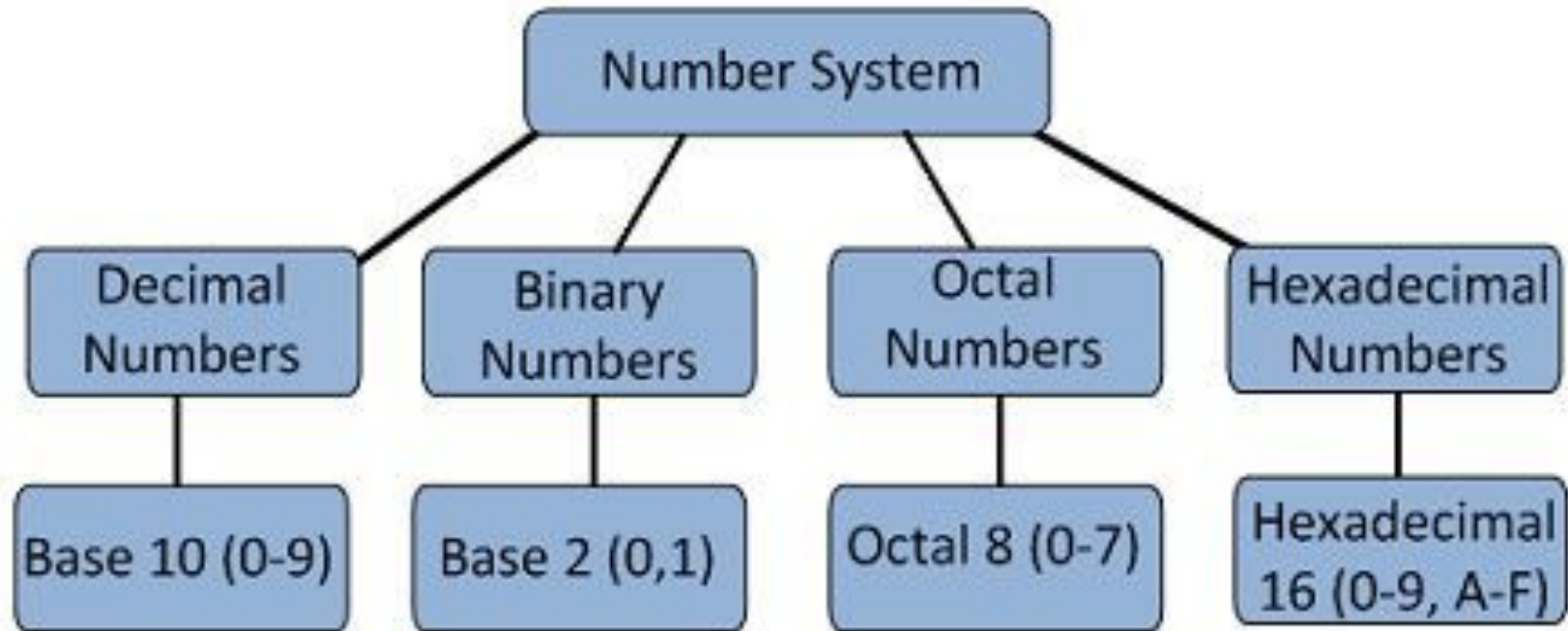# Number System

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Number System

- A digital system can understand positional number system only where there are a few symbols called digits and these symbols represent different values depending on the position they occupy in the number.

- A value of each digit in a number can be determined using the position of the digit in the number.

- The base of the number system (where base is defined as the total number of digits available in the number system).

Number Systems

DECIMAL NUMBERS

- Decimal number systems each of the ten digits, 0 through 9, represents a certain quantity.

- The position of each digit in a decimal number indicates the magnitude of the quantity represented and can be assigned a weight.

- The weights for whole numbers are positive powers of ten that increases from right to left, beginning with $10^0 = 1$ that is 10^3 10^2 10^1 10^0

- For fractional numbers, the weights are negative powers of ten that decrease from left to right beginning with 10^{-1} that is 10^2 10^1 10^0. 10^{-1} 10^{-2} 10^{-3}.

- The value of a decimal number is the sum of digits after each digit has been multiplied by its weights assign following examples

- **Express the decimal number 87 as a sum of the values of each digit**.

- The digit 8 has a weight of 10 which is 10 as indicated by its position. The digit 7 has a weight of 1 which is 10^0 as indicated by its position.
- **87 = (8 x $10^{1)}$) + (7 x 10^0)**

- **Express the decimal number 725.45 as a sum of the values of each digit.**

- **725. 45 = (7 x 10^2) + (2 x 10^1) + (5 x 10^0) + (4 x 10^{-1}) + (5 x 10^{-2}) = 700 + 20 + 5 + 0.4 + 0.05**

BINARY NUMBERS

- The binary system is less complicated than the decimal system because it has only two digits, it is a base two system.

- The two binary digits (bits) are 1 and 0.

- The position of a 1 or 0 in a binary number indicates its weight, or value within the number, just as the position of a decimal digit determines the value of that digit.

- The weights in a binary number are based on power of two as:

- ….. 2^4 2^3 2^2 2^1 2^0. 2^{-1} 2^{-2} ….

Binary

- With 4 digits position we can count from zero to 15.
- In general, with n bits we can count up to a number equal to Ķ - 1. Largest decimal number = Ķ - 1.

For example: what is the largest decimal number that can be represented by 4 bits?

- M=2powN−1=16−1=15

With 4 bits, the maximum possible number is binary 1111 or decimal 15.

