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Signals and systems

L-T-P-C: 2-1-1-4

Objectives	The objective of this course is to provide the fundamental concept of: • Signal representation, Analysis • Different types of systems and their characteristics, • Mathematical transforms and use of computer tools/programming to solve problems
Unit 1	Introduction; Classification of signals - Continuous time and discrete time, Even and odd, Periodic and non-periodic, Deterministic and Random, Energy and Power; Basic operations on signals - Scaling, Shifting, Reflection, Precedence rule for time shifting and time scaling; Elementary signals - Exponential, Sinusoidal, Step, Pulse, Impulse, Ramp, Relationship between sinusoidal and complex exponential signals, Exponentially damped sinusoid signals; Properties of systems - Stability, Memory, Causality, Invertibility, Time invariance
Unit 2	Convolution sum; Interconnection of LTI systems; Impulse response; Step response; Relationship between impulse response and system properties; Properties of LTI systems - Stability, Memory, Causality, Invertibility, Time invariance
Unit 3	Periodic signal Fourier Series - Properties of Fourier Representations, Parseval's relationships and applications
Unit 4	Aperiodic signal Fourier transform - Properties, Parseval's relation, Duality property and its applications; Hilbert transform - Pre-envelope; Phase and Groupdelay

Unit 5	Laplace transform - Eigen function property, Laplace transform representation, Convergence, S-plan, Unilateral Laplace transform, ROC, Properties
Unit 6	Sampling theory - Sampling continuous time signals, Aliasing, Reconstruction - Ideal, Practical

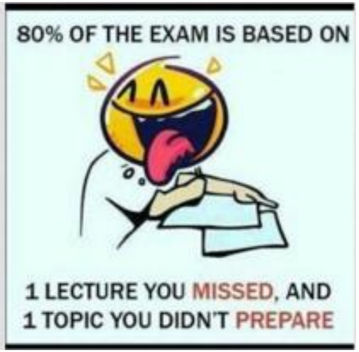
List of proposed Lab experiments:

1. Introduction to MATLAB and generation of basic signals
2. Generation and decomposing of signal onto Even and Odd components
3. Signal decimation and, energy and power of a signal
4. Signal representation (unit impulse, unit step, shifting, Folding, Multiplication, Scaling)
5. Convolution sum
6. Verification of different properties of LTI system: Commutation, Association, Distribution, Identity
7. Fourier analysis and synthesis
8. Fourier transform
9. Discrete-time Fourier Transform (DTFT) and Hilbert-transform
10. Laplace transform
11. Sampling theorem
12. Reconstruction signal from sampled value

Matlab - https://www.mathworks.com/

Text books:	1. Alan V. Oppenheim, Alan S. Willsky with S. Hamid nawab, Signal and System, Pearson Education India, 2nd edition (1 January 2015) 2. John G. Proakis, Dimitris G. Manolakis, Digital Signal Processing, Pearson Education India; 4th edition (1 January 2007)
References:	1. Luis Chaparro, Signals and Systems using MATLAB, Academic Press, 2nd Edition (2April 2014) 2. Vinay K. Ingle , John G. Proakis, Digital Signal Processing Using MATLAB: A Problem Solving Companion, Cl-Engineering, 4th Edition (1 January 2016 3. Hahn, Essential MATLAB for Engineers and Scientists, Elsevier, 5th Edition (10 January 2013)
Course Outcomes	At the end of the course, students should have the ability: • To understand and classify the mathematical representation of the continuous and discrete-time signals and systems • To apply the concept of convolution to evaluate the output of the LTI systems • To analyze both periodic and aperiodic signals and system's output in the frequency domain • To analyze the continuous-time signals and systems with the Laplace-transform • To apply the sampling theorem and signal reconstruction in signal transmission/receiving

Assessment components	Weightage (%)
Mid-sem	20
End-sem	30
Class participation/Surprise quiz	5
Scheduled Quiz	20
Lab	20
Assignments	5



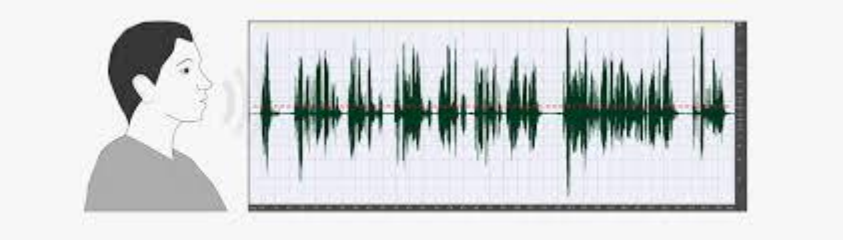
Applications of signal processing

- Speech/audio (speech recognition/synthesis etc.),
- Image/video (enhancement, coding for storage and transmission, robotic vision, animation, etc.),
- Military/space (radar processing, secure communication, missile guidance, sonar processing, etc.),
- Biomedical/health care (scanners, ECG analysis, X-ray analysis, EEG brain mappers, etc.)
- Consumer electronics (cellular/mobile phones, digital television, digital camera, Internet voice/music/video, interactive entertainment systems, etc) and many more

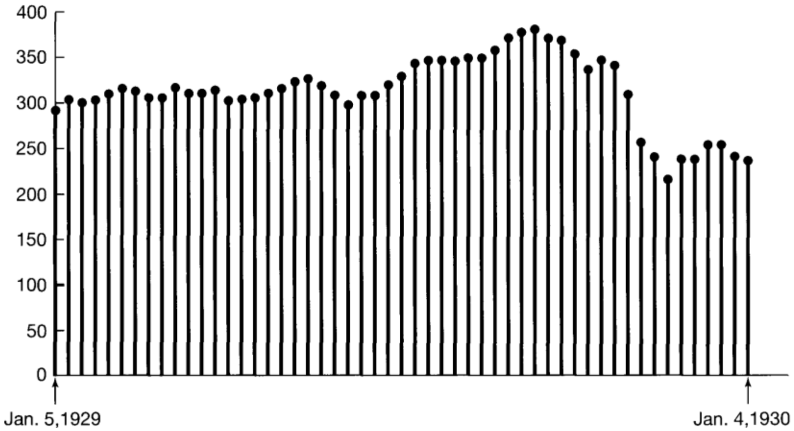
Sonar Sound Navigation

What is signal?

- Any physical quantity that varies
  - with time, space, or any other independent variable or variables (e.g. voice signal, video)



Sound as a function of time



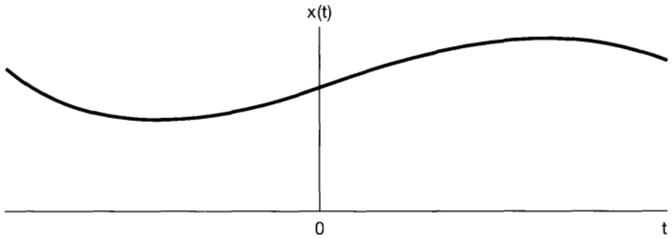
An example of a discrete-time signal: The weekly Dow-Jones stock market index from January 5, 1929, to January 4, 1930.

Examples: A telephone or a television signal, monthly sales of a corporation, pressure as a function of altitude, Radio signal

Representation of signal

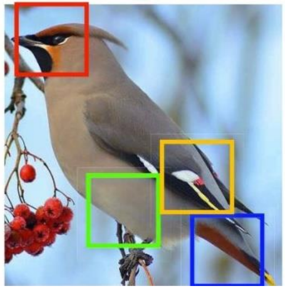
- Mathematically, a signal is represented as a function of an independent variable t.
  - Thus, a signal is denoted by $S_1(t),\ S_2(t)$

$$S_1(t) = 5t$$
$$S_2(t) = 20t^2$$



- **A signal of two independent variables** x **and** y that could represent the **two spatial** coordinates in a plane

$$s(x, y) = 3x + 2xy + 10y^2$$



- **Other examples:**

$$x(t) = \cos(2\pi t),\ x(t) = 4\sqrt{t} + t^3,\ x(m, n) = (m + n)^2$$

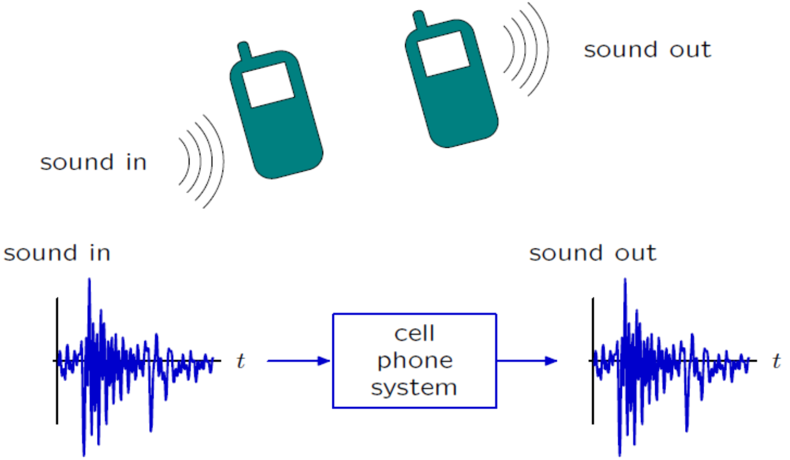
What is system?

- System is a device or algorithm which process or transforms an input signal into an desired output signal

Input Signal → system → Output Signal

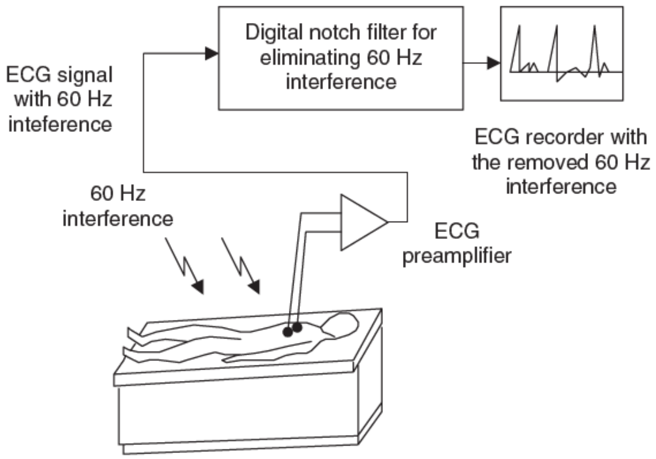
$x(t)$ $\quad$ $h(.)$ $\quad$ $y(t) = h(x(t))$

Examples: $y(t) = -4x(t)$, $\frac{dy(t)}{dt} + 3y(t) = -\frac{dx(t)}{dt} + 6x(t)$,
$y(n) - \frac{1}{2}y(n-2) = 3x(n) + x(n-2)$

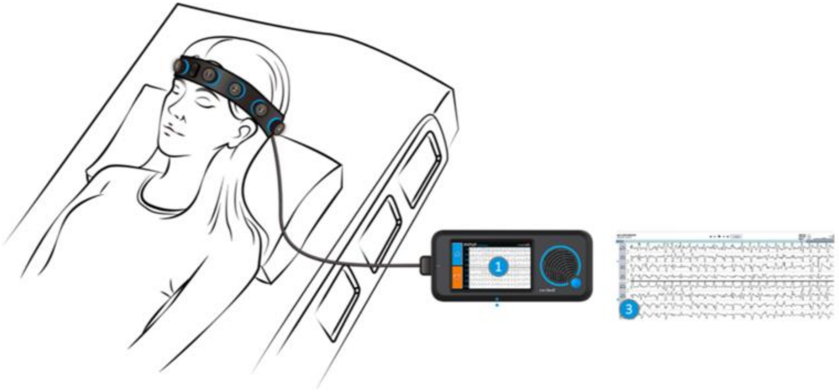


- radio receiver
- audio amplifier
- modem
- microphone
- cell telephone
- cellular metabolism
- national and global economies

[ECG recording system]



[Seizure detection system with AI]

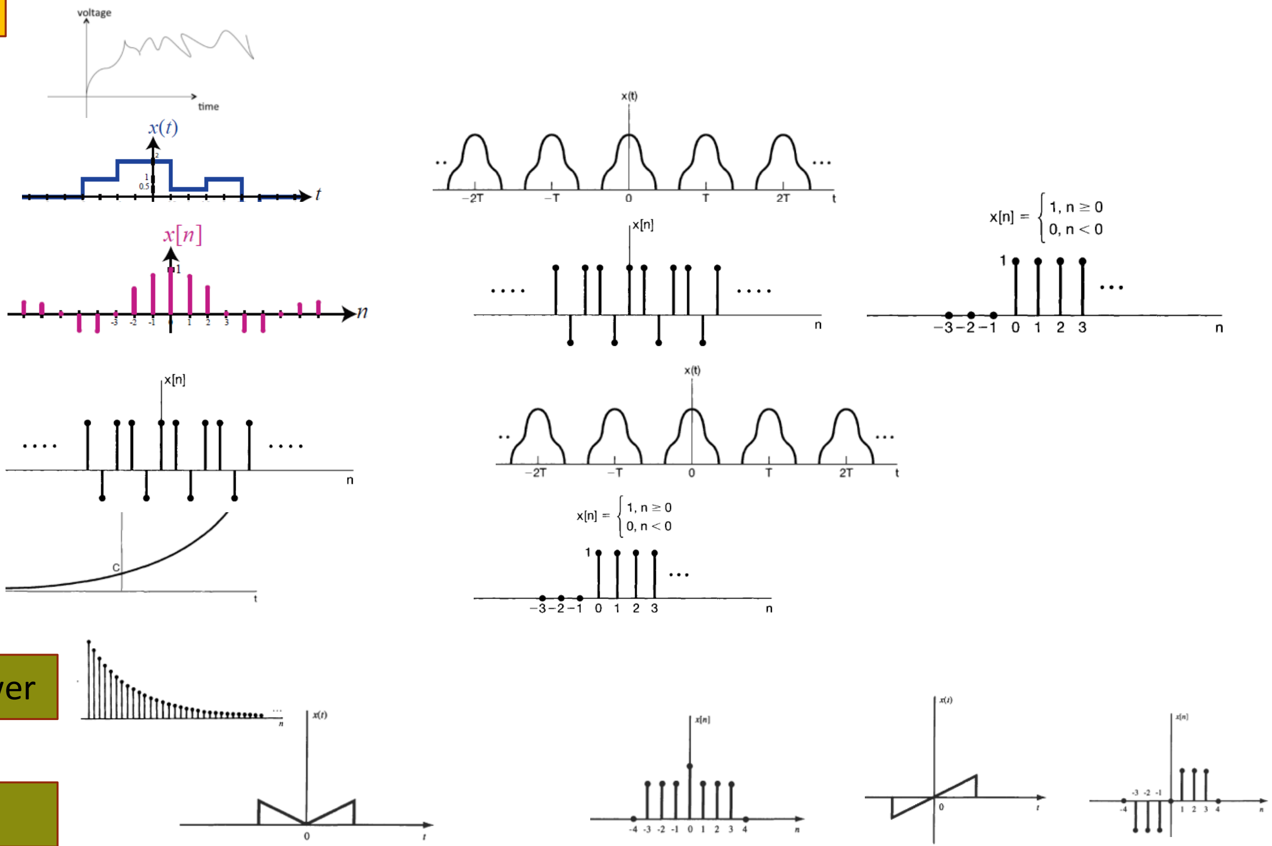


[Brain-computer interface (BCI)]

Different types of signals

- Signal
  - Deterministic
  - Random
  - Continuous
  - Discrete
  - Periodic
  - Aperiodic
  - Energy or power
  - Odd or Even

$x(t) = e^{-2t}$ x(t) = A sin (2wt) $x[n] = C\alpha^n$



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Write answer of the following question in chatbox

1. What is signal? Example of a signal
2. What are the types of signals?
3. Difference between deterministic and random signals? Example.

Deterministic Vs Random signals

Deterministic: A signal whose physical description is known completely either in

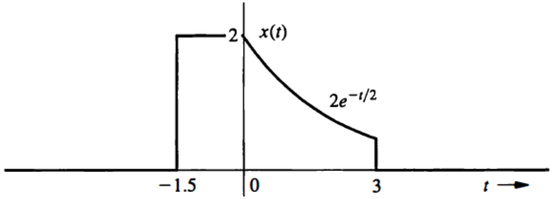
- A mathematical form or a graphical form

e.g.

$$x(t) = e^{-2t}$$

x(t) = A sin (2wt)

$$x[n] = C\alpha^n$$



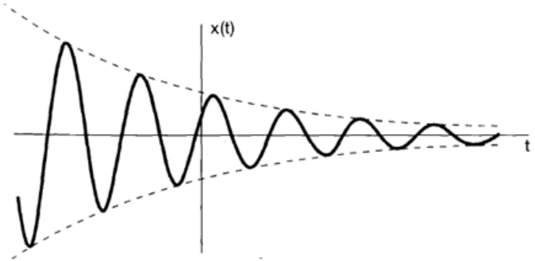
Random:

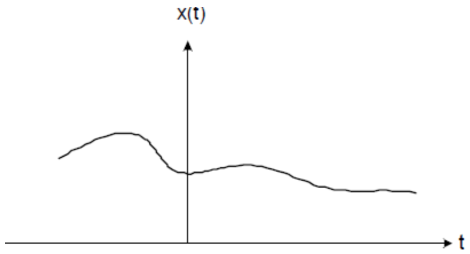
- If signal has uncertainty with respect to its value at some point of time
- Existence of signals are in random
- Signals are not able explained by a explicit mathematical equation as they are modeled in probabilistic terms

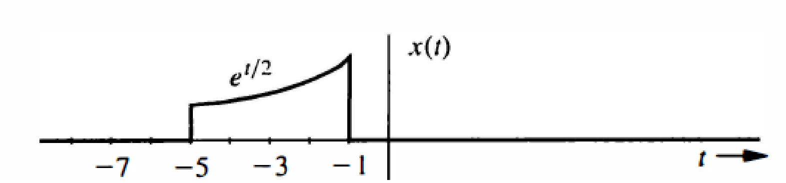
Example: Speech, thermal noise (random of movement of electron)

Continuous-time signals

- Are defined for every value of time and
- They take on values in the continuous interval $(a\,,b\,)$, *where* a *can be* $-\infty$ *and* b can be $+\infty$



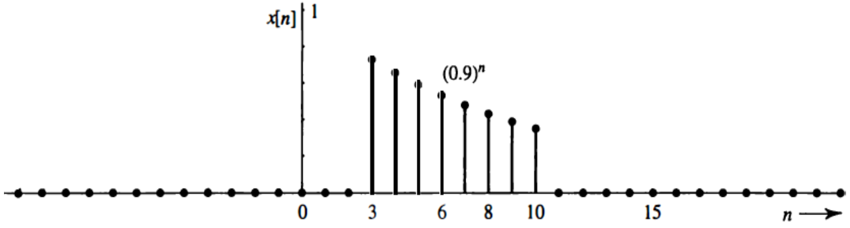




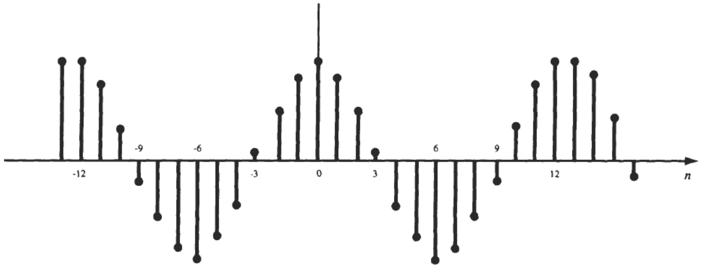
$$x(t) = \begin{cases} e^{t/2} & -1 \geq t > -5 \\ 0 & \text{otherwise} \end{cases}$$

Discrete-time signals

- Are defined **only** at **certain specific values of time**
- These time instants need not be equidistant n **(integer value)**
- In practice, they are usually taken at equally spaced intervals for computational convenience and mathematical tractability



$x[n] = (0.9)^n$ for $3 \le n \le 10$



$x[n] = \cos\left(\frac{n}{2}\right)$

Notation: $x[n]$ is analogous to $x(n)$

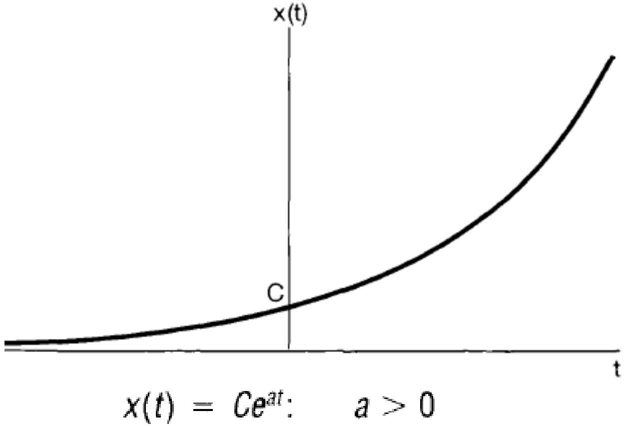
Continuous-time exponential signal

- **Continuous exponential:** $x(t) = Ce^{at}$

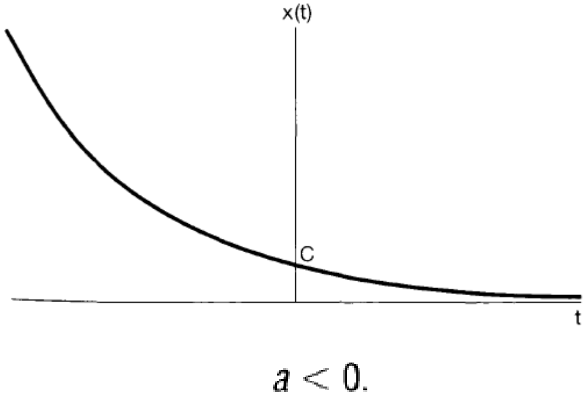
Condition-1: if C and a **are real** → real exponential

→ Two types of behavior based on value of "a"

→ (a) growing exponential or (b) decaying exponential



Growing exponential



Decaying exponential

Cont..

- **Continuous exponential**: $x(t) = Ce^{at}$

Condition-2: if C and a is inform $C = |C|e^{j\theta}$ $a = r + j\omega_0.$

$$x(t) = Ce^{at} \Rightarrow Ce^{at} = |C|e^{j\theta}e^{(r+j\omega_0)t} = |C|e^{rt}e^{j(\omega_0 t+\theta)}$$

Using Euler's relation, $Ce^{at} = |C|e^{rt}\cos(\omega_0 t + \theta) + j|C|e^{rt}\sin(\omega_0 t + \theta).$

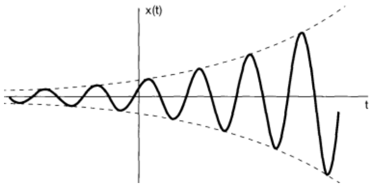
$e^{i\phi} = \cos\phi + i\sin\phi$

$\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$

$\sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$

Case 1: For r > 0 $Ce^{at} = \boxed{|C|e^{rt}}\cos(\omega_0 t + \theta) + j\boxed{|C|e^{rt}}\sin(\omega_0 t + \theta).$

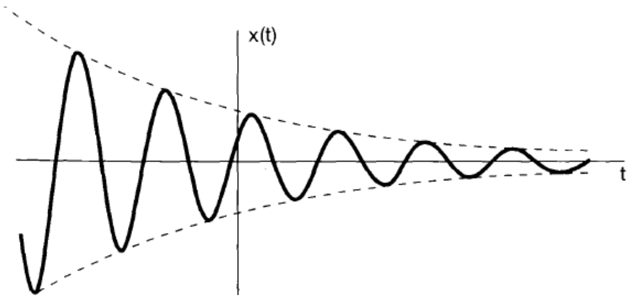
Growing sinusoid = "sinusoidal signals" X "Growing exponential"



$x(t) = Ce^{rt}\cos(\omega_0 t + \theta),$

Case 2: For **r < 0** $Ce^{at} = \boxed{|C|e^{rt}}\cos(\omega_0 t + \theta) + j\boxed{|C|e^{rt}}\sin(\omega_0 t + \theta).$

Decaying/damped sinusoid = "sinusoidal signals" x "decaying exponential"



$$x(t) = Ce^{rt}\cos(\omega_0 t + \theta),$$

Example:

Viscosity in mechanical systems (brake), automotive suspension (spring) systems,

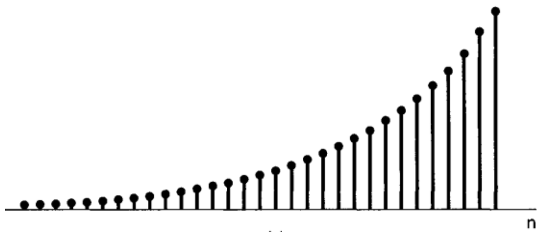
Exponential growth is unstable since nothing can *grow exponentially* forever

Discrete-time exponential signal

- **Discrete exponential signal:** $x[n] = Ce^{\beta n}$

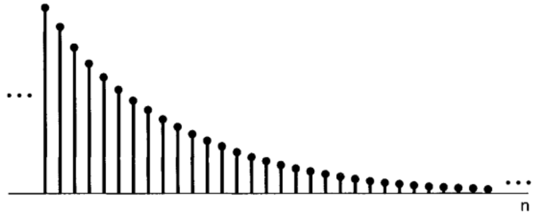
Condition-1: if C and β **are real and** $\boldsymbol{e^{\beta} > 1}$

→ the magnitude of the signal grows exponentially with $\boldsymbol{n}$



Condition-2: if C and β **are real and** $\boldsymbol{e^{\beta} < 1}$

→ the magnitude of the signal decaying exponentially with $\boldsymbol{n}$



Condition-3: if β is purely **imaginary and** $C = |C|e^{j\theta}$ →

$$x[n] = Ce^{j\omega_0 n} = |C|e^{j\theta}e^{j\omega_0 n}$$

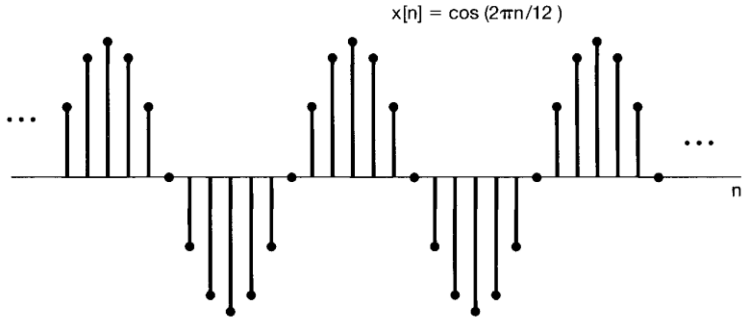
$$= |C|e^{j\theta}[\cos(\omega_0 n) + j\,sin(\omega_0 n)]$$

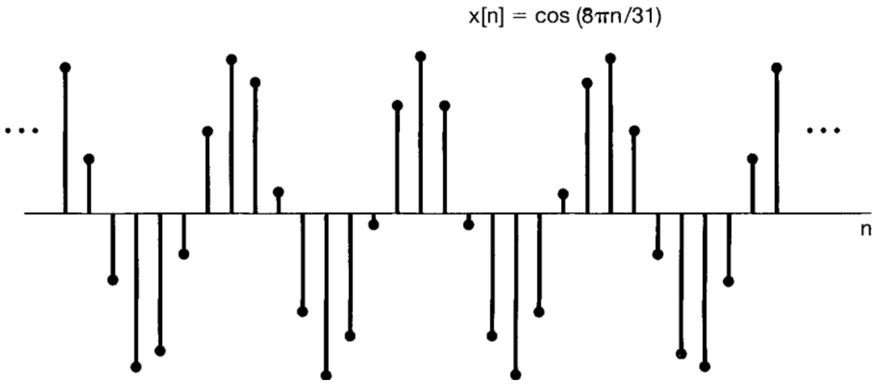
Cont..

