{F}$ and evaluate $\int_c \bar{F}.dr$ along the curve joining the points (0, 0, 0) and $(-1, 2, \pi)$. 8

4. (a) Evaluate by Green's theorem. $\int e^{-x}\sin y\,dx + e^{-x}\cos y\,dy$ where c is the rectangle whose vertices are $(0, 0)$ $(\pi, 0)$ $\left(\pi, \dfrac{\pi}{2}\right)$ and $\left(0, \dfrac{\pi}{2}\right)$. 6

(b) Find the half range sine series for the function. 6

$$f(x) = \frac{2kx}{\ell}, \quad 0 \le x \le \ell/2$$
$$= \frac{2k}{\ell}(\ell - x), \quad \frac{\ell}{2} \le x \le \ell$$

(c) Find the inverse z-transform of $\dfrac{1}{(z-3)(z-2)}$ 8

(i) $|z| < 2$ (ii) $2 < |z| < 3$ (iii) $|z| > 3$.

5. (a) Solve using Laplace transform. $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 3y = e^{-x}$, $y(0) = 1$, $y'(0) = 1$. 6

(b) Express $f(x) = \dfrac{\pi}{2}\, e^{-x}\cos x$ for $x > 0$ as Fourier sine integral and show that 6

$$\int_0^\infty \frac{w^3 \sin wx}{w^4+4}\, dw = \frac{\pi}{2} e^{-x} \cos x$$

(c) Evaluate $\iint_s \bar{F} \cdot \bar{n}\, ds$, where $\bar{F} = xi - yj + (z^2 - 1)\,k$ and s is the cylinder formed by the surface $z = 0$, $z = 1$, $x^2 + y^2 = 4$, using the Gauss - Divergence theorem. 8

6. (a) Find the inverse Laplace transform by using convolution theorem $L^{-1}\left\{\dfrac{s^2+2s+3}{(s^2+2s+5)(s^2+2s+2)}\right\}$ 6

(b) Find the directional derivative of $\phi = 4e^{2x-y+z}$ at the point $(1, 1, -1)$ in the direction towards the point $(-3, 5, 6)$. 6

(c) Find the image of the circle $x^2 + y^2 = 1$, under the transformation $w = \dfrac{5-4z}{4z-2}$ 8

MAY 2014

Con.9833-14 (REVISED COURSE) QP Code : NP-18619

(3 Hours) [Total Marks : 80]

N.B.: (1) Question No. 1 is compulsory.
(2) Attempt any three questions from Question No.2 to Question No.6
(3) Non-programmable calculator is allowed.

1. (a) Find $L^{-1}\left[\dfrac{se^{-\pi s}}{s^2+2s+2}\right]$ 5

(b) State true or false with proper justification "There does not exist an analytic function whose real part is $x^3 - 3x^2y - y^3$". 5

(c) Prove that $f_1(x) = 1$, $f_2(x) = x$, $f_3(x) = \dfrac{(3x^2-1)}{2}$ are orthogonal over $(-1, 1)$. 5

(d) Using Green's theorem in the plane, evaluate $\int_c (x^2 - y)\,dx + (2y^2 + x)\,dy$ around the boundary of the region defined by $y = x^2$ and $y = 4$. 5

2. (a) Find the fourier cosine integral representation of the function $f(x) = e^{-ax}$, $x > 0$ and hence show that 6

$$\int_0^\infty \frac{\cos ws}{1+w^2}\, dw = \frac{\pi}{2} e^{-x},\ x \ge 0.$$

(b) Verify laplaces equation for $U = \left(r + \frac{a^2}{r}\right)\cos\theta$. Also find V and f (z) 6

(c) Solve the following eqn. by using laplace transform $\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t$ given that y (0) = 1. 8

3. (a) Expand $f(x) = \begin{cases} \pi x, 0 < x < 1 \\ 0, \quad 1 < x < 2 \end{cases}$ with period 2 into a fourier series. 6

(b) A vector field is given by $\bar{F} = (x^2 + xy^2)\,i + (y^2 + x^2y)\,j$ show that $\bar{F}$ is irrotational and find its scalar potential 6

(c) Find the inverse z - transform of - $f(z) = \frac{z+2}{z^2 - 2z + 1}$, |z| >1 8

