Interval

- 'π'
  - (0, 2π)
  - (−π, π)
- 'l'
  - (0, 2l)
  - ~~(−l, l)~~ (−l, l)

I. For 'π'

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos(nx) + b_n \sin n(x) \right]$$

$$a_0 = \frac{1}{\pi} \int f(x)\, dx$$

$$a_n = \frac{1}{\pi} \int f(x) \cos(nx)\, dx$$

$$b_n = \frac{1}{\pi} \int f(x) \sin(nx)\, dx .$$

For 'l'

$$f(x) = \frac{a_0}{2} + \sum_{1=n}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right) \right]$$

$$a_0 = \frac{1}{l} \int f(x)\, dx$$

$$a_n = \frac{1}{l} \int f(x) \cos\left(\frac{n\pi x}{l}\right) dx$$

$$b_n = \frac{1}{l} \int f(x) \sin\left(\frac{n\pi x}{l}\right) dx$$

General formula:

$\sin 0 = 0$ $\quad$ $\sin 3\pi = 0$

$\sin \pi = 0$ $\quad$ $\sin n\pi = 0$

$\sin 2\pi = 0$ $\quad$ $\sin 2n\pi = 0$

$\cos 0 = 1$

$\cos\pi = (-1)^1$

$\cos 2\pi = (-1)^2$

$\cos 3\pi = (-1)^3$

$\cos n\pi = (-1)^n$

$\cos 2n\pi = (-1)^{2n}$

$= [(-1)^2]^n$

$= [1]^n$

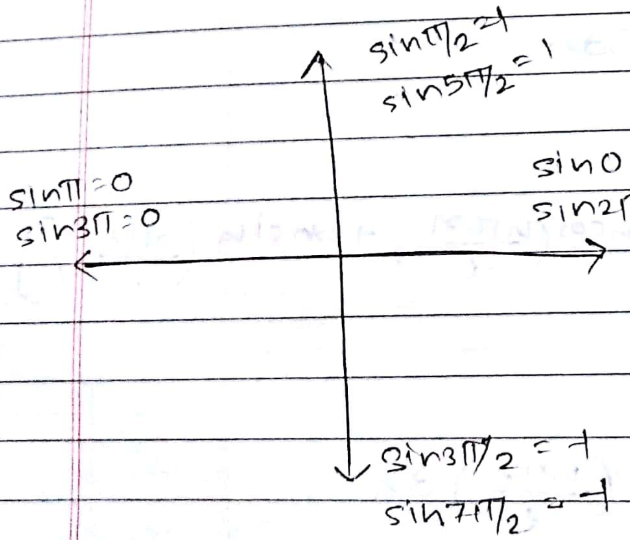
$= 1$

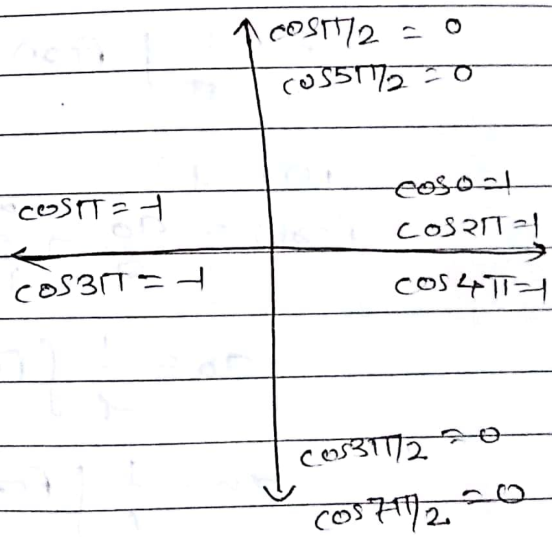
$\sin(n+1)\pi = 0$

$\sin(n-1)\pi = 0$

$\cos(n+1)\pi = (-1)^{n+1} = (-1)^n(-1)$

$\cos(n-1)\pi = (-1)^{n-1} = (-1)^n(-1)$





$\frac{d}{dx} x^n = nx^{(n-1)}$ $\qquad \int x^n dx = \frac{x^{n+1}}{n+1} + C$

$\frac{d}{dx}(\text{const}) = 0$ $\qquad \int(\text{const})dx = \text{const} \times (x) + C$

$\frac{d}{dx}(5x) = 5$ $\qquad \int uv =$

$\partial s = c$	$\int s = -c$
$\partial c = -s$	$\int c = s$

Q) $f(x) = \left[\frac{\pi - x}{2}\right]$ find fourier series also P.T

$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} - \cdots$ for interval $[0, 2\pi]$

$\longrightarrow$ $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n\cos(nx) + b_n\sin(nx))$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} \left(\frac{\pi - x}{2}\right) dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (\pi - x)\,dx$$

$$= \frac{1}{2\pi}\left[\pi x - \frac{x^2}{2}\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left\{\left[\pi \times 2\pi - \frac{2\pi^2}{2}\right] - [0]\right\}$$

$$= \frac{1}{2\pi}\left[2\pi^2 - \frac{4\pi^2}{2}\right]$$

$$= \frac{1}{2\pi}\left[2\pi^2 - 2\pi^2\right]$$

$$= 0$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos(nx)\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi}\left[\frac{\pi - x}{2}\right]\cos(nx)\,dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (\pi - x)\cos(nx)\,dx$$

$$= \frac{1}{2\pi}\left[(\pi x)\left(\frac{\sin nx}{n}\right) - (-1)\left(-\frac{\cos nx}{n^2}\right)\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[\frac{(-1)\cancel{\cos nx}\cos nx}{n^2}\right]_0^{2\pi}$$

$$= \frac{-1}{2\pi n^2}\left[\cos(nx)\right]_0^{2\pi}$$

$$= \frac{-1}{2\pi n^2}\left[\cos 2n\pi - \cos 0\right]$$

$$= \frac{-1}{2\pi n^2}\left[1-1\right]$$

$$= 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} F(x)\sin nx \, dx$$

$$= \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi x}{2}\right)\sin nx \, dx$$

$$= \frac{1}{2\pi}\left[(\pi x)\left(-\frac{\cos nx}{n}\right) - (-1)\left(-\frac{\sin nx}{n^2}\right)\right]_0^{2\pi}$$

$$= \frac{-1}{2\pi n}\left[(\pi - x)\cos nx\right]_0^{2\pi}$$

$$= \frac{-1}{2\pi n}\left[(\pi - 2\pi)\cos 2n\pi - (\pi - 0)\cos 0\right]$$

$$= \frac{-1}{2\pi n}\left[-\pi \times 1 - \pi \times 1\right]$$

$$= \frac{+1}{2\pi n}(2\pi)$$

$$= \frac{1}{n}$$

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos nx + b_n \sin nx\right)$$

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$$\left[\frac{\pi - x}{2}\right] = \sum_{n=1}^{\infty} \frac{1}{n} \sin nx$$

$$= \frac{\sin x}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \frac{\sin 4x}{4} + \frac{\sin 5x}{5}$$

$$\frac{\sin 6x}{6} + \frac{\sin 7x}{7} \cdots$$

put $x = \pi/2$

$$\left[\frac{\pi - \pi/2}{2}\right] = \sin\pi/2 + \sin 3\pi/2 + \sin 5\pi/2 + \sin 7\pi/2$$

$$\frac{\pi}{4} = 1 - 1/3 + 1/5 - 1/7 + \cdots$$

Q) $f(x) = \left[\frac{\pi - x}{2}\right]^2$ interval $[0, 2\pi]$

P.T

① $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + 1/5^2$

② $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$

③ $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$

④ $\frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots$

$\rightarrow$ $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$

$$= \frac{1}{\pi} \int_0^{2\pi} \left[\frac{\pi^2 - 2\pi x + x^2}{4}\right] dx$$

$$= \frac{1}{4\pi} \left[\pi^2 x - \frac{2\pi x^2}{2} + \frac{x^3}{3}\right]_0^{2\pi}$$

$$= \frac{1}{4\pi} \left[\pi^2 \times 2\pi - \pi(2\pi)^2 + \frac{(2\pi)^3}{3}\right]$$

$$= \frac{1}{4\pi} \left[2\pi^3 - \pi \times 4\pi^2 + \frac{8\pi^3}{3}\right]$$

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$$= \frac{1}{4\pi}\left[-\frac{2\pi^3}{1}+\frac{8\pi^3}{3}\right]$$

$$= \frac{1}{4\pi}\left[\frac{-6\pi^3+8\pi^3}{3}\right]$$

$$= \frac{1}{4\pi}\left[\frac{2\pi^3}{3}\right]$$

$$= \frac{\pi^2}{6}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx \, dx$$

$$= \frac{1}{4\pi}\int_0^{2\pi} (\pi^2 - 2\pi x + x^2)\cos nx \, dx$$

$$= \frac{1}{4\pi}\left[\cancel{(\pi^2-2\pi x+x^2)\left(\frac{\sin nx}{n}\right)} - (-2\pi+2x)\left(\frac{-\cos nx}{n^2}\right) + \cancel{2\left(\frac{-\sin nx}{n^3}\right)}\right]_0^{2\pi}$$