Condition-3: if β is purely **imaginary and** $C = |C|e^{j\theta}$ → $x[n] = Ce^{j\omega_0 n} = |C|e^{j\theta}e^{j\omega_0 n}$

$$= |C|e^{j\theta}[\cos(\omega_0 n) + j\, sin(\omega_0 n)]$$

Illustration graphically





Representation of discrete & continuous signals

1. Functional representation, such as

$$x(n) = \begin{cases} 1, & \text{for } n = 1, 3 \\ 4, & \text{for } n = 2 \\ 0, & \text{elsewhere} \end{cases}$$

2. Tabular representation, such as

n	$\cdots$	-2	-1	0	1	2	3	4	5	$\ldots$
$x(n)$	$\cdots$	0	0	0	1	4	1	0	0	$\ldots$

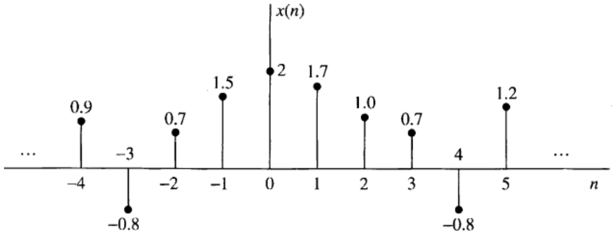
3. Sequence representation

Signal or sequence with the time origin (**n =0**) is represented by ↑ as

$$x(n) = \{\ldots 0, \underset{\uparrow}{0}, 1, 4, 1, 0, 0, \ldots\}$$

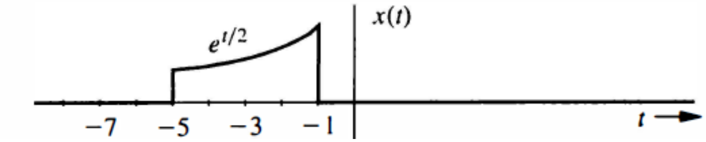
A sequence x(n), which **is zero** for **n < 0**, $x(n) = \{\underset{\uparrow}{0}, 1, 4, 1, 0, 0, \ldots\}$

4. Graphical



Continuous

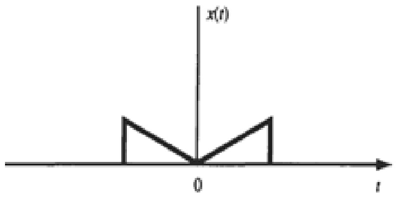
$$x(t) = \begin{cases} e^{t/2} & -1 \geq t > -5 \\ 0 & \text{otherwise} \end{cases}$$

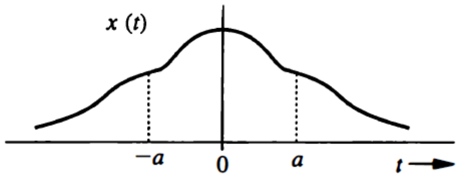


Even signals

Continuous:

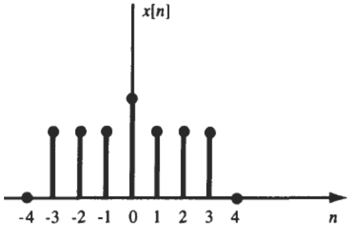
- A **continuous** signal **x(t)** is said to be an **<u>even</u>** signal if satisfy $x(-t) = x(t)$





Discrete:

- A **discrete** signal **x(n)** is said to be an **<u>even</u>** signal if satisfy $x[-n] = x[n]$



$$x[n] = \begin{cases} \frac{1}{2}, & n < 0 \\ 1, & n = 0 \\ \frac{1}{2}, & n > 0 \end{cases}$$

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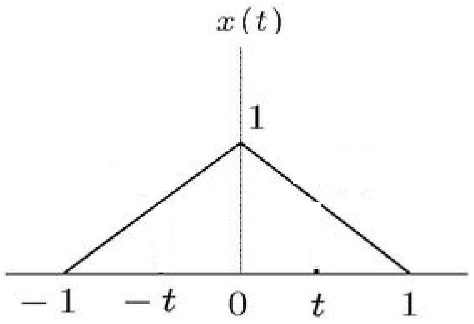
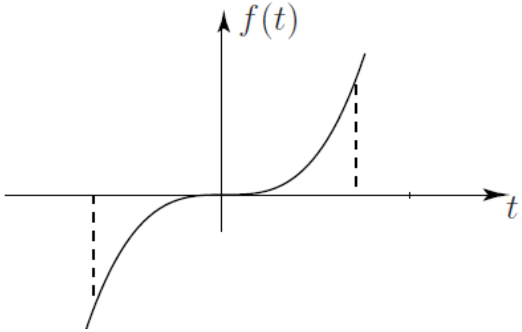
Verify signals are even or odd?

- $Sin(wt)$
- $x(t) \ = cos(wt)$
- $sin(wn)$
- $\cos(wn)$
- $x(t) = \ |t|$
- $x(t) = \ t^4$





Let $x(t) \ = \ Sin(wt),$

$x(-t) \ = \ Sin(w.-t) \ = \ Sin(-wt) = \ -Sin(wt) \ = \ -x(t)$

$x(t) \ \neq \ x(-t)$ >> **not even signal**

Representation of signal : even and odd parts

If signal is Even, x(n) + x (−n) = ? $\rightarrow \boldsymbol{x(n) + x(n) = 2x(n)}$ [as signal is even, x(-n) = x(n)]

$$x(n) + x(-n) = 2\,x(n)$$

$$\rightarrow x(n) = \frac{1}{2}[x(n) + x(-n)]$$

$$Even[x(n)] = x_e[n] = \frac{1}{2}[x(n) + x(-n)] \; \ldots\ldots \quad \ldots \; (\mathbf{1}) \quad [\text{also } x_e(n) = x_e(-n)]$$

If signal is Odd, x(n) − x(−n) = ? $\rightarrow \boldsymbol{x(n) - (-x(n)) = 2\,x(n)}$ [as signal is odd, $\boldsymbol{x(-n) = -x(n)}$]

x(n) − x(−n) = 2 x(n)

$$\boldsymbol{Odd[x(n)] = x_o(n) = \frac{1}{2}[x(n) - x(-n)]} \; \ldots \quad \ldots. \; \ldots. (\mathbf{2})$$

Using Eq. (1) & (2)

Even [x(n)] + Odd [x(n)]

$$= \frac{1}{2}[x(n) + x(-n)] + \frac{1}{2}[x(n) - x(-n)]$$

$$= \frac{1}{2}[x(n) + \cancel{x(-n)}] + \frac{1}{2}[x(n) - \cancel{x(-n)}]$$

$$= x(n)$$

$$\begin{matrix} x_e(t) & + & x_o(t) \end{matrix}$$

$$x(t) = \underbrace{\tfrac{1}{2}[x(t) + x(-t)]}_{\text{even}} + \underbrace{\tfrac{1}{2}[x(t) - x(-t)]}_{\text{odd}}$$

Sketch and label the even and odd components of the signals

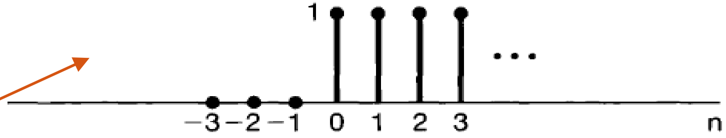
$$x[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

We know that $x(n) = x_e(n) + x_o(n)$

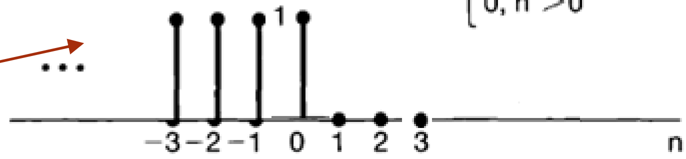
We have to determine the $x_e(n)$ and $x_o(n)$

$$x_e[n] = \frac{1}{2}[x(n) + x(-n)]$$

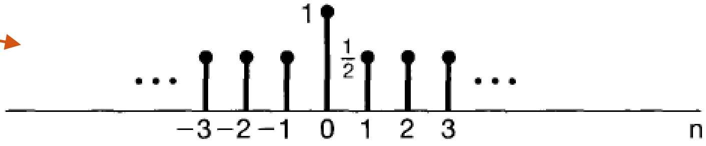
$$x[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



$$x[-n] = \begin{cases} 1, & n <= 0 \\ 0, & n > 0 \end{cases}$$



$$\mathcal{E}v\{x[n]\} = \begin{cases} \frac{1}{2}, & n < 0 \\ 1, & n = 0 \\ \frac{1}{2}, & n > 0 \end{cases}$$



Find the even and odd components of e^{jt}.

We know that x(t) = $x_e(t) + x_o(t)$

We have to determine the $x_e(t)$ and $x_o(t)$

Even:

$$x_e(t) = \tfrac{1}{2}[x(t) + x(-t)]$$

$$x_e(t) = \tfrac{1}{2}[e^{jt} + e^{-jt}] = \cos t$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

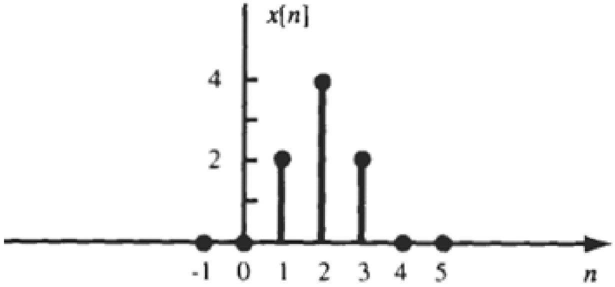
$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Odd:

$$x_o(t) = \tfrac{1}{2}[x(t) - x(-t)]$$

$$x_o(t) = \tfrac{1}{2}[e^{jt} - e^{-jt}] = j \sin t$$

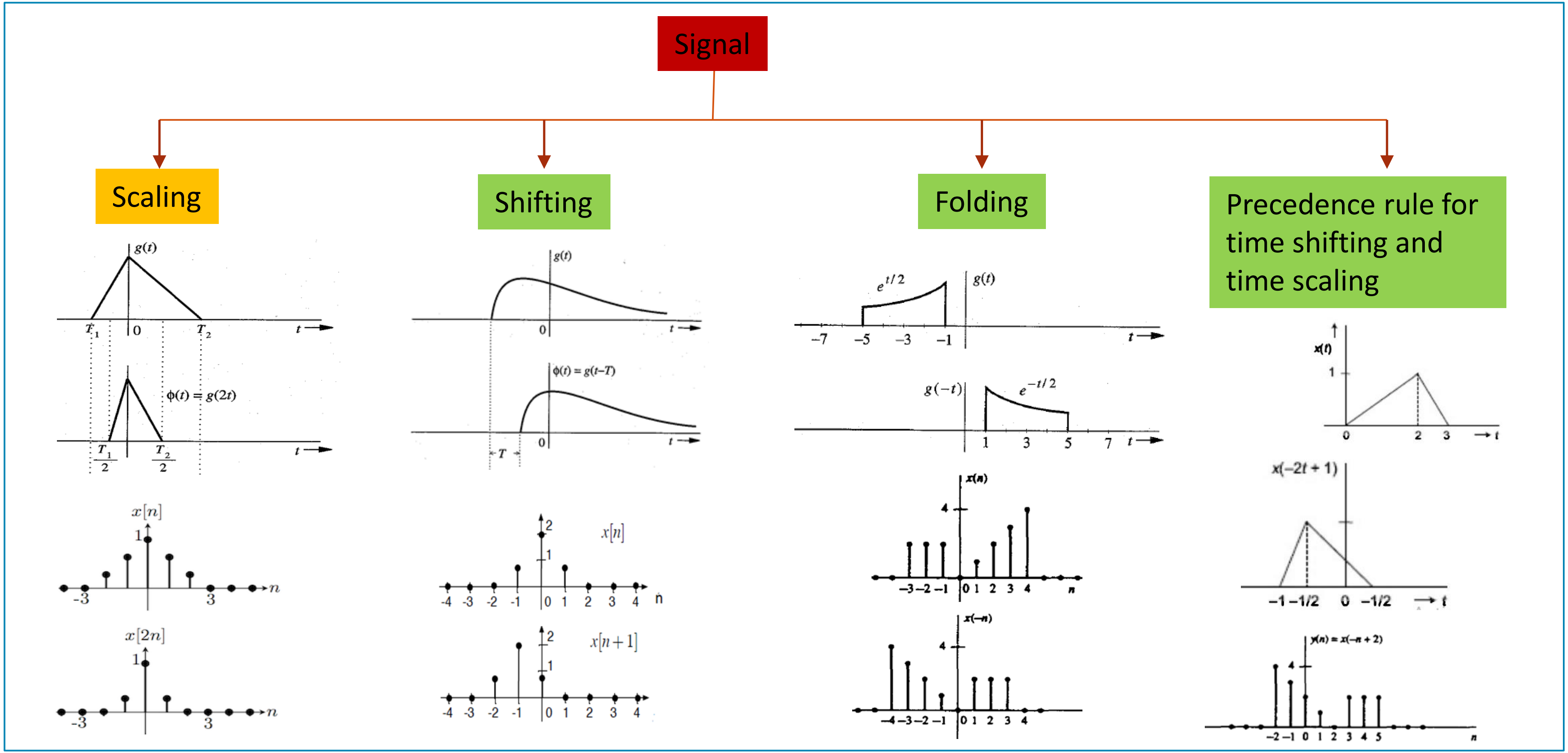
Determine the even and odd components of the signals



$(b)\ x(t) = \sin(\Omega_0 t + \frac{\pi}{4})$

$(c)\ x(n) = \mathrm{e}^{j(\omega_0 n + \frac{\pi}{2})}$

Basic operations on the signals



Scaling

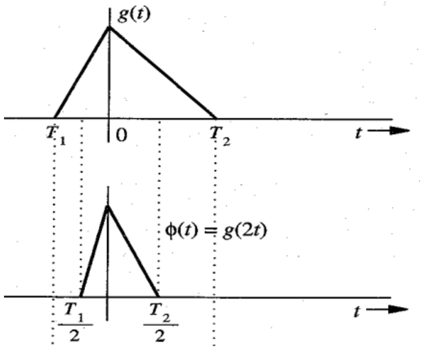
- Time scaling compresses or dilates a signal g(t) by multiplying the time variable (t) by some quantity (a)
- If quantity **(a)** is **greater than one**, the signal becomes **narrower** and the operation is called **compression.**

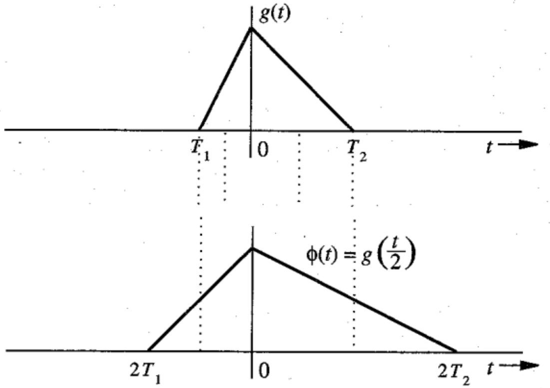
a>1

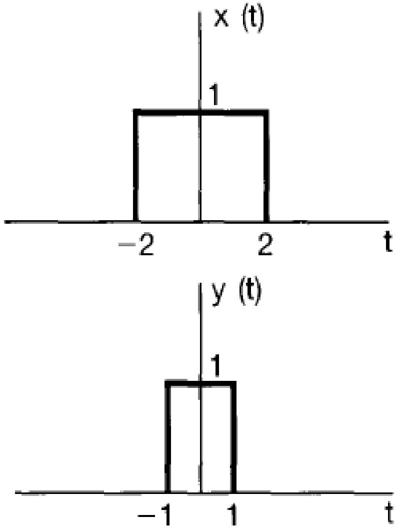
$$y(t) = x(2t)$$

$$\varphi(t) = g(at)$$

$$\varphi(t) = g(\frac{t}{a})$$



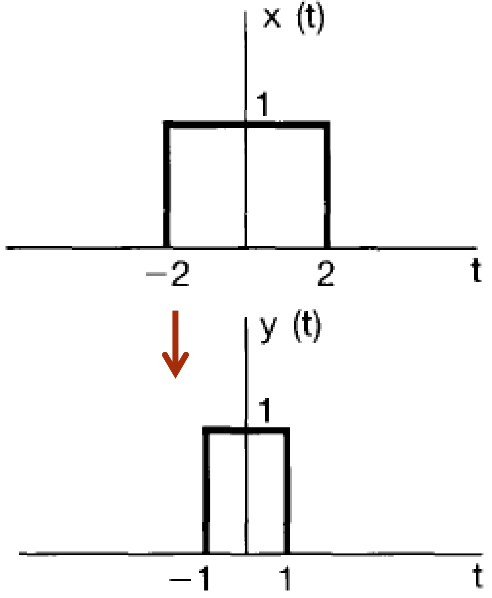




Compress

Dilates

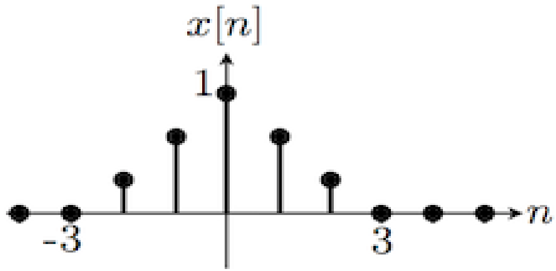
$$y(t) = x(2t)$$

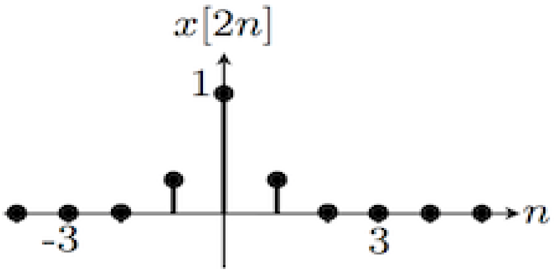


a>1

$y[n] = x\,[2n]$

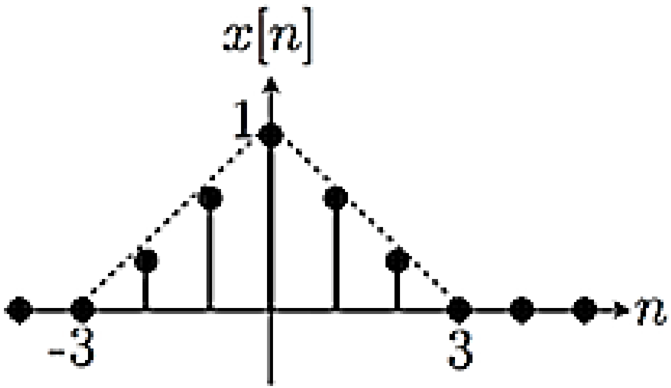
Discrete signal **only consider** the integer value of “**index**” **i.e. n**





a <1

$$y[n] = x[\frac{n}{2}]$$

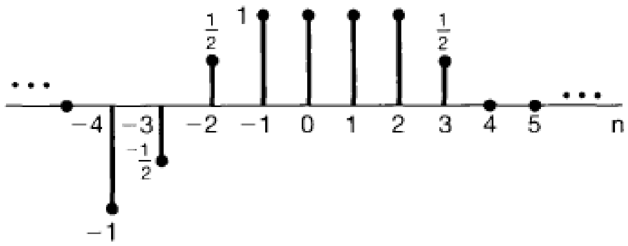


Dilates

If the quantity (a) is less than one, the signal becomes wider and is called dilation.

$(a)\ \ x\,[3n] = ?$

$x[n]$

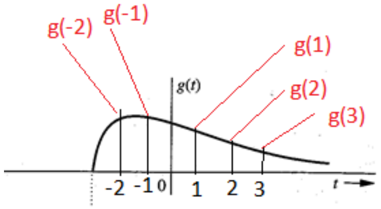


Time-Shifting

- Time shifting is the shifting of a signal in time
- This is done by **adding** or **subtracting a quantity (say. T)** of the shift to the time variable in the function

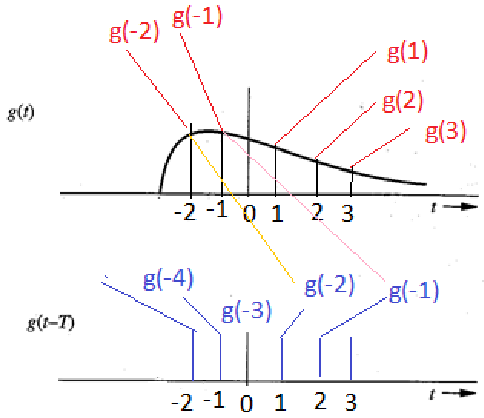
$$\varphi(t) = g(t - T) = ?$$

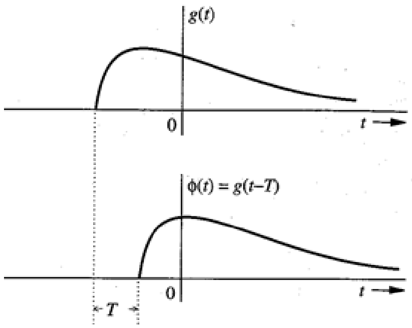
For **positive value of T >0** ----- right-shift/delay



Let's to determine signal $g(t-3)$ for $T = 3$

0. $g(t-3)\ at\ t = -1 \ \rightarrow\ g(-1-3) = g(-4)$
1. $g(t-3)\ at\ t = 0 \ \rightarrow\ g(0-3) = g(-3)$
2. $g(t-3)\ at\ t = 1 \ \rightarrow\ g(1-3) = g(-2)$
3. $g(t-3)\ at\ t = 2 \ \rightarrow\ g(2-3) = g(-1)$





For a discrete time signal $x[n]$, and an integer $n_1 > 0$

$$y[n] = x[n - n_1] \longrightarrow \text{Delay/ right-shift}$$

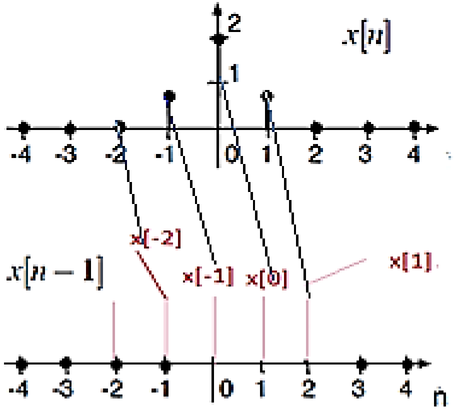
Let's given x[n], determine $x[n-1]$

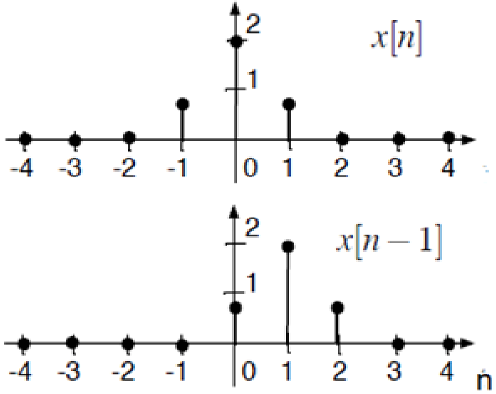
0. $x[n-1]$ at $n = -1$ $\rightarrow x[-1-1] = x[-2]$

1. $x[n-1]$ at $n = 0$ $\rightarrow x[0-1] = x[-1]$

2. $x[n-1]$ at $n = 1$ $\rightarrow x[1-1] = x[0]$

3. $x[n-1]$ at $n = 2$ $\rightarrow x[2-1] = x[1]$





Thank you

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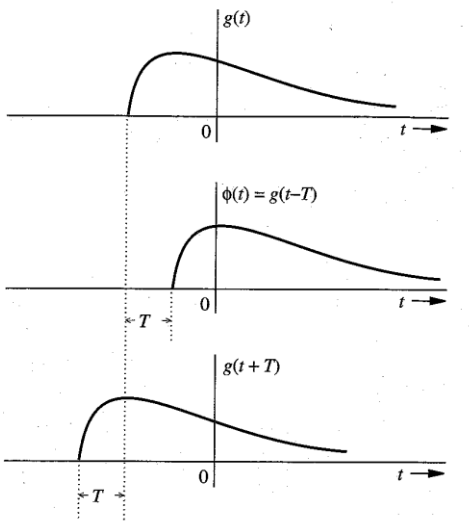
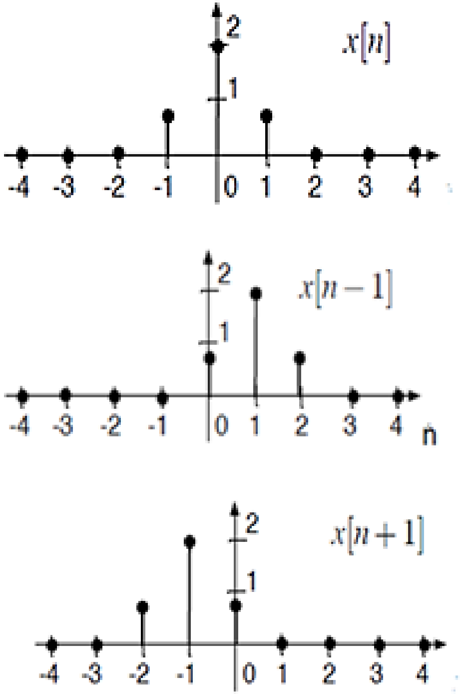
Similarly, left-shift/advance

$$\varphi(t) = g(t + T)$$

$$y[n] = x[n + n_1]$$

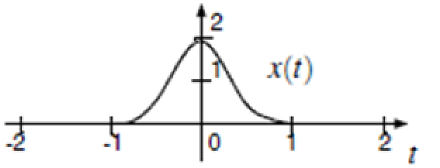


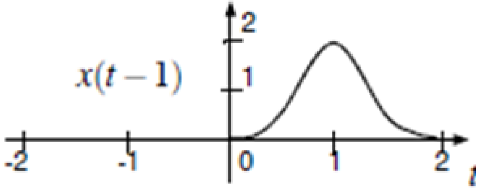


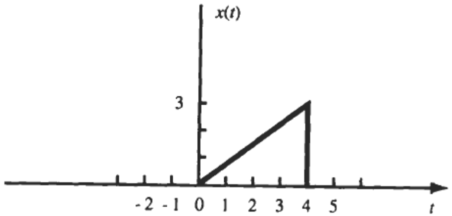
Draw the following signals

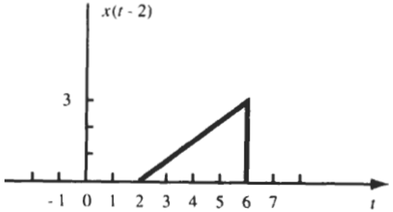
$(a) \quad y(t) = x(t-1)$

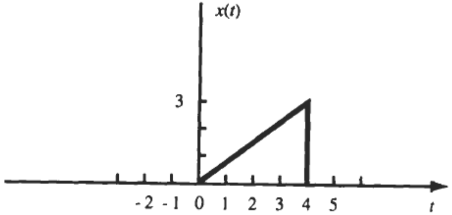
$(b)\ x(t-2) \qquad (c)\ x(2t) \qquad (d)\ x(\frac{t}{2})$

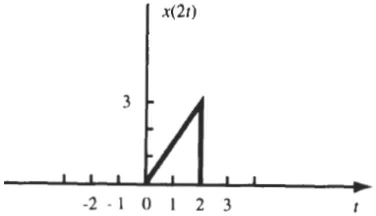


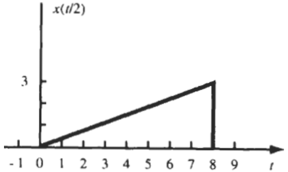




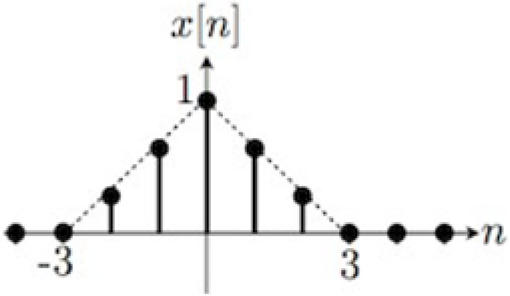




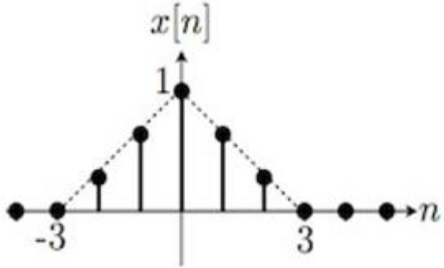


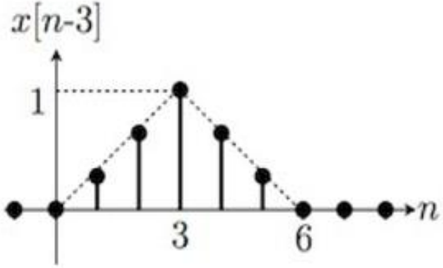


Given,



Determine the signal $y[n] = x[n-3]$?

