Discrete Structures and Matrix Algebra

Propositional Logic

Syllabus:

1. Unit – 1 [10 Hours]: Mathematical Logic - Propositions, Predicates and Quantifiers, Logical Statements, Equivalence of Statements, Converse, Contrapositive and InverseStatements, Tautology and Contradiction, Mathematical Inference, VariousProof Strategies, Disprove, Normal Forms;

2. Unit – 2 [8 Hours]: Sets - Basic Set Operations, Functions, Cardinality, Countable and Uncountable Sets, Sequence & Summations; Induction - Principle of Induction, Strong Induction; Recursion - Recursive Algorithms, Recursive Definition of Sets, Structural Induction;

3. Unit – 3 [7 Hours]: Counting techniques - Sum and Product Rule, Inclusion and Exclusion Principles, Pigeonhole Principle, Generalized Pigeonhole Principle, Permutation, Combination, Recurrence Relation, Solving Homogeneous and Non Homogeneous Recurrence Relations, Binomial Coefficients and Identities;

4. Unit – 4 [7 Hours]: Relations - Relations, Equivalence and Partial Order Relations, Partition and Equivalence Classes, Closure of Relation, Representation and Operation on Relations, Posets, Totally Ordered Sets, Well-Ordered Sets, Least and Maximum Elements, Least Upper Bound, Greatest Lower Bound, Lattice;

5. Unit – 5 [9 Hours]: Solving Linear Equations Solving Ax = b, Elimination with Matrices, Multiplication and Inverse Matrices, Factorization into A = LU, Transposes and Permutations; Vector Spaces and Subspaces - Spaces of Vectors, Column Space, Null Space, Row Space, Left Null Space, Independence, Basis, and Dimension, Rank and Row Reduced Form, Invertible Matrices;

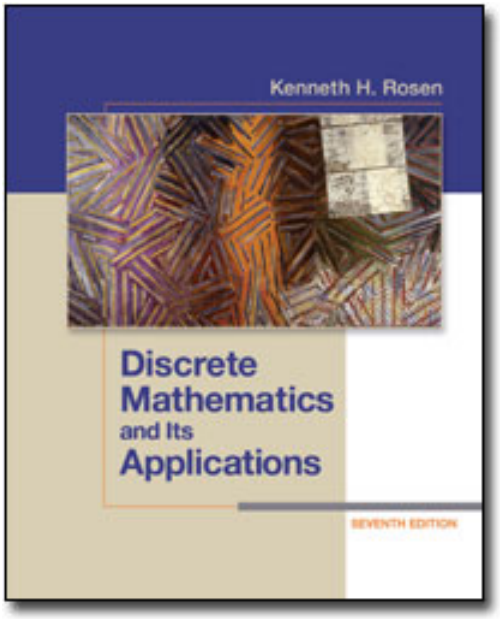
6. Unit – 6 [7 Hours]: Orthogonality – Orthogonal Vectors and Spaces, Projections, Orthogonal Bases and Gram-Schmidt; Eigen Values and Eigen vectors - Diagonalization, Spectral Decomposition, Symmetric Matrices, Positive Definiteness, Singular Value Decomposition

Text Book

Now 8th Edition may be available.

PDF for free download of the 7th edition is available in the Internet (search for it).

Textbook:



Kenneth H. Rosen. *Discrete Mathematics and Its Applications, 7th Edition*. McGraw Hill, 2012.

Evaluation Plan

- Class participation quizzes – 15%
- Scheduled quizzes – 25%
- Assignments – 10%
- Mid – 20%
- End – 30%

- This may be modified as per the CC meeting.

Objectives: Discrete Mathematics

We will focus on two major goals:

- Basic tools and techniques in discrete mathematics
  - Propositional logic
  - Set Theory
  - Simple algorithms
  - Induction, recursion
  - Counting techniques (Combinatorics)
- Precise and rigorous mathematical reasoning
  - Writing proofs

Unit 1 Syllabus

- **Unit – 1** [10 Hours]: Mathematical Logic - Propositions, Predicates and Quantifiers, Logical Statements, Equivalence of Statements, Converse, Contrapositive and Inverse Statements, Tautology and Contradiction, Mathematical Inference, Various Proof Strategies, Disprove, Normal Forms;

To do well you should:

- Study with pen and paper
- Ask for help immediately
- Practice, practice, practice…
- Ask questions in class
- Keep up with the class
- Read the book, not just the slides

Logic and reasoning?

- Logic is the basis of all mathematical reasoning,
- It has practical applications to the design of computing machines,
- Applied to many from artificial intelligence, to computer programming, to programming languages, and to other areas of computer science, as well as to many other fields of study.

Reasoning about problems

- Is the number of primes finite?
- There exists integers a,b,c that satisfy the equation $a^2+b^2 = c^2$
- The program below that I wrote works correctly for all possible inputs…..

why Proofs?

Everyone knows that proofs are important throughout mathematics, but many people find it surprising how important proofs are in computer science.

- In fact, proofs are used to verify that computer programs produce the correct output for all possible input values,
- to show that algorithms always produce the correct result,
- to establish the security of a system, and
- to create artificial intelligence.

Tools for reasoning: Logic

Ch. 1: Introduction to Propositional Logic

- Truth values, truth tables
- Boolean logic: $\vee$ $\wedge$ $\neg$
- Implications: $\rightarrow$ $\leftrightarrow$

Why study propositional logic?

- A formal mathematical “language” for precise reasoning.
- Start with propositions.
- Add other constructs like negation, conjunction, disjunction, implication etc.
- All of these are based on ideas we use daily to reason about things.

Example of a formal language: Arithmetic

E.g., the language of arithmetic

- $x+2 \geq y$ is a sentence;
- $2x+y > \{\}$ is not a sentence
- $x+2 \geq y$ is true iff the number $x+2$ is no less than the number y
- $x+2 \geq y$ is true in a world where $x = 7, y = 1$
- $x+2 \geq y$ is false in a world where $x = 0, y = 6$

Sentence, statement are interchangeably used. In the following slides it becomes clear.