4. (a) Find the constants 'a' and 'b' so that the surface $ax^2 - byz = (a + 2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at (1, –1, 2) 6

(b) Given $L(\text{erf}\sqrt{t}) = \frac{1}{S\sqrt{S+1}}$, evaluate $\int_0^\infty t.e^{-t}\,\text{erf}(\sqrt{t})\,dt$ 6

(c) Obtain the expansion of f (x) = x (π – x), 0 < x < π as a half-range cosine series. 8

Hence show that - (i) $\sum_1^\infty \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$ (ii) $\sum_1^\infty \frac{1}{n^4} = \frac{\pi^4}{90}$

5. (a) If the imaginary part of the analytic function W= f (z) is $V = x^2 - y^2 + \frac{x}{x^2 + y^2}$ find the real part U. 6

(b) If $f(k) = 4^k U(K)$ and $g(k) = 5^k U(K)$, then find the z- transform of f(k) . g(k) 6

(c) Use Gauss's Divergence theorem to evaluate $\iint_s \bar{N} \cdot \bar{F}\,ds$ where $\bar{F} = 4xi + 3yj - 2z\bar{k}$ and S is the surface bounded by x = 0, y = 0, z = 0 and 2x + 2y + z = 4. 8

6. (a) Obtain complex form of Fourier series for f(x) = cos h 3x + sin h 3x in (–3, 3). 6

(b) Find the inverse Laplace transform of $L^{-1}\frac{(s-1)^2}{(s^2 - 2s + 5)^2}$ 6

(c) Find the bilinear transformation under which 1, i, –1 from the z-plane are mapped onto 0, 1, ∞ of w-plane. Also show that under this transformation the unit circle in the w-plane is mapped onto a straight line in the z-plane. Write the name of this line. 8

DECEMBER 2014

GN.Con.6452-14 (REVISED COURSE) QP Code : 14544

(3 Hours) [Total Marks : 80]

N.B.: (1) Question no. 1 is compulsory.
(2) Attempt any three from the remaining.
(3) Figures to the right indicate full marks.

1. (a) Find the Laplace Transform of sint cos2t cosht. 5
(b) Find the Fourier series expansion of $f(x) = x^2$ (– π , π) 5

(c) Find the z-transform of $\left(\frac{1}{3}\right)^{|k|}$ 5

(d) Find the directional derivative of $4xz^2 + x^2yz$ at $(1, -2, -1)$ in the direction of $2\bar{i} - \bar{j} - 2\bar{k}$ 5

2. (a) Find an analytic function f(z) whose real part is $e^x(x\cos y - y\sin y)$ 6

(b) Find inverse Laplace Transform by using convolution theorem, $\frac{1}{(s-3)(s+4)^2}$ 6

(c) Prove that $\bar{F} = (6xy^2 - 2z^3)\,\bar{i} + (6x^2y + 2yz)\,\bar{j} + (y^2 - 6z^2x)\,\bar{k}$ is a conservative field. Find the scalar potential Φ such that $\nabla\phi = \bar{F}$. Hence find the work done by $\bar{F}$ in displacing a particle from A(1, 0, 2) to B(0, 1, 1) along AB. 8

3. (a) Find the inverse z-transform of $f(z) = \frac{z^3}{(z-3)(z-2)^2}$ 6

(i) $2 < |z| < 3$ (ii) $|z| > 3$

(b) Find the image of the real axis under the transformation $w = \frac{2}{z+i}$ 6

(c) Obtain the Fourier series expansion of $f(x) = \pi x;\ 0 \le x \le 1$ 8

$= \pi(2 - x);\ 1 \le x \le 2$

Here deduce That $\frac{1}{1^2} + \frac{1}{3^2} + \ldots = \frac{\pi^2}{8}$

4. (a) Find the Laplace Transform of $f(t) = E;\ 0 \le t \le p/2$ 6

$f(t + p) = f(t)$

$= -E;\ p/2 \le t \le p,$

(b) Using Green's theorem evaluate $\int_c \frac{1}{y}dx + \frac{1}{x}dy$ where c is the boundary of the region bounded by $x = 1, x = 4, y = 1, y = \sqrt{x}$ 6

