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ASSIGNMENT-1

Submitted by →
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Section -B

Negative Binomial distribution →

1) PMF:

$$P\{X=x\} = {}^{x-1}C_{r-1}\, p^{r-1}(1-p)^{x-r}\, p$$

here,
x = No. of trials required to obtain r ~~total~~ successes

⇒ r = No. of successes,
p = Probability of success ⇒ $(1-p)$ = Probability of failure

2) CDF:

$$\sum_{x=r}^{X} \left({}^{x-1}C_{r-1}\right) p^{r}(1-p)^{x-r} = \sum_{r}^{X} \left({}^{X}C_{r}\right) p^{n}(1-p)^{x-r}$$

⇒ ~~$P\{X \leq x\}$~~ $P\{X \leq x\} = \sum_{r}^{x} \left({}^{x-1}C_{r-1}\right) p^{r}(1-p)^{x-r}$

$$= 1 - \sum_{0}^{r-1} {}^{X}C_{x}\, p^{x}(1-p)^{X-x}.$$

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$$= \sum_{r}^{x} {}^{x-1}C_{r-1}\, p^r (1-p)^{x-r}$$

$$+ \sum_{x+1}^{\infty} {}^{x-1}C_{r-1}\, p^r (1-p)^{x-r}$$

Simply,

$$CDF = \begin{cases} 0 & x < 0 \\ p^n \sum\limits_{j=0} {}^{n+j-1}C_{n-1} (1-p)^j & x \geq 0 \end{cases}$$

where n = no. of successes less than or equal to x

3) Mean:

$$P\{X=x\} = {}^{x-1}C_{r-1}\, p^r (1-p)^{x-r}$$

~~Let~~ Put $Y = x - r$

$$P\{Y=y\} = \left({}^{r+y-1}C_y\right) p^r (1-p)^y$$

$$E(Y) = \sum_{y=0}^{\infty} y \left({}^{r+y-1}C_y\right) p^r (1-p)^y$$

$$= \sum_{y=1}^{\infty} y \left[\frac{(y+r-1)!}{y!\,(r-1)!}\right] p^r (1-p)^y$$

$$= \sum_{y=1}^{\infty} \frac{r(1-p)}{p} \frac{(y+r-1)!}{(y-1)!\,r(r-1)!}\, p^r \frac{p(1-p)^y}{(1-p)}$$

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$$= \sum \frac{r(1-p)}{p} \sum_{y=1}^{\infty} \frac{(r+y-1)!}{(y-1)!\, r!} p^{r+1} (1-p)^{r-1}$$

let $y-1=z$

$y=1 \Rightarrow \underline{z=0}$

$$\Rightarrow \frac{r(1-p)}{p} \sum_{z=0}^{\infty} \frac{(r+z)!}{r!\, z!} p^{r+1} (1-p)^{z}$$

Since $\displaystyle\sum_{k=0}^{\infty} \frac{(k+r)!}{r!\, k!} p^{r} (1-p)^{k} = (p+(1-p))^{k+r} = 1$,

$$\Rightarrow E(y) = \frac{r(1-p)}{p}$$

$$\Rightarrow E(x) = E(y+r) = E(y) + E(r)$$

$$= \frac{r(1-p)}{p} + r\left(\frac{p}{p}\right)$$

$$= \frac{r}{p}.$$

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Variance:

$$Var(X) = E(x^2) - (E(x))^2$$

$$\sum x \left({}^{x-1}C_{r-1}\right) p^r (1-p)^{x-r} = \frac{r}{p}$$

$$\frac{\partial}{\partial p}\left(\sum x \binom{x-1}{r-1} p^r (1-p)^{x-r}\right) = \frac{\partial}{\partial p}\left(\frac{r}{p}\right)$$

$$\sum x \binom{x-1}{r-1}\left[r p^{r-1}(1-p)^{x-r} - p^r (x-r)(1-p)^{x-r-1}\right]$$

$$= \frac{-r}{p^2}$$

$$\frac{r}{p}\sum_x x f(x) - \frac{1}{1-p}\sum_x x^2 f(x) + \frac{r}{1-p}\sum x f(x) = \frac{-r}{p^2}$$

$$\Rightarrow \frac{1}{1-p} E(x^2) = \frac{r}{p^2} + \frac{r^2}{p^2} + \frac{r^2}{p(1-p)}$$

