① $\{x_i\}$ – фикс. выборка

$$\prod_i p(x_i) \to \max_{\mu,\sigma}, \quad p(x) := \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\exists\, i = \overline{1,\ell} \Rightarrow \prod_{i=1}^{\ell} p(x_i) = (2\pi\sigma^2)^{-\frac{\ell}{2}} e^{-\sum_{i=1}^{\ell} \frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$\log P := \log\left(\prod_{i=1}^{\ell} p(x_i)\right) = \sum_1^{\ell} \log p(x_i) = -\frac{\ell}{2}\log 2\pi\sigma^2 - \sum \frac{(x_i-\mu)^2}{2\sigma^2}$$

Усл. на максимум: $\partial_\mu \log P = \partial_{\sigma^2} \log P = 0$;

$$\partial_\mu \log P = \frac{1}{2\sigma^2}\sum_1^{\ell}(x_i-\mu) = 0 \Rightarrow \ell\mu = \sum x_i \Rightarrow \mu = \langle x_i \rangle$$

$$\partial_{\sigma^2} \log P = /\sigma^2 = S/ = -\frac{\ell}{2}\partial_S \log S + \frac{1}{2}\sum(x_i-\mu)^2 \cdot \frac{1}{S^2} = 0$$

$$-\frac{\ell}{2S} + \frac{\frac{1}{2}\sum(x_i-\mu)^2}{S^2} = 0$$

$$\ell S = \sum_1^{\ell}(x_i-\mu)^2 \Rightarrow S = \frac{\sum(x_i - \langle x_i \rangle)^2}{\ell} = \operatorname{var} x$$

② $P_\lambda(n) = \frac{\lambda^n}{n!}e^{-\lambda}, \quad P(\lambda) = P_0$

a) $$P(\lambda|m) = \frac{P(m|\lambda)P(\lambda)}{P(m)} = \frac{P(m|\lambda)P(\lambda)}{\int d\lambda\, P(m|\lambda)P(\lambda)} = \frac{P_\lambda(m)}{\int_0^\infty d\lambda\, P_\lambda(m)} = P_\lambda(m) \Rightarrow$$

$$P(\lambda|m) = \frac{\lambda^m}{m!}e^{-\lambda}$$

2) Теперь, $P(\lambda) = P_\lambda(m)$ – априор.

$$P(\lambda|m,m') = \frac{P(m'|\lambda)P(\lambda)}{P(m')} = \frac{P(m'|\lambda)P(\lambda)}{\int d\lambda\, P(m'|\lambda)P(\lambda)} = \frac{P_\lambda(m')P_\lambda(m)}{\int_0^\infty P_\lambda(m')P_\lambda(m)} \Rightarrow$$

$$P(\lambda|m,m') = \frac{2^{1+m+m'}\lambda^{m+m'}}{(m+m')!}e^{-2\lambda}$$

③ $\exists$ A – болезнь; B – тест положит.

$P(A) = 10^{-5}$; $P(B|A) = 0{,}99$; $P(B|\neg A) = 0{,}01$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\neg A)P(\neg A)} = \frac{0{,}99\cdot 10^{-4}}{0{,}99\cdot 10^{-5} + 0{,}01\cdot(1-10^{-5})} = 9{,}89\cdot 10^{-4}$$

При 2^x тестах необходима $P(B)$

④ $$p_0(\bar{x}) = \frac{1}{z}\exp\left(-\frac{x^T A x}{2}\right) = \frac{1}{z}\exp\left(-\frac{1}{2}x^i A_{ij} x^j\right) = \frac{1}{z}\prod_{i,j=1}^{n}\exp\left(\frac{x_i A_{ij} x_j}{-2}\right)$$

$$P(A|\bar{x}_1) = \frac{P(x_1|A)P(A)}{P(x_1)} = \frac{P(x_1|A)P(A)}{\int dA\, P(x_1|A)P(A)} = \frac{P_0 \prod \exp\left(-\frac{x_i A_{ij} x_j}{2}\right)}{\prod \int_0^\infty e^{-\frac{x_i A_{ij} x_{1j}}{2}} P_0\, dA_{ij}} = \frac{\prod x_{1i}^{n^2}}{2^{n^2}}\exp\left(-\frac{x_1^i A_{ij} x_1^j}{2}\right)$$

$$\Rightarrow P(\bar{x}) = \int_0^\infty P(\bar{x}|A)P(A|\bar{x}_1)\,dA = \frac{\prod x_{1i}^{n^2}}{2^{n^2} z}\prod\int_0^\infty e^{-\frac{x_i A_{ij} x_j}{2}} e^{-\frac{x_{1i} A_{ij} x_{1j}}{2}} dA_{ij}$$

$$P(\bar{x}) = \frac{\prod_1^n x_{1i}^{n^2}}{z}\prod_1^n \frac{1}{x_i x_j + x_{1i} x_{1j}}$$

⑦ $\mathcal{L} = \|X\omega - y\|^2 \to \min_\omega$, $\sum_\alpha |\omega_\alpha| < C$

$$\mathcal{L}' = \|X\omega - y\|^2 + \mu\left(\sum_\alpha |\omega_\alpha| + C\right)$$

Дос. усл-я Каруша–Куна–Таккера

$$\begin{cases} \min_\omega \mathcal{L}'(\omega) = \mathcal{L}'(\hat{\omega}) \\ \mu\left(\sum_\alpha |\hat{\omega}_\alpha| - C\right) = 0 \\ \mu > 0 \end{cases}$$

L_1 рег: $\mathcal{L}'' = \|X\omega - y\|^2 + \mu\sum_\alpha |\omega_\alpha| \to \min_\omega \Rightarrow$ при $\omega_\alpha = \hat{\omega}_\alpha$: $\mathcal{L}'' = \mu C \to \min$

ЧТД

⑧ $\exists\, y = f(x) + \varepsilon$; $E\varepsilon = 0$; $\operatorname{var}\varepsilon = E_{x,y}(y - E(y|x))^2$

$$E_{x,y} y = E_{x,y} f(x) = f(x)$$

$$\operatorname{var} y = E_{x,y}(y - E_{x,y} y)^2 = E_{x,y}(y - f)^2 = E_{x,y}\varepsilon^2 = \operatorname{var}\varepsilon$$

$$E_{x,y}(y - \check{y})^2 = E_{x,y}(y^2 + \check{y}^2 - 2y\check{y}) = \operatorname{var} y + E_{x,y} y^2 + \operatorname{var}\check{f} + E_{x,y}\check{f}^2 - 2f E_{x,y}\check{y}$$

$$E_{x,y}(y-\breve{y})^2 = \operatorname{var} y + \operatorname{var} \breve{f} + (f^2 - 2f\,E_{x,y}\breve{y} + E_{x,y}\breve{y}^2) = \underbrace{\operatorname{var} y}_{\text{noise}} + \underbrace{\operatorname{var} \breve{y}}_{\text{variance}} + \underbrace{(f - E_{x,y}\breve{y})^2}_{\text{bias}}$$

Усред. по обуч. выб-е ⇒ ЧТД