$$= \frac{1}{4\pi}\left[(-2\pi+2x)\frac{\cos nx}{n^2}\right]_0^{2\pi}$$

$$= \frac{1}{4\pi n^2}\left[(-2\pi+2x)\cos nx\right]_0^{2\pi}$$

$$= \frac{1}{4\pi n^2}\left[(-2\pi+2(2\pi))\cos 2n\pi - (-2\pi)\cos 0\right]$$

$$= \frac{1}{4\pi n^2}\left[2\pi\times 1 + 2\pi\times 1\right]$$

$$= \frac{1}{4\pi n^2}\times 4\pi$$

$$= 1/n^2$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx \, dx$$

$$= \frac{1}{4\pi}\int_0^{2\pi} (\pi^2 - 2\pi x + x^2) \sin nx \, dx$$

$$= \frac{1}{4\pi}\left[(\pi^2 - 2\pi x + x^2)\left(\frac{-\cos nx}{n}\right) - (-2\pi + 2x)\left(\frac{-\sin nx}{n^2}\right) + 2\left(\frac{\cos nx}{n^3}\right)\right]_0^{2\pi}$$

$$= \frac{-1}{4\pi n}\left[(\pi^2 - 2\pi x + x^2) \cos nx \right]_0^{2\pi} + \frac{2}{4\pi n^3}\left[\cos nx\right]_0^{2\pi}$$

$$= \frac{-1}{4\pi n}\left[(\pi^2 - 2\pi \times 2\pi + 4\pi^2) \cos 2n\pi - (\pi^2)\cos 0 \right] + \frac{1}{2\pi n^3}\left[\cos 2n\pi - \cos 0\right]$$

$$= -\frac{1}{4\pi n}\left[\pi^2 \times 1 - \pi^2 \times 1 \right]$$

$$= \frac{-1}{4\pi n}[0]$$

$$= 0$$

(I) $$\left[\frac{\pi - x}{2}\right]^2 = \frac{\frac{\pi^2}{6}}{2} + \sum_{n=1}^{\infty} \frac{1}{n^2}\cos nx \qquad (x = 0)$$

$$\left[\frac{\pi - 0}{2}\right]^2 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$$

$$\frac{\pi^2}{4} - \frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{(2)^2} + \frac{1}{3^2}$$

(II) $$\left(\frac{\pi - x}{2}\right)^2 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{\cos nx}{n^2} \qquad (x = \pi)$$

$$\int (ax+b)^n = \frac{(ax+b)^{n+1}}{(n+1)a}$$

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$$\left[\frac{\pi-\pi}{2}\right]^2 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^2}$$

$$0 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

$$\frac{-\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

$$\frac{-\pi^2}{12} = \frac{-1}{(1)^2} + \frac{1}{(2)^2} + \left(\frac{-1}{(3)^2}\right) + \frac{1}{4^2} - \cdots$$

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2}$$

(III) $$\left[\frac{\pi^2}{6} + \frac{\pi^2}{12}\right] = \left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} \cdots\right) + \left(\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2}\right)$$

$$\frac{\pi^2}{4} = \frac{2}{1^2} + \frac{2}{(3)^2} + \frac{2}{5^2}$$

$$\frac{\pi^2}{4} = 2\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2}\right)$$

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2}$$

(IV) $$\int_0^{2\pi} [F(x)]^2\, dx = \pi\left[\frac{a_0^2}{2} + \sum_{n=1}^{\infty} (a^2n + bn^2)\right]$$

$$\int_0^{2\pi} \left(\frac{\pi-x}{2}\right)^4 dx = \pi\left[\frac{(\pi^2/6)^2}{2} + \sum_{n=1}^{\infty} \left(\frac{1}{n^2}\right)^2\right]$$

$$\frac{1}{16}\int_0^{2\pi} (\pi-x)^4\, dx = \pi\left[\frac{\pi^4/36}{2} + \sum_{n=1}^{\infty} \frac{1}{n^4}\right]$$

$$\frac{1}{16}\left[\frac{(\pi-x)^5}{5(-1)}\right]_0^{2\pi} = \pi\left[\frac{\pi^4}{72} + \sum_{n=1}^{\infty} \frac{1}{n^4}\right]$$

$$-\frac{1}{80}\left[(-\pi)^5 - (\pi)^5\right] = \pi\left[\frac{\pi^4}{72} + \sum_{n=1}^{\infty} \frac{1}{n^4}\right]$$

$$-\frac{1}{80}\left[-\pi^5-\pi^5\right]=\pi\left[\frac{\pi^4}{72}+\sum_{n=1}^{\infty}\frac{1}{n^4}\right]$$

$$\frac{1}{40}(\pi^5)=\pi\left[\frac{\pi^4}{72}+\sum_{n=1}^{\infty}\frac{1}{n^4}\right]$$

$$\frac{\pi^4}{40}=\frac{\pi^4}{72}+\sum_{n=1}^{\infty}\frac{1}{n^4}$$

$$\pi^4\left[\frac{1}{40}-\frac{1}{72}\right]=\sum_{n=1}^{\infty}\frac{1}{n^4}$$

$$\frac{\pi^4}{90}=\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}$$

Q) $f(x)=x^2$ $(-\pi,\pi)$ Find F.S also P.T

I) $\frac{\pi^2}{6}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}$

II) $\frac{\pi^2}{12}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}$

III) $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}$

→ If interval $[-,+]$ then check whether Fun is even or odd

Fun is even $b_n=0$

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}(f(x))\,dx$$

$$=\frac{2}{\pi}\int_0^{\pi}f(x)\,dx$$

$$=\frac{2}{\pi}\int_0^{\pi}x^2\,dx$$

$$=\frac{2}{\pi}\left[\frac{x^3}{3}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{\pi^3}{3}\right]$$

$$a_0 = \frac{2\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos nx \, dx$$

$$= \frac{2}{\pi}\left[\cancel{x^2\left(\frac{\sin nx}{n}\right)} - 2x\left(\frac{-\cos nx}{n^2}\right) + \cancel{2\left(\frac{-\sin nx}{n^3}\right)}\right]_0^{\pi}$$

$$= \cancel{2\pi}\ \frac{2}{\pi} \times \frac{2}{n^2}\left[x\cos nx\right]_0^{\pi}$$

$$= \frac{4}{\pi n^2}\left[\pi\cos n\pi - 0\right]$$

$$= \frac{4}{\pi n^2}\pi(-1)^n = \frac{4}{n^2}(-1)^n$$

$$= \frac{4(-1)^n}{n^2}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$\pi^2 - \frac{\pi^2}{3} = 4 \sum_{1=n}^{\infty} \frac{(-1)^n (-1)^n}{n^2}$$

$$\frac{3\pi^2 - \pi^2}{3} = 4 \sum_{n=1}^{\infty} \frac{(1)^n}{n^2}$$

$$\frac{2\pi^2}{12} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2}$$

$$\frac{\pi^2}{6} = \frac{1}{(1)^2} + \frac{1}{(2)^2} + \frac{1}{(3)^2} + \cdots \quad \text{(B)}$$

put $x = 0$

$$0 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n \cos 0}{n^2}$$

$$\frac{-\pi^2}{3} = 4 \sum_{n=1}^{\infty} \frac{(-1)^n 1}{n^2}$$

$$\frac{-\pi^2}{12} = \cancel{\frac{+1}{1^2}} - \frac{1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2}$$

$$\frac{\pi^2}{12} = \frac{1}{(1)^2} - \frac{1}{(2)^2} + \frac{1}{(3)^2} - \frac{1}{(4)^2} \text{ ——— (A)}$$

Adding A & B

$$\frac{\pi^2}{48} = \frac{2}{(1)^2} + \frac{2}{(3)^2} + \frac{2}{(5)^2}$$

$$= 2\left(\frac{1}{(1)^2} + \frac{1}{(3)^2} + \frac{1}{(5)^2}\right)$$

$$\frac{\pi^2}{8} = \frac{1}{(1)^2} + \frac{1}{(3)^2} + \frac{1}{(5)^2} \cdots$$

Q) $f(x) = 1 + \frac{2x}{\pi}$; $-\pi \leq x \leq 0$

$= 1 - \frac{2x}{\pi}$; $0 \leq x \leq \pi$

Find F.S

P.T. $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2}$

$f(-x) = 1 - \frac{2x}{\pi}$; $-\pi \le (-x) \le 0$

$= 1 + \frac{2x}{\pi}$; $0 \le (-x) \le \pi$

$f(-x) = 1 - \frac{2x}{\pi}$; $\pi \ge x \ge 0$

$= 1 + \frac{2x}{\pi}$; $0 \ge x \ge -\pi$

$f(-x) = 1 - \frac{2x}{\pi}$; $0 \le x \le \pi$

$= 1 + \frac{2x}{\pi}$; $-\pi \le x \le 0$

f(x) is even

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$= \frac{2}{\pi} \int_{0}^{\pi} f(x)\, dx$$

$$= \frac{2}{\pi} \int_{0}^{\pi} \left(1 - \frac{2x}{\pi}\right) dx$$

$$= \frac{2}{\pi} \left[x - \frac{\cancel{2}}{\pi} \frac{x^2}{\cancel{2}} \right]_0^{\pi}$$

$$= \frac{2}{\pi} [\pi - \pi]$$

$$= 0$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx$$

$$= \frac{2}{\pi} \int_{0}^{\pi} \left(1 - \frac{2x}{\pi}\right) \cos nx\, dx$$

$$= \frac{2}{\pi} \left[\left(1 - \frac{2x}{\pi}\right)\left(\frac{\sin nx}{n}\right) - \left(\frac{-2}{\pi}\right)\left(\frac{-\cos nx}{n^2}\right) \right]_0^{\pi}$$

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$$= \frac{2}{\pi}\left[\frac{(-2)}{\pi n^2}\cos nx\right]_0^{\pi}$$

$$= \frac{-4}{\pi^2 n^2}\left[\cos nx\right]_0^{\pi}$$

$$= \frac{-4}{\pi^2 n^2}\left[\cos n\pi - \cos 0\right]$$

$$= \frac{-4}{\pi^2 n^2}\left[(-1)^n - 1\right]$$

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$\left(1 - \frac{2x}{\pi}\right) = \frac{0}{2} + \sum_{n=1}^{\infty} \frac{(-4)}{n^2\pi^2}\left[(-1)^n - 1\right]\cos nx$$