$$\Rightarrow E(x^2) = \frac{r(1-p)}{p^2}\left[r+1 + \frac{rp}{1-p}\right]$$

$$= \frac{r}{p^2}\left[r - \cancel{rp} + 1 - p + \cancel{rp}\right]$$

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$$Var(x) = E(x^2) - [E(x)]^2$$

$$= \frac{r^2}{p^2} + \frac{r}{p^2} - \frac{r}{p} - \left(\frac{r}{p}\right)^2$$

$$= \frac{r}{p}\left(\frac{1}{p} - 1\right)$$

$$\Rightarrow Var(x) = \frac{r(1-p)}{p^2}$$

4) Moment generating function:

$$M_x(t) = E(e^{tx}) = \sum e^{tx} f(x)$$

and

$$f(x) = \binom{k-1}{n-1} p^n (1-p)^{k-n}$$

$$\Rightarrow M_x(t) = \sum_{k=n}^{\infty} e^{tk} \binom{k-1}{n-1} p^n (1-p)^{k-n}$$

Let $i = k-n$

$$\Rightarrow \sum_{i=0}^{\infty} e^{t(n+i)} \frac{(n+i-1)!}{(n-1)!\, i!} p^n (1-p)^i$$

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$$= \frac{e^{tn} p^n}{(n-1)!} \sum_{i=0}^{\infty} \frac{(n+i-1)!}{i!} e^{ti} (1-p)^i$$

$x = e^t(1-p)$

$$\Rightarrow \frac{e^{tn} p^n}{(n-1)!} \sum_{i=0}^{\infty} \frac{d^{n-1} x^{n+i-1}}{dx^{n-1}}$$

$$= \frac{e^{tn} p^n}{(n-1)!} \frac{d^{n-1}}{dx^{n-1}} \sum_{i=0}^{\infty} x^{n+i-1}$$

Now, $\sum_{i=0}^{\infty} x^{n+i-1} = \sum_{i=0}^{\infty} x^{i}$

Since ~~e^{tn}~~ the $(n-1)$th derivative will remove any term x^k with $k < n-1$

$$\Rightarrow \frac{e^{tn} p^n}{(n-1)!} \frac{d^{n-1}}{dx^{n-1}} \sum_{i=0}^{\infty} x^{i} \Big|_{x=e^t(1-p)}$$

$$= \frac{e^{tn} p^n}{(n-1)!} \frac{d^{n-1}}{dx^{n-1}} \frac{1}{1-x} \Big|_{x=e^t(1-p)}$$

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$$= \frac{e^{tn} p^n}{(n-1)!} (n-1)! \left(\frac{1}{1-x}\right)\Bigg|_{x = e^t(1-p)}$$

$$= \left[\frac{e^t p}{1-e^t(1-p)}\right]^n$$

where n denotes no. of successes

5) Skewness and Kurtosis :

Let $q = 1-p$

$$M_1 = \mu_1 = M'(s)\Big|_{s=0} = \text{Mean} = \frac{r}{p}$$

$$M'(s) = p^r rq \left[e^s (1-qe^s)^{-r-1}\right]$$

$$M_2 = M''(s)\Big|_{s=0} = rq\left[\frac{(r+1)q}{p^2} + \frac{1}{p}\right]$$

$$M''(s) = p^r rq \left[e^s(-r-1)(1-qe^s)^{-r-2}(-qe^s) + (1-qe^s)^{-r-1}e^s\right]$$

$$\mu_2 = M_2 - (M_1)^2 = \frac{rq}{p^2}$$

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$$M_3 = M'''(s)\Big|_{s=0}$$

$$M'''(s) = p^r r q^2 (r+1)\left[(1-qe^s)^{-r-2} e^{2s}(2) + e^{2s}(-r-2)(1-qe^s)^{-r-3}(-qe^s)\right]$$

$$+ p^r r q\left[e^s(-r-1)(1-qe^s)^{-r-2}(-qe^s) + (1-qe^s)^{-r-1} e^s\right]$$

$$M'''(s)\Big|_{s=0} = M_3 = p^r r q^2 (r+1)\left[2(1-q)^{-r-2} + q(r+2)(1-q)^{-r-3}\right]$$
$$+ p^r r q\left[q(r+1)(1-q)^{-r-2} + (1-q)^{-r-1}\right]$$