(c) Find the Fourier integral for $f(x) = 1 - x^2,\ 0 \le x \le 1$ 8

$= 0 \quad x > 1$

Hence evaluate $\int_0^\infty \frac{\lambda\cos\lambda - \sin\lambda}{\lambda^3}\cos\left(\frac{\lambda}{2}\right)d\lambda$

5. (a) If $\bar{F} = x^2\,\bar{i} + (x-y)\,\bar{j} + (y+z)\,\bar{k}$ moves a particle from A (1, 0, 1) to B(2, 1, 2) along line AB. Find the work done. 6

(b) Find the complex form of fourier series f(x) = sinhax $(-\ell, \ell)$ 6

(c) Solve the differential equation using Laplace Transform. $(D^2 + 2D + 5)\,y = e^{-t}\sin t$, $y(0) = 0$, $y'(0) = 1$ 8

6. (a) If $\int_0^\infty e^{-2t}\sin(t+\alpha)\cos(t-\alpha)\,dt = \frac{3}{8}$ find the value of α. 6

(b) Evaluate $\iint_s (y^2 z^2 \bar{i} + z^2 x^2 \bar{j} + z^2 y^2 \bar{k}).\bar{n}\, ds$ where s is the hemisphere $x^2 + y^2 + z^2 = 1$ above xy-plane and bounded by this plane. 6

(c) Find Half range sine series for $f(x) = \ell x - x^2 \ (0, \ell)$. Hence prove that $\frac{1}{1^6} + \frac{1}{3^6} + \ldots = \frac{\pi^6}{960}$ 8

MAY 2015

JP-Con.8899-15 (REVISED COURSE) QP Code : 4827

(3 Hours) [Total Marks : 80]

N.B.: (1) Question No.1 is compulsory.
(2) Attempt any three from the remaining six questions.
(3) Figures to the right indicate full marks.

1. (a) Find Laplace Transform of $\frac{\sin t}{t}$ 20

(b) Prove that $f(z) = \sinh z$ is analytic and find its derivative

(c) Find Fourier Series for $f(x) = 9 - x^2$ over $(-3, 3)$

(d) Find $Z\,[f(k)*g(k)]$ if $f(k) = \frac{1}{3^k}$.$g(k) = \frac{1}{5^k}$

2. (a) Prove that $\bar{F} = ye^{xy} \cos z i + xe^{xy} \cos z j - e^{xy} \sin z k$ is irrotational. Find Scalar Potential for $\bar{F}$ Hence evaluate $\int_C \bar{F}.d\bar{r}$ along the curve C joining the points (0, 0, 0) and $(-1, 2, \pi)$ 6

(b) Find the Fourier series for $f(x) = \frac{\pi - x}{2}$, $0 \le x \le 2\pi$ 6

(c) Find Inverse Laplace Transform of (i) $\frac{s+29}{(s+4)(s^2+9)}$ (ii) $\frac{e^{-2s}}{s^2+8s+25}$ 8

3. (a) Find the Analytic function $f(z) = u + iv$ if $u + v = \frac{x}{x^2+y^2}$ 6

(b) Find Inverse Z transform of $\frac{1}{(z-1/2)(z-1/3)}$, $1/3 < |z| < 1/2$ 6

(c) Solve the Differential Equation $\frac{d^2y}{dt^2} + y = t$, $y(0) = 1$, $y'(0) = 0$, using Laplace Transform 8

4. (a) Find the Orthogonal Trajectory of $3x^2y - y^3 = k$ 6

(b) Using Greens theorem evaluate $\int_c (xy + y^2)\, dx + x^2 dy$, C is closed path formed by $y = x$, $y = x^2$ 6

(c) Find Fourier Integral of $f(x) = \begin{cases} \sin x & 0 \le x \le \pi \\ 0 & x > \pi \end{cases}$. Hence show that $\int_0^\infty \frac{\cos(\lambda \pi/2)}{1-\lambda^2}\, d\lambda = \frac{\pi}{2}$ 8

5. (a) Find Inverse Laplace Transform using Convolution theorem $\dfrac{s}{(s^4+8s^2+16)}$ 6

(b) Find the Bilinear Transformation that maps the points z = 1, i, –1 into w = i, 0, – i. 6

(c) Evaluate $\int_C \bar{F}.d\bar{r}$ where C is the boundary of the plane 2x + y + z = 2 cut off by co-ordinate planes and $\bar{F} = (x+y)i + (y+z)j - xk$. 8