$x = 0$

$$1 = \frac{-4}{\pi^2}\sum_{n=1}^{\infty}\left[\frac{(-1)^n - 1}{n^2}\right]\cos 0$$

$$\frac{-\pi^2}{4} = \sum_{n=1}^{\infty}\left[\frac{(-1)^n - 1}{n^2}\right]\times 1$$

$$\frac{-\pi^2}{4} = \frac{-2}{1^2} - \frac{2}{(3)^2} - \frac{2}{(5)^2}$$

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2}$$

Q> $F(x) = -\pi \qquad -\pi < x < 0$

$= x \qquad 0 < x < \pi$

F.S for given fun

$P.T = \frac{\pi^2}{8}$

$f(-x) = -\pi \qquad -\pi < -x < 0$

$= -x \qquad 0 < -x < \pi$

$= -\pi \qquad \pi > -x > 0$

$= -x \qquad 0 > -x > \pi$

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$$= -\pi \qquad 0 < x < \pi$$
$$\quad -x \qquad -\pi < x < 0$$

Fun is NENO

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} F(x)\, dx$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} (-\pi)\, dx + \int_{0}^{\pi} x\, dx \right]$$

$$= \frac{1}{\pi} \left[-\pi [x]_{-\pi}^{0} + \left[\frac{x^2}{2} \right]_0^{\pi} \right]$$

$$= \frac{1}{\pi} \left[-\pi [0 - (-\pi)] + \left(\frac{\pi^2}{2} - 0 \right) \right]$$

$$= \frac{1}{\pi} \left[(-\pi)\pi + \frac{\pi^2}{2} \right]$$

$$= \frac{1}{\pi} \left[-\pi^2 + \frac{\pi^2}{2} \right]$$

$$= \frac{1}{\pi} \left[\frac{-2\pi^2 + \pi^2}{2} \right]$$

$$= \frac{1}{\pi} \left[\frac{-\pi^2}{2} \right]$$

$$a_0 = -\pi/2$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} F(x) \cos nx\, dx$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} (-\pi) \cos nx\, dx + \int_{0}^{\pi} x (\cos nx)\, dx \right]$$

$$= \frac{1}{\pi} \left[(-\pi) \left(\frac{\sin nx}{n} \right)_{-\pi}^{0} + \left[x \left(\frac{\sin nx}{n} \right) - 1 \left(-\frac{\cos nx}{n^2} \right) \right]_0^{\pi} \right\}$$

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$$= \frac{1}{\pi}\left[\frac{\cos nx}{n^2}\right]_0^{\pi}$$

$$= \frac{1}{n^2\pi}\left[\cos n\pi - \cos 0\right]$$

$$a_n = \frac{1}{\pi n^2}\left[(-1)^n - 1\right]$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0}(-\pi)\sin nx\,dx + \int_0^{\pi} x\sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left[(-\pi)\left(\frac{-\cos nx}{n}\right)_{-\pi}^{0} + \left[x\left(\frac{-\cos nx}{n}\right) - 1\left(\frac{-\sin nx}{n^2}\right)\right]_0^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi}{n}(\cos nx)_{-\pi}^{0} - \frac{1}{n}\left[x\cos nx\right]_0^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi}{n}\left[\cos 0 - \cos(-n\pi)\right] - \frac{1}{n}\left[\pi\cos n\pi - 0\right]\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi}{n}\left[1 - \cos n\pi\right] - \frac{\pi}{n}\cos n\pi\right]$$

$$= \frac{1}{\cancel{\pi}} \times \frac{\cancel{\pi}}{n}\left[1 - (-1)^n - (-1)^n\right]$$

$$= \frac{1}{n}\left[1 - 2\times(-1)^n\right]$$

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)$$

$$= \frac{-\pi}{2}\times\frac{1}{2} + \sum_{n=1}^{\infty}\frac{1}{\pi n^2}\left[(-1)^n - 1\right]\cos nx + \frac{1}{n}\left[1 - 2\times(-1)^n\right]$$

$$\sin nx$$

$$= F(x) = \frac{1}{2}\left[\lim_{x\to c^-} f(x) + \lim_{x\to c^+} f(x)\right]$$

$$= \frac{1}{2}\left[\lim_{x\to 0}(-\pi) + \lim_{x\to 0} x\right]$$

$= -\pi/2$

$$-\pi/2 = \frac{-\pi}{4} + \sum_{n=1}^{\infty}\left[\frac{1}{\pi n^2}[(-1)^n - 1]\cos nx + \frac{1}{n}[1-2(-1)^n] \sin nx\right]$$

$$-\frac{\pi}{2} = \frac{-\pi}{4} + \sum_{n=1}^{\infty}\left[\frac{1}{\pi n^2}[(-1)^n - 1]\cos 0 + \frac{1}{n}[1-2(-1)^n]\sin nx\right]$$

$$-\frac{\pi}{2} + \frac{\pi}{4} = \frac{1}{\pi}\sum_{n=1}^{\infty}\left[\frac{(-1)^n - 1}{n^2}\right] \times 1$$

$$\frac{-2\pi + \pi}{4} = \frac{1}{\pi}\sum_{n=1}^{\infty}\left[\frac{(-1)^n - 1}{n^2}\right]$$

$$\frac{-\pi^2}{4} = -\frac{2}{1^2} - \frac{2}{3^2} - \frac{2}{5^2}$$

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2}$$

Q) $f(x) = \sqrt{1-\cos x}$, $[0, 2\pi]$

find F.S given fun

P.T $\frac{1}{2} = \sum_{n=1}^{\infty} \frac{1}{(4n^2-1)}$

→ $f(x) = \sqrt{1 - \cos x}$

$= \sqrt{2\sin^2 x/2}$

$= \sqrt{2}\ \sin x/2$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$

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$$= \frac{1}{\pi}\int_0^{2\pi} \sqrt{2}\,\sin x/2 \, dx$$

$$= \frac{\sqrt{2}}{\pi}\left[\frac{-\cos x/2}{1/2}\right]_0^{2\pi}$$

$$= \frac{2\sqrt{2}}{\cancel{2}\,\pi}\left[-\cos\pi + \cos 0\right]$$

$$= \frac{2\sqrt{2}}{\cancel{\sqrt{2}}\pi}\left[+1+1\right]$$

$$\cancel{= \frac{1}{\sqrt{2}\pi}} \cancel{[0]} [2]$$

$$= \frac{4\sqrt{2}}{\pi}$$

$$\cancel{= 0}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx \, dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} \sqrt{2}\sin x/2 \cos nx \, dx$$

$$= \frac{\sqrt{2}}{\pi}\int_0^{2\pi} \sin x/2 \cos nx \, dx$$

$$= \frac{\sqrt{2}}{2\pi}\int_0^{2\pi} 2\sin x/2 \cos nx \, dx$$

$$= \frac{\sqrt{2}}{2\pi}\int_0^{2\pi}\left[\sin\left(n+\tfrac{1}{2}\right)x - \sin\left(n-\tfrac{1}{2}\right)x\right] dx$$

$$= \frac{\sqrt{2}}{2\pi}\int_0^{2\pi}\left[\sin\left(\frac{2n+1}{2}\right)x - \sin\left(\frac{2n-1}{2}\right)x\right] dx$$

$$= \frac{\sqrt{2}}{2\pi}\left[\frac{-\cos\left(\frac{2n+1}{2}\right)x}{\left(\frac{2n+1}{2}\right)} + \frac{\cos\left(\frac{2n-1}{2}\right)x}{\left(\frac{2n-1}{2}\right)}\right]_0^{2\pi}$$

$$= \frac{\sqrt{2}}{2\pi}\,\frac{(-2)}{(2n+1)}\left[\cos\left(\frac{2n+1}{2}\right)x\right]_0^{2\pi}$$

$$+ \frac{\sqrt{2}}{2\pi}\,\frac{(2)}{(2n-1)}\left[\cos\left(\frac{2n-1}{2}\right)x\right]_0^{2\pi}$$

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$\cos(2n\pi + \pi) = \cos(\pi) = (-1)$
$\cos(2n\pi - \pi) = \cos\pi = (-1)$
$\sin(2n\pi + \pi) = \sin\pi =$
$\sin(2n\pi - \pi) = -\sin\pi = 0$

$= \frac{\sqrt{2}}{\pi(2n+1)} \left[\cos(2n+1)\pi - \cos 0\right] + \frac{\sqrt{2}}{\pi(2n-1)}$
$\left[\cos(2n-1)\pi - \cos 0\right]$

$= \frac{-\sqrt{2}}{\pi(2n+1)} [-1-1] + \frac{\sqrt{2}}{\pi(2n-1)} [-1-1]$

$= \frac{2\sqrt{2}}{\pi(2n+1)} - \frac{2\sqrt{2}}{\pi(2n-1)} = \frac{2\sqrt{2}}{\pi} \left[\frac{1}{(2n+1)} - \frac{1}{(2n-1)}\right]$