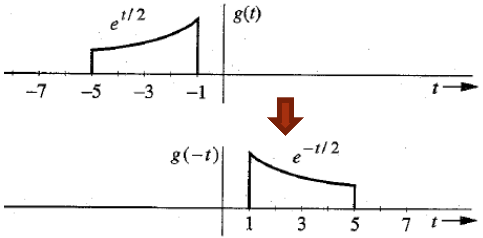


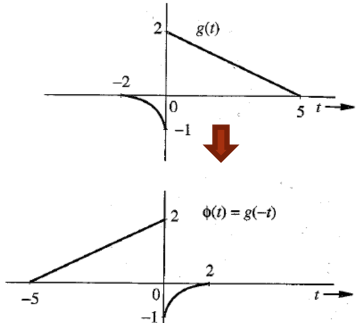


Reflection (Time Reversal/Inversion/folding)

- Generate an **mirror image of signal** with respect to Y-axis (at t =0)

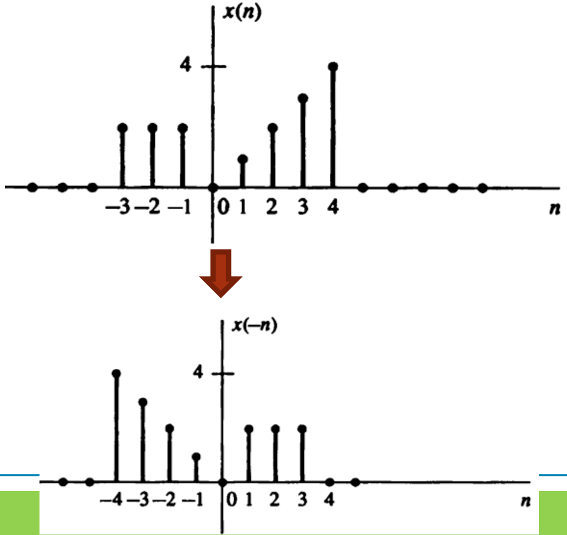
$$\phi(t) = g(-t)$$





Mathematically,

$$g(-t)\ at\ t = -1 \ \rightarrow g(1)$$
$$g(-t)\ at\ t = 0 \ \rightarrow g(0)$$
$$g(-t)\ at\ t = 1 \ \rightarrow g(1)$$



For discrete: Ø(n) = x(-n) (at time n=0 with respect to the Y- axis)

Mathematically,

$$x(-n)\ at\ n = -1 \ \rightarrow x(1)$$
$$x(-n)\ at\ n = 0 \ \rightarrow x(0)$$
$$x(-n)\ at\ n = 1 \ \rightarrow x(-1)$$

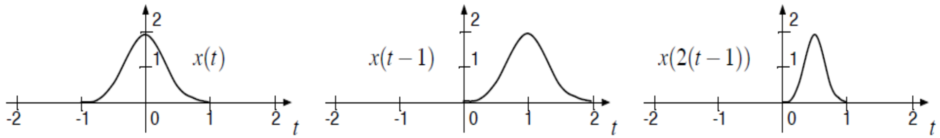
Shifting and scaling

$y(t) = x(2t\ - 2)$

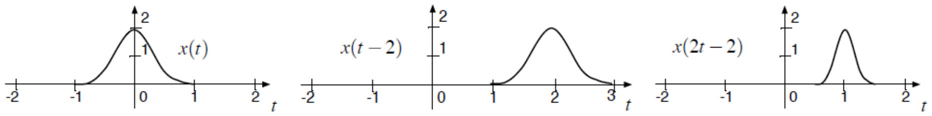
scaling shifting

The following which one is correct form of $x(2t-2)$?

Incorrect



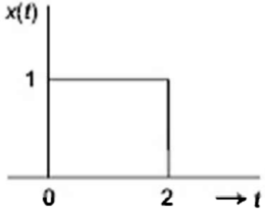
Correct



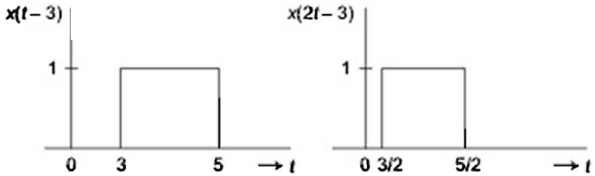
Precedence rule of time shifting and Scaling

$y(t) = x(2t-2) \rightarrow$ 1. $x(t-2)$ - First shifting
2. $x(2t-2)$ – Second scaling

Given $x(t)$:



Draw signal $x(2t-3)$



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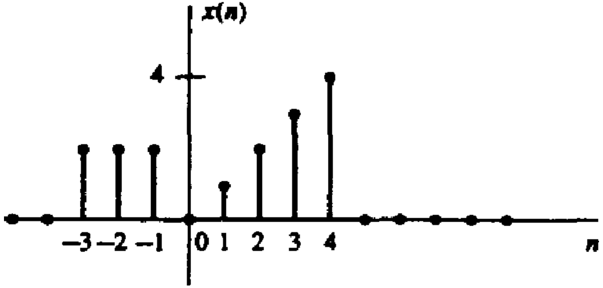
Solve the following questions:

Q1. $$x(t) = \begin{cases} e^{t/2} & -1 \geq t > -5 \\ 0 & \text{otherwise} \end{cases}$$

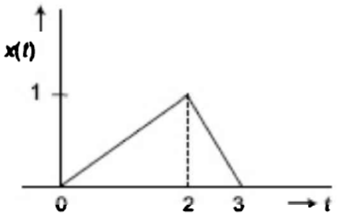
$$x(-t) = \begin{cases} e^{-t/2} & -1 \geq -t > -5 \quad \text{or} \quad 1 \leq t < 5 \\ 0 & \text{otherwise} \end{cases}$$

Evaluate the signal $x[n+2], x[-n]$



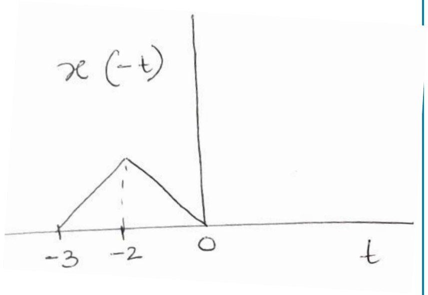
Precedence rule for time shifting and time scaling

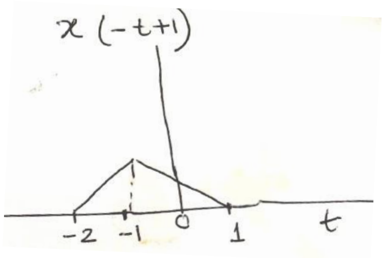
Given, $x(t)$



Precedence rule of time shifting and Scaling

1. Reflection/folding : $x(-t)$
2. Shifting: $x(-t+1)$ >> delay [right]
3. Scaling : $x(-2t+1)$





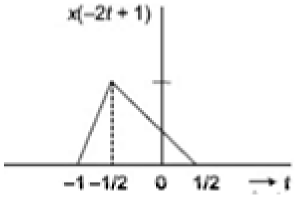
Draw the signal $\boldsymbol{x(-2t+1)}$

After rewriting $x(-2t+1) = x(2(-t) + 1)$

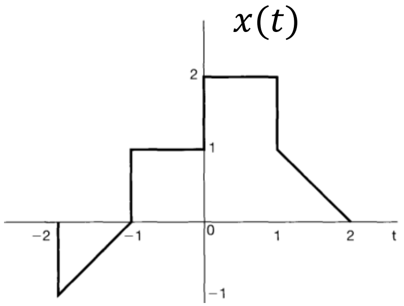
Scaling

Folding

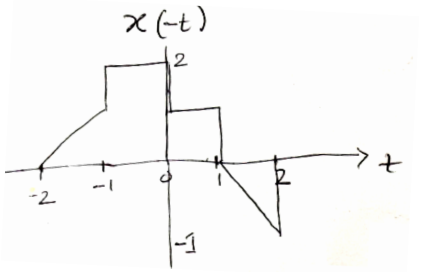
Shifting

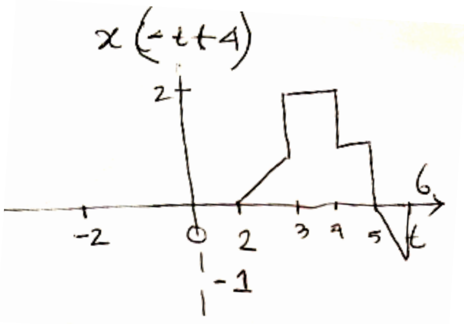


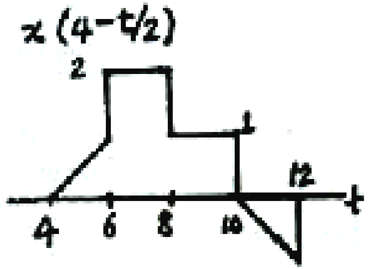
Given,



Plot the signal $x(4 - t/2)$?



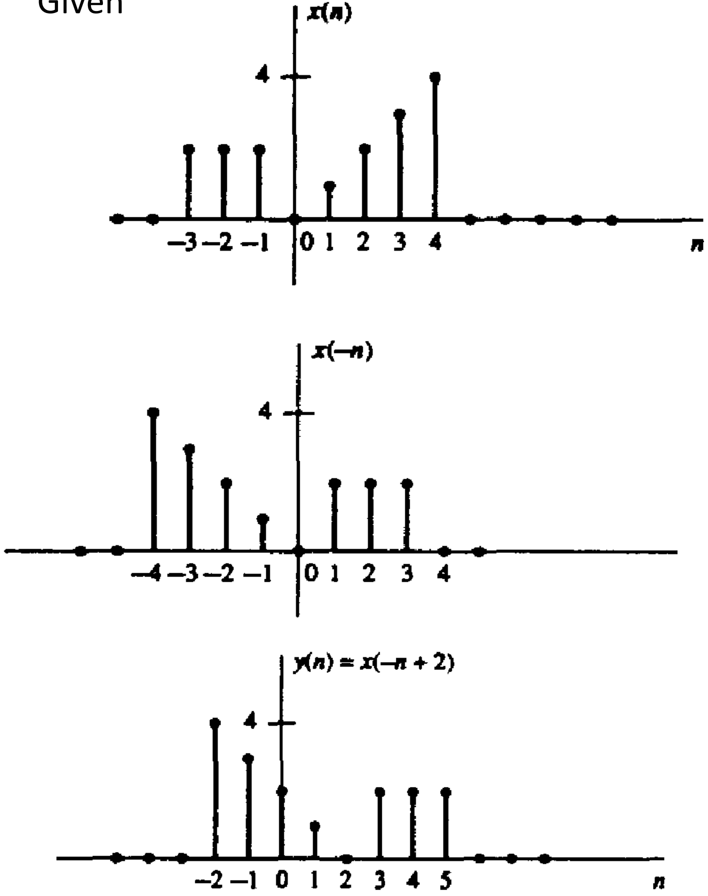




As per the Precedence rule of time shifting and Scaling

1. Reflection/folding : $x(-t)$
2. Shifting: $x(-t + 4)$ >> delay [right]
3. Scaling : $x(-t/2 + 4)$

Given



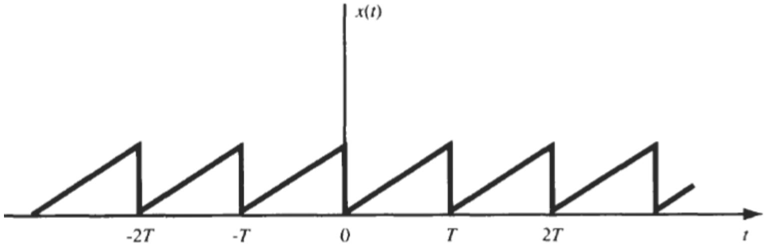
Evaluate the signal $x[-n+2]$

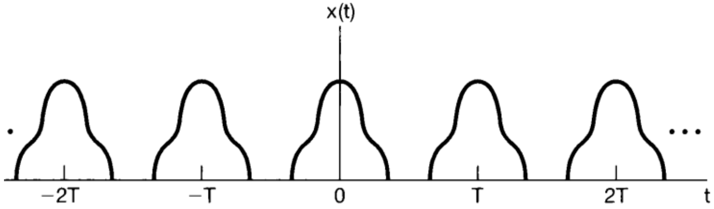
Periodic signal (Continuous signal)

- **Continuous signal**

A continuous-time signal $x(t)$ is said to be *periodic with period* T if there is a positive nonzero value of T for which

$$x(t+T) = x(t) \qquad \text{all } t$$





The *fundamental period* (say, T_0) $of\ x(t)$ → $is\ the$ smallest positive value of $\boldsymbol{T}$

Thank you

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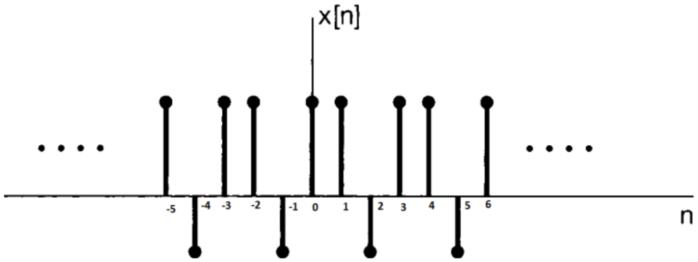
Periodic discrete signal

- **Discrete signal:** a discrete time signal x(n) *is* periodic *with period N*

$$x(n) = x(n+N) \quad \text{for all values of } n$$

where N is a ***positive integer****, if it is unchanged* by a time shift of ***N***, *i.e.*



The **fundamental period** N_0 is **the smallest positive value of *N*** for which it holds the above relation.

What is the fundamental period of the above given signal?

The fundamental period $\boldsymbol{N_0 = 3}$

Is the signal is periodic $x(t) = e^{j\omega t}$? If so what is fundamental period of the signal?

Using the relation for period $of\ a\ signal$, $x(t) = x(t+T)$

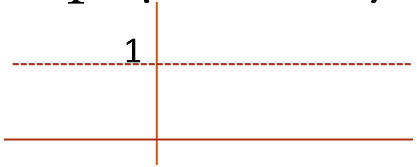
$$\rightarrow e^{j\omega t} = e^{j\omega(t+T)}$$

$$\rightarrow e^{j\omega t} = e^{j\omega t}.e^{j\omega T}$$

$$\rightarrow e^{j\omega T} = 1 \quad = e^{j.0.T}$$

Case 1: $\omega = 0$, $e^{j.0.t} = \cos(0.t) + j\sin(0.t) = 1$

$\rightarrow$ $x(t) = e^{j\omega=0t} = 1$ **periodic** for **any value** of **T**



Case 2:

$$e^{j\omega T} = 1$$

$$\rightarrow e^{j\omega T} = 1 = e^{j2\pi m}$$

(m integer)

Short-cut

$$\rightarrow j\omega T = j2\pi m$$

$$\rightarrow T = \frac{2\pi m}{\omega}$$

$$\omega = 2\pi$$

$$e^{j.2\pi.m} = \cos(2\pi.m) + j\sin(2\pi.m)$$

$$e^{j2\pi m} = \cos(4.\frac{\pi}{2}m + 0) + j\sin(4.\frac{\pi}{2}m + 0)$$

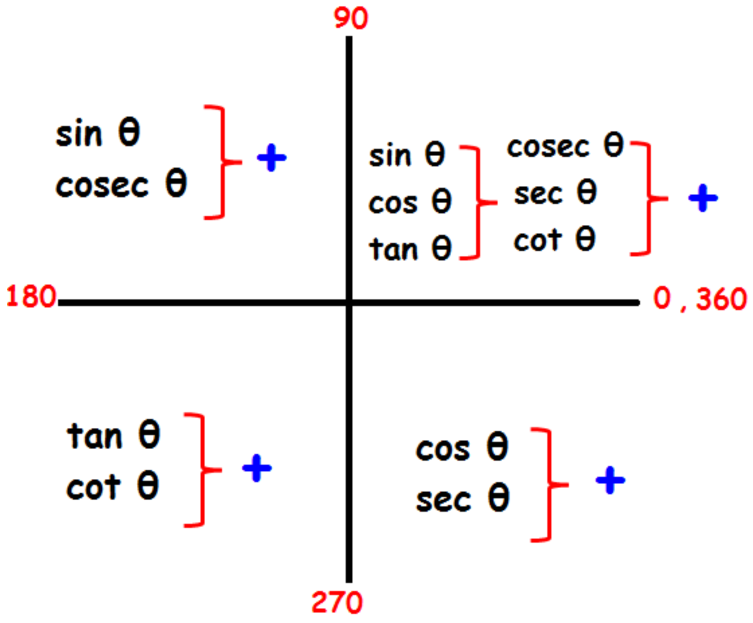
$$e^{j2\pi m} = \cos(0) + j\sin(0) = 1$$

The *fundamental period* (say, T_0) $of\ x(t)$

$\rightarrow$ $is\ the$ smallest positive value of $\boldsymbol{T}$

$$\boldsymbol{T_0 = \frac{2\pi(m=1)}{|\omega|} = \frac{2\pi}{|\omega|}}$$

Should be rational



$$\boxed{\sin 2A} = \sin(A + A) = \sin A\cos A + \cos A\sin A$$
$$= 2\sin A\cos A$$
$$= 2\sin A\sqrt{1 - \sin^2 A}$$

$$\boxed{\cos 2A} = \cos(A + A) = \cos A\cos A - \sin A\sin A$$
$$= \cos^2 A - \sin^2 A = \cos^2 A - (1 - \cos^2 A)$$
$$= 2\cos^2 A - 1$$

$$\boxed{\tan 2A} = \tan(A + A) = \frac{\tan A + \tan A}{1 - \tan A\tan A} = \frac{2\tan A}{1 - \tan^2 A}$$

or

$$\boxed{\tan 2A} = \frac{\sin 2A}{\cos 2A} = \frac{2\sin A\cos A}{\cos^2 A - \sin^2 A} = \frac{2\,\dfrac{\sin A\cos A}{\cos^2 A}}{\dfrac{\cos^2 A}{\cos^2 A} - \dfrac{\sin^2 A}{\cos^2 A}}$$

$$= \frac{2\tan A}{1 - \tan^2 A}$$

Evaluate the fundamental period of signal $x(t) = e^{-j\omega t}$

Hints.

$$x(t) = x(t+T)$$
$$\rightarrow\ e^{-j\omega t} = e^{-j\omega(t+T)}$$
$$\rightarrow\ e^{-j\omega t} = e^{-j\omega t}.e^{-j\omega T}$$
$$\rightarrow\ e^{-j\omega T} = 1$$

1. For ω=0, x(t) = 1 → **periodic for** any value of **T**
2. For ω=2πm

$$e^{-j\omega T} = 1 = e^{j2\pi m}$$
$$\rightarrow\ -j\omega T = j2\pi m$$
$$\rightarrow T = \frac{2\pi m}{-\omega}$$

Short-cut

As per the definition, the *fundamental period* (say, T_0) $of\ x(t)$

→ *is the* smallest positive value *of* $\boldsymbol{T}$

$$T_0 = \frac{2\pi(m=1)}{|-\omega|} = \frac{2\pi}{|\omega|}$$

- Determine whether the following signal are periodic? If so, what is the value of period?

$$(a)\cos(\frac{\pi n}{2})$$

$$(b)\sin(\frac{\pi n}{8})$$

Short-cut:

$$\frac{\pi N}{2} = 2\pi k$$
$$\rightarrow N = 2 \times 2k$$
N_0 = 2x2 =4 (k=1)

(a)

$$x(n) = x(n+N)$$

N *is a* positive integer

$$\rightarrow \cos(\frac{\pi n}{2}) = \cos(\frac{\pi}{2}(n+N)) = \cos(\frac{\pi}{2}n)\cos(\frac{\pi}{2}N) - \sin(\frac{\pi n}{2})\sin(\frac{\pi N}{2}) = \cos(\frac{\pi}{2}n)$$

If $\cos(\pi N/2) = 1$, i.e. $(\boldsymbol{\pi N/2})$ **should have** $2\pi k$ **form for integer value of $\boldsymbol{k}$**

$$\frac{\pi N}{2} = 2\pi k$$
$$\rightarrow N = 2 \times 2k$$

Fundamental period N_0 = 2x2(k=1)=4

- The **fundamental period** N_0 is **the smallest positive value of N** for which it holds
$$x(n) = x(n+N)$$

- Determine whether the following signal is periodic? If so, what is the value of period?

$$x(n) = e^{j(2\pi/3)n} + e^{j(3\pi/4)n}$$

We can write, $\boldsymbol{x(n) = x_1(n) + x_2(n)}$

For the fundamental period of $x_1(n)$, it should satisfy

$$x(n) = x(n+N) \quad \text{for all values of } n \quad \textit{where } N \textit{ is a positive integer}$$

Now, we can get $\quad x(n) = x(n+N)$

$$\rightarrow e^{j\left(\frac{2\pi}{3}\right)n} = e^{\frac{j2\pi}{3}(n+N)} = e^{j\left(\frac{2\pi}{3}\right)n} . e^{j\left(\frac{2\pi}{3}\right)N}$$

$$\rightarrow e^{j\left(\frac{2\pi}{3}\right)N} = 1$$

$$\rightarrow e^{j\left(\frac{2\pi}{3}\right)N} = 1 = e^{j2\pi m} \text{ (m integer)} \longrightarrow$$

$$\omega = 2\pi m$$
$$e^{j.2\pi.m} = \cos(2\pi.m) + j\sin(2\pi.m)$$
$$e^{j2\pi m} = \cos(4.\frac{\pi}{2}m + 0) + j\sin(4.\frac{\pi}{2}m + 0)$$
$$e^{j2\pi m} = \cos(0) + j\sin(0) = 1$$

$$\rightarrow j\left(\frac{2\pi}{3}\right)N = j2\pi m$$

$$\rightarrow N = \frac{2\pi m}{\left(\frac{2\pi}{3}\right)} = 2\pi m \times \left(\frac{3}{2\pi}\right) = 3m = 3 \ (m = 1)$$

The fundamental period of $x_1(n)$ is **3**

Similarly, for the $\boldsymbol{x_2(n) = e^{j\left(\frac{3\pi}{4}\right)n}}$

$$x(n) = x(n+N)$$

$$\rightarrow e^{j\left(\frac{3\pi}{4}\right)n} = e^{\frac{j3\pi}{4}(n+N)} = e^{j\left(\frac{3\pi}{4}\right)n} . e^{j\left(\frac{3\pi}{4}\right)N}$$

$$\rightarrow e^{j\left(\frac{3\pi}{4}\right)N} = 1$$

$$\rightarrow e^{j\left(\frac{3\pi}{4}\right)N} = 1 = e^{j2\pi m}$$

$\rightarrow j\left(\frac{3\pi}{4}\right)N = j2\pi m$ for m integer

$$\rightarrow N = \frac{2\pi m}{\left(\frac{3\pi}{4}\right)} = 2\pi m \times \left(\frac{4}{3\pi}\right) = 8m/3$$

$$= 8\ (m = 3)$$

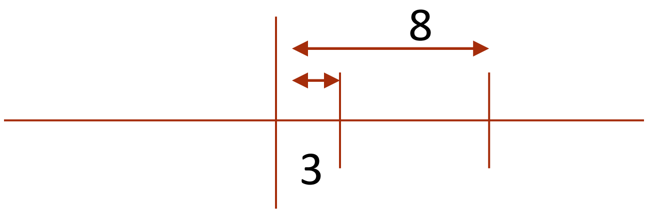
The fundamental period of $x_2(n)$ is 8

The fundamental period of $x_1(n)$ is **3**

Short-cut:

$$\rightarrow j\left(\frac{3\pi}{4}\right)N = j2\pi m$$

$$\rightarrow N = \frac{2\pi m}{\left(\frac{3\pi}{4}\right)} = 2\pi m \times \left(\frac{4}{3\pi}\right) = 8m/3 = 8\ (m = 3)$$



What is the period of x[n]? LCM(3,8) = 24

What is the Least Common Multiple of 4 and 6?