6. (a) Find the Directional derivative of $\phi = x^2 + y^2 + z^2$ in the direction of the line $\dfrac{x}{3} = \dfrac{y}{4} = \dfrac{z}{5}$ at (1, 2, 3) 6

(b) Find Complex Form of Fourier Series for e^{2x}; 0 < x < 2 6

(c) Find Half Range Cosine Series for $f(x) = \begin{cases} kx; & 0 \le x \le \ell/2 \\ k(\ell - x); & \ell/2 \le x \le \ell \end{cases}$ hence find $\dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \dots$ 8

DECEMBER 2015

MD-Con.7526-15 (REVISED COURSE) QP Code : 5067

(3 Hours) [Total Marks : 80]

Instructions:
(1) Question No. 1 is compulsory.
(2) Attempt any THREE of the remaining.
(3) Figures to the right indicate full marks.

1. (a) Find Laplace of $\{t^5 \cosh t\}$ 5

(b) Find Fourier series for $f(x) = 1 - x^2$ in (–1, 1) 5

(c) Find a, b, c, d, e if, $f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i(4x^3y - exy^3 + 4xy)$ is analytic 5

(d) Prove that $\nabla\left(\dfrac{1}{r}\right) = -\dfrac{\bar{r}}{r^3}$ 5

2. (a) If f (z) = u + iv is analytic and $u + v = \dfrac{3\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$, find f(z) 6

(b) Find inverse Z-transform of $f(z) = \dfrac{z+2}{z^2 - 2z + 1}$ for |z| > 1 6

(c) Find Fourier series for $f(x) = \sqrt{1 - \cos x}$ in (0, 2π) Hence, deduce that $\dfrac{1}{2} = \sum_1^\infty \dfrac{1}{4n^2 - 1}$ 8

3. (a) Find $L^{-1}\left\{\dfrac{1}{(s-2)+(s+3)}\right\}$ Using Convolution theorem 6

(b) Prove that $f_1(x) = 1$, $f_2(x) = x$, $f_3(x) = (3x^2 - 1)/2$ are orthogonal over (–1, 1) 6

(c) Verify Green's theorem for $\int_c \bar{F}.\overline{dr}$ where $\bar{F} = (x^2 - y^2)i + (x+y)j$ and c is the triangle with vertices (0, 0), (1, 1), (2, 1) 8

4. (a) Find Laplace Transform of $f(t) = |\sin pt|, t \ge 0$ 6

(b) Show that $\bar{F} = (y\sin z - \sin x)\,i + (x\sin z + 2yz)\,j + (xy\cos z + y^2)\,k$ is irrotational. Hence, find its scalar potential. 6

(c) Obtain Fourier expansion of $f(x) = x + \frac{\pi}{2}$ where $-\pi < x < 0$ 8

$$= \frac{\pi}{2} - x \text{ where } 0 < x < \pi$$

Hence, deduce that (i) $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$ (ii) $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots$

5. (a) Using Gauss Divergence theorem to evaluate $\iint_S \bar{N}.\bar{F}\,ds$ where $\bar{F} = 4xi - 2y^2 j + z^2 k$ and S is the region bounded by $x^2 + y^2 = 4$, $z = 0$, $z = 3$. 6

(b) Find $Z\,[2^k \cos(3k + 2)\}$, $k \geq 0$ 6

(c) Solve $(D^2 + 2D + 5)\,y = e^{-t}\sin t$, with $y(0) = 0$ and $y'(0) = 1$ 8

6. (a) Find $L^{-1}\left\{\tan^{-1}\left(\frac{z}{s^2}\right)\right\}$ 6

(b) Find the bilinear transformation which maps the points 2, i, –2 onto points 1, i, –1 by using cross-ratio property. 6

(c) Find Fourier Sine integral representation for $f(x) = \frac{e^{-ax}}{x}$ 8

MAY 2016

FW-Con.9413-16 (REVISED COURSE) QP Code : 30557

(3 Hours) [Total Marks : 80]

N.B.: (1) Question No. 1 is compulsory.
(2) Attempt any THREE of the remaining.
(3) Figures to the right indicate full marks.