$= \frac{2\sqrt{2}}{\pi} \left[\frac{(2n-1)-(2n+1)}{(2n+1)(2n-1)}\right]$

$= \frac{2\sqrt{2}}{\pi} \left[\frac{2n-1-2n-1}{4n^2-1}\right]$

$a_n = \frac{-4\sqrt{2}}{\pi} \times \frac{1}{(4n^2-1)}$

$b_n = \frac{1}{\pi} \int_0^{2\pi} F(x) \sin nx \, dx$

$= \frac{1}{\pi} \int_0^{2\pi} \sqrt{2} \sin x/_2 \sin nx \, dx$

$= \frac{\sqrt{2}}{2\pi} \left[\frac{\cos\cancel{(n+1/2)}x}{\cancel{(n+1/2)}} \frac{(n-1/2)x}{(n-1/2)} - \frac{\cos(n+1/2)x}{(n+1/2)} \right]_0^{2\pi}$

$$= \frac{\sqrt{3}}{2\pi}[0]$$

$$= 0$$

$$F(x) = a_0/2 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

$$\sqrt{1-\cos x} = \frac{2\sqrt{2}}{\pi} \quad \frac{4\sqrt{2}}{2\pi} + \sum_{n=1}^{\infty}\left(\frac{-4\sqrt{2}}{\pi}\right) - \frac{1}{(4n^2-1)} \cos nx$$

$$= \frac{2\sqrt{2}}{\pi} - \frac{4\sqrt{2}}{\pi}\sum_{n=1}^{\infty}\frac{\cos nx}{4n^2-1}$$

put $x = 0$

$$0 = \frac{2\sqrt{2}}{\pi} - \frac{4\sqrt{2}}{\pi}\sum_{n=1}^{\infty}\frac{\cos 0}{4(0)-1}$$

$$\frac{-2\sqrt{2}}{\cancel{\pi}} = \frac{-4\sqrt{2}}{\cancel{\pi}}\sum_{n=1}^{\infty}\frac{1}{4n^2-1}$$

$$\frac{1}{2} = \sum_{n=1}^{\infty}\frac{1}{4n^2-1}$$

Q) $F(x) = |\cos x|$; $[-\pi, \pi]$

$$F(x) = -\cos x \qquad -\pi < x < 0$$
$$= \cos x \qquad 0 < x < \pi$$

$$F(-x) = -\cos(-x)$$
$$= -\cos x \qquad -\pi < x < 0$$

$$= \cos(-x)$$
$$= \cos x \qquad 0 < x < \pi$$

$= -\cos x \qquad 0 < x < \pi$

$= \cos x \qquad -\pi < x < 0$

Fun is odd

$a_n = 0$

$a_0 = 0$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx \, dx$$

$$= \frac{2}{\pi}\int_{0}^{\pi} f(x)\sin nx \, dx$$

$$= \frac{2}{\pi}\int_{0}^{\pi} \cos x \sin nx \, dx$$

$$= \frac{2}{2\pi}\int_{0}^{\pi} (2\sin nx \cos x)\, dx$$

$$= \frac{1}{\pi}\int_{0}^{\pi} \left[\sin(nx+x) + \sin(nx-x)\right] dx$$

$$= \frac{1}{\pi}\int_{0}^{\pi} \left[\sin(n+1)x + \sin(n-1)x\right] dx$$

$$= \frac{1}{\pi}\left[\frac{-\cos(n+1)x}{(n+1)} - \frac{\cos(n-1)x}{(n-1)}\right]_0^{\pi}$$

$$= \frac{-1}{\pi(n+1)}\left[\cos(n+1)x\right]_0^{\pi} - \frac{1}{\pi(n-1)}\left[\cos(n-1)x\right]_0^{\pi}$$

$$= \frac{-1}{\pi(n+1)}\left[\cos(n+1)\pi - \cos 0\right] - \frac{1}{\pi(n-1)}\left[\cos(n-1)\pi - \cos 0\right]$$

$$= \frac{-1}{\pi(n+1)}\left[(-1)^n(-1) - 1\right] - \frac{1}{\pi(n-1)}\left[(-1)^n(-1) - 1\right]$$

$$= \frac{1}{\pi(n+1)}\left[(-1)^n+1\right] + \frac{1}{\pi(n-1)}\left[(-1)^n(-1)-1\right]\Bigg]$$

$$= \frac{\left[(-1)^n+1\right]}{\pi}\left[\frac{1}{n+1} + \frac{1}{n-1}\right]$$

$$= \left[\frac{(-1)^n+1}{n}\right]\left[\frac{n-1+n+1}{n^2-1}\right]$$

$$= \frac{2n}{(n^2-1)}\left[\frac{(-1)^n+1}{\pi}\right] \qquad n \geq 2$$

$$b_1 = \frac{1}{\pi}\int_0^\pi 2\sin x\cos x$$

$$= \frac{1}{\pi}\int_0^\pi \sin 2x \, dx$$

$$= \frac{1}{\pi}\left[\frac{-\cos 2x}{2}\right]_0^\pi$$

$$= \frac{-1}{2\pi}\left[\cos 2x\right]_0^\pi$$

$$= \frac{-1}{2\pi}\left[1-1\right]$$

$$b_1 = 0$$

$$F(x) = \sum_{n=1}^\infty b_n \sin nx$$

$$= b_1 \sin x + \sum_{n=2}^\infty b_n \sin nx$$

$$= \sum_{n=2}^\infty \frac{2n}{(n^2-1)} \frac{\left[(-1)^n+1\right]}{\pi} \sin nx$$

Q) $F(x) = x\sin x \quad [-\pi, \pi]$

$\to$ $F(-x) = -x(\sin(-x))$

$= x\sin x$

$F(x)$ is even

$\boxed{b_n = 0}$

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\, dx$$

$$= \frac{2}{\pi}\int_{0}^{\pi} x\sin x\, dx$$

$$= \frac{2}{\pi}\left[x(-\cos x) - 1(-\sin x)\right]_0^{\pi}$$

$$= \frac{-2}{\pi}\left[x\cos x\right]_0^{\pi}$$

$$= \frac{-2}{\pi}\left[\pi\cos\pi - 0\right]$$

$$= \frac{-2}{\pi}\left[\pi(-1)\right]$$

$$a_0 = 2$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\cos nx$$

$$= \frac{2}{\pi}\int_{0}^{\pi} x\sin x\cos nx\, dx$$

$$= \frac{2}{2\pi}\int_{0}^{\pi} x(2\cos nx\sin x)\, dx$$

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$$= \frac{1}{\pi}\int_0^{\pi} x\left[\frac{\sin(n+1)x - \sin(n-1)x}{}\right] dx$$

$$= \frac{1}{\pi}\left[x\left[\frac{-\cos(n+1)x}{(n+1)} + \frac{\cos(n-1)x}{(n-1)}\right] - 1\right.$$

$$\left.\left[\frac{-\sin(n+1)x}{(n+1)^2} + \frac{\sin(n-1)x}{(n-1)^2}\right]\right]_0^{\pi}$$

$$= \frac{1}{\cancel{\pi}}\left[\cancel{\pi}\left[\frac{-\cos(n+1)\pi}{(n+1)} + \frac{\cos(n-1)\pi}{(n-1)}\right]\right]$$

$$\left[\frac{(-1)(-1)^n(-1)}{(n+1)} + \frac{(-1)(-1)^n}{(n-1)}\right]$$

$$= (-1)^n\left[\frac{1}{(n+1)} - \frac{1}{(n-1)}\right]$$

$$= (-1)^n\left[\frac{(n-1)-(n+1)}{(n-1)(n+1)}\right] = \frac{(-1)^n}{(n^2-1)}\left[n-1-n-1\right]$$

$$a_n = \frac{(-2)(-1)^n}{(n^2-1)} \qquad n \geq 2$$

$$a_1 = \frac{1}{\pi}\int_0^{\pi} x \sin 2x \, dx$$

$$= \frac{1}{\pi}\left[x\left(\frac{-\cos 2x}{2}\right) - 1\cancel{\left(\frac{-\sin 2x}{4}\right)}\right]_0^{\pi}$$

$$= \frac{-1}{2\pi}\left[x\cos 2x\right]_0^{\pi}$$

$$= \frac{-1}{2\pi}\left[\cancel{\pi - c}\ \pi\cos 2\pi - 0\right]$$

$$= \frac{-1}{2\cancel{\pi}}\left[\cancel{\pi}\right]$$

$$= -1/2$$

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$$= \frac{1}{\pi}\left[\pi\left[\frac{-\cos(n+1)\pi}{(n+1)} - \frac{\cos(n-1)\pi}{(n-1)}\right]\right]$$

$$= \frac{(-1)(-1)^n(-1)}{(n+1)} - \frac{(-1)^n(-1)}{(n-1)}$$

$$= \frac{(-1)^n}{n+1} - 1\frac{(-1)^n}{(n-1)}$$

$$= (-1)^n\left[\frac{1}{n+1} + \frac{1}{n-1}\right]$$

$$= (-1)^n\left[\frac{2n}{n^2-1}\right]$$

$$= \frac{(-1)^n(2n)}{(n^2-1)} \quad n \geq 2$$

$$b_1 = \frac{1}{\pi}\int_0^\pi x(2\sin x \cos x)$$

$$= \frac{1}{\pi}\int_0^\pi x(\sin 2x)$$

$$= \frac{1}{\pi}\left[x\left(\frac{-\cos 2x}{2}\right) - 1\left(\frac{-\sin 2x}{4}\right)\right]_0^\pi$$

$$= \frac{-1}{2\pi}\left[x\cos 2x\right]_0^\pi$$

$$= \frac{-1}{2\pi}\left[\pi\cos 2\pi - 0\right]$$

$$= \frac{-1}{2\pi}[\pi]$$

$$b_1 = -1/2$$

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$$= \frac{1}{\pi}\left[\pi\left[\frac{-\cos(n+1)\pi}{(n+1)} - \frac{\cos(n-1)\pi}{(n-1)}\right]\right]$$