Multiples of 4 are:

4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, ...

and the multiples of 6 are:

6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, ...

Common multiples of 4 and 6 are simply the numbers that are in both lists:

12, 24, 36, 48, 60, 72, ...

So, from this list of the first few common multiples of the numbers 4 and 6,

their *least common multiple* is 12.

Determine whether the following signal is periodic?

$$\boldsymbol{x(t) = 2sin\left(\frac{2}{3}\right)t + 3\,cos\left(\frac{2\pi}{5}\right)t}$$

We express $x(t) = x_1(t) + x_2(t)$

For $x_1(t) = x_1(t+T)$

$$\rightarrow 2\sin\left(\frac{2}{3}\right)t = 2\sin(\frac{2}{3})(t+T)$$

With short-cut

$$\rightarrow \left(\frac{2}{3}\right)T = 2\pi m,\ \text{(m integer)}$$

$$\rightarrow T = 2\pi m \times \left(\frac{3}{2}\right) = 3\pi m$$

$$\rightarrow T_0 = 3\pi\ (\text{for smallest positive value of T, m} = 1)$$

The fundamental period of $x_1(t) = 2\sin\left(\frac{2}{3}\right)t = 3\pi$

Similarly, for $x_2(t) = x_2(t+T)$

$$\rightarrow \left(\frac{2\pi}{5}\right)T = 2\pi m,\ \text{(m integer)}$$

$$\rightarrow T = 2\pi m \times \left(\frac{5}{2\pi}\right) = 5m$$

$$\rightarrow T_0 = 5\ (\text{for smallest positive value of T, m} = 1)$$

The fundamental period of $x_2(t) = 2\cos\left(\frac{2\pi}{5}\right)t$ =5

The fundamental period of $x(t)$ *is*
LCM$(3\pi, 5) = ?$ (*does not exist*)
$\rightarrow$ *Signal Aperiodic*

Determine whether the following signal is periodic?

$$y(t) = 3\,sin\,(t) + 5\,cos\,\left(\frac{4}{3}\right)t$$

The period of $3\,sin\,(t)$ is

$\rightarrow T = 2\pi m,$ (m integer)

$\rightarrow\ T_0 = 2\pi$ (for smallest positive value of T, m = 1)

Method-1:

The period of $5\,cos\,(4/3)t$ is

$\rightarrow (\frac{4}{3})T = 2\pi m,$ (m integer)

$\rightarrow T = 2\pi m \times \left(\frac{3}{4}\right) = (3\pi m/2)$

$\rightarrow\ T_0 = 3\pi/2$ (for smallest positive value of T, m = 1)

The period of $y(t)$
LCM$(2\pi, 3\pi/2)$
$= 6\pi$

Method-2: The ratio of fundamental periods of both signal $= \frac{T_1}{T_2} = \frac{2\pi}{\frac{3\pi}{2}} = 2\pi \times \frac{2}{3\pi} = \frac{4}{3}$, a rational number

$\rightarrow$ y(t)is periodic

A rational number in the form $\frac{p}{q}$; where p and q are integers and q ≠0

e.g. $\frac{3}{5}, \frac{-3}{10}, \frac{11}{-15}$

e.g. not rational $\frac{0}{0}$

Thank you

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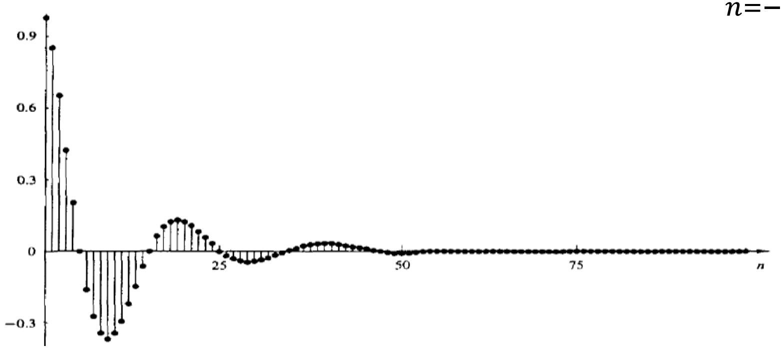
Energy signal

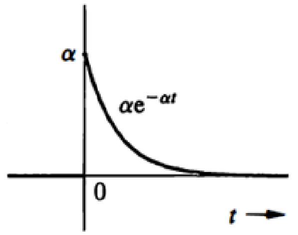
- The energy (E) of a signal can be **finite or infinite**.
- A **signal** with **finite energy** is an energy signal (if E *is finite* $(0 < E < \propto)$, *the signal is called an energy signal)*

Discrete signal: The energy of a signal $x(n)$ is defined as $$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$



Continuous signal: The energy of continuous-time signal x(t) is defined as

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$



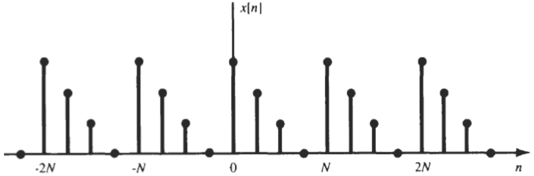
Power signal

- Signal with **finite and nonzero power** is a power signal.
- Many signals that possess infinite energy, have a finite average power
- The signal power is the time average of its energy

Discrete signal: The average power of a discrete-time signal $x(n)$ *is defined as*

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

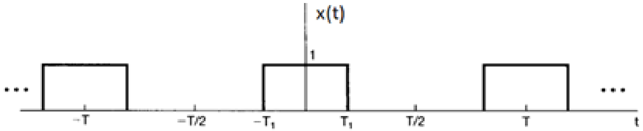
$$= \lim_{N\to\infty} \frac{1}{2N+1} E$$



N is the fundamental period of signal

Continuous signal: The average power of a continuous-time signal $x(t)$ *is* defined as

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$



T is the fundamental period of the signal

Check whether the following signals are energy/power signals?

$(a)\quad x(t) = \begin{cases} A\,; 0 < t < T_0 \\ 0; \quad otherwise \end{cases}$ $\qquad (b)\quad x[n] = 2e^{j3n}$ $\qquad (c)\quad x(t) = \text{A}\cos(\omega_0 t + \theta)$

Energy of the signal:

$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

$$E = \int_{-\infty}^{0} |x(t)|^2\, dt + \int_{0}^{T_0} |x(t)|^2 dt + \int_{T_0}^{\infty} |x(t)|^2 dt$$

$$= 0 + \int_{0}^{T_0} |A|^2 dt + 0 = A^2[t]_0^{T_0}$$

$$= A^2[T_0 - 0]$$

$$= A^2 T_0$$

Energy has finite value

Let's calculate Power of the signal:

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\, dt$$

$$P = \lim_{T\to\infty} \frac{1}{T} [\int_{-T/2}^{0} |x(t)|^2\, dt + \int_{0}^{T_0} |x(t)|^2\, dt + \int_{T_0}^{T/2} |x(t)|^2\, dt]$$

$$= \lim_{T\to\infty} \frac{1}{T} [\int_{-T/2}^{0} |0|^2\, dt + \int_{0}^{T_0} |A|^2\, dt + \int_{T_0}^{T/2} |0|^2\, dt]$$

$$= \lim_{T\to\infty} \frac{1}{T} [\int_{0}^{T_0} |A|^2\, dt = A^2 T_0]$$

$$= \lim_{T\to\infty} \frac{1}{T} A^2\, T_0 = 0$$

Power =0

Cont..

$(b)\quad x[n] = 2e^{j3n}$

Power of the signal:

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|2e^{j3n}\right|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |2|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1}\ 4\ \sum_{n=-N}^{N} 1$$

$$= \lim_{N\to\infty} \frac{1}{2N+1}\ 4\,(2N+1) = 4 < \infty$$

$$\left|e^{j3n}\right| = |\cos 3n + j \sin 3n|$$
$$= \sqrt{\cos^2 3n + \sin^2 3n}$$
$$= 1$$

Signal has finite power → power signal

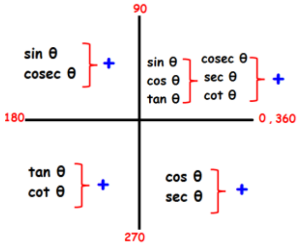
Cont..

$(c)\quad x(t) = A\cos(\omega_0 t + \theta)$

The signal is periodic. So it is a power signal

Let's calculate the fundamental period for the justification

By definition of periodic of a signal, it should satisfy: $x(t) = x(t+T)$



$$A\cos(\omega_0 t + \theta) = A\cos(\omega_0(t+T) + \theta)$$
$$\rightarrow A\cos(\omega_0 t + \theta) = A\cos((\omega_0 t + \theta) + \omega_0 T)$$
$$= A[\cos(\omega_0 t + \theta).\cos(\omega_0 T) - \sin(\omega_0 t + \theta).\sin(\omega_0 T)]$$

If $(\omega_0 T)$ is in the form of $(2\pi k)$ for a integer value **of k**
$\rightarrow \cos(2\pi k) = 1 \; and \; \sin(2\pi k) = 0$

$$= A\cos(\omega_0 t + \theta)$$

$$\omega_0 T = 2\pi k$$
$$T = \frac{2\pi k}{\omega_0}$$

Fundamental period is the smallest value of **T** =(2π.k=1)/ω_0

$$i.e.\;\; T_0 = \frac{2\pi}{|\omega_0|}$$

Cont..

(c) $x(t) = \text{A}\cos(\omega_0 t + \theta)$ $\qquad T = \dfrac{2\pi}{|\omega_0|}$

Using the definition power of continuous-time signal:

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\,dt \quad = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |A\cos(\omega_0 t + \theta)|^2\,dt$$

$[1 + \cos 2A = 2\cos^2 A]$

$\int \cos A = \sin A$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} A^2 cos^2(\omega_0 t + \theta)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \frac{A^2}{2}[1 + cos2\,(\omega_0 t + \theta)]\quad dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \frac{A^2}{2}\,dt + \lim_{T\to\infty} \frac{A^2}{2T}\int_{-T/2}^{T/2} \cos 2\,(\omega_0 t + \theta)]\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\,\frac{A^2}{2}[t]_{-T/2}^{T/2} + \lim_{T\to\infty} \frac{1}{4\omega_0 T}[\sin\, 2(\omega_0 t + \theta)]\,_{-T/2}^{T/2}$$

$$= \lim_{T\to\infty} \frac{1}{T}\,\frac{A^2}{2}T + 0 = \frac{A^2}{2} < \infty$$

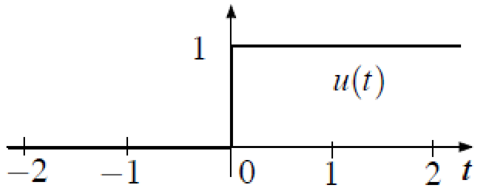
Signal has finite power

Step function

Continuous:

The *unit step function* $u(t)$ is defined as

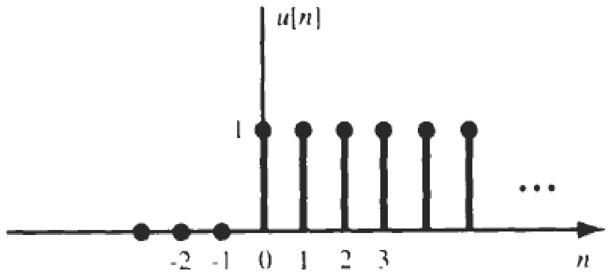
$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



Discrete:

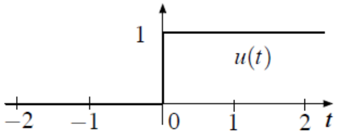
The *unit step* sequence $u[n]$ is defined as

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



Q1. Express the following function using $u(t)$

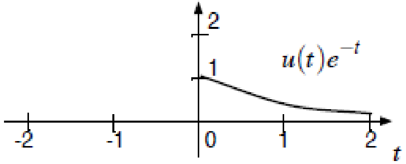
$$x(t) = \begin{cases} e^{-t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



Step function

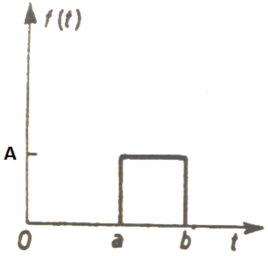
can be written as

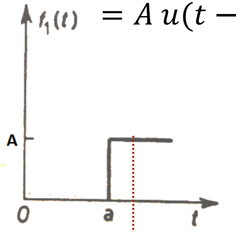
$$x(t) = u(t)e^{-t}$$



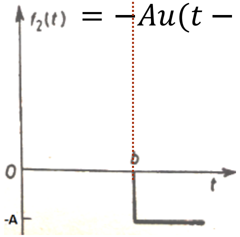
Q2. Represent the following function using $u(t)$

$$f(t) = \begin{cases} 0 & t < a \\ A & a < t < b \\ 0 & t > b \end{cases}$$



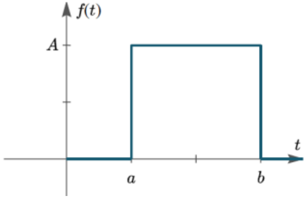


$= A\,u(t-a)$



$= -Au(t-b)$

$$f(t) = A \cdot [u(t-a) - u(t-b)]$$



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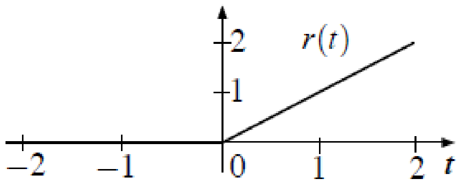
Ramp function

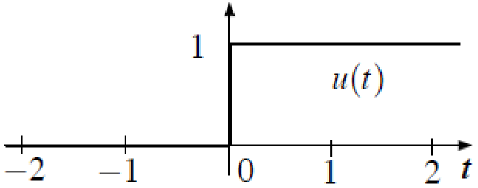
Continuous-time signal: The *unit ramp* is defined as

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



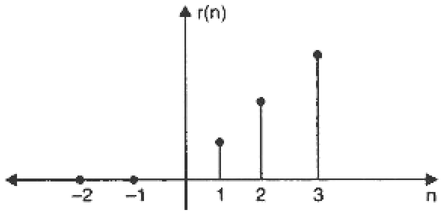
The unit ramp is the integral of the unit step,

$$r(t) = \int_{-\infty}^{t} u(\tau)d\tau$$

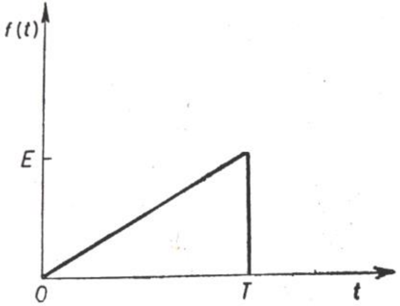


Discrete-time signal: The unit –ramp signal is defined as,

$$r(n) = \begin{cases} 0, & n < 0 \\ n, & n \geq 0 \end{cases}$$

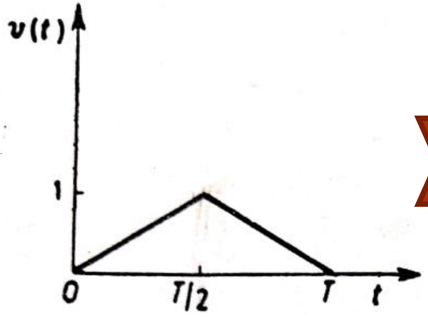


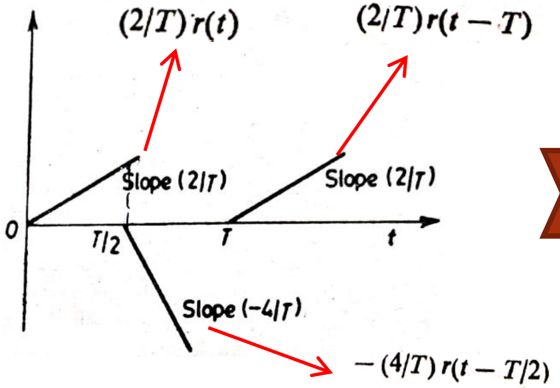
Q1. Express the following signal using $u(t)$



$$f(t) = \frac{E}{T}t[u(t) - u(t-T)]$$

Q2. Express the following signal using ramp r(t)





$$v(t) = (2/T)\,r(t) - (4/T)\,r(t - T/2) + (2/T)\,r(t - T)$$

Determine whether signals are energy/power/zero signal

$Q1.\ \ x(t) = e^{-at}\, u(t), \qquad a > 0$

We know, $u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$

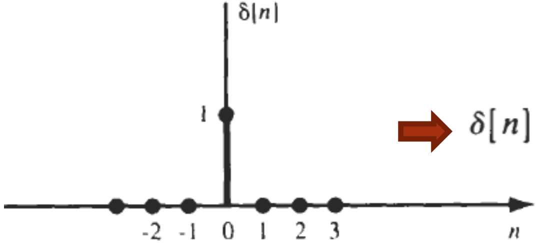
$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_0^{\infty} e^{-2at}\, dt = \frac{1}{2a} < \infty$$

$Q2.\ \ x[n] = (-0.5)^n u[n]$

Impulse function

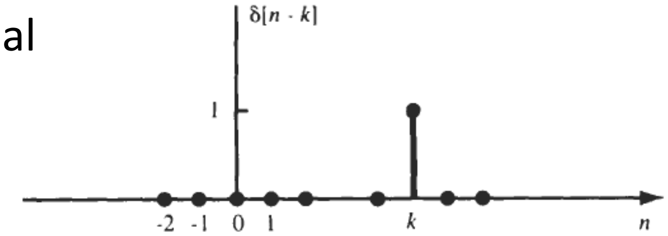
Discrete: The unit-impulse signal is called "unit-sample signal" and is defined as

$$\delta(n) = \begin{cases} 1, n = 0 \\ 0, n \neq 0 \end{cases}$$



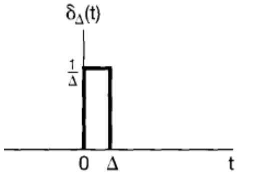
$\delta[n] = u[n] - u[n-1]$

The delayed of "k" unit of unit-impulse signal



Continuous: The unit-impulse function in the continues-time signal is defined as

$\delta_\Delta(t)$ is a short pulse, of duration Δ and with unit area for any value of Δ.
As $\Delta \to 0$, $\delta_\Delta(t)$ becomes narrower and higher, maintaining its unit area.



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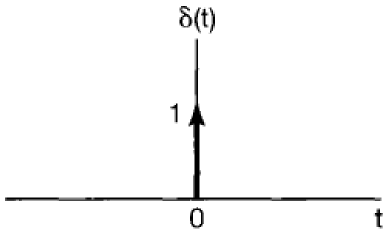
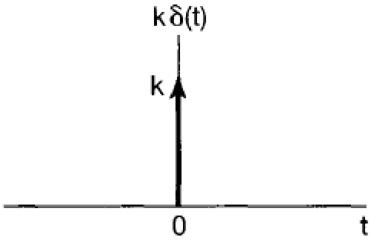
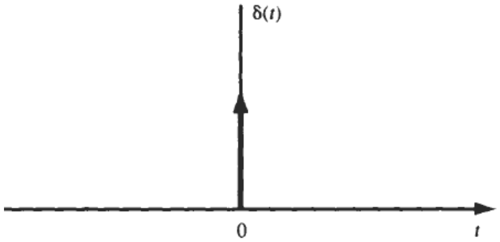


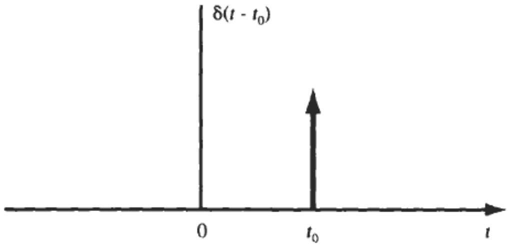


$$\int_{-\infty}^{t} k\delta(t)dt = k$$

In general, we can write:

$$\boxed{\int_{-\infty}^{\infty} \emptyset(t)\delta(t)dt = \emptyset(0)}$$



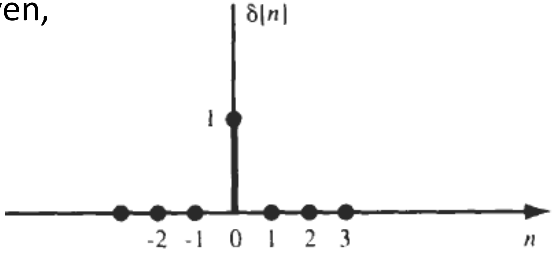


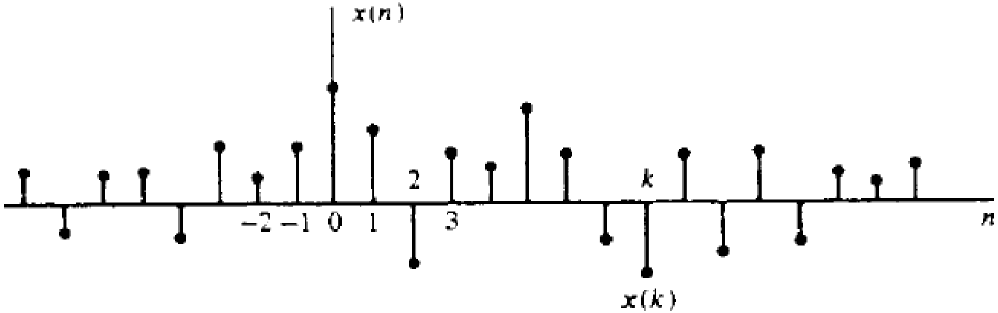
(a) $\mathrm{t}\delta(\mathrm{t}) = ?$

(b) $\sin \mathrm{t}\, \delta(\mathrm{t}) = ?$

(c) $\cos \mathrm{t}\delta(\mathrm{t} - \pi) = ?$

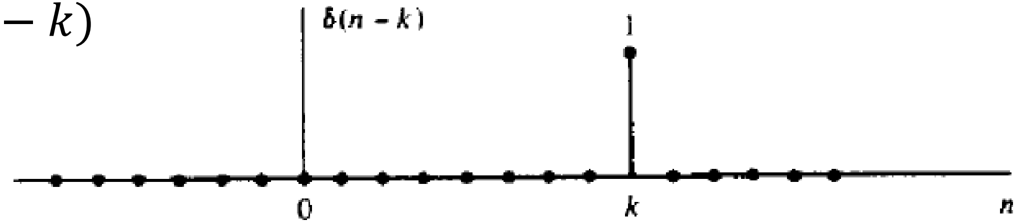
Given,

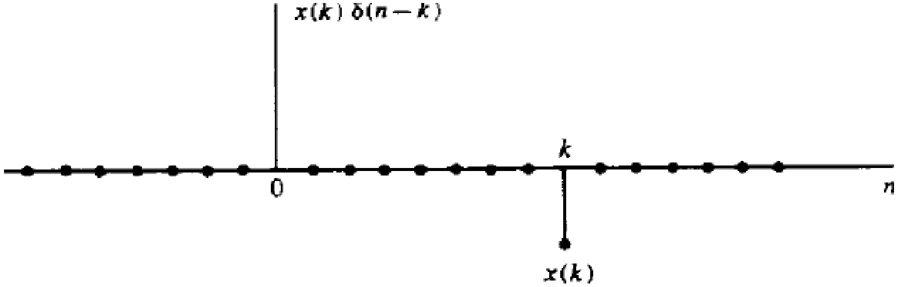




Evaluate the signal,

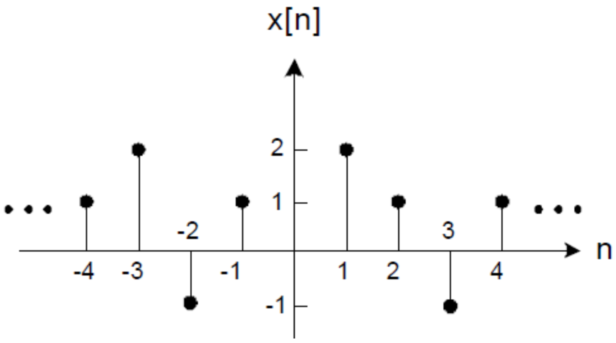
$x(n)\delta(n-k) = ? \quad = \; x(k)\delta(n-k)$

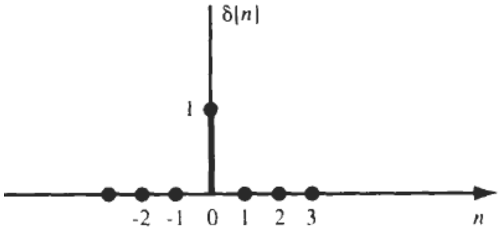




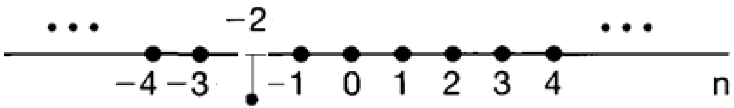
Representation of an arbitrary discrete signal

- A discrete-time signal can be decomposed into a **sequence of individual impulses**



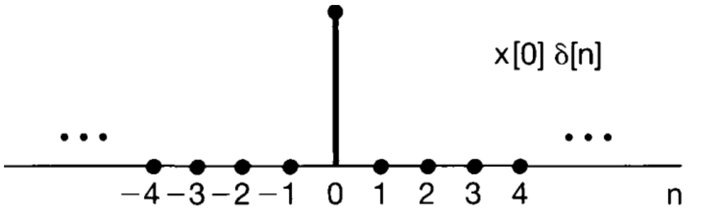


x[−2] δ[n + 2]

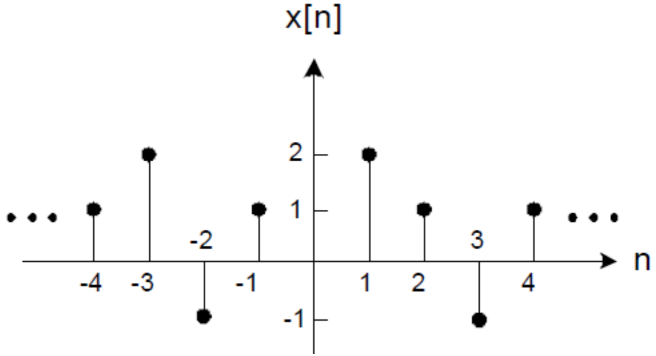


$$x[-1]\delta[n+1] = \begin{cases} x[-1], & n = -1 \\ 0, & n \neq -1 \end{cases}$$

$$x[0]\delta[n] = \begin{cases} x[0], & n = 0 \\ 0, & n \neq 0 \end{cases}$$



Cont..



