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LAB - REPORT - 8

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Q1 (a) $\cos(\omega t) \cdot u(t)$

(b) $t\cos(\omega t)\, u(t)$

(c) $t\sin(\omega t)\, u(t)$

find $X(s)$

a) $$\cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$X(s) = \int_{-\infty}^{\infty} f(\omega t)\, e^{-st} dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \left(e^{j\omega t} + e^{-j\omega t}\right) e^{-st}\, u(t)\, dt$$

$$= \frac{1}{2}\int_{0}^{\infty} \left[e^{(j\omega - s)t} + e^{(-j\omega - s)t}\right] dt$$

$$= \frac{1}{2}\, \frac{e^{(j\omega - s)t}}{(j\omega - s)}\Bigg|_0^{\infty} + \frac{1}{2}\, \frac{e^{(-j\omega - s)t}}{(-j\omega - s)}\Bigg|_0^{\infty}$$

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$$= \frac{1}{2}\left[\frac{-1}{j\omega-s} + \frac{1}{(j\omega+s)}\right]$$

$$= \frac{j\omega+s+j\omega-s}{2(j^2\omega^2-s^2)}$$

$$= \frac{-2s}{2(-\omega^2-s^2)}$$

$$X(s) = \left(\frac{s}{\omega^2+s^2}\right)$$

b) $X(s) = \frac{1}{2}\int_{-\infty}^{\infty} t\,(e^{j\omega t} + e^{-j\omega t})\, u(t)\, e^{-st}\,dt$

$$= \frac{1}{2}\int_{0}^{\infty} t\, e^{(j\omega-s)t}\,dt$$

$$+ \frac{1}{2}\int_{0}^{\infty} t\, e^{(-j\omega-s)t}\,dt$$

(3)

Say $(j\omega - s) = k$

let $I = \int t e^{(j\omega-s)t} dt$

$\Rightarrow I = \int t e^{kt} dt$

$= t \frac{e^{kt}}{k} - \int \frac{e^{kt}}{k} dt$

$= \left(\frac{te^{kt}}{k} - \frac{e^{kt}}{k^2} \right)$

$\Rightarrow \int t e^{(j\omega-s)t} dt = \frac{t e^{(j\omega-s)t}}{(j\omega - s)} - \frac{e^{(j\omega-s)t}}{(j\omega-s)^2}$

Similarly

$\int t e^{(-j\omega-s)t} dt = \frac{t e^{(-j\omega-s)t}}{(-j\omega-s)} - \frac{e^{(-j\omega-s)t}}{(-j\omega-s)^2}$

$\cancel{X(s) = \frac{1}{2}\left[\frac{t e^{(j\omega-s)t}}{} - \frac{e^{(j\omega-s)t}}{} \right]_{}^{\infty} + \frac{1}{2}\left[t e^{(-j\omega-s)t} - e^{(-j\omega-s)t} \right]^{\infty}}$

(4)

$$X(s) = \frac{1}{2}\left[\frac{t\,e^{(j\omega-s)t}}{(j\omega-s)} - \frac{e^{(j\omega-s)t}}{(j\omega-s)^2}\right]_0^\infty + \frac{1}{2}\left[\frac{t\,e^{(-j\omega-s)t}}{(-j\omega-s)} - \frac{e^{(-j\omega-s)t}}{(-j\omega-s)^2}\right]_0^\infty$$

$$= \frac{1}{2}\left[0 - \left(0 - \frac{1}{(j\omega-s)^2}\right)\right]$$

$$+ \frac{1}{2}\left[0 - \left(0 - \frac{1}{(-j\omega-s)^2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{(j\omega-s)^2} + \frac{1}{(j\omega+s)^2}\right]$$

c) $x(t) = t\sin(\omega t)\,u(t)$

$\cancel{X(s)}$ $\sin(\omega t) = \dfrac{e^{j\omega t} - e^{-j\omega t}}{2j}$

$$\Rightarrow X(s) = \int_0^\infty t\,\frac{(e^{j\omega t} - e^{-j\omega t})}{2j}\,e^{-st}\,dt$$

From previous result

$$\int t\,e^{kt}\,dt = \frac{t e^{kt}}{k} - \frac{e^{kt}}{k^2}$$

(5)

$$X(s) = \frac{1}{2j}\int_0^\infty t e^{(j\omega - s)t} dt - \frac{1}{2j}\int_0^\infty t e^{(-j\omega - s)t} dt$$

$$= \frac{1}{2j}\left[\frac{t e^{(j\omega - s)t}}{(j\omega - s)} - \frac{e^{(j\omega - s)t}}{(j\omega - s)^2}\right]_0^\infty$$

$$- \frac{1}{2j}\left[\frac{t e^{(-j\omega - s)t}}{(-j\omega - s)} - \frac{e^{(-j\omega - s)t}}{(-j\omega - s)^2}\right]_0^\infty$$

$$= \frac{1}{2j}\left[\frac{1}{(j\omega - s)^2} - \frac{1}{(j\omega + s)^2}\right]$$