1. (a) If $\int_0^\infty e^{-2t} \sin(t + \alpha)\cos(t - \alpha)dt = \frac{1}{4}$ find α. 5

(b) Find half range Fourier cosine series for $f(x) = x$, $0 < x < 2$ 5

(c) If u (x, y) is a harmonic function then prove that $f(z) = u_x - iu_y$ is an analytic function. 5

(d) Prove that $\nabla f(r) = f'(r)\frac{\bar{r}}{r}$ 5

2. (a) If $v = e^x \sin y$, prove that v is a harmonic function. Also find the corresponding analytic function. 6

(b) Find Z-transform of $f(k) = b^k$, $k \geq 0$ 6

(c) Obtain Fourier series for $f(x) = \frac{3x^2 - 6x\pi + 2\pi^2}{12}$ in $(0, 2\pi)$, where $f(x + 2\pi) = f(x)$. 8

Hence deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$

3. (a) Find inverse Laplace of $\frac{(s+3)^2}{(s^2+6s+5)^2}$ using Convolution theorem 6

(b) Show that the set of functions {sin x, sin 3x, sin 5x,... } is orthogonal over [0, π/2]. Hence construct orthonormal set of functions. 6

(c) Verify Green's theorem for $\int_c \frac{1}{y}dx + \frac{1}{x}dy$ where C is the boundary of region defined by x = 1, x = 4, y = 1 and y = $\sqrt{x}$ 8

4. (a) Find $Z\{k^2 a^{k-1} U(k-1)\}$ 6

(b) Show that the map of the real axis of the z- plane is a circle under the transformation $w = \frac{2}{z+i}$. Find its centre and the radius. 6

(c) Express the function $f(x) = \begin{cases} \sin x & |x| < \pi \\ 0 & |x| > \pi \end{cases}$ as Fourier sine Integral. 8

5. (a) Using Gauss Divergence theorem evaluate $\iint_s \bar{N}.\bar{F}ds$ where $\bar{F} = x^2 i + zj + yzk$ and S is the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0, z = 1 6

(b) Find inverse Z-transform of $F(z) = \frac{z}{(z-1)(z-2)}$, $|z| > 2$ 6

(c) Solve $(D^2 + 3D + 2)y = e^{-2t}\sin t$, with y(0) = 0 and y'(0) = 0 8

6. (a) Find Fourier expansion of $f(x) = 4 - x^2$ in the interval (0,2) 6

(b) A vector field is given by $\bar{F} = (x^2 + xy^2)i + (y^2 + x^2 y)j$. Show that $\bar{F}$ is irrotational and find its scalar potential. 6

(c) Find (i) $L^{-1}\left\{\tan^{-1}\left(\frac{a}{s}\right)\right\}$ (ii) $L^{-1}\left(\frac{e^{-\pi s}}{s^2 - 2s + 2}\right)$ 8

DECEMBER 2016

(REVISED COURSE) QP Code : 540701

(3 Hours) [Total Marks : 80]

N.B.: (1) Question No. 1 is compulsory.
(2) Attempt any three of the remaining.
(3) Figures to the right indicate full marks.

1. (a) Find the Laplace transform of $te^{3t}\sin 4t$. 5

(b) Find half-range cosine series for $f(x) = e^x$, $0 < x < 1$. 5

(c) Is $f(z) = \frac{z}{\bar{z}}$ analytic? 5

(d) Prove that $\nabla \times (\bar{a} \times \nabla \log r) = 2\frac{(\bar{a}.\bar{r})\bar{r}}{r^4}$, where $\bar{a}$ is a constant vector. 5

2. (a) Find the inverse Z-transform of $\dfrac{1}{(z-5)^3}$ if $|z| < 5$ 6

(b) If $V = 3x^2y + 6xy - y^3$, show that V is harmonic & find the corresponding analytic function. 6

(c) Obtain Fourier series for the function $f(x) = \begin{cases} 1 + \dfrac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \dfrac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$ 8

hence deduce that $\dfrac{\pi^2}{8} = \dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \dots$

3. (a) Find $L^{-1}\left[\dfrac{(s+2)^2}{(s^2+4s+8)^2}\right]$ using convolution theorem. 6

(b) Show that the set of functions 1, $\sin\left(\dfrac{\pi x}{L}\right), \cos\left(\dfrac{\pi x}{L}\right), \sin\left(\dfrac{2\pi x}{L}\right), \cos\left(\dfrac{2\pi x}{L}\right), \dots\dots$ 6

Form an orthogonal set in $(-L, L)$ and construct an orthonormal set .