The signal in Fig. can **be expressed** as a **sum of the *shifted impulses***

$$x[n] = \ldots + x[-3]\delta[n+3] + x[-2]\delta[n+2] + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + x[2]\delta[n-2] + \ldots$$

or in a **more compact** form

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

Right-hand side represents an arbitrary signal x(n)   as weighted (scaled) sum of shifted impulse sequence

Evaluate the following integrals

$(a)\ \int_{-1}^{1}(3t^2+1)\delta(t)\ dt$

$(b)\ \int_{1}^{2}(3t^2+1)\delta(t)dt$

$(c)\ \int_{-\infty}^{\infty}(t^2+\cos(\pi t))\delta(t-1)dt$

$(d)\ \int_{-\infty}^{\infty}e^{-t}\delta(2t\ -2)\ dt$

$(e)\ \int_{-\infty}^{\infty}e^{-t}\ \dot{\delta}\ (\mathrm{t})\ \mathrm{dt}$

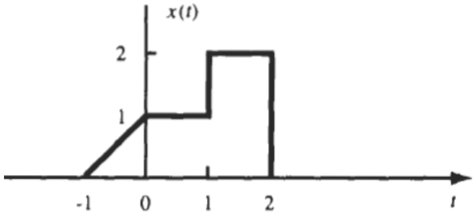
$$\int F(x)G'(x)\,dx = F(x)G(x) - \int F'(x)G(x)\,dx + C.$$

$$\int_{-\infty}^{\infty}\emptyset(t)g^{n}(t)dt = (-1)^{n}\int_{-\infty}^{\infty}\emptyset^{n}(t)\ g(t)\ dt$$

- Draw the following signal for given x(t)

$(a)\ x(t)\delta\left(t-\frac{3}{2}\right)$

$(b)\ x(t)u(1-t)$



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Evaluate the following integrals

$(a)\ \int_{-1}^{1}(3t^2+1)\delta(t)\,dt$

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$(c)\ \int_{-\infty}^{\infty}(t^2+\cos(\pi t))\delta(t-1)dt$

$(d)\ \int_{-\infty}^{\infty}e^{-t}\delta(2t\ -2)\,dt$

$(e)\ \int_{-\infty}^{\infty}e^{-t}\,\dot{\delta}\,(\mathrm{t})\,\mathrm{dt}$

(a) $\int_{-1}^{1}(3t^2+1)\delta(t)dt=(3t^2+1)|_{t=0}$

$$(b)\int_{1}^{2}(3t^2+1)\delta(t)dt=0$$

$$(c)\int_{-\infty}^{\infty}(t^2+\cos\quad \pi t)\,\delta(t-1)dt$$

$$=(t^2+\cos\pi t)|_{t=1}$$

$$=1+\cos\pi=1\ -1=0$$

$$\int F(x)G'(x)\,dx=F(x)G(x)-\int F'(x)G(x)\,dx+C.$$

$$\int_{-\infty}^{\infty}\emptyset(t)g^n(t)dt=\ (-1)^n\int_{-\infty}^{\infty}\emptyset^n(t)\ g(t)\ dt$$

- Draw the following signal for given x(t)

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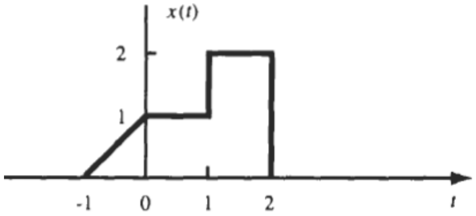
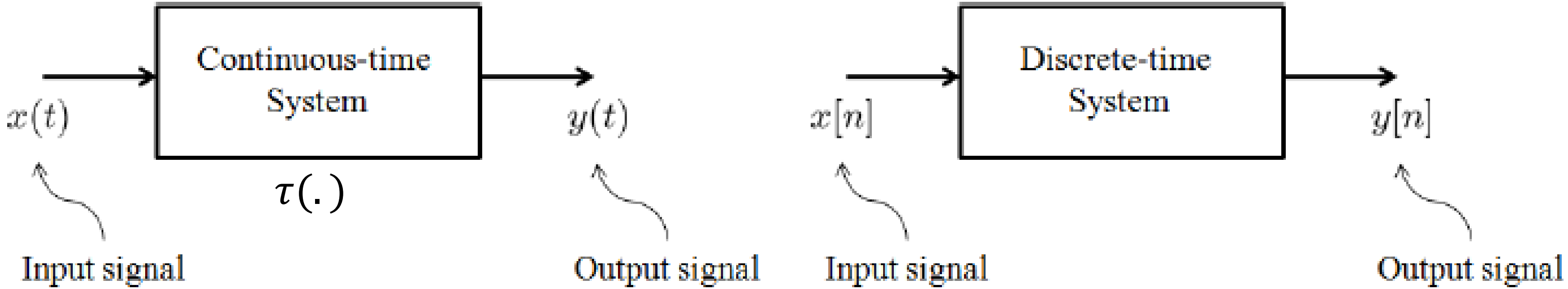


Types of systems

- System is a device or algorithm which process or transforms an input signal into an desired output signal

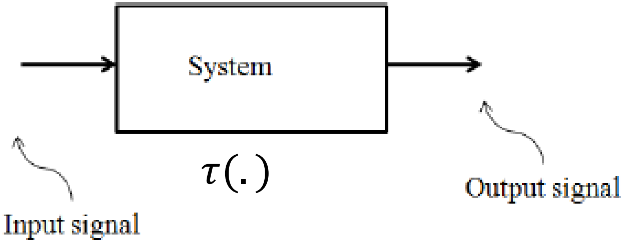


A system is an operator $\mathcal{T}$ which maps input into output:

$$y(t) = \mathcal{T}\{x(t)\} \quad \text{or} \quad y[n] = \mathcal{T}\{x[n]\}$$

Linearity- property

- A *linear system, in* ***continuous time or discrete time****, is a system that possesses the important* property of **additivity, Scaling or superposition**:



a) **Additivity**: Given that $\tau\{x_1\} = y_1$ and $\tau\{x_2\} = y_2$

b) **Homogeneity (or Scaling):** $\tau\{\alpha x\} = \alpha y$

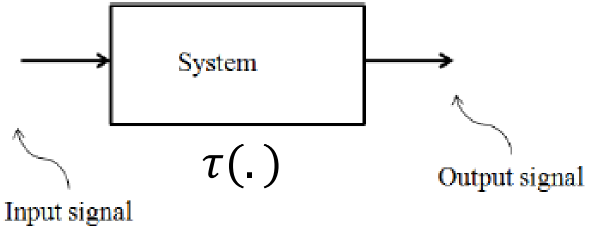
- [ ] **(a) & (b) Superposition:**

$$\tau\{x_1 + x_2\} = y_1 + y_2$$
$$\tau\{\alpha_1 x_1 + \alpha_2 x_2\} = \alpha_1 y_1 + \alpha_2 y_2$$

where α_1 and α_2 are arbitrary scalars

Time invariance

Continuous system: A system is called time-invariant if a time shift (delay or advance) in the input signal causes the **same time shift** in the output signal



Mathematically, $\tau\{x(t-\eta)\} = y(t-\eta)$ for any real value of η

Discrete system: The system is time-invariant (or shift-invariant) if

$$\tau\{x[n-k]\} = y[n-k]$$ for any **integer *k***

If a system which does not satisfy the above relation is called time-varying system

Determine whether systems are linear or non-linear

$(a)\ y(t) = t\ x(t)$

$(b)\ y[n] = 2\ x[n] + 3$

$(c)\ y[n] = Re\{x[n]\}$

$(d)\ y(n) = x(n^2)$

(a) Let's considers two arbitrary inputs $x_1(t)$ & $x_2(t)$

$$x_1(t) \rightarrow y_1(t) = tx_1(t)$$
$$x_2(t) \rightarrow y_2(t) = tx_2(t)$$

$$Let\ , x_3\ (t) = ax_1(t) + b\ x_2(t)$$

an inputs linear combination $\boldsymbol{of}\ x_1\ (t)\ and\ x_2(t)$

$$x_3(t) \rightarrow y_3(t) = tx_3(t)$$
$$= t(ax_1(t) + b\ x_2(t))$$
$$= atx_1(t) + bt\ x_2(t)$$
$$= ay_1(t) + b\ y_2(t)$$

As per the superposition law of linearity, it satisfies it i.e.

$$\tau\{\alpha_1 x_1(t) + \ \alpha_2 x_2(t)\} = \alpha_1 y_1(t) + \alpha_2 y_2(t)$$

The system (a) is linear

Cont..

$(b)\ y[n] = 2\,x[n] + 3$

Let's considers two arbitrary inputs $x_1[n]$ & $x_2[n]$

$$x_1[n] \to y_1[n] = 2x_1[n] + 3$$
$$x_2[n] \to y_2[n] = 2x_2[\mathrm{n}] + 3$$

$$Let\ , x_3\,[n] = ax_1[n] + b\,x_2[n]$$

another inputs linear combination $\boldsymbol{of}\ x_1[n] and\ x_2[n]$

$$\begin{aligned} x_3[n] \to y_3[n] &= 2x_3[n] + 3 \\ &= 2[ax_1[n] + b\,x_2[n]] + 3 \\ &= a\,2x_1[n] + b\,2x_2[n] + 3 \\ &\neq a\,y_1[n] + b\,y_2[n] \end{aligned}$$

It does not satisfy the superposition law for the linearity

The system (b) is not linear.

Cont..

$(c)\ y[n] = Re\{x[n]\}$

Let's input $x_1[n] = r[n] + js[n]$

$$x_1[n] \rightarrow y_1[n] = \mathrm{Re}\{x_1[n]\} = \mathrm{Re}\{r[n] + js[n]\}$$

Let's another input scaled of version $x_2[n] = ax_1[n]$ (scaling property)

$$x_2[n] \rightarrow y_2[n] = \mathrm{Re}\{x_2[n]\}$$
$$= \mathrm{Re}\{a(r[n] + js[n])\}$$
$$= \mathrm{Re}\{ar[n] + jas[n]\}$$
$$= \mathrm{Re}\{jr[n] - s[n]\}$$ If a=j
$$= -s[n]$$

$$ax_1[n] \rightarrow\neq ay_1[n]$$

System is not linear

Cont..

$(d)\ y(n) = x(n^2)$

Let's considers two arbitrary inputs $x_1[n]$ & $x_2[n]$

$$x_1[n] \rightarrow y_1[n] = x_1[n^2]$$
$$x_2[n] \rightarrow y_2[n] = x_2[n^2]$$

$$Let\ , x_3\ [n] = ax_1[n] + b\ x_2[n]$$

an inputs linear combination $\boldsymbol{of}$ $x_1[n] and\ x_2[n]$

$$x_3[n] \rightarrow y_3[n] = x_3[n^2]$$
$$= ax_1[n^2] + b\ x_2[n^2]$$
$$= a\ y_1[n] + b\ y_2[n]$$

As per the superposition law of **linearity, it satisfies it** i.e.

$$\tau\{\alpha_1 x_1[n] + \alpha_2 x_2[n]\} = \alpha_1 y_1[n] + \alpha_2 y_2[\mathrm{n}]$$

The system (d) is linear

Verify whether the following systems are time invariant

$(a)\ y(t) = x(t)\cos(\omega_c t)$ $(b)\ y(n) = n\,x(n)$ $(c)\ y(n) = x(n) - x(n-1)$

(a) $Let's,$ $\boldsymbol{x(t)}$ be an **arbitrary input** *to* this system, and ***output***

$$y(t) = \tau\{x(t)\} = x(t)\cos\omega_c t$$

Let's a second input obtained by shifting the x(t) by t_0 unit in time

$$x_1(t) = x(t-t_0)$$

$$x_1(t) \rightarrow y_1(t) = \tau\{x_1(t)\} = \tau\{x(t-t_0)\} = x(t-t_0)\cos\omega_c t \ \ldots (1)$$

If, we delay directly the output $y(t)$ *by* t_0 *unit in time*, we get $y(t-t_0) = x(t-t_0)\cos\,\omega_c(t-t_0)\ \ldots\ (2)$

As per the definition of time-invariance, it should satisfy $\tau\{x(t-\eta)\} = y(t-\eta)$

The equation $(1) \neq (2)$ System is time-variant

Cont..

$(b)\ y(n) = nx(n)$ **(Eq. 0)**

Let $x(n)$ *be an* **arbitrary input** to this system, and **output** $y(n) = \tau\{x(n)\} = nx(n)$

Let, a **second input** obtained by **shifting *x(n)* by n_0** *in time* $x_1(n) = x(n - n_0)$

The output corresponding to the input $x_1(n)$ *is* $x_1(n) \rightarrow y_1(n) = \tau\{x_1(n)\} = \tau\{x(n - n_0)\}$

$$= nx(n - n_0) \quad \textbf{(1)}$$

If, we **delay directly the output** $y(t)$ **[Eq.0]** $by\ n_0\ unit\ in\ time,$ we get

$$y(n - n_0) = (n - n_0)x(n - n_0)$$

$$= nx(n - n_0) - n_0 x(n - n_0) \quad \textbf{(2)}$$

As per the definition of time-invariance, it should satisfy $\tau\{x(t - \eta)\} = y(t - \eta)$

The equation (1) $\neq$ (2) System is **time-variant**

Cont..

$(c)\ y(n) = x(n) - x(n-1)$ ... (0)

Let $x(n)$ *be an* **arbitrary input** to this system, and **output** $y(n) = \tau\{x(n)\} = x(n) - x(n-1)$

Let a **second input** obtained by **shifting** $\boldsymbol{x_1(n)}$ ***by*** $\boldsymbol{n_0}$ *in time* $x_1(n) = x(n-n_0)$

$$x_1(n) \to y_1(n) = \tau\{x_1(n)\} = \tau\{x(n-n_0)\}$$
$$= x(n-n_0) - x(n-1-n_0)$$
$$= x(n-n_0) - x(n-n_0-1)$$
... (1)

If, we delay directly the output $y[n]$ **[Eq. 0]** $by\ n_0\ unit\ in\ time,$ we get

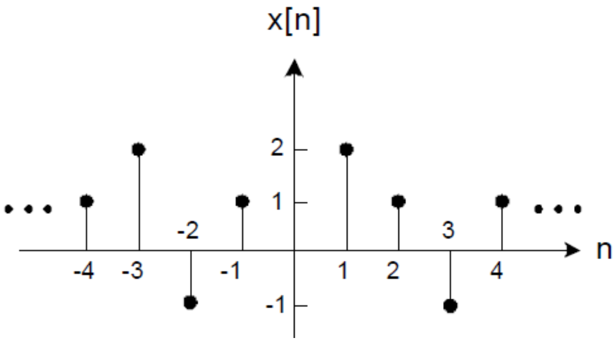
$$y(n-n_0) = x(n-n_0) - x(n-n_0-1) \text{ ... (2)}$$

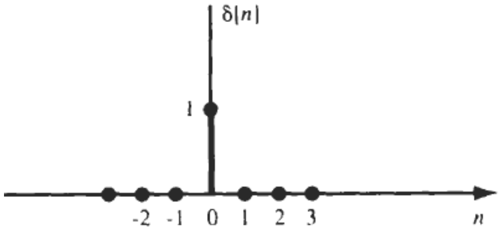
As per the definition of time-invariance, it should satisfy $\tau\{x(t-\eta)\} = y(t-\eta)$

The equation (1) $=$ (2) System is **time-invariant**

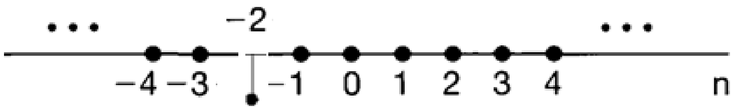
Representation of an arbitrary discrete signal

- A discrete-time signal can be decomposed into a **sequence of individual impulses**



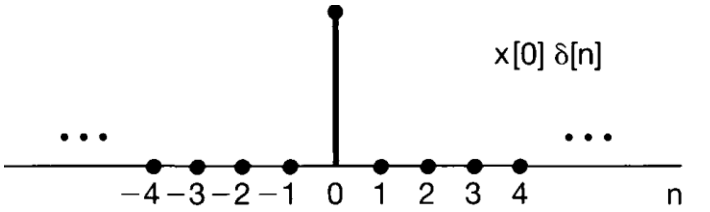


x[−2] δ[n + 2]



$$x[-1]\delta[n+1] = \begin{cases} x[-1], & n = -1 \\ 0, & n \neq -1 \end{cases}$$

$$x[0]\delta[n] = \begin{cases} x[0], & n = 0 \\ 0, & n \neq 0 \end{cases}$$



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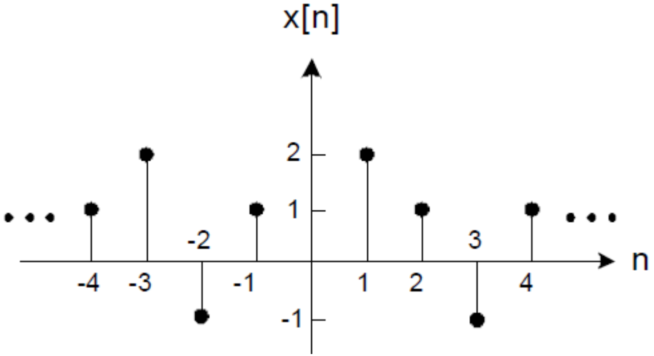


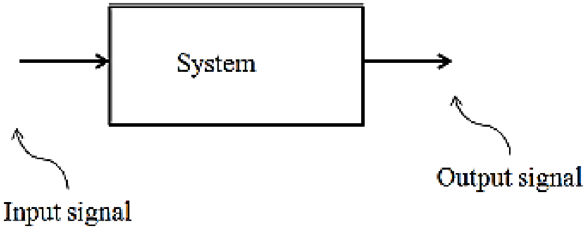
The signal in Fig. can **be expressed** as a **sum of the** ***shifted impulses***

$$x[n] = \ldots + x[-3]\delta[n+3] + x[-2]\delta[n+2] + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + x[2]\delta[n-2] + \ldots$$

or in a **more compact** form

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

Right-hand side represents an arbitrary signal x(n) as weighted (scaled) sum of shifted impulse sequence



Let, $x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \longrightarrow y[n] = \mathrm{T}\{x[n]\}$

Amplitude of signal at "k"

Now, **system response y(n)** for the **input x(n)** can be written as,

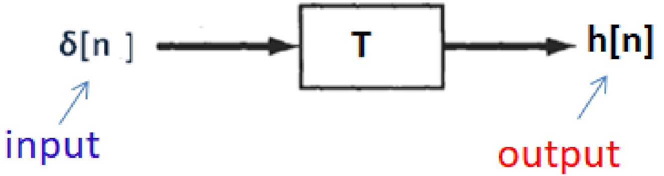
$$y(n) = T\{x(n)\}$$
$$= T\left\{\sum_{k=-\propto}^{\propto} x(k)\delta(n-k)\right\}$$
$$= \sum_{k=-\propto}^{\propto} x(k)\, T\{\delta(n-k)\}$$
$$= \sum_{k=-\propto}^{\propto} x(k)h(n,k)$$

If **system is "Linear"** then it will hold the "**superposition rule**" (output is weighted combination of input)

Cont..

For time-invariant system, we know that it holds $\tau\{x[n-k]\} = y[n-k]$ for any **integer** ***k***

$$if \;\; T\{\delta(n)\} = h(n)$$



If the system **T** is time-invariant $\rightarrow T\{\delta(n-k)\} = h(n-k)$ for any **integer** ***k***

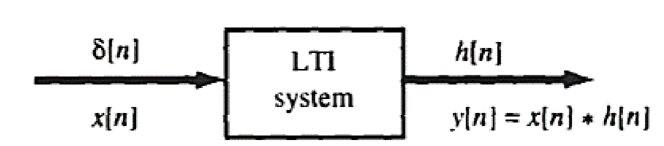
$$y(n) = T\{x(n)\} = T\left\{\sum_{k=-\infty}^{\infty} x(k)\delta(n-k)\right\} = \sum_{k=-\infty}^{\infty} x(k)\, T\{\delta(n-k)\} = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

h[n] called "**impulse response ($\boldsymbol{\delta[n]}$) of LTI system**"

Input-output relation of an "LTI" system (discrete)

$$y(n) = T\{x(n)\} = \sum_{k=-\propto}^{\propto} x(k)h(n-k) = x(n) * h(n) = h(n) * x(n)$$

- This is called convolution sum/ response of system
- $h[n]$ is called Impulse response of LTI system



❑ **Suppose**, we wish to compute the output of system at **some time** n=n_0,

$$y(n_0) = \sum_{k=-\propto}^{\propto} x(k)h(n_0-k)$$

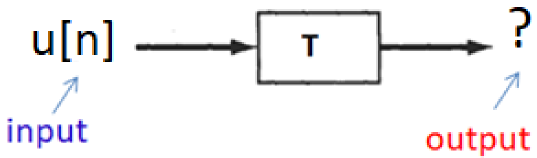
In continuous-time system: **(Convolution integral)**

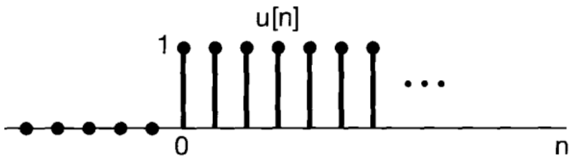
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \quad = h(t) * x(t) = \int_{-\propto}^{\propto} h(\tau)x(t-\tau)d\tau$$

Step response of LTI system

We have to calculate the response (output) of the system for input

$$x[n] = u[n]$$





$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$

As per definition of convolution sum,

$$x[n] \rightarrow y[n] = s[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]\, u[n-k]$$

$u[n-k] = 1 \quad if \ [\boldsymbol{n} - \boldsymbol{k}] \geq \boldsymbol{0}$,

$\rightarrow \ n - k \geq 0$

$\rightarrow k \leq n$

$$s[n] = \sum_{k=-\infty}^{n} h[k]$$

By induction method,

$$s[n-1] = \sum_{k=-\infty}^{n-1} h[k]$$

$$s[n] - s[n-1] = \sum_{k=-\infty}^{n} h[k] - \sum_{k=-\infty}^{n-1} h[k] = h[n]$$

Thank you

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$$y(n) = T\{x(n)\} = \sum_{k=-\infty}^{\infty} x(k)h(n-k) = x(n) * h(n) = h(n) * x(n) = \sum_{k=-\infty}^{\infty} x(n-k)h(k)$$

for all value of n

Let, $m = n - k\ ,\ k = n - m$

$$y(n) = \sum_{m=-\infty}^{\infty} x(n-m)h(m)$$

As "m" is a dummy index, we can again replace the m by k

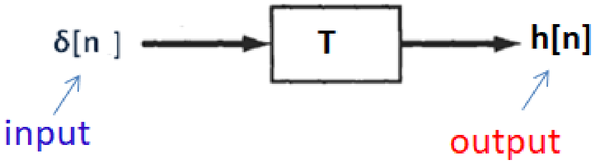
$$y(n) = \sum_{k=-\infty}^{\infty} x(n-k)h(k)$$

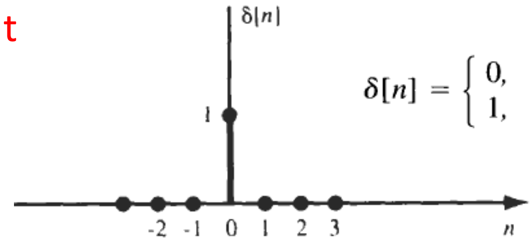
$$y(n) = T\{x(n)\} = x(n) * h(n) = h(n) * x(n)$$

Impulse response of the LTI

We have to calculate the response (output) of the system for input

$$x[n] = \delta[n]$$





$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

As per definition of convolution sum,

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]\, \delta[n-k]$$

$$= h[n]$$

$\delta(n-k) = 1,\ if\ \boldsymbol{n - k = 0}$

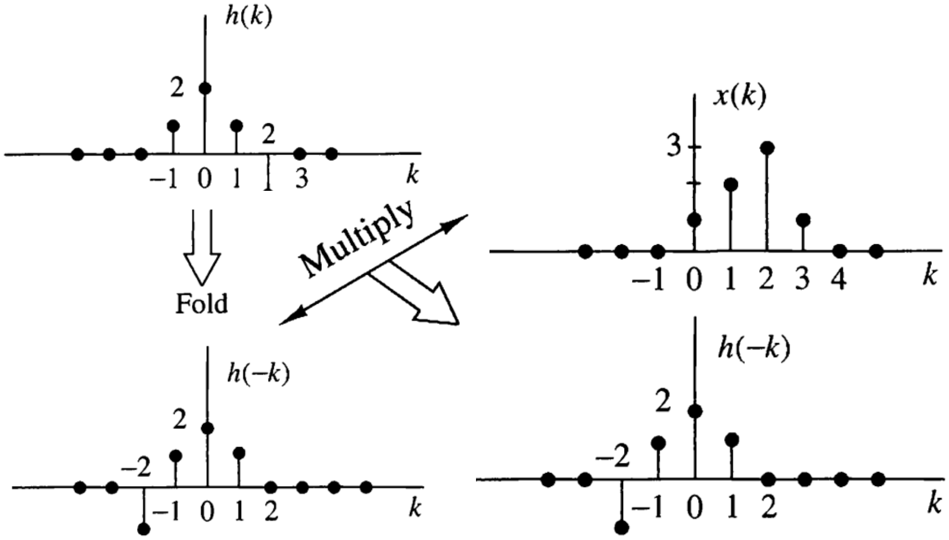
$\rightarrow k = n$

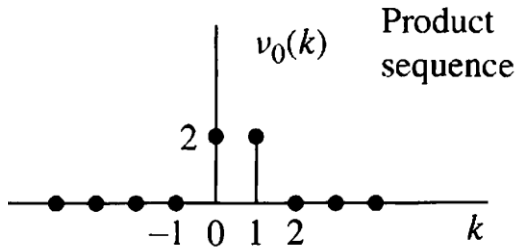
Determine the response of the Linear-time invariant system to the input signal $x(n) = \{\underset{\uparrow}{1}, 2, 3, 1\}$

The impulse response of a linear time-invariant system is $h(n) = \{1, \underset{\uparrow}{2}, 1, -1\}$

We know for the LTI system, $y(n) = \sum_{k=-\propto}^{\propto} x(k)h(n-k)$ for all value of n