$$= p^r r q^2 (r+1)\left[2p^{-r-2} + q(r+2)p^{-r-3}\right] + p^r r q\left[q^{r+1} p^{-r-2} + p^{-r-1}\right]$$

$$M_3 = \frac{3rq^2(r+1)}{p^2} + \frac{rq^3(r+1)(r+2)}{p^3} + \frac{rq}{p}$$

$$\mu_3 = m_3 - 3m_1 m_2 + \cancel{3m_1^2}\, 2m_1^3$$

$$\mu_3 = \cancel{\frac{3rq^2(r+1)}{p^2}} + \cancel{\frac{rq^3(r+1)(r+2)}{p^3}} + \frac{rq}{q} - 3\left(\frac{rq}{p}\right)$$

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$$\mu_3 = \frac{3rq^2(r+1)}{p^2} + \frac{rq^3(r+1)(r+2)}{p^3} + \frac{rq}{p} - 3\left(\frac{rq}{p}\right)\left[\frac{rq^2(r+1)}{p^2} + \frac{rq}{p}\right] + 2\left(\frac{rq}{p}\right)^3$$

$$\Rightarrow \mu_3 = \frac{3rq^3(r+1)}{p^2} + \frac{rq^3(r+1)(r+2)}{p^3} + \frac{rq}{p} - \frac{3r^2q^3(r+1)}{p^3} - \frac{3r^2q^2}{p^2} + \frac{2r^3q^3}{p^3}$$

$$\text{Skewness } (\alpha_3) = \frac{\mu_3}{(\mu_2)^{3/2}}$$

$$\Rightarrow \alpha_3 = \frac{3pq(r+1) + (r+1)(r+2)q^2 + p^2 - 3(r+1)q^2r - 3pqr + 2r^3q^2}{\sqrt{qr}}$$

$$\cancel{\alpha_3 = \frac{3pq + r^2q^2 + 3rq^2 + 2q^2 + p^2 - 3q^2r^2 + 2r^2q^2 - 3q^2r(3)}{\sqrt{qr}}}$$

$$\alpha_3 = \frac{3pq + \cancel{r^2q^2} + \cancel{3rq^2} + 2q^2 + p^2 - \cancel{3q^2r} + \cancel{2r^2q^2} - \cancel{3rq^2}}{\sqrt{qr}}$$

$$\Rightarrow \boxed{\alpha_3 = \frac{3pq + 2q^2 + p^2}{\sqrt{qr}}}$$

Substituting $q = 1-p$

$$\boxed{\alpha_3 = \frac{2-p}{\sqrt{pr}}}$$

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Kurtosis:

$$\alpha_4 = \frac{\mu_4}{(\mu_2)^2}$$

$$M^4(s) = 2p^r q^2 r(r+1)\left[e^{2s}(1-qe^s)^{-r-2}\right] + p^r r q^3 (r+1)(r+2)\left(e^{3s}(1-qe^s)^{-r-3}\right)$$

$$+ p^r r q^2 (r+1)\{e^{2s}(1-qe^s)^{-r-2} + p^r r q\, e^s (1-qe^s)^{-r-1}$$

$$M^4(s)\Big|_{s=0} = 2p^r r q^2 (r+1)(r+2) q (1-q)^{-r-3}$$

$$+ p^{-r-2}(4) p^r q^2 r(r+1)$$

$$+ p^r r q^3 (r+1)(r+2)(r+3) p^{-r-4} q + 3p^{-r-3} p^r q^3 (r+1)(r+2)$$

$$+ p^r r q^2 (r+1)(r+2) p^{-r-3} q + 2p^r r q^2 (r+1) p^{-r-2} + p^r q r p^{r-1}$$

$$+ p^r r q (r+1) p^{-r-2} q$$

$$\Rightarrow M^4(s)\Big|_{s=0} = m_4 = r(r+1)(r+2)(r+3)\left(\frac{q}{p}\right)^4 + 6r(r+1)(r+2)\left(\frac{q}{p}\right)^3$$

$$+ 7r(r+1)\left(\frac{q}{p}\right)^2 + \left(\frac{rq}{p}\right)$$

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$$\Rightarrow m_4 = \frac{q[rp^3 + 7p^2q(r)_1 + 6pq^2(r)_2 + q^3(r)_3]}{p^4}$$