(c) Verify Green's theorem for $\int_c \{(e^{2x} - xy^2)\, dx + (ye^x + y^2)\, dy$ Where C is the closed curve bounded by 8

$y^2 = x$ & $x^2 = y$.

4. (a) Find Laplace transform of $f(x) = K\dfrac{t}{T}$ for $0 < t < T$ & $f(t) = f(t + T)$. 6

(b) Show that the vector, $\overline{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ is irrotational and hence, find ϕ such 6

that $\overline{F} = \nabla \phi$

(c) Find Fourier series for f(x) in $(0, 2\pi)$, $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 2\pi - x, & \pi \le x \le 2\pi \end{cases}$ hence deduce that 8

$\dfrac{\pi^4}{96} = \dfrac{1}{1^4} + \dfrac{1}{3^4} + \dfrac{1}{5^4} + \dots\dots\dots$

5. (a) Use Gauss's Divergence theorem to evaluate $\iint_s \overline{N}.\overline{F}\, ds$ where $\overline{F} = 2xi + xyj + zk$ over the region 6

bounded by the cylinder $x^2 + y^2 = 4$, $z = 0$, $z = 6$.

(b) Find inverse Z - transform of $f(x) = \dfrac{z}{(z-1)(z-2)}$, $|z| > 2$ 6

(c) (i) Find $L^{-1}\left[\log\left(\dfrac{s+1}{s-1}\right)\right]$ (ii) $L^{-1}\left[\dfrac{s+2}{s^2-4s+13}\right]$ 8

6. (a) Solve $(D^2 + 3D + 2)\, y = 2(t^2 + t + 1)$ with $y(0) = 2$ & $y'(0) = 0$. 6

(b) Find the bilinear transformation which maps the points 0, i, –2i of z-plane onto the points –4i, ∞, 0 respectively of w-plane. Also obtain fixed points of the transformation. 6

(c) Find Fourier sine integral of $f(x)=\begin{cases} x, & 0<x<1 \\ 2-x, & 1<x<2 \\ 0, & x>2 \end{cases}$ 8

MAY 2017

(REVISED COURSE) QP Code : 540702

(3 Hours) [Total Marks : 80]

N.B.: (1) Question No. 1 is compulsory

(2) Attempt any three from the remaining six questions

(3) Figures to thc right indicate full marks

1. (a) Find the Laplace Transform of $e^{-t}\int_0^t u\cos 2u\,du$ 20

(b) Prove that $f(z) = \sinh z$ is analytic and find its derivative

(c) Obtain Half range Sine Series for $f(x) = x + 1$ in $(0, \pi)$

(d) Find a unit vector normal to the surface $x^2y + 2xz = 4$ at $(2, -2, 3)$

2. (a) Prove that $\overline{F}=\left(2xy^2+yz\right)i+\left(2x^2y+xz+2yz^2\right)j-\left(2y^2z+xy\right)k$ is Irrotational. Find Scalar Potential for $\overline{F}$ 6

(b) Find the inverse Laplace Transform using Convolution theorem $\dfrac{(s-1)^2}{(s^2-2s+5)^2}$ 6

(c) Find Fourier Series of $f(x)=\begin{cases} \pi x; 0\le x\le 1 \\ \pi(2-x); 1\le x\le 2 \end{cases}$ 8

3. (a) Find the Analytic function $f(z) = u + iv$ if $v=\dfrac{x}{x^2+y^2}+\cosh x\cos y$ 6

(b) Find Inverse Z transform of $\dfrac{\left(3z^2-18z+26\right)}{(z-2)(z-3)(z-4)}$, $3 < |z| < 4$ 6

(c) Solve the Differential Equation $\dfrac{d^2y}{dt^2}+2\dfrac{dy}{dx}+2y=5\sin t$, $y(0) = 0$, $y'(0) = 0$ using LaplaceTransform 8

4. (a) Find the Orthogonal Trajectory of $3x^2y - y^3 = k$ 6

(b) Find the z-transform of $2^K \sinh 3K$, $K \ge 0$ 6

(c) Express the function $f(x)=\begin{cases} 1 : |x|<1 \\ 0 : |x|>1 \end{cases}$ as Fourier Integral. Hence evaluate $\int_0^\infty \dfrac{\sin\lambda}{\lambda}.\cos(\lambda x)d\lambda$ 8