$$= \frac{(-1)(-1)^n(-1)}{(n+1)} - \frac{(-1)^n(-1)}{(n-1)}$$

$$= \frac{(-1)^n}{n+1} + \frac{(-1)^n}{(n-1)}$$

$$= (-1)^n\left[\frac{1}{n+1} + \frac{1}{n-1}\right]$$

$$= (-1)^n\left[\frac{2n}{n^2-1}\right]$$

$$= \frac{(-1)^n(2n)}{(n^2-1)} \quad n \geq 2$$

$$b_1 = \frac{1}{\pi}\int_0^\pi x(2\sin x\cos x)$$

$$= \frac{1}{\pi}\int_0^\pi x(\sin 2x)$$

$$= \frac{1}{\pi}\left[x\left(\frac{-\cos 2x}{2}\right) - 1\left(\frac{-\sin 2x}{4}\right)\right]_0^\pi$$

$$= \frac{-1}{2\pi}\left[x\cos 2x\right]_0^\pi$$

$$= \frac{-1}{2\pi}\left[\pi\cos 2\pi - 0\right]$$

$$= \frac{-1}{2\pi}\left[\pi\right]$$

$$b_1 = -1/2$$

$$F(x) = b_1 \sin nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$= \frac{\cancel{\sin 2x} - \sin nx}{2} + \sum_{n=1}^{\infty} \frac{(2n)(-1)^n}{(n^2-1)}$$

$$= \frac{-\sin x}{2} + \sum_{n=1}^{\infty} \frac{(2n)(-1)^n \sin nx}{(n^2-1)}$$

(2) $f(x) = x - x^2 \quad (-\pi, \pi)$

$\rightarrow$ $f(-x) = -x - (-x)^2$

$= -x \cancel{+x} - x^2$

$f(x)$ IS NENO

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \cancel{a_n \sin} \; a_n \cos nx + b_n \sin nx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$= \cancel{\frac{1}{\pi} 2 \int_0^{\pi}} \; x - x^2 \, dx$$

$$= \cancel{\frac{1}{\pi}} \frac{1}{\pi} \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi} \left\{ \left[\frac{\pi^2}{2} - \frac{\pi^3}{3} \right] - \left[\frac{-\pi^2}{2} - \left(\frac{-\pi^3}{3} \right) \right] \right\}$$

$$= \frac{1}{\pi} \left\{ \left[\cancel{\frac{\pi^2}{2}} - \frac{\pi^3}{3} - \cancel{\frac{\pi^2}{2}} - \frac{\pi^3}{3} \right] \right\}$$

$$= \frac{1}{\pi} \left\{ \left[\frac{-2\pi^3}{3} \right] \right\}$$

$$a_0 = \frac{-2\pi^2}{3}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} (x-x^2)\cos nx \, dx$$

$$= \frac{1}{\pi}\left[(x-x^2)\left(\frac{\sin nx}{n}\right) - (1-2x)\left(\frac{-\cos nx}{n^2}\right) + 2\left(\frac{-\sin nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[(1-2x)\left(\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi n^2}\left[(1-2x)(\cos nx\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi n^2}\left[((1-2\pi)(\cos n\pi) - (1+2\pi)(\cos(-n\pi))\right]$$

$$= \frac{1}{\pi n^2}\left[\left[(1-2\pi)(-1)^n\right] - \left[(1+2\pi)(-1)^n\right]\right]$$

$$= \frac{(-1)^n}{\pi n^2}\left[\cancel{1}-2\pi-\cancel{1}-2\pi\right]$$

$$= \frac{(-1)^n}{\cancel{\pi} n^2}\left[-4\cancel{\pi}\right]$$

$$= \frac{(-4)(-1)^n}{n^2}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx \, dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} (x-x^2)\sin nx \, dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi}\left[(x-x^2)\left(-\frac{\cos nx}{n}\right) - (1-2x)\cancel{\left(\frac{\sin nx}{n^2}\right)} + (-2)\left(\frac{\cos nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

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$$= \frac{1}{n\pi}\left[(x-x^2)\cos nx\right]_{-\pi}^{\pi} - \frac{2}{\pi n^3}\left[\cos nx\right]_{-\pi}^{\pi}$$

$$= \frac{-1}{n\pi}\left[(\pi-\pi^2)\cos n\pi - (-\pi-(-\pi^2)\cos(-\pi n))\right]$$

$$- \frac{2}{\pi n^3}\left[\cos n\pi - \cos(-n\pi)\right]$$

$$= \frac{-1}{n\pi}\left[(\pi-\pi^2)(-1)^n + (\pi + \pi^2)\cos(n\pi)\right]$$

$$= \frac{(-1)^n}{n\pi}\left[\pi - \cancel{\pi^2} + \pi + \cancel{\pi^2}\right]$$

$$= \frac{(-1)^n(-2\cancel{\pi})}{n\cancel{\pi}}$$

$$b_n = \frac{(-1)^n(2)}{n}$$

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)$$

$$= \frac{-2\pi^2}{6} + \sum_{n=1}^{\infty}\left(\frac{(-4)(-1)^n\cos nx}{n^2} + \frac{(-1)^n(-2)\sin nx}{n}\right)$$

Q) $F(x) = 2x-1$; $[0, 3]$

Find f.S

$\rightarrow$ $(0, 3) \longrightarrow (0, 2l)$

$2l = 3$

$l = 3/2$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos\left(\frac{n\pi x}{l}\right) + b_n\sin\left(\frac{n\pi x}{l}\right)$$

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$$a_0 = \frac{1}{l}\int_0^3 f(x)\,dx$$

$$= \frac{1}{l}\int_0^3 (2x-1)\,dx$$

$$= \frac{2}{3}\int_0^3 (2x-1)\,dx$$

$$= \frac{2}{3}\left[\frac{2x^2}{2} - x\right]_0^3$$

$$= \frac{2}{3}\left[9-3\right]$$

$$= 4$$

$$a_n = \frac{1}{l}\int_0^3 f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{2}{3}\int_0^3 (2x-1)\cos\left(\frac{2n\pi x}{3}\right)dx$$

$$= \frac{2}{3}\left[(2x-1)\left[\frac{\sin\left(\frac{2n\pi x}{3}\right)}{\left(\frac{2n\pi}{3}\right)}\right] - 2\left[\frac{-\cos\left(\frac{2\pi n x}{3}\right)}{\left(\frac{2\pi n}{3}\right)}\right]\right]_0^3$$

$$= \frac{2}{3} \times \frac{2\times 9}{4n^2\pi^2}\left[\cos\left(\frac{2n\pi x}{3}\right)\right]_0^3$$

$$= \frac{3}{n^2\pi^2}\left[\cos(2n\pi) - \cos 0\right]$$

$$= \frac{3}{n^2\pi^2}\left[1-1\right]$$

$$= \frac{3}{n^2\pi^2}\left[0\right]$$

$$= 0$$

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$$b_n = \frac{1}{l}\int_0^3 F(x)\sin\left(\frac{n\pi x}{l}\right) dx$$

$$= \frac{2}{3}\int_0^3 (2x-1)\sin\left(\frac{2n\pi x}{3}\right) dx$$

$$= \frac{2}{3}\left[(2x-1)\left[\frac{-\cos\left(\frac{2n\pi x}{3}\right)}{\frac{2n\pi}{3}}\right] - 2\left[\frac{-\sin\left(\frac{2n\pi x}{3}\right)}{\left(\frac{2n\pi}{3}\right)}\right]\right]_0^3$$

$$= \frac{2}{3}\frac{(-3)}{2n\pi}\left[(2x-1)\cos\left(\frac{2n\pi x}{3}\right)\right]_0^3$$

$$= \frac{-1}{n\pi}\left[5\cos(2n\pi) - (-1)\cos 0\right]$$

$$= \frac{-1}{n\pi}\left[5\times 1 + 1\right]$$

$$= \frac{-6}{n\pi}$$

$$F(x) = \frac{4}{2} + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right) dx$$

$$x - x^2 = 2 + \sum_{n=1}^{\infty}\left[\frac{-6}{n\pi}\sin\left(\frac{2n\pi x}{3}\right)\right]$$

Q) $F(x) = 1 - x^2$; $[-2, 2]$

Find F.S

$\rightarrow$ $F(-x) = 1 - x^2$

$= 1 - (-x)^2$

$= 1 - x^2$

$F(x)$ is even

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$\boxed{b_n = 0}$

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx$$

$$= \frac{2}{l}\int_{0}^{l} f(x)\,dx$$

$$= \frac{2}{l}\int_{0}^{2} (1-x^2)\,dx$$

$$= \frac{\cancel{2}}{\cancel{2}}\left[x - \frac{x^3}{3}\right]_0^2$$

$$= \left[2 - \frac{8}{3}\right]$$

$$= \frac{6-8}{3}$$

$$a_0 = -2/3$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{2}{2}\int_{0}^{2} (1-x^2)\cos\left(\frac{n\pi x}{2}\right)dx$$

$$= \int_0^2 (1-x^2)\frac{\left(\sin\left(\frac{n\pi x}{2}\right)\right)}{\left(\frac{n\pi}{2}\right)} - (-2x)\left(\frac{-\cos\left(\frac{n\pi x}{2}\right)}{\left(\frac{n\pi}{2}\right)^2}\right)$$

$$-2\left(\frac{-\sin\left(\frac{n\pi x}{2}\right)}{\left(\frac{-n\pi^2}{4}\right)}\right)$$

$$\left(\frac{n\pi}{2}\right)^3$$

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$$= \frac{-8}{n^2\pi^2}\left[x\cos\left(\frac{n\pi x}{2}\right)\right]_0^2$$

$$= \frac{-8}{n^2\pi^2}\left[2\cos(n\pi)\right]$$

$$= \frac{-8\times 2(-1)^n}{n^2\pi^2}$$

$$= \frac{(-16)(-1)^n}{n^2\pi^2}$$

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$$

$$= -2/6 + \sum_{n=1}^{\infty} \frac{(-16)(-1)^n}{n^2\pi^2} \cos\left(\frac{n\pi x}{2}\right)$$

$$= -1/3 + \sum_{n=1}^{\infty} \frac{(-16)(-1)^n}{n^2\pi^2} \cos\left(\frac{n\pi x}{2}\right)$$