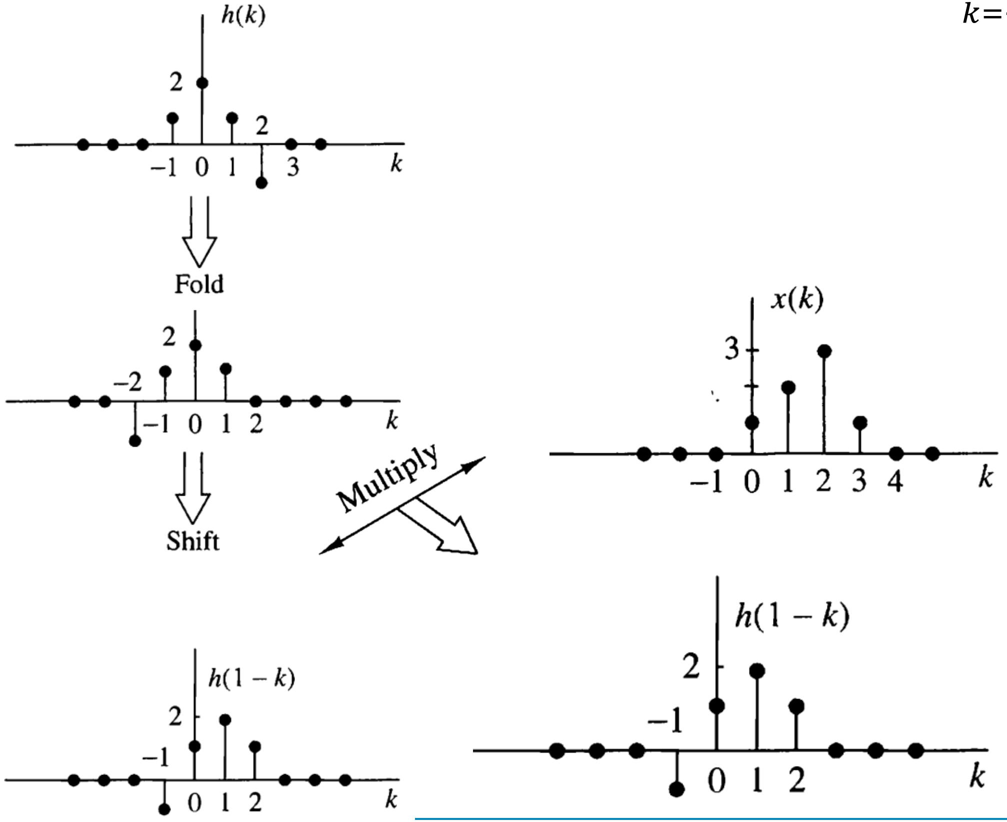
$n = 0 \rightarrow y(0) = \sum_{k=-\propto}^{\propto} x(k)\, h(0-k) = \sum_{k=-\propto}^{\propto} x(k)\, h(-k) = 4$

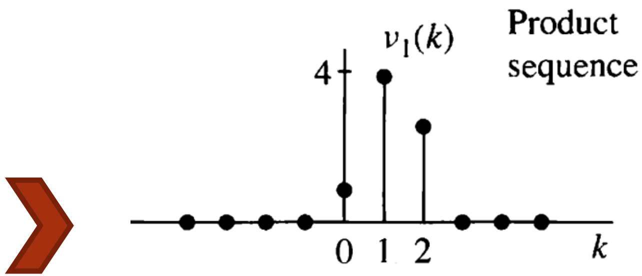




$$y(0) = \sum_{k=-\infty}^{\infty} v_0(k) = 4$$

For $\boldsymbol{n} = \mathbf{1}$, $y(1) = \sum_{k=-\infty}^{\infty} x(k)h(1-k) = 8$

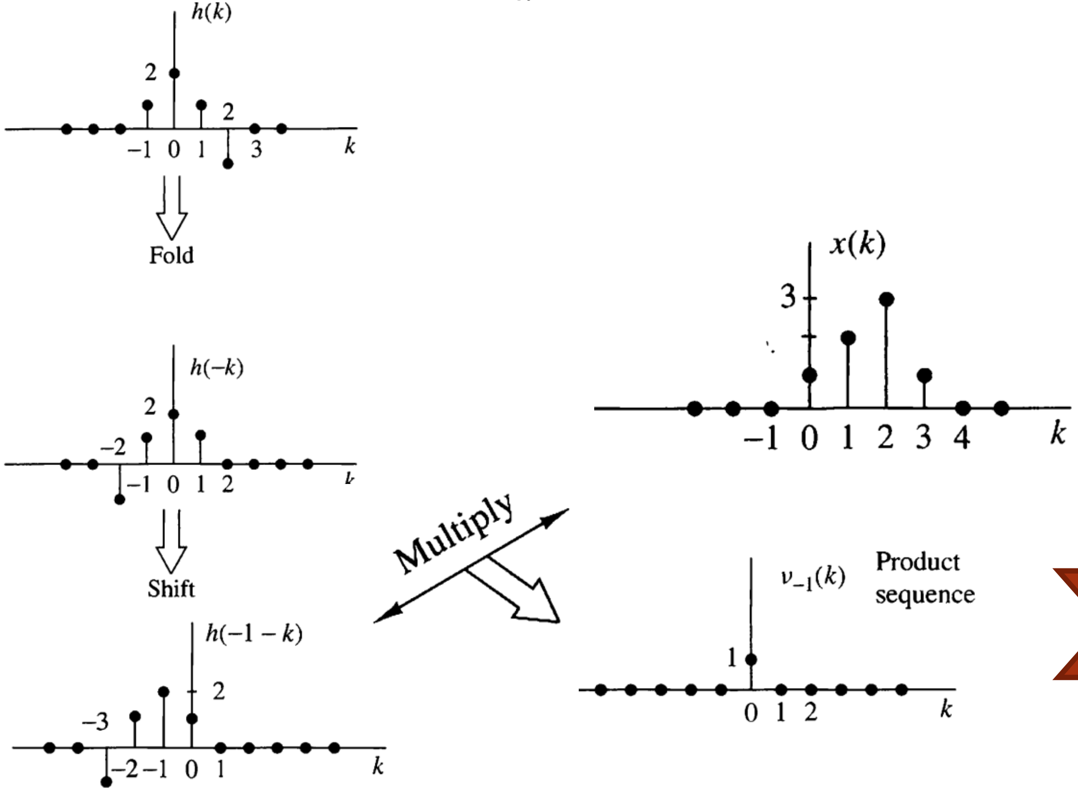




$$y(1) = \sum_{k=-\infty}^{\infty} v_1(k) = 8$$

For $\boldsymbol{n} < \mathbf{0}$, $\boldsymbol{n} = -\mathbf{1}$

$$y(-1) = \sum_{k=-\propto}^{\propto} x(k)h(-1-k) = 1$$



$$y(-1) = \sum_{k=-\infty}^{\infty} v_{-1}(k) = 1$$

Similarly for $-\propto < n < \propto$

$$y(n) = \{\ldots, 0, 0, \underset{\uparrow}{1}, 4, 8, 8, \ldots\}$$

Determine the response of the Linear-time invariant system to the input signal $x(n) = \{\underset{\uparrow}{1}, 2, 3, 1\}$

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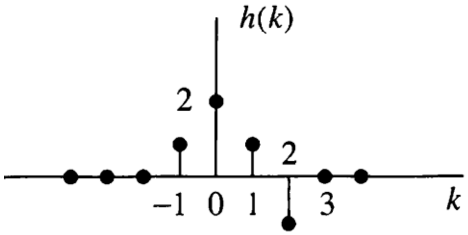
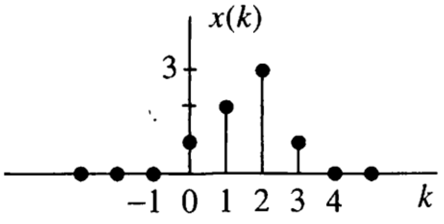




Method-2:

$n = 0 \rightarrow y(0) = \sum_{k=-\infty}^{\infty} x(k)\, h(0-k) = \sum_{k=-\infty}^{\infty} x(k)\, h(-k)$

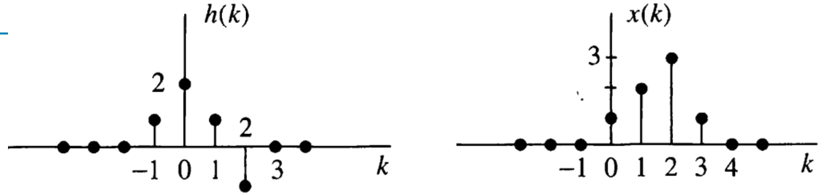
$= \ldots + x(-1)h(-(-1) + x(0)\, h(-0) + x(1)\, h(-1) + x(2)\, h(-2) + x(3)\, h(-3) + \ldots$

$= \ldots + x(-1)\, h(1) + x(0)\, h(0) + x(1)\, h(-1) + x(2)\, h(-2) + x(3)\, h(-3) + \ldots$

$= \ldots + 0 * 1 + 1 * 2 + 2 * 1 + 3 * 0 + 1 * 0 + \ldots$

$= \ldots + 0 * 1 + 1 * 2 + 2 * 1 + 3 * 0 + 1 * 0 + \ldots$

$= 4$



n =-1 $\rightarrow y(-1) = \sum_{k=-\propto}^{\propto} x(k)\, h(-1-k) \;= \sum_{k=-\propto}^{\propto} x(k)\, h(-1-k)$

$= \;...+x(-1)h(-1+1)+x(0)\, h(-1-0)+\; x(1)\, h(-1-1) \;+\; x(2)\; h(-1-2)$
$+\; x(3)\, h(-1-3)+\; ...$

$= \;...+x(-1)\, h(0)+x(0)\, h(-1)+\; x(1)\, h(-2) \;+\; x(2)\; h(-3)\; +\; x(3)\, h(-4)+\; ...$

$= \;...+0*2+1\; *1+2\; *0\; +3\; *\; \; 0\; +1\; *0+\; ...$

$= 1$

Practice problems

Q1. Suppose given $x[n] = \delta[n] + 2\,\delta[n-1] - \delta[n-3]$ and $h[n] = 2\delta[n+1] + 2\delta[n-1]$.
Evaluate the following convolutions:

$(a)\ y[n] = x[n] * h[n]$
$(b)\ y[n] = x[n+2] * h[n]$
$(c)\ y[n] = x[n] * h\,[n+2]$

Q2. Compute the output of the a LTI system where $x[n] = \begin{cases} 1, & 3 \le n \le 8 \\ 0, & \text{otherwise} \end{cases}$ $\quad \mathrm{h}[n] = \begin{cases} 1, & 4 \le n \le 15 \\ 0, & \text{otherwise} \end{cases}$

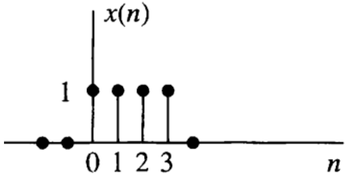
Q3. Show that $(a)\ \delta(n) = u(n) - u(n-1)$
$(b)\ u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$

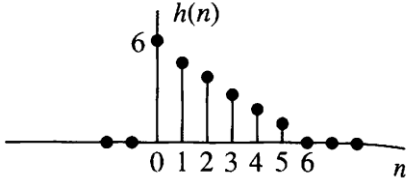
Q4. Compute the convolution $y(n) = x(n) * h(n)$

$(a) \quad x(n) = \begin{cases} 1, & n = -2, 0, 1 \\ 2, & n = -1 \\ 0, & else \end{cases}$

$h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$

(b)





Thank you

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Determine the response of the Linear-time invariant system to the input signal $x(n) = \{\underset{\uparrow}{1}, 2, 3, 1\}$

The impulse response of a linear time-invariant system is $h(n) = \{1, \underset{\uparrow}{2}, 1, -1\}$

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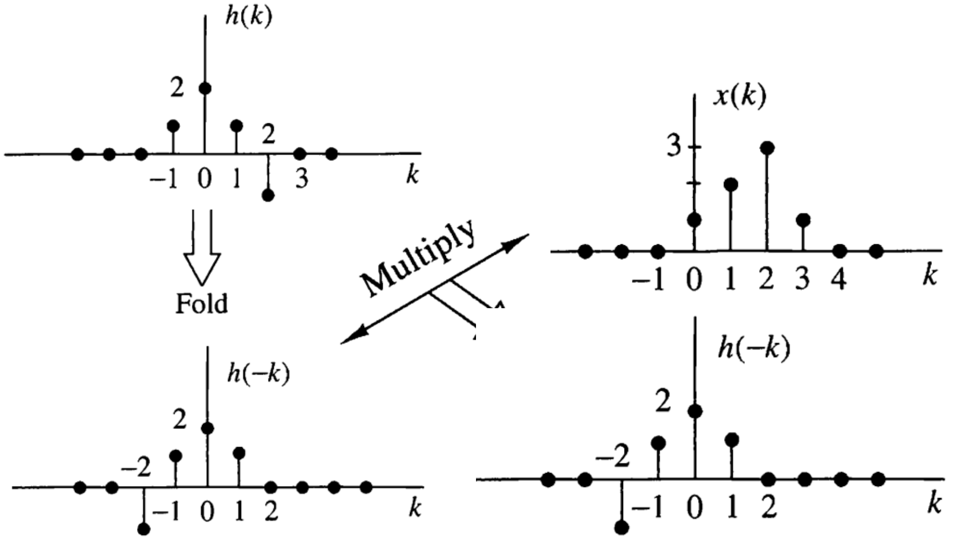
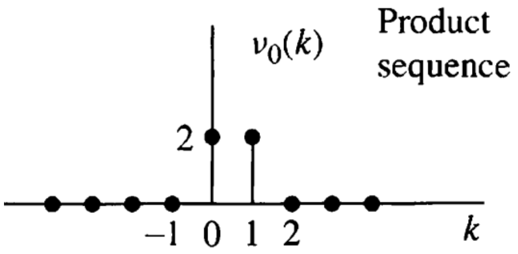
$n = 0 \rightarrow y(0) = \sum_{k=-\infty}^{\infty} x(k)\, h(0-k) = \sum_{k=-\infty}^{\infty} x(k)\, h(-k) = 4$





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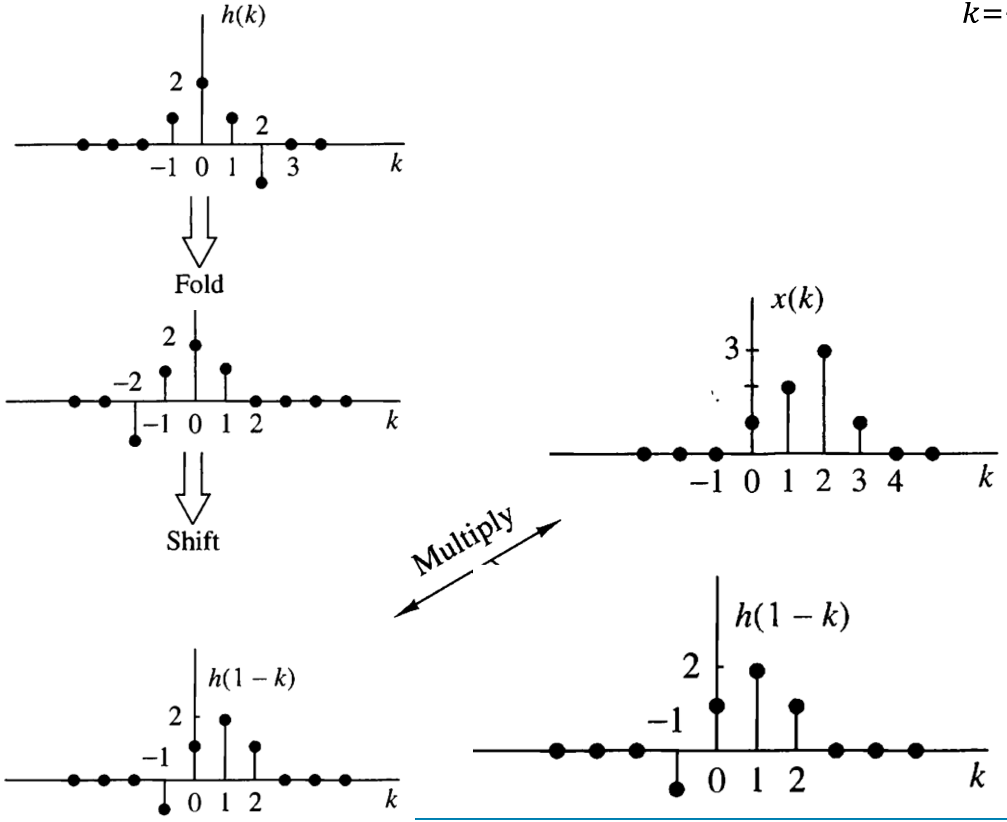
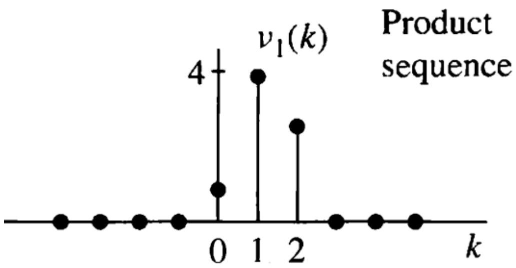
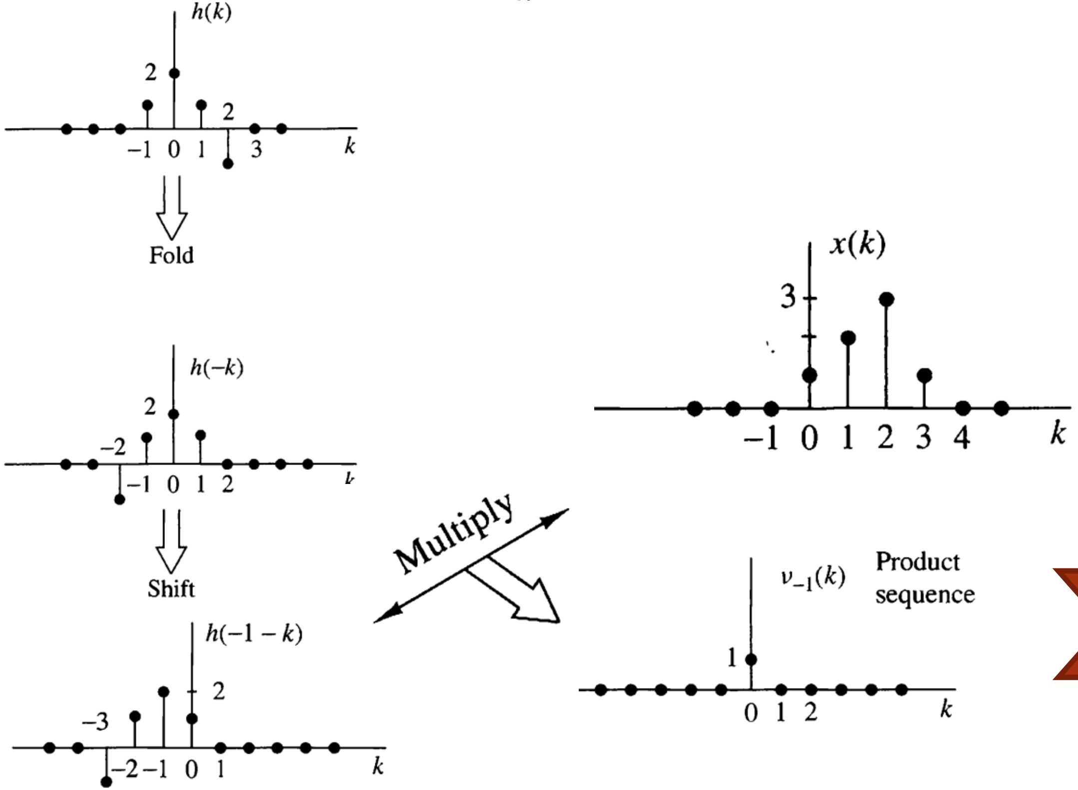




$$y(1) = \sum_{k=-\infty}^{\infty} v_1(k) = 8$$

For $n < 0$, $n = -1$

$$y(-1) = \sum_{k=-\propto}^{\propto} x(k)h(-1-k) = 1$$



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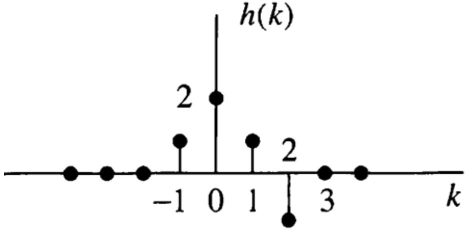

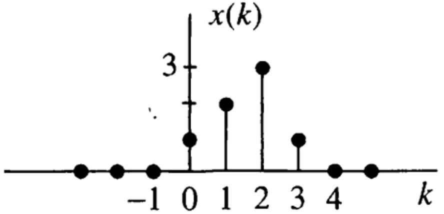

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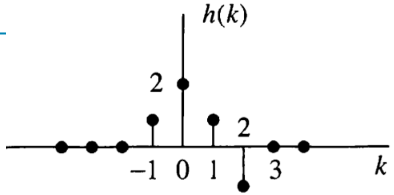
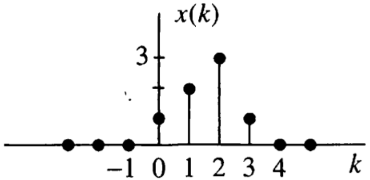
$= 4$





$$n = -1 \rightarrow y(-1) = \sum_{k=-\infty}^{\infty} x(k)\, h(-1-k) = \sum_{k=-\infty}^{\infty} x(k)\, h(-1-k)$$

$$= \ldots + x(-1)h(-1+1) + x(0)\, h(-1-0) + x(1)\, h(-1-1) + x(2)\, h(-1-2) + x(3)\, h(-1-3) + \ldots$$

$$= \ldots + x(-1)\, h(0) + x(0)\, h(-1) + x(1)\, h(-2) + x(2)\, h(-3) + x(3)\, h(-4) + \ldots$$

$$= \ldots + 0 * 2 + 1 * 1 + 2 * 0 + 3 * 0 + 1 * 0 + \ldots$$

$$= 1$$

Practice problems

Q1. Suppose given $x[n] = \delta[n] + 2\,\delta[n-1] - \delta[n-3]$ and $h[n] = 2\delta[n+1] + 2\delta[n-1]$. Evaluate the following convolutions:

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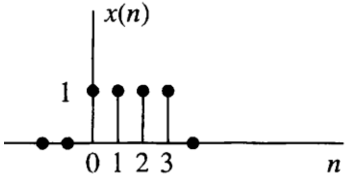
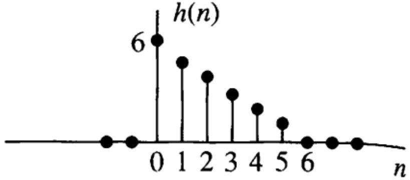
(b)



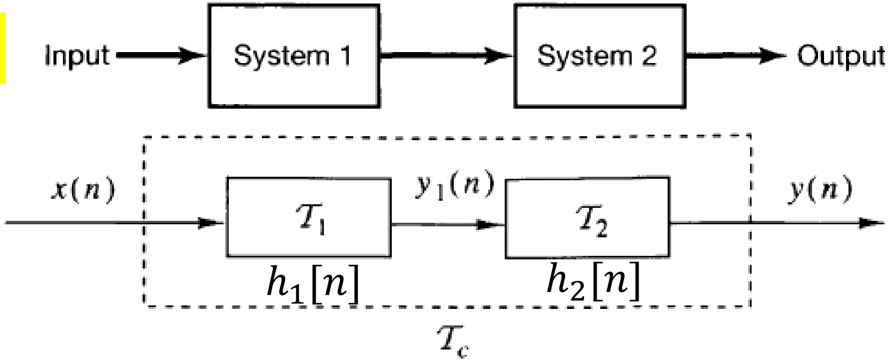


Inter connection of LTI systems

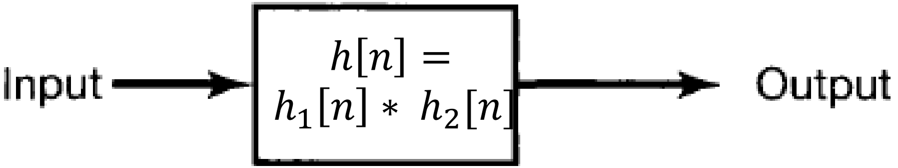
Commutative:

$$y[n] = x[n] * h[n] = h[n] * x[n]$$

Associative:

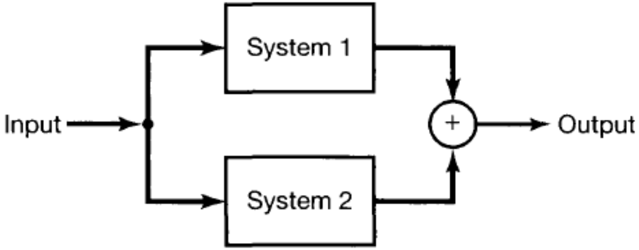


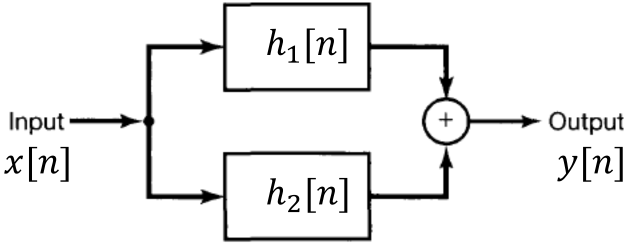
$$y[n] = \{x[n] * \ h_1[n]\} * \ h_2[n]$$
$$= x[n] * \ \{h_1[n] * h_2[n]\}$$

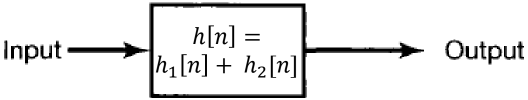


(Proof yourself)

Distributive:

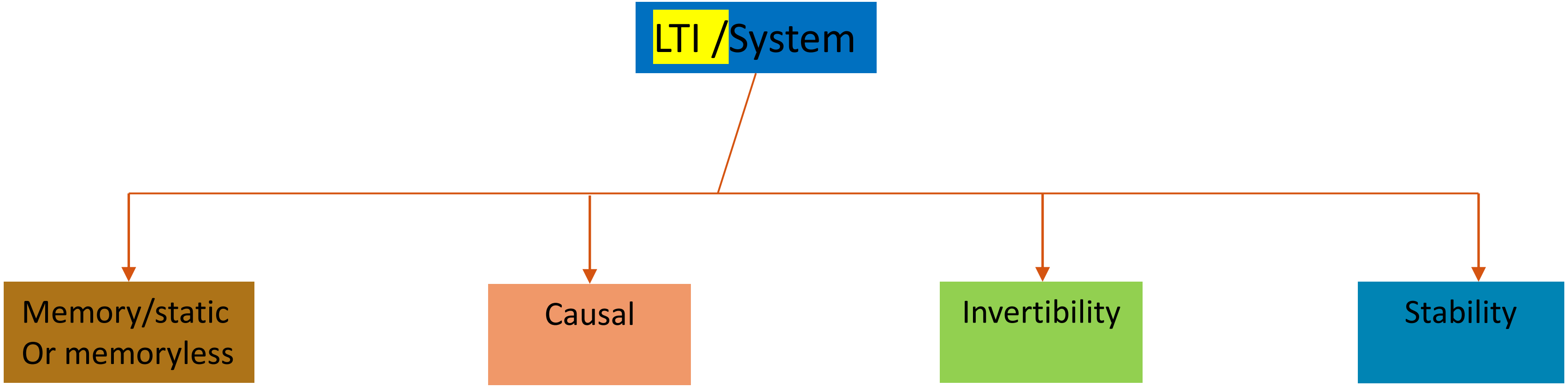






$$y[n] = x[n] * \{h_1[n] + h_2[n]\}$$
$$= x[n] * h_1[n] + x[n] * h_2[n]$$

Property of the LTI/systems



Memory/static or memoryless

A system is said to be *memory-less:*

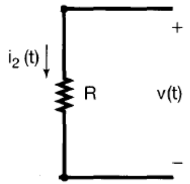
if the output at any time depends on ***only the input*** at **that same time**

