

DSMA Assignment 2 Questions

1. Determine whether these statements are true or false. Prove your answer with sound, precise and simple reasoning. {Complicated proofs can attract negative marks}

a) $\emptyset \in \{\emptyset\}$

b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$

c) $\{\emptyset\} \in \{\emptyset\}$

d) $\{\emptyset\} \in \{\{\emptyset\}\}$

e) $\{\emptyset\} \subseteq \{\emptyset, \{\emptyset\}\}$

f) $\{\{\emptyset\}\} \subseteq \{\emptyset, \{\emptyset\}\}$

g) $\{\{\emptyset\}\} \subseteq \{\{\emptyset\}, \{\emptyset\}\}$

h) $\emptyset \subseteq \{1, 2, 3\} \rightarrow \emptyset \not\subseteq \{(1,2), (2,3), (1,3)\}$

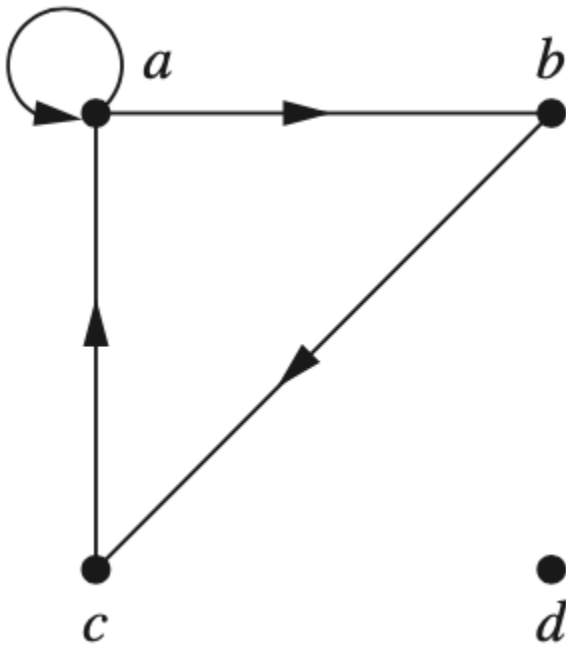
i) $A=\{1,2\} \rightarrow A \times \emptyset \neq \emptyset$ [note, \times means cartesian product]

j) $\emptyset \times \emptyset \neq \emptyset$

2. Let the strings be composed from the alphabet = $\{0,1,2\}$. For example, 00122 is a string. These are called ternary strings (similar to binary strings which are over alphabet = $\{0,1\}$).

- a) Find a recurrence relation for the number of ternary strings of length n that contain either two consecutive 0's or two consecutive 1's or two consecutive 2's.
- b) What are the initial conditions?
- c) How many ternary strings of length six contain two consecutive 0's or two consecutive 1's or two consecutive 2's? Is this same as your recurrence relation?
- d) Give a closed form solution.

3. Prove that there is a positive multiple of 3333 which is entirely made of 0s and 1s. (For example: 110000011; note that we don't need to find the number. We just need to prove that there exists such a number)
4. Draw the directed graph of the reflexive and symmetric closure of the relation with the directed graph shown below.



5. Let $R = \{(0,1),(0,2),(1,1),(1,3),(2,2),(3,0)\}$ be a relation defined on $A = \{0,1,2,3\}$. Find the zero-one matrix of transitive closure of R .
6. The population of City A is 8,000,000 at the end of the year 2020. The number of immigrants is 25,000n at the end of year n. The population of city increases at the rate of 8% per year. Use recurrence relation to determine the population of the city at the end of 2030.