Q2 $x(t) = e^{-at}\cos(\omega t)\,u(t)$, $X(s) = ?$

$$X(s) = \int_0^\infty e^{-at}\left(\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right) e^{-st} dt$$

$$= \frac{1}{2}\int_0^\infty \left[e^{(j\omega - s - a)t} + e^{(-j\omega - s - a)t}\right] dt$$

$$= \frac{1}{2}\left.\frac{e^{(j\omega - a - s)t}}{(j\omega - a - s)}\right|_0^\infty + \frac{1}{2}\left.\frac{e^{(-j\omega - a - s)t}}{(-j\omega - a - s)}\right|_0^\infty$$

⑥

$$= \frac{-1}{2(j\omega - a - s)} - \frac{1}{2(-j\omega - a - s)}$$

$$= \frac{-1}{2(j\omega - (a+s))} + \frac{1}{2(j\omega + (a+s))}$$

$$= \frac{1}{2}\left[\frac{-j\omega - a - s + j\omega - a - s}{(j\omega)^2 - (a+s)^2}\right]$$

$$X(s) = \frac{(a+s)}{\omega^2 + (a+s)^2}$$

Q3 $x(t) = u(t) - u(t-1)$, $X(s) = ?$

$$\Rightarrow x(t) = \begin{cases} 1 & 0 \le t < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= \int_0^1 1 \cdot e^{-st}\, dt + 0$$

$$= \frac{e^{-st}}{-s}\Big|_0^1 = \frac{e^{-s}-1}{-s} = \left(\frac{1-e^{-s}}{s}\right)$$

Q4 $x(t) = \delta(t) - 3u(t) + 5e^{-2t}u(t)$

$$X(s) = \int_{-\infty}^{\infty} \delta(t)\, e^{-st}\,dt - 3\int_{-\infty}^{\infty} \cancel{e^{-st}u(t)\,dt}\; e^{-st}u(t)\,dt + 5\int_{-\infty}^{\infty} e^{-2t} e^{-st} u(t)\,dt$$

Since $\delta(t) = 1$ for $t = 0$,

$$\int_{-\infty}^{\infty} \delta(t)\, e^{-st}\,dt = e^{-st}\Big|_{t=0} = 1$$

$$X(s) = 1 - 3\int_0^{\infty} e^{-st}\,dt + 5\int_0^{\infty} e^{(-2-s)t}\,dt$$

$$= 1 - 3\,\frac{e^{-st}}{-s}\Big|_0^{\infty} + 5\,\frac{e^{(-2-s)t}}{(-2-s)}\Big|_0^{\infty}$$

$$= 1 - \frac{3}{s} + \frac{5}{s+2}$$

(8)

Q5 $x(t) = \left(\cos 3t + e^{-5t}\right)u(t)$

$$X(s) = \int_0^\infty \cos(3t)\, e^{-st}\,dt + \int_0^\infty e^{-5t} e^{-st}\,dt$$

$$= \cancel{\int_0^\infty e^{(3j-s)t} + e^{(-3j-s)}}$$

$$= \int_0^\infty \left(\frac{e^{3jt} + e^{-3jt}}{2}\right) e^{-st}\,dt$$

$$+ \cancel{\int_0^\infty e^{-(5-s)}}$$

$$+ \int_0^\infty e^{(-5-s)t}\,dt$$

$$= \int_0^\infty \frac{e^{(3j-s)t}}{2}\,dt + \int_0^\infty \frac{e^{(-3j-s)t}}{2}\,dt$$

$$+ \left.\frac{e^{-5-s}}{(-5-s)}\right|_0^\infty$$

$$\cancel{= \frac{1}{2}\left[\frac{1}{(3j-s)} + \frac{1}{3j+s}\right.}$$

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$$= \frac{1}{2}\left[\frac{e^{(3j-s)t}}{(3j-s)}\Big|_0^{\infty} + \frac{e^{(-3j-s)t}}{(-3j-s)}\Big|_0^{\infty} \right]$$

$$+ \frac{1}{s+5}$$

$$\cancel{= \frac{1}{2}\left[\frac{}{9j^2 -} \right.}$$

$$= \frac{1}{2}\left[\frac{-1}{(3j-s)} + \frac{1}{(3j+s)} \right] + \frac{1}{s+5}$$

$$X(s) = \frac{1}{2}\left[\frac{-2s}{-9-s^2} \right] + \frac{1}{s+5} = \left[\frac{1}{s+5} + \frac{s}{9+s^2} \right]$$

Q6 $x(t) = u(t) - u(t-2)$

$y(t) = x(t) * x(t)$, $Y(s) = ?$

$$Y(s) = X(s) \cdot X(s)$$

$$x(t) = \begin{cases} 1 & 0 \le t < 2 \\ 0 & \text{otherwise} \end{cases}$$

(10)

$$X(S) = \int_0^2 e^{-st}\,dt = \frac{e^{-st}}{-s}\Big|_0^2$$

$$= \frac{e^{-2s} - 1}{-s}$$

$$= \frac{1 - e^{-2s}}{s}$$

$$\Rightarrow Y(S) = \left(\frac{1 - e^{-2s}}{s}\right)^2$$