$$\mu_4 = m_4 - 4m_1m_3 + 6m_1^2m_2 - 3m_1^4$$

$$-4m_1m_2 = -12r^2(r+1)\left(\frac{q}{p}\right)^3 - 4r^2(r+1)(r+2)\left(\frac{q}{p}\right)^4 - 4\left(\frac{qr}{p}\right)^2$$

$$6m_1^2m_2 = 6r^3(r+1)\left(\frac{q}{p}\right)^4 + 6r^3\left(\frac{q}{p}\right)^3$$

$$-3m_1^4 = -3r^4\left(\frac{q}{p}\right)^4$$

$$\Rightarrow \mu_4 = \frac{r(1-p)[6-6p+p^2+3r-3pr]}{p^4}$$

$$\underset{(\text{Kurtosis})}{\alpha_4} = \frac{\mu_4}{(\mu_2)^2} = \frac{\left[r(1-p)(6-6p+p^2+3r-3pr)\right]}{p^4}\frac{(p^4)}{r^2(1-p)^2}$$

$$\Rightarrow \alpha_4 = \frac{p^2-6p+6}{r(1-p)}$$

$$\Rightarrow \boxed{\alpha_4 = \frac{p^2-6p+6}{rq}}$$

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Exponential distribution:

PDF:

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

CDF: $F(a) = P\{X \leq a\} = \int_0^a \lambda e^{-\lambda x} dx = \frac{\lambda e^{-\lambda x}}{-\lambda}\Big|_0^a$

$$\Rightarrow F(x) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Mean: $E(x) = \int_0^\infty x \lambda e^{-\lambda x} dx$

$$= x\int_0^\infty \lambda e^{-\lambda x} dx - \int_0^\infty \int_0^\infty \lambda e^{-\lambda x} dx\, dx$$

$$= x \frac{\lambda e^{-\lambda x}}{-\lambda}\Big|_0^\infty - \int_0^\infty \frac{\lambda e^{-\lambda x}}{-\lambda} dx$$

$$= -x e^{-\lambda x}\Big|_0^\infty + \int_0^\infty e^{-\lambda x} dx$$

$$= -x e^{-\lambda x}\Big|_0^\infty + \frac{e^{-\lambda x}}{-\lambda}\Big|_0^\infty = \frac{1}{\lambda}$$

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Variance: $Var(x) = E(x^2) - (E(x))^2$

$$E(x^2) = \int_0^\infty x^2 \lambda e^{-\lambda x}\, dx$$

$$= x^2 \int_0^\infty \lambda e^{-\lambda x} dx - \int_0^\infty 2x \int_0^\infty \lambda e^{-\lambda x} dx\, dx$$

$$= \lambda x^2 \frac{e^{-\lambda x}}{-\lambda}\Big|_0^\infty - \int_0^\infty \frac{2x\, \lambda e^{-\lambda x}}{-\lambda}\, dx$$

$$= \int_0^\infty 2x e^{-\lambda x} dx = \frac{2}{\lambda}\int_0^\infty x\lambda e^{-\lambda x} dx$$

$$= \frac{2}{\lambda} E(x) = \frac{2}{\lambda^2}$$

$$Var(x) = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

Moment generating function:

$$M(s) = \int_0^\infty e^{sx} \lambda e^{-\lambda x} dx = \int_0^\infty \lambda e^{-(s+\lambda)x} dx$$

$$= \lambda \frac{e^{-(-s+\lambda)x}}{-(-s+\lambda)}\Big|_0^\infty = \frac{\lambda}{\lambda - s}$$

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Skewness: $\alpha_3 = \frac{\mu_3}{(\mu_2)^{3/2}}$

$m_1 = \frac{1}{\lambda}$, $m_2 = \frac{2\lambda}{(\lambda-s)^3}\Big|_{s=0} = \frac{2\lambda}{\lambda^3} = \frac{2}{\lambda^2}$

$m_3 = \frac{6\lambda}{(\lambda-s)^4}\Big|_{s=0} = \frac{6}{\lambda^3}$, $m_4 = \frac{24\lambda}{(\lambda-s)^5}\Big|_{s=0} = \frac{24}{\lambda^4}$