Q) $F(x) = x \quad 0 < x < \pi$

$= 2\pi - x \quad \pi < x < 2\pi$

→ $F(-x) = -x \quad 0 < -x < \pi$

$= 2\pi + x \quad \pi < -x < 2\pi$

$= -x \quad \pi < x < 2\pi$

$= 2\pi + x \quad 0 < x < \pi$

NENO

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$$= \frac{1}{\pi}\left[\frac{\cos nx}{n^2}\right]_0^{\pi} + \frac{1}{\pi}\left[\frac{\cos nx}{n^2}\right]_{\pi}^{2\pi}$$

$$= \frac{1}{\pi n^2}\left[(-1)^n - 1\right] - \frac{1}{\pi n^2}\left[1 - (-1)^n\right]$$

$$= \frac{1}{\pi n^2}\left[(-1)^n - \cancel{1} - \cancel{1} + (-1)^n\right]$$

$$= \frac{1}{\pi n^2}\left[(-2) + (-2)^n\right]$$

$$a_n = \frac{1}{\pi n^2}(-2)+(2)(-1)^n$$

$$= \frac{2\left[(-1)^n \cancel{(2)} - 1\right]}{\pi n^2}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\, dx$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} x\sin nx\, dx + \int_{\pi}^{2\pi} (2\pi - x)\sin nx\, dx\right]$$

$$= \frac{1}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - 1\left(\cancel{\frac{-\sin nx}{n^2}}\right)\right]_0^{\pi} +$$

$$\left[(2\pi - x)\left(-\frac{\cos nx}{n}\right) - 1\left(\cancel{\frac{-\sin nx}{n^2}}\right)\right]_{\pi}^{2\pi}$$

$$= \frac{1}{\pi}\left[\left[-x\left(\frac{\cos nx}{n}\right)\right]_0^{\pi} - \left[\frac{\cos nx}{n}(2\pi - x)\right]_{\pi}^{2\pi}\right]$$

$$= \frac{1}{n\pi}\left[\left[-\pi(-1)^n - 0\right] - \left[0 - (-1)^n(\pi)\right]\right]$$

$$= \frac{1}{\pi n}\left[(-1)^n(-\pi) + (-1)^n(\pi) \right]$$

$$= \frac{1}{\pi n} \cancel{(-1)^n \pi(-1)} [0]$$

$$= 0$$

Forier half range series:

I) No need to find even & odd fun

II) Title change formula change

III) In the forier half range series interval $(0, l)$ is half range limit, then actual limit is $(-l, l)$ while calculating value of l it must be compare with $(-l, l)$

Forier half range cosine series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{2}{\pi} \int f(x)\, dx$$

$$a_n = \frac{2}{\pi} \int f(x) \cos nx$$

Forier half range sin series:

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{2}{\pi} \int f(x) \sin nx\, dx$$

Q) Find F.h.r. cosine series for

$$f(x) = (\pi x - x^2) \qquad [0, \pi]$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1} a_n \cos nx \, dx$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\, dx$$

$$= \frac{2}{\pi}\int_0^{\pi} (\pi x - x^2)\, dx$$

$$= \frac{2}{\pi}\left[\frac{\pi x^2}{2} - \frac{x^3}{3}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{\pi^3}{2} - \frac{\pi^3}{3}\right]$$

$$= \frac{2}{\pi}\left[\frac{\pi^3}{6}\right]$$

$$= \frac{2\pi^2}{6}$$

$$= \frac{\pi^2}{3}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} (\pi x - x^2) \cos nx \, dx$$

$$= \frac{2}{\pi}\left[\cancel{(\pi x - x^2)\left(\frac{\sin nx}{n}\right)} - (\pi - 2x)\left(\frac{-\cos nx}{n^2}\right) + \cancel{(-2)\left(\frac{-\sin nx}{n^3}\right)}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[(\pi - 2x)\left(\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2}{n^2\pi}\left[(-\pi)(-1)^n - (\pi)(1)\right]$$

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$= \frac{-2}{\pi n^2}\left[\pi(-1)^n + 1\right]$

$= \cancel{\frac{(-2)(-1)^n - (2)}{n^2}} \quad \frac{(-2)\left[(-1)^n + 1\right]}{n^2}$

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \, dx$

$= \frac{\pi^2}{6} + \sum_{n=1}^{\infty} \cancel{\frac{(-2)(-1)^n - 2}{n^2}} \frac{(-2)\left[(-1)^n + 1\right]}{n^2} \cos nx$

Q) $f(x) = 1 - x^2$; $[0,1]$ find H.R.S series

$\rightarrow$ $f(x) = \sum_{l=n}^{\infty} b_n \sin \cancel{nx\, dx} \left(\frac{n\pi x}{l}\right) dx$

$[0,1]$ is half range limit

~~bn~~ $\therefore$ Full range limit is $[-1,1]$

compare with $[-l, l]$

i.e $l = 1$

$b_n = \frac{2}{l}\int_0^l f(x) \sin\left(\frac{n\pi x}{l}\right) dx$

$= \frac{2}{l}\int_0^1 (1-x^2) \sin\left(\frac{n\pi x}{l}\right) dx$

$= \frac{2}{l}\left[(1-x^2)\left(\frac{-\cos(n\pi x)}{n\pi}\right) - (-2x)\left(\frac{-\sin(n\pi x)}{(n\pi)^2}\right)\right.$

$\left. + (-2)\left(\frac{+\cos(n\pi x)}{(n\pi)^3}\right)\right]_0^1$

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$$= \frac{-2}{n\pi}\left((1-x^2)(\cos n\pi x)\right)_0^1 - \frac{2}{(n\pi)^3}\left[\cos n\pi x\right]_0^1$$

$$= \frac{-2}{n\pi}\left[0-(1)\right] - \frac{2}{(n\pi)^3}\left[(-1)^n - 1\right]$$

$$= \frac{2}{n\pi} - \frac{2}{(n\pi^3)}\left[(-1)^n - 1\right]$$

$$= \frac{2}{n\pi} - \frac{2}{(n\pi)^3}\left[(-1)^n - 1\right]$$

$$= \frac{2}{n\pi}\left[1 - \frac{1}{(n\pi)^2}\left[(-1)^n - 1\right]\right]$$

$F(x) = (-2x - x^2)$ $[0, 1]$

Find forier half range sine series.

Also P.T

$$\frac{\pi^3}{32} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}$$

$$= \frac{2}{l}\left[(lx - x^2)\left[\frac{-\cos\left(\frac{n\pi x}{l}\right)}{\frac{n\pi}{l}}\right] - \cancel{(l-2x)\left[\frac{-\sin\left(\frac{n\pi x}{l}\right)}{\frac{n^2\pi^2}{l^2}}\right]}\right.$$

$$\left. + (-2)\left[\frac{\cos\left(\frac{n\pi x}{l}\right)}{\frac{n^3\pi^3}{l^3}}\right]\right]_0^l$$

$$= \frac{2}{l} \times \frac{(-2)\, l^3}{n^3\pi^3}\left[\cos\left(\frac{n\pi x}{l}\right)\right]_0^l$$

$$= \frac{-4l^2}{n^3\pi^3}\left[\cos n\pi - \cos 0\right]$$

$\sin\left(\frac{\pi x}{l}\right) = 1$

$\frac{\pi x}{l} = \sin^{-1}(1) = \frac{\pi}{2}$

$x = l/2$

$$= \frac{-4l^2}{n^3\pi^3}\left[(-1)^n - 1\right]$$

$$F(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$lx - x^2 = \sum_{n=1}^{\infty} \frac{-4l^2}{n^3\pi^3}\left[(-1)^n - 1\right] \sin\left(\frac{n\pi x}{l}\right)$$

$$lx - x^2 = \frac{(-4l^2)}{\pi^3} \sum_{n=1}^{\infty} \frac{[(-1)^n - 1]}{n^3} \sin\left(\frac{n\pi x}{l}\right)$$

put $x = l/2$

$$l \times \frac{l}{2} - \frac{l^2}{4} = \frac{-4l^2}{\pi^3}\sum_{n=1}^{\infty} \frac{[(-1)^n - 1]}{n^3} \sin\left(\frac{n\pi}{l} \times \frac{l}{2}\right)$$

$$\frac{l^2}{4} = \frac{-4l^2}{\pi^3} \sum_{n=1}^{\infty} \frac{[(-1)^n - 1]}{n^3} \sin n\pi/2$$

$$\frac{-\pi^3}{16} = \frac{-2\sin\pi/2}{1^3} - \frac{2\sin 3\pi/2}{3^3} - \frac{2\sin 5\pi/2}{5^3}$$

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(2) $F(x) = \frac{2k}{l} \times x \quad ; \quad 0 < x < l/2$

$= \frac{2k}{l}(l-x) \quad ; \quad l/2 < x < l$

$\rightarrow \quad f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$

$b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx$

$= \frac{2}{l}\left[\int_0^{l/2} \frac{2kx}{l}\sin\left(\frac{n\pi x}{l}\right)dx + \int_{l/2}^{l} \frac{2k(l-x)}{l}\right.$