$(\boldsymbol{y}(\boldsymbol{n}))$ at any particular time n_0 depends only on the value of *x[n]* at that **time** $\boldsymbol{n_0}$)

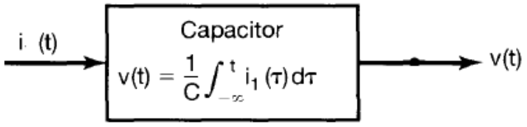
In memoryless: no need to store any of the past inputs or outputs in order to compute the present output

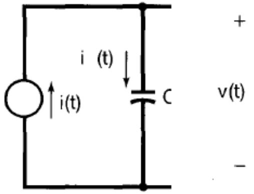
Otherwise, the system is said **to have *memory***

e.g. a memory less system is a resistor R with the input *x(t)* *taken as the voltage and current at* the output *y(t)*



A **system with memory** is a capacitor ***C with the current as the input*** $\boldsymbol{x}(\boldsymbol{t})$ and the voltage as the output $\boldsymbol{y}(\boldsymbol{t})$





Evaluate the response of the system

$$x[n] = \left(\frac{1}{2}\right)^n u[n] + 2^n u[-n]$$

$$\mathrm{h}[n] = u[n]$$

As per definition of convolution sum, $y[n] = x[n] * h[n]$

Let, $x_1[n] = \left(\frac{1}{2}\right)^n u[n]$ and $x_2[n] = 2^n u[-n]$

then, $y[n] = (x_1[n] + x_2[n]) * h[n]$

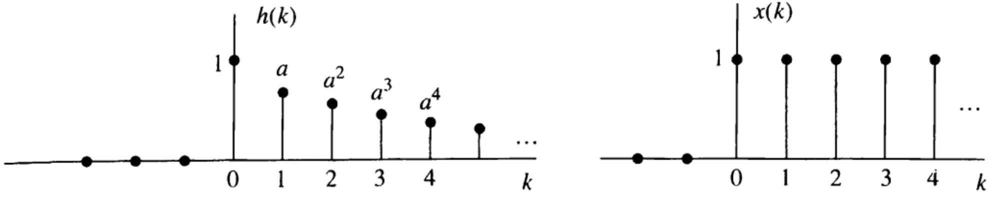
$= y_1[n] + y_2[n]$ (using distribution property)

$$y_1[n] = x_1[n] * h[n]$$

$$y_2[n] = x_2[n] * h[n]$$

$$h(n) = a^n u(n), \qquad |a| < 1$$

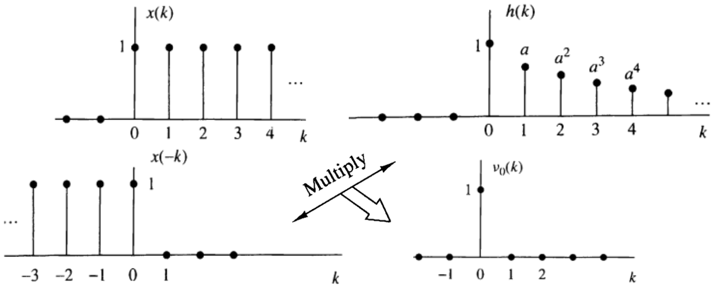
$$x(n) = u(n)$$



We know for the LTI system, $y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$ for all value of n

For $\boldsymbol{n = 0}$ $\quad y(0) = \sum_{k=-\infty}^{\infty} h(k)\, x(0-k) = \sum_{k=-\infty}^{\infty} h(k)x(-k)$

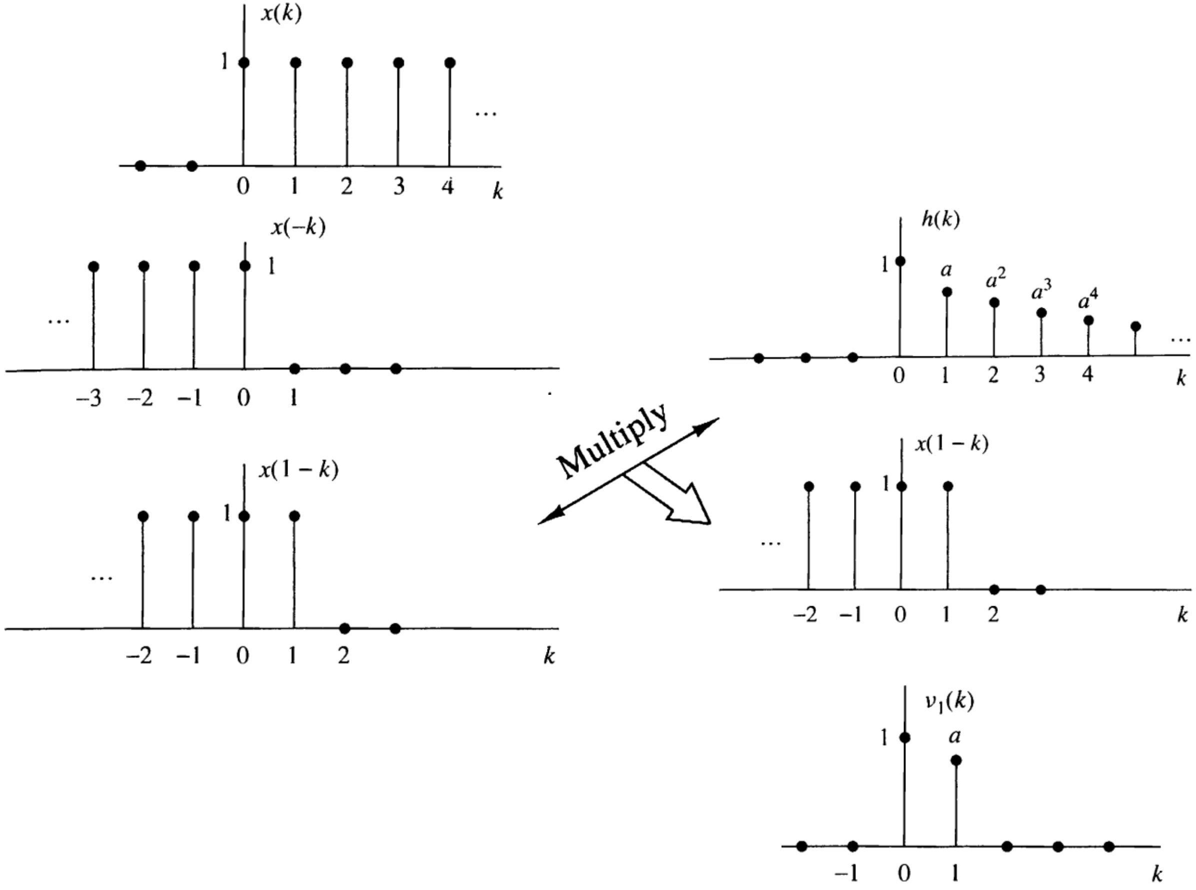
$= 1$



For $\boldsymbol{n = 1}$,

$$y(1) = \sum_{k=-\infty}^{\infty} h(k)\, x(1-k)$$

$$= 1 + a$$



Similarly,

$$y(0) = 1$$

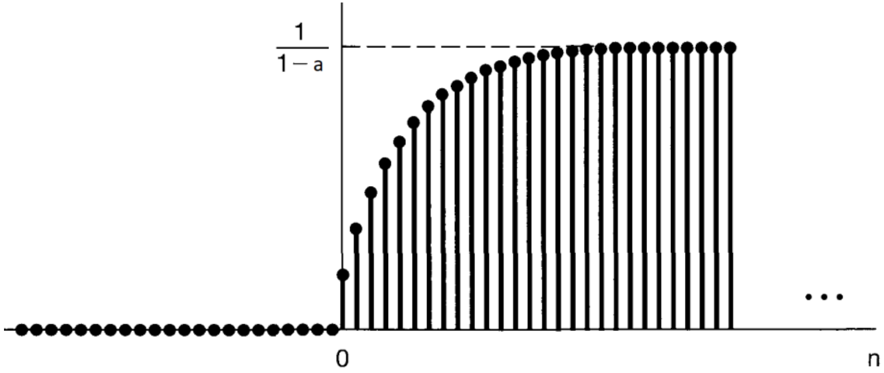
$$y(1) = 1 + a$$

$$y(2) = 1 + a + a^2$$

$$y(n) = 1 + a + a^2 + \ldots\ldots + a^n$$

$$= \frac{1 - a^{n+1}}{1-a}$$

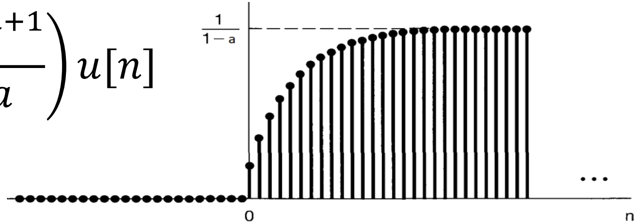
$$y[n] = \left(\frac{1 - a^{n+1}}{1 - a}\right) u[n]$$



$x_1[n] = a^n u(n), a = 1/2$

$h[n] = u[n]$

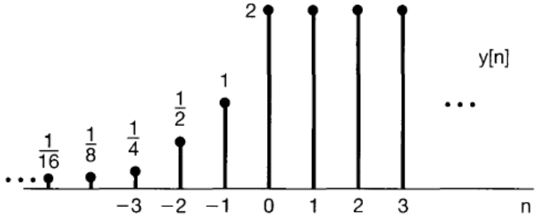
$$y_1[n] = \left(\frac{1 - a^{n+1}}{1 - a}\right) u[n]$$



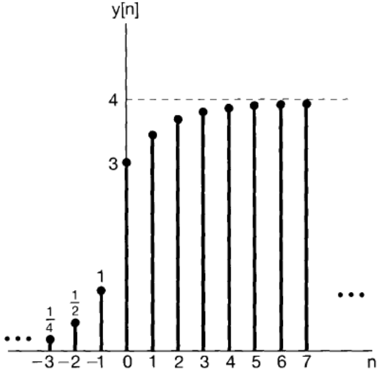
$x_2[n] = 2^n u[-n]$

$h[n] = u[n]$

$$y_2[n] = 2^{n+1}$$



$$\boldsymbol{y[n] = y_1[n] + y_2[n]}$$



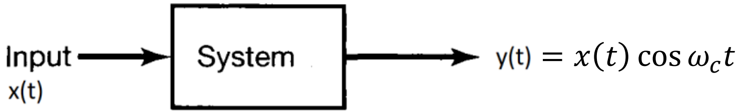
Determine whether systems are memoryless

$(a)\ \ y(t) = x(t)\cos\omega_c\, t$

$(b)\ y[n] = x[n-1]$

(a)

$y(t) = x(t)\cos\omega_c\, t$ ➡



Both same time index

$$
\begin{aligned}
t &= -1 \quad \rightarrow y(-1) = x(-1)\cos(\omega_c.-1) \\
t &= 0 \quad\ \ \rightarrow y(0) \ = x(0)\cos(\omega_c.0) \\
t &= 1 \quad\ \ \rightarrow y(1) = x(1)\cos(\omega_c.1)
\end{aligned}
$$

....

Both same time index

Since, the value of the output $y(t)$ *depends*

⬇

on only the current values of the input $x(t)$

the system ***is* memoryless**

Condition for LTI system to be memoryless

We know for the **LTI system input-output** is related

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

Output time-instant

$$= \sum_{k=0}^{\infty} h[k]\, x[n-k] + \sum_{k=-\infty}^{-1} h[k]\, x[n-k]$$

Past inputs

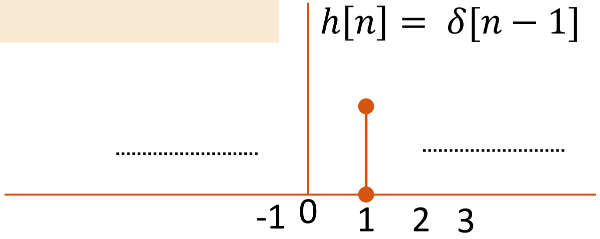
$$= (h[0]\, x[n] + h[1]\ x[n-1] + h[2]\, x[n-2]\ + \ldots\ldots)$$

$$+(\underbrace{h[-1]\, x[n+1] + h[-2]\ x[n+2] +}_{\text{Future inputs}} \ldots\ldots)$$

As per definition of memory less system: **output** at **any time-instant depends only on the value of the input** at **that same time-instant**

For **memoryless LTI system,** $h[n] = 0; n \neq 0$

Q1. The impulse response of a LTI system is expressed as $h[n] = \delta[n-1]$. Determine whether system is memoryless.



For **memoryless LTI system**, $h[n] = 0; n \neq 0$

The given $h[n]$ has 1 value at $n = 1$. Therefore, it does not satisfy the condition for memoryless. The system has memory.

Q2. The input-output of a LTI system is given by $y(n) = x(n-1)$. Verify whether system is memoryless.

For **memoryless LTI system**, $h[n] = 0; n \neq 0$

We need to calculate the $h[n]$ which is *the output of system* for $x(n) = \delta(n)$

$$\mathrm{h}(n) = \tau\{x(n) = \delta(n)\} = \delta(n-1)$$

The given $h[n]$ has 1 value at $n = 1$. Therefore, it does not satisfy the condition for memoryless. The system has memory.

Thank you

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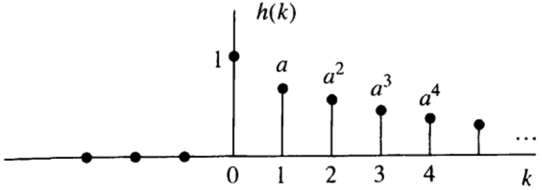
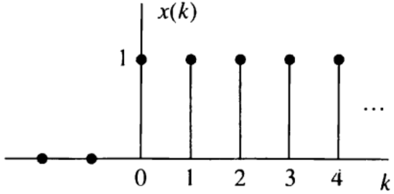
$$h(n) = a^n u(n), \qquad |a| < 1$$

$$x(n) = u(n)$$

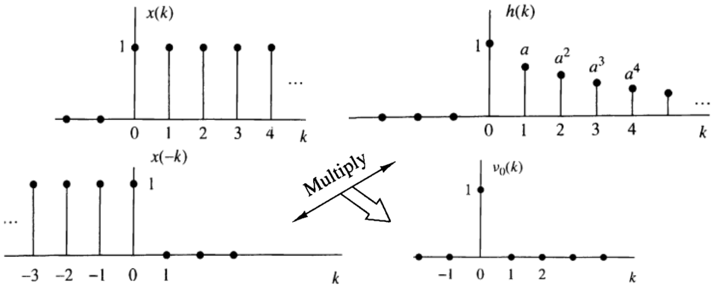




We know for the LTI system, $y(n) = \sum_{k=-\propto}^{\propto} h(k)\, x(n-k)$ for all value of n

For $\boldsymbol{n = 0}$ $y(0) = \sum_{k=-\propto}^{\propto} h(k)\, x(0-k) = \sum_{k=-\propto}^{\propto} h(k) x(-k)$

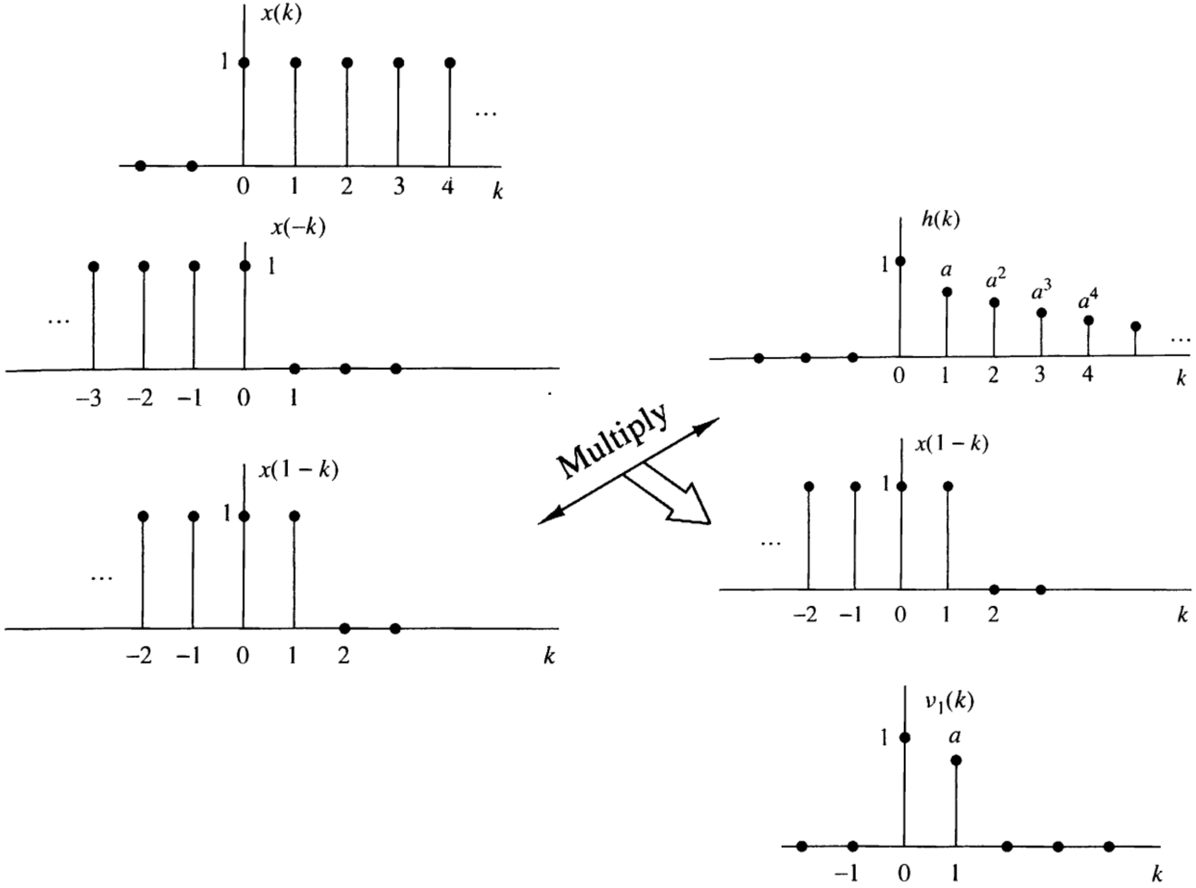
$= 1$



For $\boldsymbol{n = 1}$,

$$y(1) = \sum_{k=-\infty}^{\infty} h(k)\, x(1-k)$$

$$= 1 + a$$



Similarly,

$$y(0) = 1$$

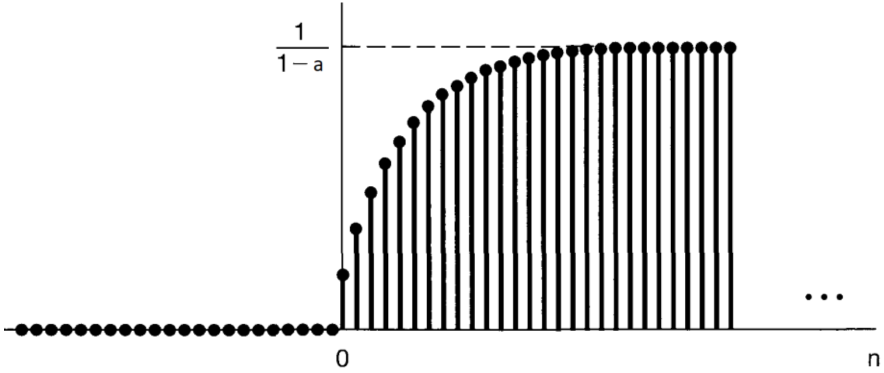
$$y(1) = 1 + a$$

$$y(2) = 1 + a + a^2$$

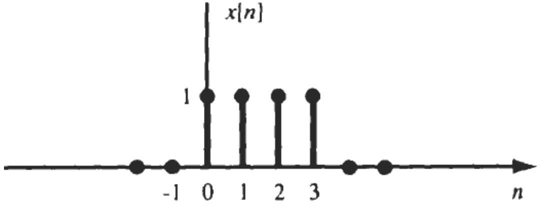
$$y(n) = 1 + a + a^2 + \ldots\ldots + a^n$$

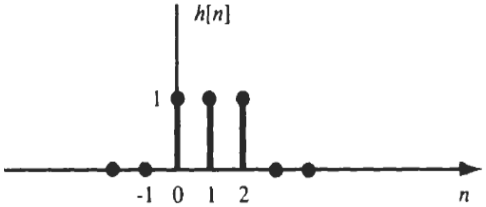
$$= \frac{1 - a^{n+1}}{1-a}$$

$$y[n] = \left(\frac{1 - a^{n+1}}{1 - a}\right) u[n]$$



- Evaluate $y[n] = x(n) * h(n)$, where





$$x[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$$

$$h[n] = \delta[n] + \delta[n-1] + \delta[n-2]$$

$$x[n] * h[n] = x[n] * \{\delta[n] + \delta[n-1] + \delta[n-2]\}$$

$$= x[n] * \delta[n] + x[n] * \delta[n-1] + x[n] * \delta[n-2]\}$$

$$= x[n] + x[n-1] + x[n-2]$$

$$y[n] = \{\underset{\uparrow}{1}, 2, 3, 3, 2, 1\}$$

Causality

- A system is *causal* if the output at any time depends only on values of the input at the present time and in the past (**does not depend** on future inputs)

Mathematically,

Output: $y(n)$ at time − instant $n = n_0$ **depends only on value** of $x(n)$ for $n \le n_0$

All **memory less** systems are causal.

Example of non-causal/anti-causal systems:

$(a)\ y(n) = x(-n)$

$(b)\ y(t) = x(t+1)$

Output depending on future value of input-time

For $n = -1$, $y(-1) = x(-(-1)) = x(1)$

$n = 0$, $y(0) = x(0)$

$n = 1$, $y(1) = x(-1)$

Condition for LTI system to be Causal

We know for the LTI system input-output is related

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

Let's estimate output for $n = n_0$

$$y[n_0] = \sum_{k=-\infty}^{\infty} h[k]x[n_0-k] \quad = \sum_{k=0}^{\infty} h[k]\, x[n_0-k] + \sum_{k=-\infty}^{-1} h[k]\, x[n_0-k]$$

$$= (h[0]\ \ x[n_0] + h[1]\, x[n_0-1] + h[2]\ \ x[n_0-2] + \ldots\ldots)$$

$$+(h[-1]\ \ x[n_0+1] + h[-2]\, x[n_0+2] + \ldots\ldots)$$

Input at **past** time-instant

Input at **present** time-instant

Input at **future** time-instant

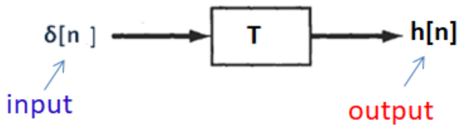
For causal system, output at time n = n_0 can depend only on the present and past inputs. Therefore, the causality condition for a discrete-time LTI system is

$$h[n] = 0, \qquad n < 0$$

Is the following system causal?

$$y[n] = \sum_{k=-\propto}^{n} 2^{k-n} x[k+1]$$

We need to calculate **the $h[n]$** which is *the **output of system*** for $x[n] \; = \; \delta[n]$



Impulse response of the system can be written as $h[n] = T\{\delta[n]\} = y[n] = \sum_{k=-\propto}^{n} 2^{k-n} \delta[k+1]$

For causality of LTI system, it should satisfy the condition $h[n] = 0, \qquad n < 0$

$$h[n] = \sum_{k=-\propto}^{n} 2^{k-n} \delta[k+1]$$

$$= \sum_{m=-\propto}^{n+1} 2^{m-1-n} \delta[m]$$

changing the variable, $k + 1 \; = \; m$

$$= 2^{-(n+1)} \sum_{m=-\propto}^{n+1} 2^{m} \delta[m]$$

Let's consider, $n = -1, \quad h[-1] \; \neq 0$

System is not causal

Determine whether systems are causal or anti-causal

$$(a)\ \ h[n] = \left(\frac{1}{2}\right)^n u[n]$$

$$(b)\ \ h[n] = \left(\frac{1}{2}\right)^n u[n-1]$$

$$(c)\ \ h[n] = \left(\frac{1}{2}\right)^{|n|}$$

$$(d)\ \ h[n] = u[n+2] - u[n-2]$$

$$(e)\ \ h[n] = \left(\frac{1}{3}\right)^n u[n] + \ 3^n\, u[-n-1]$$

$|x| = x$ if x is positive,
$|x| = -x$ if x is negative (in which case $-x$ is positive),
$|0| = 0$

Thank you

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Condition for LTI system to be Stable

We know for the LTI system input-output is related

$$y[n] = \sum_{k=-\propto}^{\propto} x[k]\, h[n-k] = \sum_{k=-\propto}^{\propto} h[k]\, x[n-k]$$

$$|y[n]| = \left|\sum_{k=-\propto}^{\propto} h[k]\, x[n-k]\right|$$

$$\leq \sum_{k=-\propto}^{\propto} |h[k]||x[n-k]|$$

$$\leq M_x \sum_{k=-\propto}^{\propto} |h[k]|$$

If $x[n]$ is **bounded (finite)** say, $M_x \rightarrow\ x[n-k]$ will bounded

For the bounded value of $y[n]$, $\sum_{k=-\propto}^{\propto} |h[k]|$ should be (finite) $< \propto$

Therefore, a LTI system is **stable if** its impulse response $h[n]$ is absolutely summable

- Determine the range of values of the parameter a *for which the linear time-invariant* system with impulse response is stable.

$$h(n) = a^n u(n)$$

For the stability of LTI system, it should satisfy $\displaystyle\sum_{k=-\propto}^{\propto} |h[k]| < \propto$

Using the relation,

$$\sum_{k=-\propto}^{\propto} |h(k)| = \sum_{k=-\propto}^{\propto} \left|a^k u(k)\right| = \sum_{k=0}^{\propto} \left|a^k\right| < \propto$$