A binary number is a weighted number. The right-most bit is the least significant bit (LSB) in a binary whole number and has a weight of 2^0 =1.

- The weights increase from right to left by a power of two for each bit.
- The left-most bit is the most significant bit (MSB); its weight depends on the size of the binary number.

Decimal to Binary Conversion

1. $(42)_{10} = ($? $)_2$

Solution:

$(42)_{10} = (\underline{})_2$

```
2     42
2     21    0    ↑
2     10    1    ↑
2      5    0    ↑
2      2    1    ↑
2      1    0    ↑
0      1         ↑
```

∴ $(42)_{10} = (10101\underline{0})_2$

BINARY-TO-DECIMAL CONVERSION

- The decimal value of any binary number can be found by adding the weights of all bits that are 1 and discarding the weights of all bits that are 0.

- Example

Weight: 2^5 2^4 2^3 2^2 2^1 2^0
X

| Binary no: | 1 | 0 | 1 | 1 | 0 | 1 |
| Value | 32 | 0 | 8 | 4 | 0 | 1 |

Sum = 45

binary number (1110)2 to a decimal number:

Hexadecimal Numbers

- The hexadecimal number system has sixteen digits and is used primarily as a compact way of displaying or writing binary numbers.

- Hexadecimal is widely used in computer and microprocessor applications.

- The hexadecimal system has a base of sixteen; it is composed of 16 digits and alphabetic characters.

- The maximum 3-digits hexadecimal number is FFF or decimal 4095 and maximum 4-digit hexadecimal number is FFFF or decimal 65.535

Hexadecimal Numbers

DECIMAL	HEX	BINARY
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

Hexadecimal to Binary Conversion

- To convert a hexadecimal number to a binary number, convert each hexadecimal digit to its four digit equivalent. For example, consider the hexadecimal number 9AF which is converted into a binary digit. The conversions are explained below.

- Therefore the equivalent binary number is 1001 1010 1111

Hexadecimal number :	9	A	F
Binary Number :	1001	1010	1111

Circuit Globe

BINARY-TO-HEXADECIMAL CONVERSION

- Simply break the binary number into 4-bit groups, starting at the right-most bit and replace each 4-bit group with the equivalent hexadecimal symbol as in the following example.

- **Convert the binary number to hexadecimal: 1100101001010111 Solution:**

- **1100 1010 0101 0111**

- **C   A   5   7   = CA57**

- **HEXADECIMAL-TO-DECIMAL CONVERSION**

- One way to find the decimal equivalent of a hexadecimal number is to first convert the hexadecimal number to binary and then convert from binary to decimal.

- Convert the hexadecimal number 1C to decimal:

$$1 \quad C$$
$$0001 \quad 1100 = 2^4 + 2^3 + 2^2 = 16 + 8 + 4 = 28$$

DECIMAL-TO-HEXADECIMAL CONVERSION

- Repeated division of a decimal number by 16 will produce the equivalent hexadecimal number, formed by the remainders of the divisions. The first remainder produced is the least significant digit (LSD). Each successive division by 16 yields a remainder that becomes a digit in the equivalent hexadecimal number. When a quotient has a fractional part, the fractional part is multiplied by the divisor to get the remainder.

Contd..

- Convert the decimal number 650 to hexadecimal by repeated division by 16

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- **650 /16 = 40.625 0.625 x 16 = 10   = A (LSD)**

- **40 /16= 2.5 0.5 x 16 = 8 =   8**

- **2/16= 0.125   0.125 x 16 = 2 = 2 (MSD)**

- **The hexadecimal number is 28A**

OCTAL NUMBERS

- The octal system provides a convenient way to express binary numbers and codes.

- It is used less frequently than hexadecimal in conjunction with computers and microprocessors to express binary quantities for input and output purposes.

- The octal system is composed of eight digits, which are: 0, 1, 2, 3, 4, 5, 6, 7

- To count above 7, begin another column and start over: 10, 11, 12, 13, 14, 15, 16, 17, 20, 21 and so on. Counting in octal is similar to counting in decimal, except that the digits 8 and 9 are not used.

OCTAL TO DECIMAL

Decimal	Octal	Binary
0	0	000
1	1	001
2	2	010
3	3	011
4	4	100
5	5	101
6	6	110
7	7	111

OCTAL-TO-DECIMAL CONVERSION

- Octal number system has a base of eight, each successive digit position is an increasing power of eight, beginning in the right-most column with 8^0.

- The evaluation of an octal number in terms of its decimal equivalent is accomplished by multiplying each digit by its weight and summing the products.

OCTAL-TO-DECIMAL CONVERSION

- Let's convert octal number 2374 in decimal number.

- Weight: 8^3 8^2 8^1 8^0
- Octal number: 2   3   7 4
- **2374 = (2 x 8^3) + (3 x 8^2) + (7 x 8^1) + (4 x 8^0) =1276**

OCTAL-TO-BINARY CONVERSION

- Because each octal digit can be represented by a 3-bit binary number, it is very easy to convert from octal to binary.

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- **Octal/Binary Conversion**
- Octal Digit 0   1   2   3   4   5   6   7
- Binary   000   001   010   011   100   101   110   111
-

- Let's convert the octal numbers 25 and 140.
- Octal Digit 2   5   1   4   0
- Binary   010   101   001   100   000

BINARY-TO-OCTAL CONVERSION

- Conversion of a binary number to an octal number is the reverse of the octal-to-binary conversion.

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- Let's convert the following binary numbers to octal:

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- 1 1 0 1 0 1 1 0 1 1 1 1 0 0 1