5. (a) Evaluate using Stoke's theorem $\int_c (2x-y)dx - yz^2dy - y^2zdz$ where C is the circle $x^2 + y^2 = 1$ corresponding to the sphere $x^2 + y^2 + z^2 = 1$ above the XY plane 6

(b) Show that $w = \dfrac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ into straight line $4u + 3 = 0$ 6

(c) Find Inverse Laplace Transform (i) $e^{-s} \tanh^{-1} s$ (ii) $\dfrac{6}{(2s+1)^3}$ 8

6. (a) Find the Laplace transform of $f(t) = \dfrac{2t}{3}, 0 \le t \le 3, f(t+3) = f(t)$ 6

(b) Find Complex Form of Fourier Series for $\sin(\alpha x)$; $(-\pi, \pi)$, α is not an integer 6

(c) Verify Green's theorem for $\int_c (2x^2 - y^2)dx + (x^2 + y^2)dy$ where C is the boundary of the surface enclosed by lines $x = 0$, $y = 0$, $x = 2$, $y = 2$. 8

APPLIED MATHEMATICS III

DECEMBER 2017

(REVISED COURSE) QP Code : 24395

(3 Hours) [Total Marks : 80]

N.B.: (1) Question no 1 is compulsory.
(2) Figures to the right indicate full marks.
(3) Attempt any three from Q2 to Q6

1. (a) If any 14 integers from l to 26 are chosen then show that at least one of them is a multiple of another. 5

(b) Functions f and g are defined as follows: 5
$f : R \rightarrow R$, $g : R \rightarrow R$ $f(x) = 2x + 3$, $g(x) = 3x - 4$. Find fog and gofog.

(c) $L\left(\frac{d}{dt}\frac{\sin 3t}{t}\right)$ 5

(d) Show that there does not exist an analytic function whose real part is $3x^2 - 2x^2y + y^2$. 5

2. (a) Evaluate $\int_0^{\infty} e^{-t}\left(\frac{\cos 3t - \cos 2t}{t}\right)dt$ 6

(b) Evaluate $L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)(s^2+9)}\right\}$ 6

(c) Find bilinear transformation which maps the points Z = 1, i ,–1 into points W = i, 0, –i . 8
Hence find fixed pts of transformation and the image of $|z| < 1$,

3. (a) If A, B ,C are of subbsets of universal set U, then prove that 6
$A \times (B \cup C) = (A \times B) \cup (A \times C)$

(b) Let A = {1,2,3,6}, B = {1,2,3,6,7,14,21,42} and R be the relation 'is divisible by'. Draw Hasse Diagram for two sets. Show that are posets. 6

(c) Find Laplace transform of following functions. 8
(i) $e^{-2t}\sqrt{1-\sin t}$ (ii) $te^{-2t}H(t-1)$

4. (a) In how many different ways can 4 ladies and 6 gentlemen be seated in a row, so no ladies sit together, 6

(b) Find analytic function whose real part is $\frac{\sin 2x}{\cosh 2y + \cos 2x}$ 6

(c) Evaluate inverse Laplace Transform of following functions 8
(i) $\frac{1}{(s-3)(s+4)^2}$ by convolution theorem (ii) $\log\left(1+\frac{a^2}{s^2}\right)$

5. (a) Solve the following equation by using Laplace transform $\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t$, given that y(0) = 1 6

(b) Find p such that the function $\frac{1}{2}\log(x^2 + y^2) + i\tan^{-1}\frac{px}{y}$ is analytic. 6

(c) For $x, y \in Z$, xRy if and if only 2x + 5y is divisible by 7 8
is R an equivalence relation ? Find equivalence relation.

6. (a) Each coefficient of the equation $ax^2 + bx + c = 0$ is determined by throwing an ordinary die. Find the probability that the equation will have real roots. 6

(b) A certain test for particular cancer is known to be 95% accurate. A person submits to the test and result is positive. Suppose that a person comes from a population of the 1,00,000 where 2000 people suffer from disease . What can we conclude about the probability that person under test has particular cancer? 6

(c) (i) If five points are taken in a square of side 2 units. Show that at least two of them are no more than $\sqrt{2}$ units apart. 4

(ii) How many friends must you have to guarantee that at least five of them have their birthday in same month. 4

MAY 2018

(REVISED COURSE) QP Code : 24396

(3 Hours) [Total Marks : 80]

N.B.: (1) Question no.1 is compulsory.