$\left.\sin\left(\frac{n\pi x}{l}\right)dx\right]$

$= \frac{2}{l}\frac{2k}{l}\int_0^{l/2} x\sin\left(\frac{n\pi x}{l}\right)dx + \frac{2}{l}\times\frac{2k}{l}\int_{l/2}^{l}$

$(l-x)\sin\left(\frac{n\pi x}{l}\right)dx$

$= \frac{4k}{l^2}\left[x\left(\frac{-\cos\left(\frac{n\pi x}{l}\right)}{\frac{n\pi}{l}}\right) - 1\left(\frac{-\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2}\right)\right]_0^{l/2} + \frac{4k}{l^2}$

$\left[(l-x)\left(\frac{-\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)}\right) - (-1)\left(\frac{-\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2}\right)\right]_{l/2}^{l}$

$= \frac{4k}{l^2}\left[\frac{-xl}{n\pi}\cos\left(\frac{n\pi x}{l}\right) + \frac{l^2}{n^2\pi^2}\sin\left(\frac{n\pi x}{l}\right)\right]_0^{l/2} + \frac{4k}{l^2}$

$\left[\frac{(-l)(l-x)}{n\pi}\cos\left(\frac{n\pi x}{l}\right) - \frac{l^2}{n^2\pi^2}\sin\left(\frac{n\pi x}{l}\right)\right]_0^{l}$

$= \frac{4k}{l^2}\left[\frac{-l^2/2}{n\pi}\cos\left(n\pi/2\right) + \frac{l^2}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)\right] - \frac{4k^2}{l^2}$

$\left\{[0] - \left[\frac{(-l)l/2}{n\pi}\cos\left(\frac{n\pi}{2}\right) - \frac{l^2}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)\right]\right\}$

$$+\frac{\ell^2}{2n\pi}\cos\left(\frac{n\pi}{2}\right)+\frac{\ell^2}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$$

$$=\frac{4K}{\ell^2}\frac{+2\ell^2}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$$

$$=\frac{8K}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$$

Complex form of forier series:

π | ℓ

$$f(x)=\sum_{n=-\infty}^{\infty} c_n e^{inx}\,dx \qquad f(x)=\sum_{n=-\infty}^{\infty} c_n e^{i\left(\frac{n\pi x}{\ell}\right)}$$

$$c_n=\frac{1}{2\pi}\int f(x)\,e^{-inx}\,dx \qquad c_n=\frac{1}{2\ell}\int f(x)\,e^{-i\left(\frac{n\pi x}{\ell}\right)}\,dx$$

$$\sinh ax=\frac{e^{ax}-e^{-ax}}{2} \qquad e^{i\theta}=\cos\theta+i\sin\theta$$

$$e^{-i\theta}=\cos\theta-i\sin\theta$$

$$\cosh ax=\frac{e^{ax}+e^{-ax}}{2}$$

Note : no need of

$\int e^{ax} = \frac{e^{ax}}{a} + c$

(a) $F(x) = e^{ax}$ ~~For~~ ; $[0, 2\pi]$

$\longrightarrow F(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$

$C_n = \frac{1}{2\pi} \int_0^{2\pi} f(x)\, e^{-inx}\, dx$

$= \frac{1}{2\pi} \int_0^{2\pi} e^{ax} \times e^{-inx}\, dx$

$= \frac{1}{2\pi} \int_0^{2\pi} e^{(a-in)x}\, dx$

$= \frac{1}{2\pi} \left[\frac{e^{(a-in)x}}{(a-in)} \right]_0^{2\pi}$

$= \frac{1}{2\pi(a-in)} \left[e^{(a-in)x} \right]_0^{2\pi}$

$= \frac{1}{2\pi(a-n)} \left[e^{(a-in)2\pi} - e^0 \right]$

$= \frac{1}{2\pi(a-n)} \left[e^{2a\pi}\, e^{-2in\pi} - 1 \right]$

$= \frac{1}{2\pi(a-in)} \left[e^{2a\pi} \times 1 - 1 \right]$

$= \frac{[e^{2a\pi} - 1]}{2\pi} \frac{(a+in)}{(a-in)(a+in)}$

$= \left[\frac{e^{2a\pi} - 1}{2\pi} \right] \frac{(a+in)}{(a^2+n^2)}$

$f(x) = \sum_{n=-\infty}^{\infty} \frac{[e^{2a\pi} - 1]}{2\pi} \frac{(a+in)\, e^{inx}}{(a^2+n^2)}$

(c) $f(x) = e^{-ax}$ $[-\pi, \pi]$

→ $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx} \, dx$

$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) \, e^{-inx} \, dx$

$= \frac{1}{2\pi} 2 \int_{0}^{\pi} e^{-ax} e^{-inx} \, dx$

$= \frac{1}{\pi} \int_{0}^{\pi} e^{-(a+in)x} \, dx$

$= \frac{1}{\pi} \int_{0}^{\pi} e^{-(a+in)x} \, dx$

$= \frac{1}{\pi} \left[\frac{e^{-(a+in)x}}{-(a+in)} \right.$

$= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-ax} e^{-inx} \, dx$

$= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-(a+in)x} \, dx$

$= \frac{1}{2\pi} \left[\frac{e^{-(a+in)x}}{-(a+in)} \right]_{-\pi}^{\pi}$

$= \frac{1}{2\pi(-a+in)} \left[e^{-(a+in)x} \right]_{-\pi}^{\pi}$

$= \frac{1}{2\pi(-a-in)} \left[e^{-(a+in)\pi} - e^{+(a+in)\pi} \right]$

$= \frac{1}{2\pi(-a+in)} \left[e^{-a\pi} e^{-in\pi} - e^{+a\pi} e^{in\pi} \right]$

$$= \frac{1}{-2\pi(a+in)}\left[e^{-a\pi}(-1)^n - e^{a\pi}(-1)^n\right]$$

$$= \frac{1\ (-1)^n}{-2\pi(a+in)}\left[e^{-a\pi} - e^{a\pi}\right]$$

$$= \frac{(-1)^n}{\pi(a+in)}\left[\frac{e^{a\pi} - e^{-a\pi}}{2}\right]$$

$$= \frac{(-1)^n}{\pi(a+in)} \sinh a\pi$$

$$= \frac{(-1)^n}{\pi} \sinh a\pi \ \frac{(a-in)}{(a+in)(a-in)}$$

$$C_n = \frac{(-1)^n}{\pi} \sinh a\pi \ \frac{(a-in)}{a^2+n^2}$$

$$F(x) = \sum_{n=-\infty}^{\infty} \frac{(-1)^n \sinh(a\pi)}{\pi} \frac{(a+in)}{a^2+n^2} e^{inx} \, dx$$

Q) $F(x) = \cosh 3x + \sinh 3x \qquad [0,3]$

$$[0,2l] \longrightarrow [0,3]$$

$$2l = 3$$

$$l = 3/2$$

$$= \left(\frac{e^{3x} + \cancel{e^{-3x}}}{2}\right) + \left(\frac{e^{3x} - \cancel{e^{-3x}}}{2}\right)$$

$$= \frac{2e^{3x}}{2}$$

$$f(x) = e^{3x} \qquad [0,3]$$

$$l = 3/2$$

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{i\left(\frac{n\pi x}{l}\right)}$$

$$c_n = \frac{1}{3}\int_0^3 e^{3x} e^{-i\left(\frac{n\pi x}{l}\right)} dx$$

$$= \frac{1}{3}\int_0^3 e^{3x} e^{-i\left(\frac{2n\pi x}{3}\right)} dx$$

$$= \frac{1}{3}\int_0^3 e^{\left(3-\frac{i2n\pi}{3}\right)x} dx$$

$$= \frac{1}{3}\left[\frac{e^{\left(3-\frac{2ni\pi}{3}\right)x}}{\left(3-\frac{2ni\pi}{3}\right)}\right]_0^3$$

$$= \frac{1}{\cancel{3}} \times \frac{\cancel{3}}{(9-2ni\pi)}\left[e^{\left[\frac{9-2ni\pi}{3}\right]x}\right]_0^3$$

$$= \frac{1}{(9-2ni\pi)}\left[e^{[9-2ni\pi]} - e^0\right]$$

$$= \frac{1}{(9-2ni\pi)}\left[e^9 e^{-2ni\pi} - e^0\right]$$

$$= \frac{1}{(9-2ni\pi)}\left[\cancel{e^9 - 1}\right]\left[e^9 - 1\right]$$

$$= \frac{[e^9-1]\,[9+2ni\pi]}{(9-2ni\pi)(9+2ni\pi)}$$

$$= \frac{[e^9-1]\,[9+2ni\pi]}{(81+4n^2\pi^2)}$$

$$f(x) = \sum_{n=-\infty}^{\infty} \frac{[e^9-1]\,[9+2ni\pi]}{(81+4n^2\pi^2)} e^{i\left(\frac{n\pi x}{l}\right)}$$

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Q) $F(x) = 2x \quad [0, 2\pi]$

$\rightarrow F(x) = \sum_{n=-\infty}^{\infty} C_n e^{inx} \, dx$

$C_n = \frac{1}{2\pi} \int_0^{2\pi} F(x) e^{-inx} \, dx$

$= \frac{1}{2\pi} \int_0^{2\pi} (2x) e^{-inx} \, dx$

$= \frac{1}{2\pi} \left[(2x) \frac{e^{-(inx)}}{-in} - 2 \frac{e^{-(inx)}}{(-in)^2} \right]_0^{2\pi}$

$= \frac{2}{(-in)2\pi} \left[(2\pi) e^{-(2ni\pi)} - 0 \right] - \frac{1}{\pi(-in)^2}$