If **|a| < 1** i.e. **sum able value will decreases as k→ ∞**

Therefore, the system is stable if $|a| < 1$

- Determine the range of values of the parameter a, b *for which the linear time-invariant* system with impulse response is stable

$$h(n) = \begin{cases} a^n, n \geq 0 \\ b^n, \quad n < 0 \end{cases}$$

For the stability of LTI system, it should satisfy $$\sum_{k=-\propto}^{\propto} |h[k]| < \propto$$

Using the relation, $$\sum_{k=-\propto}^{\propto} |h(k)| = \sum_{k=0}^{\propto} |a|^k + \sum_{k=-\propto}^{-1} |b|^k = \sum_{k=0}^{\propto} |a|^k + \sum_{k=\propto}^{1} \frac{1}{|b|^k} < \propto$$

First-term: if $|\boldsymbol{a}| < \mathbf{1},$ i.e. sum able value will decreases as $k \rightarrow \infty$

Second-term: if $\dfrac{1}{|b|} < 1$ i.e. sum able value will decreases as $k \rightarrow \infty$

i.e. $|\boldsymbol{a}| < \mathbf{1}$

$\dfrac{\mathbf{1}}{|\boldsymbol{b}|} < \mathbf{1}$

$$|a| < 1$$

$$\frac{1}{|b|} < 1 \rightarrow |b| > 1$$

$$|a| < 1$$
$$|b| > 1$$

For stability,

$$|a| < 1$$
$$|b| > 1$$

- Determine whether the following systems are stable or not

$$(a)\ h[n] = 4^n u[n]$$
$$(b)\ h[n] = u[n] - u[n-10]$$
$$(c)\ h[n] = 3^n u[-n-1]$$

Thank you

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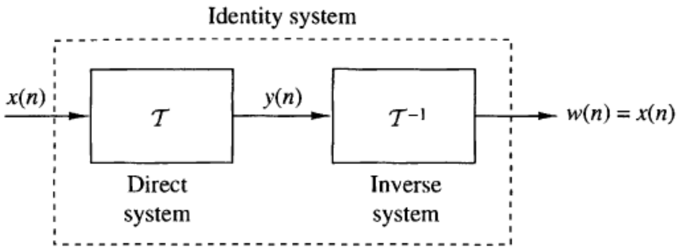
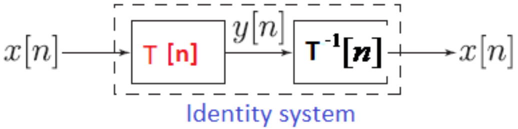
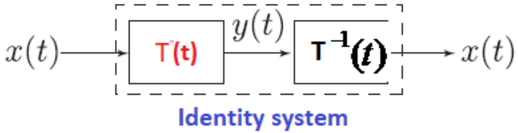
Invertibility

- A system is **invertible**, if an inverse system exists that when cascaded with the original system yields an output equal to input
- A system is invertible if <u>**distinct inputs lead**</u> to <u>**distinct outputs**</u>





Continuous case:



Determine if the following systems are invertible or not

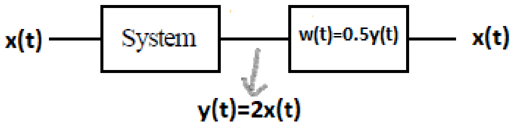
$(a)\ y(t) = 2\,x(t)$

$(b)\ y(t) = x^2(t)$

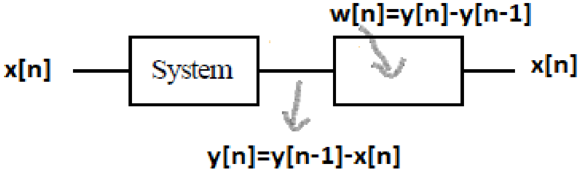
$(c)\ y[n] = \sum_{k=-\infty}^{n} x[k]$

$(d)\ y[n] = 0$

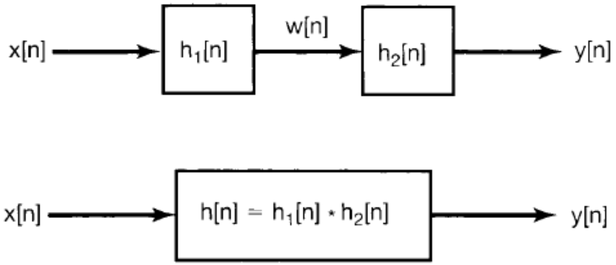
(a)



$$(c)\quad y[n] = \sum_{k=-\infty}^{n} x[k] = [\cdots x[n-2] + x[n-1]] + x[n] = y[n-1] + x[n]$$



Invertibility – LTI system



$$h[n] * h_1[n] = \delta[n]$$

(identity)

- Consider an LTI system with impulse response $h[n] = u[n]$. Determine whether inverse system of it is exist.

We know for the LTI system input-output is related

$$y[n] = \sum_{k=-\propto}^{\propto} x[k]\, h[n-k] = \sum_{k=-\propto}^{\propto} x[k]\, u[n-k] \quad = \sum_{k=-\propto}^{n} x[k] \quad \ldots (\mathbf{1})$$

for $(n-k) > 0\,, u[n-k] = 1$

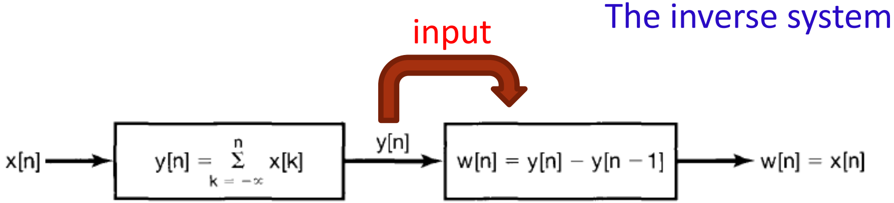
$$y[n] = \sum_{k=-\infty}^{n} x[k] \quad \ldots(\mathbf{1})$$

$$y[n-1] = \sum_{k=-\infty}^{n-1} x[k] \quad \ldots(\mathbf{2})$$

Using Eq. (1) & (2)

$$y[n] - y[n-1] = \sum_{k=-\infty}^{n} x[k] - \sum_{k=-\infty}^{n-1} x[k] \quad = x[n]$$

The inverse system is $y[n] - y[n-1] = x[n]$

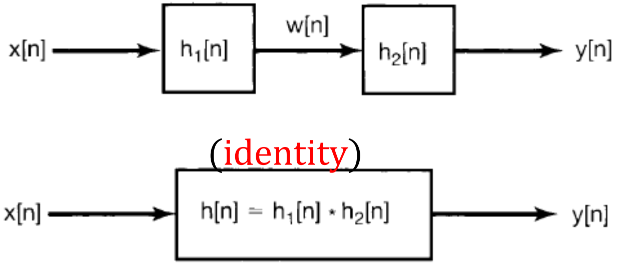


System   Inverse

To evaluate the impulse response of **the inverse system**, consider input to the system $y[n] = \delta[n]$

$$h_1(n) = \tau\{x(n) = \delta(n)\} = \sum_{k=-\infty}^{n} \delta(k) = u(n)$$

$$h_2[n] = \tau\{x(n) = \delta(n)\} = \tau\{\delta[n]\} = \delta[n] - \delta[n-1]$$



Cross-check:

$$h_I[n] = h_1[n] * h_2[n] \quad = u[n] * (\delta[n] - \delta[n-1]) \quad = u[n] * \delta[n] - u[n] * \delta[n-1]$$

$$= u[n] * \delta[n] - u[n] * \delta[n-1]$$

1 for $[n-k] = 0$,
$\rightarrow$ k=n

1 for $[n-k-1] = 0$,
$\rightarrow$ $k = n - 1$

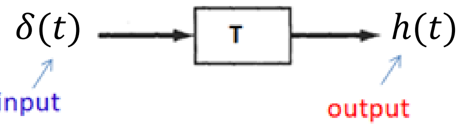
$$= \sum_{k=-\infty}^{\infty} u[k]\, \delta[n-k] \;-\; \sum_{k=-\infty}^{\infty} u[k]\, \delta[n-k-1]$$

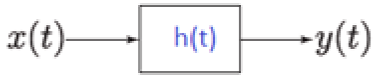
$$= u[n] - u[n-1]$$

$$= \delta[n]$$

$h_I[n] = h_1[n] * h_2[n] = \delta\,[n]$

Response of LTI systems to complex exponentials (continuous-time signal)





From convolution integral of LTI continuous-time system, we can write

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau$$

Let's consider complex exponential input to the system $x(t) = e^{st}$

Then

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, e^{s(t-\tau)}\,d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\, e^{st}e^{-s\tau}\,d\tau \quad = e^{st}\int_{-\infty}^{\infty} h(\tau)e^{-s\tau}\,d\tau$$

$$y(t) = e^{st}H(s)$$

(response in the form of e^{st})

Eigen function

Eigen value

$$H(s) = \int_{-\infty}^{\infty} h(\tau)\, e^{-s\tau}\,d\tau$$

Shows that **complex exponentials** are **eigenfunctions** of LTI systems

Thank you

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The input-output relation of a continuous-time LTI system is given by

$$y(t) = \int_{-\infty}^{\infty} e^{-(t-\tau)} x(\tau) d\tau$$

(a) Evaluate the impulse response $h(t)$ of the system

(b) Show that complex exponential function e^{st} is an eigen function of the system

(c) Evaluate the eigen value of the system for e^{st} using the impulse response obtained in (a)

Impulse-response nothing but the output of the system for input $x(t) = \delta(t)$

(a)

$$x(t) = \delta(t) \rightarrow y(t) = h(t) = \int_{-\infty}^{\infty} e^{-(t-\tau)} \delta(t)\, d\tau \;=\; e^{-(t-\tau)}|_{\tau=0}$$

$$= e^{-t}, \quad (say\ for\ t \geq 0)$$

$$h(t) = e^{-t} u(t)$$

(b) Let $x(t) = e^{st}$

Using convolution integral, we know $y(t) = \displaystyle\int_{-\infty}^{\infty} h(\tau) x(t-\tau)\, d\tau \;=\; \int_{-\infty}^{\infty} e^{-\tau}\, u(\tau)\, e^{s(t-\tau)}\, d\tau$

$$y(t) = \int_0^\infty e^{-\tau} e^{st} e^{-s\tau}\, d\tau$$

$$= e^{st} \int_0^\infty e^{-(1+s)\tau}\, d\tau$$

$$= e^{st} . \frac{e^{-(1+s)\tau}}{-(1+s)} |_0^{\tau=\infty}$$

$$= \frac{e^{st}}{-(1+s)} [e^{-(1+s).\infty} - e^{-(1+s).0}]$$

$$= \frac{e^{st}}{-(1+s)} . [0 -1]$$

$$= e^{st} . \underbrace{(\frac{1}{1+s})}_{\text{Eigen-value}}$$

(c) Using the relation

$$H(s) = \int_{-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau$$

$$Since, \qquad h(t) = e^{-t} u(t)$$

$$= \int_{-\infty}^{\infty} e^{-\tau} u(\tau) . e^{-s\tau} d\tau$$

$$= \int_0^\infty e^{-\tau} . e^{-s\tau}\, d\tau$$

$$= \int_0^\infty e^{-(1+s)\tau}\, d\tau$$

$$= \frac{1}{1+s}$$

Decomposing of signals in terms of eigenfunctions

Let's $x(t)$ is a linear combination of three exponential signals

$$x(t) = x_1(t) + x_2(t) + x_3(t)$$
$$= a_1 e^{s_1 t} + a_2 e^{s_2 t} + a_3 e^{s_3 t}$$
$$= \sum_{k=1}^{3} a_k\, e^{s_k t}$$

If $x(t)$ is applied to a LTI system, the response of the system ?

Using the eigen function concept of continuous LTI system, we can write

$$x_1(t) = a_1 e^{s_1 t} \rightarrow y_1(t) = e^{s_1 t}\, a_1\, H(s_1)$$
$$x_2(t) = a_2 e^{s_2 t} \rightarrow y_2(t) = e^{s_2 t}\, a_2\, H(s_2)$$
$$x_3(t) = a_3 e^{s_3 t} \rightarrow y_3(t) = e^{s_3 t}\, a_3\, H(s_3)$$

Using the superposition property

$$x(t) \rightarrow y(t) = y_1(t) + y_2(t) + y_3(t)$$
$$= a_1 H(s_1) e^{s_1 t} + a_2 H(s_2) e^{s_2 t} + a_3 H(s_3) e^{s_3 t} = \sum_{k=1}^{3} a_k H(s_k)\, e^{s_k t}$$

Observation:

If the **input** to a continuous-time LTI system is a linear combination of complex exponentials i.e.

$$x(t) = \sum_k a_k\, e^{s_k t}$$

$$\Downarrow$$

Output:

$$y(t) = \sum_k a_k H(s_k)\, e^{s_k t}$$

(a linear combination of the input complex exponential signals)

An **interesting fact on discrete exponential** signal as,

$$e^{j\omega_0 n} = e^{j(\omega_0+2\pi)n} = e^{j\omega_0 n}.e^{j2\pi n}$$

$$e^{i\phi} = \cos\phi + i\sin\phi$$

$$\begin{aligned} e^{j.2\pi.n} &= \cos(2\pi.n) + j\sin(2\pi.n) \\ &= \cos(4.\frac{\pi}{2}n + 0) + j\sin(4.\frac{\pi}{2}n + 0) \\ &= \cos(0) + j\sin(0) = 1 \end{aligned}$$

Since **n** can be only “integer” value in “discrete-time” signals

$$e^{j\omega_0 n} = e^{j(\omega_0+2\pi)n} = e^{j\omega_0 n}.1$$

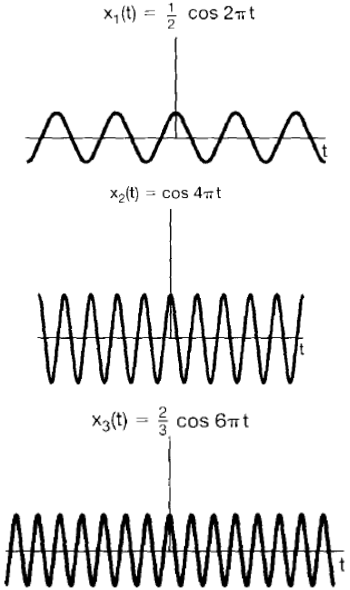
- The discrete **exponential** at frequency $\omega_0 + 2\pi$ is the *same as that* at frequency ω_0
- The signal with frequency ω_0 is identical to the signals with frequencies $\omega_0 \pm 2\pi, \omega_0 \pm 4\pi$, and so on.

Harmonically Related Complex Exponentials

Let's consider sinusoidal signal $x(t) = e^{j\omega_0 t}$

The fundamental period of the signal = ? $T = \frac{2\pi}{\omega_0}$

Fundamental angular frequency ω_0



Let's consider a signal,

$$\emptyset_k(t) = e^{jk\omega_0 t}, \quad k = 0, \pm 1, \pm 2, \ldots$$

$$k = 0 \rightarrow \emptyset_0(t) = e^{j.0.\omega_0 t}$$

$$k = 1 \rightarrow \emptyset_1(t) = e^{j.1.\omega_0 t}$$

$$k = 2 \rightarrow \emptyset_2(t) = e^{j.2.\omega_0 t}$$

Each of these signals has a fundamental frequency that is a **multiple of ω_0** → **called harmonically related**

$|k| \geq 2$, the fundamental period of $\emptyset_k(t)$ is fraction of $\boldsymbol{T}$

Fourier-series (FS) representation (continuous periodic signal)

- A **linear combination of harmonically** related **complex exponentials can be written as,**

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

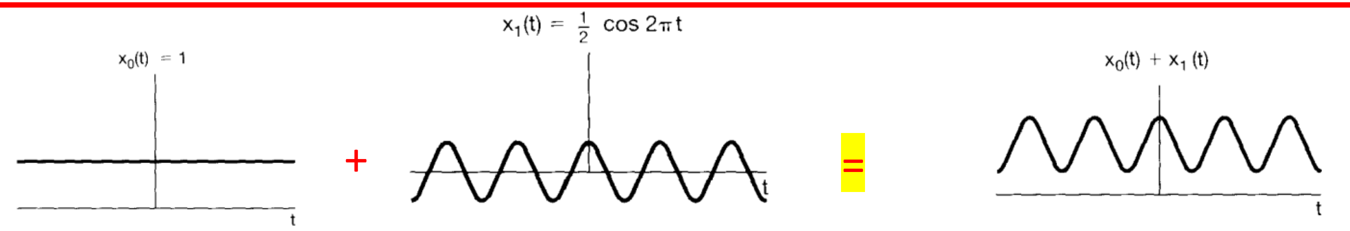
, ω_0 is the fundamental frequency and $T = \frac{2\pi}{\omega_0}$

$$= \sum_{k=-\infty}^{\infty} a_k e^{jk\left(\frac{2\pi}{T}\right)t}$$

(called synthesis equation)

$$= \ \ldots + a_{-1}e^{j.-1.\left(\frac{2\pi}{T}\right)t} + a_0 e^{j.0.\left(\frac{2\pi}{T}\right)t} + a_1 e^{j.1.\left(\frac{2\pi}{T}\right)t} + a_2 e^{j.2.\left(\frac{2\pi}{T}\right)t} + \ \ldots\ldots$$

a_k are called the *Fourier series coefficients*



Thank you

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Fourier-series (FS) representation (continuous periodic signal)

- A linear combination of harmonically related **complex exponentials can be written as,**

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$, \omega_0$ is the fundamental frequency and $T = \frac{2\pi}{\omega_0}$

$$= \sum_{k=-\infty}^{\infty} a_k e^{jk\left(\frac{2\pi}{T}\right) t}$$

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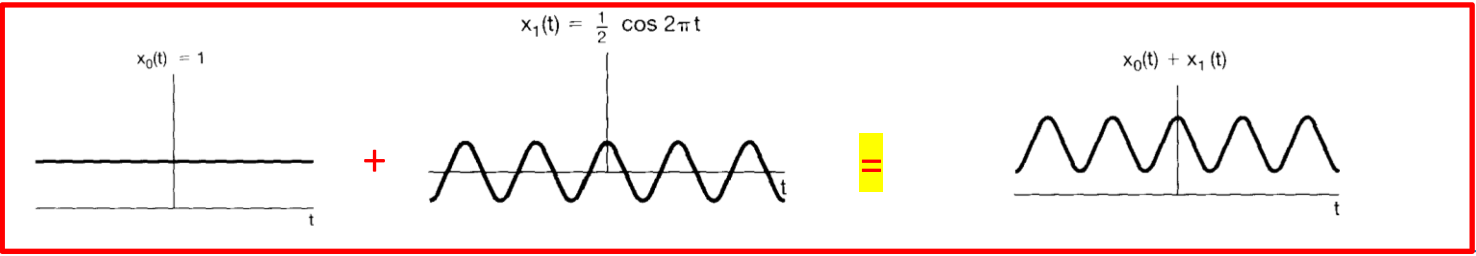
$$= \ldots + a_{-1} e^{j.-1.\left(\frac{2\pi}{T}\right)t} + a_0 e^{j.0.\left(\frac{2\pi}{T}\right)t} + a_1 e^{j.1.\left(\frac{2\pi}{T}\right)t} + a_2 e^{j.2.\left(\frac{2\pi}{T}\right)t} + \ldots\ldots$$

$\boldsymbol{a_k}$ are called the *Fourier series coefficients*



- Determine the Fourier series coefficient $x(t) = \sin \omega_0 t$

From **Fourier-series synthesis equation**, we know $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$

$$= \ldots + a_{-1} e^{j.-1.\omega_0 t} + a_0 e^{j.0.\omega_0 t} + a_1 e^{j\omega_0 t} + \ldots..$$

$$= \ldots + a_{-1} e^{-j.\omega_0 t} + a_0 e^{0} + a_1 e^{j\omega_0 t} + \ldots..$$

Expanding the given equation,

$$x(t) = \sin \omega_0 t$$

$$= \frac{1}{2j}[e^{j\omega_0 t} - e^{-j\omega_0 t}]$$

$$= \frac{1}{2j} e^{j\omega_0 t} - \frac{1}{2j} e^{-j\omega_0 t}$$

Comparing two equations

$$a_{-1} = -\frac{1}{2j}, a_1 = \frac{1}{2j}, \quad a_k = 0, \qquad k \neq \pm 1$$

Determine the Fourier series coefficient

$$x(t) = 1 + \sin\omega_0 t + 2\cos\omega_0 t + \cos(2\omega_0 t + \frac{\pi}{4})$$

$e^{i\phi} = \cos\phi + i\sin\phi$

$\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$

$\sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$

From **Fourier-series synthesis equation**, we know

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$= \ldots + a_{-1}e^{-j.\omega_0 t} + a_0 e^0 + a_1 e^{j\omega_0 t} + \ldots\ldots$$

Expanding the given equation $x(t)$

Comparing two equations

$$x(t) = 1 + \frac{1}{2j}\left[e^{j\omega_0 t} - e^{-j\omega_0 t}\right] + \left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right] + \frac{1}{2}\left[e^{j(2\omega_0 t + \pi/4)} + e^{-j(2\omega_0 t + \frac{\pi}{4})}\right]$$

$$= 1 + \left(1 + \frac{1}{2j}\right)e^{j\omega_0 t} + \left(1 - \frac{1}{2j}\right)e^{-j\omega_0 t} + \left(\frac{1}{2}e^{\frac{j\pi}{4}}\right)e^{j2\omega_0 t} + \left(\frac{1}{2}e^{\frac{-j\pi}{4}}\right)e^{-j2\omega_0 t}$$

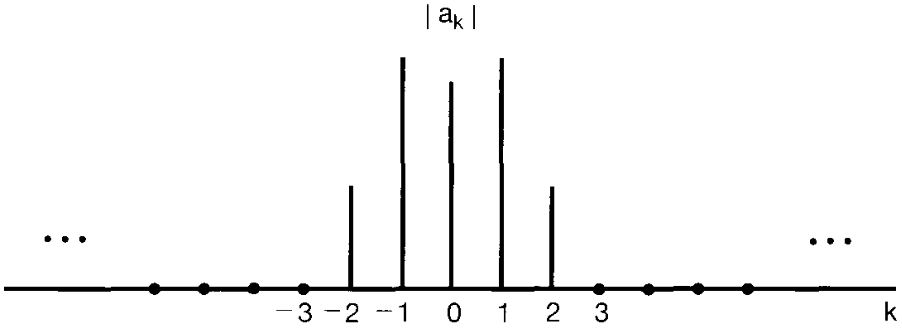
$$a_0 = 1,\ a_1 = \left(1 + \frac{1}{2j}\right), a_{-1} = \left(1 - \frac{1}{2j}\right), a_2 = \frac{1}{2}e^{j\pi/4}, a_{-2} = \frac{1}{2}e^{-j\pi/4},\ a_k = 0,\ \text{for } |k| > 2$$

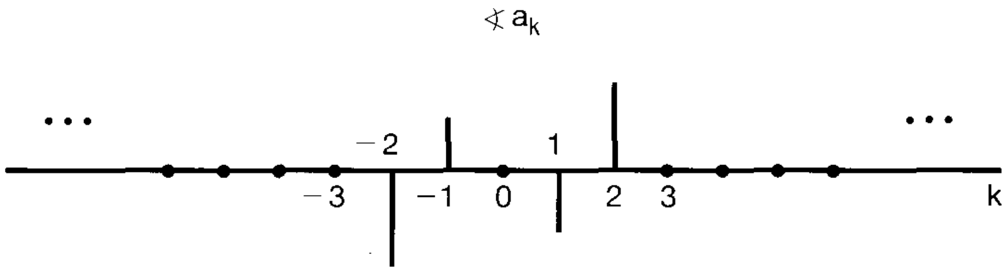
Cont..

- Magnitude and phase plot of a_k

$$a_0 = 1,$$
$$a_1 = \left(1 + \frac{1}{2j}\right),$$
$$a_{-1} = \left(1 - \frac{1}{2j}\right),$$
$$a_2 = \frac{1}{2} e^{j\pi/4},$$
$$a_{-2} = \frac{1}{2} e^{-j\pi/4},$$

$$a_k = 0, \text{ for } |k| > 2$$





$$z = x + jy$$
$$Magnitude\ |z| = \sqrt{x^2 + y^2}$$
$$Phase\ \measuredangle z = \tan^{-1}\frac{y}{x}$$

Fourier-series (analysis equation)

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\left(\frac{2\pi}{T}\right)t}$$

(synthesis equation)

$$a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt = \frac{1}{T}\int_T x(t)e^{-jk\left(\frac{2\pi}{T}\right)t}dt = \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-jk.2\pi f_0 t}dt$$

(Analysis equation)

Derivation:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\left(\frac{2\pi}{T}\right)t} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$\rightarrow x(t)\, e^{-jn\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}.\, e^{-jn\omega_0 t}$$

(multiplying both side $e^{-jn\omega_0 t}$)

$$\rightarrow \int_0^T x(t)\, e^{-jn\omega_0 t}\, dt = \int_0^T \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}.\, e^{-jn\omega_0 t}\, dt$$

(Integrating both side from 0 $to\ T$)

$$= \sum_{k=-\infty}^{\infty} a_k \int_0^T e^{j(k-n)\omega_0 t}dt$$

$$\rightarrow \int_0^T x(t)\, e^{-jn\omega_0 t}\, dt = \int_0^T \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} . e^{-jn\omega_0 t}\, dt = \sum_{k=-\infty}^{\infty} a_k \int_0^T e^{j(k-n)\omega_0 t} dt$$

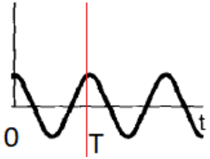
$$= \sum_{k=-\infty}^{\infty} a_k \left(\int_0^T \cos(k-n)\omega_0 t\ dt + j \int_0^T \sin(k-n)\omega_0 t\ dt \right)$$

For $k = n$ $$\left(\int_0^T \cos(k-n)\omega_0 t\ dt + j \int_0^T \sin(k-n)\omega_0 t\ dt \right) = T$$

For $k \neq n$ $$\left(\int_0^T \cos(k-n)\omega_0 t\ dt + j \int_0^T \sin(k-n)\omega_0 t\ dt \right) = 0$$

(integration -> area under function)

$$\rightarrow \int_0^T x(t)\, e^{-jn\omega_0 t}\, dt = \sum_{k=-\infty}^{\infty} a_k \int_0^T e^{j(k-n)\omega_0 t} dt = a_n T$$



In one period,
$+ve$ and $-ve$ value ,
area will zero

$$a_n = \frac{1}{T} \int_0^T x(t)\, e^{-jn\omega_0 t}\, dt$$

(**Analysis equation**)

Thank you