(2) Attempt any three questions out of remaining five questions.

1. (a) Determine the constants a, b, c, d so that the function $f(z) = x^2 + axy + by^2 + i(cx^2 + dxy + y^2)$ is analytic. 5

(b) Let A = {1, 2, 3, 4}, B = {1, 2, 3, 4} and "aRb if and only if a is not equal lo b". Find Rand its digraph. 5

(c) For the sets A, B, C given that $A \cap B = A \cap C$ and $\overline{A} \cap B = \overline{A} \cap C$ is it necessary that B = C? Justify. 5

(d) Find Laplace transform of 5

$f(t) = t$ for $0 < t < 1$

$= 0$ for $1 < t < 2$, $f(t + 2) = f(t)$.

2. (a) 75 Children went to an amusement park where they can ride on the merry-go-round, roller coaster and ferris wheel. It is known that 20 of them have taken all 3 rides. and 55 of them have taken at least two of the 3 rides. Each ride costs 0.50 Rs and the total receipt of the amusement park was 70 Rs. Determine the number of children who did not try any of the rides. 6

(b) Evaluate $\int_0^{\infty} te^{-3t} J_0(4t)\,dt = \frac{3}{125}$ if $L\{J_0(t)\} = \frac{1}{\sqrt{s^2+1}}$ 6

(c) (i) Functions f, g and h are defined as follows: 4

$f : R \to R$, $g: R \to R$, $h: R \to R$, $f(x) = x + 4$, $g(x) = x - 4$

$h(x) = 4x$ for $x \in R$, where R is the set or real numbers. Compute $f \circ g$; $g \circ f$; $f \circ g \; h$; $h \circ h$.

(ii) Show that using Venn diagram $P \cap (Q - R) = (P \cap Q) - (P \cap R)$.

3. (a) If f(z) and |f(z)| are both analytic then show that f(z) is constant. 6

(b) Let R be a binary relation on the set of positive integers such that R = {(a,b)/a – b is an odd positive integer}. Is R reflexive? Symmetric? Antisymmetric? Transitive? An equivalence relation? A partial ordering set? 6

(c) Evaluate (i) $L[te^{3t} \sin 4t]$ (ii) $L\left[\int_0^t\int_0^t\int_0^t t \sin t\,dt\,dt\,dt\right]$ 8

4. (a) Evaluate using Convolution theorem $L^{-1}\left[\frac{(s+2)}{(s^2+4s+8)^2}\right]$ 6

(b) Find the transitive closure of R where R be the relation represented by $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ 6

(c) Find analytic function $f(z) = u + iv$ where $v = e^x (x\sin y + y\cos y)$ 8

5. (a) Solve $\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t$ with $y(0) = 1$. 6

(b) Find bilinear transformation which maps the points $z = 1, i, -1$ onto $w = 0, 1, \infty$. Further show that under this transformation the unit circle In w plane is mapped onto a straight line in the z plane. 6

(c) In a bolt factory machines A, B, and C manufacture respectively 25%, 35% and 40% of the total of their output 5, 4, 2 percent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, Band C? 8

6. (a) It is known that at the university 60% of the professors play tennis. 50% of them play bridge. 70% jog, 20% play tennis and bridge. 30% play tennis and jog, 40% play bridge and jog. If someone claimed that 20% of the professors jog and play bridge and tennis. would you believe this claim? Why? 6

(b) Suppose repetitions are not permitted. 6

(i) How many four- digit numbers can be formed from the digits 1, 2, 3, 5, 7, 8?

(ii) How many of the numbers in part (a) are less than 4000?

(iii) How many of the numbers in part (a) are odd ?

(iv) How many of the numbers in part (a) are multiples of 5?

(c) Evaluate (i) $L^{-1}[2\tanh^{-1} s]$ (ii) $L^{-1}\left[\frac{e^{4-3s}}{(s+4)^{\frac{5}{2}}}\right]$ 8

ACADEMY

SET TIME SET TARGET

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