$\left[e^{-(in2\pi)} - e^0 \right]$

$= \frac{2}{(-in)\cancel{\pi}} \left[e^{-(2ni\pi)} \right] - \frac{1}{\pi(-in)^2} \left[1 - 1 \right]$

$= \frac{2}{(-in)} (1)$

$= \frac{2(in)}{-n^2}$

$= \frac{2i}{n}$

Q) $F(x) = (x \cdot x^2) \cdot \quad [-\pi, \pi]$

$\rightarrow$ ~~F(x)~~ $C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} F(x) e^{-inx} \, dx$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} (x - x^2)\, e^{-inx}\, dx$$

$$= \frac{1}{2\pi} \left[(x-x^2) \frac{e^{(-inx)}}{(-in)} - (1-2x) \frac{e^{(-inx)}}{(-in)^2} + (-2) \frac{e^{(-inx)}}{(-in)^3} \right]_{-\pi}^{\pi}$$

$$= \frac{-1}{2\pi i n} \left[(x-x^2)\, e^{-inx} \right]_{-\pi}^{\pi} + \frac{1}{2\pi n^2} \left[(1-2x)\, e^{-inx} \right]_{-\pi}^{\pi} - \frac{2}{2\pi i n^3} \left[e^{-inx} \right]_{-\pi}^{\pi}$$

$$= \frac{-1}{2\pi i n} \left[(\pi-\pi^2)\, e^{-in\pi} - (-\pi-\pi^2)\, e^{in\pi} \right] + \frac{1}{2\pi n^2} \left[(1-2\pi)\, e^{-in\pi} - (1+2\pi)\, e^{in\pi} \right] - 0$$

$$= \frac{-1}{2\pi i n} (-1)^n \left[\pi - \cancel{\pi^2} + \pi + \cancel{\pi^2} \right] + \frac{1}{2\pi n^2} (-1)^n \left[(\cancel{1} - 2\pi - \cancel{1} - 2\pi) \right]$$

$$= \frac{-1 (-1)^n (\cancel{2\pi})}{\cancel{2\pi} i n} + \frac{1}{\cancel{2\pi} n^2} (-1)^n (-\cancel{4\pi})$$

$$= \frac{-1(-1)^n}{in} + \frac{1}{n^2} (-1)^n (-2)$$

$$= \frac{-1(-i)(-1)^n}{n} + \frac{1}{n^2} (-1)^n (-2)$$

$$= \frac{(i)(-1)^n}{n} + \frac{1}{n^2} (-1)^n (-2)$$

$$= (-1)^n \left[\frac{i}{n} + \frac{(-2)}{n^2} \right]$$

$= (-1)^n \left[\frac{ni - 2}{n^2} \right]$

$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$

$= \sum_{n=-\infty}^{\infty} (-1)^n \left[\frac{ni-2}{n^2} \right] e^{inx}$

Orthogonal ortho-normal ;

~~If $f(mx)$, $f(nx)$~~

(I) If $f_m(x)$, $f_n(x)$ are orthogonal over the range (a, b) then

$\int_a^b f_m(x) f_n(x) dx = 0$ (different fun)

(II) If $f_m(x)$, $f_n(x)$ are orthonormal over the rang $[a, b]$ then

$\int_a^b f_m(x) f_m(x) dx = 1$

$\int_a^b f_n(x) f_n(x) dx = 1$

(same fun)

S.T the set of fun

$f_1(x) = 1$

$f_2(x) = x$

$f_3(x) = \frac{(3x^2 - 1)}{2}$ over rang $[-1, 1]$

→ (I) $= \int_{-1}^{1} f_1(x) f_2(x) dx$

$\int_{-1}^{1} 1 \cdot x \, dx$

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$$\left[\frac{x^2}{2}\right]_{-1}^{1}$$

$$\frac{1}{2}[1-1]$$

$$= 0 \quad \text{——— (I)}$$

$$\text{(II)} = \int_{-1}^{1} F_2(x)\, F_3(x)$$

$$= \int_{-1}^{1} x\left(\frac{3x^2-1}{2}\right) dx$$

$$= \frac{1}{2}\int_{-1}^{1} 3x^3 - x \; dx$$

$$= \frac{1}{2}\left[\frac{3x^4}{4} - \frac{x^2}{2}\right]_{-1}^{1}$$

$$= \frac{3}{2}\times\frac{1}{4}[1-1] - \frac{1}{4}[1-1]$$

$$= 0 - 0$$

$$= 0 \quad \text{——— (II)}$$

$$\text{(III)} = \int_{-1}^{1} F_1(x)\, F_3(x)\, dx$$

$$= \int_{-1}^{1} 1\left(\frac{3x^2-1}{2}\right) dx$$

$$= \frac{1}{2}\left[\frac{\cancel{3}x^3}{\cancel{3}} - x\right]$$

$$= \frac{1}{2}\left[(1^3-1) - ((-1)^3 - (-1))\right]$$

$$= \frac{1}{2}\left[0 - (-1)^3 + 1\right]$$

$$= 1/2\left[0 - (1 - \cancel{\theta}1)\right]$$

From (I) (II) & (III)

set of fun.

S.T $\phi_1(x) = 1$ are orthogonal over range
$\phi_2(x) = x$ $[-1, 1]$
also find out value of a, & b if
$\phi_1(x)$ & $\phi_2(x)$ is orthogonal $\phi_3(x)$
$\phi_3(x) = 1 + ax + bx^2$

$$= \int_{-1}^{1} \phi_1(x)\, \phi_2(x)\, dx$$

$$= \int_{-1}^{1} (1 \cdot x)\, dx$$

$$= \left[\frac{x^2}{2} \right]_{-1}^{1}$$

$$= \frac{1}{2}[1-1]$$

$$= 0$$

since
ϕ_1 & ϕ_3 are orthogonal

$$\int_{-1}^{1} \phi_1(x)\, \phi_3(x)\, dx = 0$$

$$\int_{-1}^{1} 1 \times (1 + ax + bx^2)\, dx = 0$$

$$\left[x + \frac{ax^2}{2} + \frac{bx^3}{3} \right]_{-1}^{1} = 1$$

$$\left[1 + \frac{a}{2} + \frac{b}{3}\right] - \left[-1 + a \times \frac{1}{2} - b/3\right] = 0$$

$$2 + \frac{2b}{3} = 0$$

$$b = -3$$

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Since $\phi_2(x)$ & $\phi_3(x)$ are orthogonal

$$\int_{-1}^{1} \phi_2(x)\,\phi_3(x)\,dx = 0$$

$$\int_{-1}^{1} x(1+ax+bx^2)\,dx = 0$$

$$\int_{-1}^{1} (x+ax^2+bx^3)\,dx = 0$$

$$\left[\frac{x^2}{2}+\frac{ax^3}{3}+\frac{bx^4}{4}\right]_{-1}^{1} = 0$$

$$\left[\frac{1}{2}+\frac{a}{3}+\frac{b}{4}\right]-\left[\frac{1}{2}-\frac{a}{3}+\frac{b}{4}\right] = 0$$

$$\frac{1}{2}+\frac{a}{3}+\frac{b}{4}+\frac{a}{3}-\frac{b}{4} = 0$$

$$\frac{2a}{3} = 0$$

$$a = 0$$

Q) If $\phi_1(x), \phi_2(x)\ \phi_3(x)$ are orthogonal as well as orthonormal over $[a,b]$

$$\int_a^b [F(x)]^2\,dx = c_1^2+c_2^2+c_3^2$$

where $F(x) = c_1\phi_1(x)+c_2\phi_2(x)+c_3\phi_3(x)$

→ Since $\phi_1(x), \phi_2(x), \phi_3(x)$ are orthogonal as well as orthonormal

$$\left.\begin{aligned}
&\int_a^b \phi_1(x)\,\phi_2(x)\,dx = 0\\
&\int_a^b \phi_2(x)\,\phi_3(x)\,dx = 0\\
&\int_a^b \phi_1(x)\,\phi_3(x)\,dx = 0
\end{aligned}\right\} \longrightarrow \text{(I)}$$

$$\left.\begin{aligned} &\int_a^b \phi_1(x)\,\phi_1(x)\,dx = 1 \\ &\int_a^b \phi_2(x)\,\phi(2x)\,dx = 1 \\ &\int_a^b \phi_3(x)\,\phi_3(x)\,dx = 1 \end{aligned}\right\} \longrightarrow (2)$$

$$F(x) = c_1\phi_1(x) + c_2\phi_2(x) + c_3\phi_3(x)$$

$$[F(x)]^2 = [c_1\phi_1(x) + c_2\phi_2(x) + c_3\phi_3(x)]^2$$

$$= c_1^2\phi_1^2(x) + c_2^2\phi_2^2(x) + c_3^2\phi_3^2(x) + 2c_1c_2\phi_1(x)\phi_2(x) + 2c_2c_3\phi_2(x)\phi_3(x) + 2c_1c_3\phi_1(x)\phi_3(x).$$

$$\int_a^b (F(x))^2 = c_1^2\int_a^b \phi_1(x)\phi_1(x)\,dx + c_2^2\int_a^b \phi_2(x)\phi_2(x)\,dx + c_3^2\int_a^b \phi_3(x)\phi_3(x)\,dx$$

$$+ 2c_1c_2\int_a^b \phi_1(x)\phi_2(x)\,dx + 2c_2c_3\int_a^b \phi_2(x)\phi_3(x) + 2c_1c_3\int_a^b \phi_1(x)\phi_3(x)$$

$$= c_1^2\times 1 + c_2^2\times 1 + c_3^2\times 1 + 0$$

$$= c_1^2 + c_2^2 + c_3^2$$