$\mu_2 = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$

$\mu_3 = \frac{6}{\lambda^3} - 3\left(\frac{1}{\lambda}\right)\left(\frac{2}{\lambda^2}\right) + 2\left(\frac{1}{\lambda}\right)^3 = \frac{2}{\lambda^3}$

$\alpha_3 = \frac{\left(\frac{2}{\lambda^3}\right)}{\left(\frac{1}{\lambda^2}\right)^{3/2}} = \boxed{2}$

Kurtosis:

$\mu_4 = m_4 - {}^4C_1 m_1 m_3 + {}^4C_2 m_1^2 m_2 - {}^4C_3 m_1^3 m_1 + m_1^4$

$= \frac{24}{\lambda^4} - \frac{24}{\lambda^4} + \frac{12}{\lambda^4} - \frac{3}{\lambda^4} = \frac{9}{\lambda^4}$

$\alpha_4 = \frac{\mu_4}{(\mu_2)^2} = \frac{(9/\lambda^4)}{(1/\lambda^4)} = \boxed{9}$

$K_4 = \cancel{\alpha - \beta}\ 9 - 3 = 6$

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Normal distribution :

PDF: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ $-\infty < x < \infty$

CDF: $F(x) = \int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$

let $\frac{x-\mu}{\sigma} = t \Rightarrow x = \sigma t + \mu$

$\Rightarrow dx = \sigma dt$

$\Rightarrow \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\, t^2} \sigma dt = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt$

Mean: $E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = 1$$

$$\frac{d}{d\mu} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = \frac{d}{d\mu}(1)$$

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \left(-2\frac{(x-\mu)}{2\sigma}\right)(-1)\,dx = 0$$

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$$\Rightarrow \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} (x-\mu)\, dx = 0$$

$$\Rightarrow \int_{-\infty}^{\infty} x f(x)\, dx - \int_{-\infty}^{\infty} \mu f(x)\, dx = 0$$

$$\int_{-\infty}^{\infty} x f(x)\, dx = \mu \int_{-\infty}^{\infty} f(x)\, dx = \mu$$

<u>Variance:</u>

$$\int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = \sqrt{2\pi\sigma^2}$$

$$\frac{d}{d\sigma^2} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}} dx = \frac{d}{d\sigma^2} \sqrt{2\pi\sigma^2}$$

$$\int_{-\infty}^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \left(-\frac{1}{2}(x-\mu)^2\right)\left(-\frac{1}{\sigma^4}\right) dx$$

$$= \sqrt{2\pi}\, \frac{1}{2} (\sigma^2)^{-1/2}$$

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (x-\mu)^2 e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \sigma^2$$

$$\Rightarrow E\left((x-\mu)^2\right) = \sigma^2$$

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MGF :

$$M(s) = \int_{-\infty}^{\infty} e^{sx} f(x)\, dx$$

$$= \int_{-\infty}^{\infty} e^{sx} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Say $\frac{x-\mu}{\sigma} = z \Rightarrow dx = \sigma dz$

$\cancel{\frac{x-\mu}{\sigma} =}$

$$\Rightarrow \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{s(\sigma z+\mu)} e^{-\frac{1}{2}z^2} \cancel{\sigma} dz$$

$$= \frac{e^{s\mu}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{s\sigma z} e^{-z^2/2} dz$$

$$= \frac{e^{s\mu}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-1/2\,(z-\sigma s)^2} e^{\sigma^2 s^2/2} dz$$

$$= \frac{2 e^{s\mu} * e^{\sigma^2 s^2/2}}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-1/2\,(z-\sigma s)^2} dz$$

let $\frac{1}{2}(z-\sigma s)^2 = t \Rightarrow dt = (z-\sigma s) dz$

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$$dz = \frac{dt}{z \cdot \sigma s} = \frac{dt}{\sqrt{2t}}$$

$$\Rightarrow \frac{e^{su + \sigma^2 s^2/2}}{\sqrt{\pi}} \int_0^\infty t^{-1/2} e^{-t} dt = \frac{e^{su+\sigma^2 s^2/2}}{\sqrt{\pi}} \Gamma\frac{1}{2}$$

$$\Rightarrow m(s) = e^{su + \sigma^2 s^2/2}$$

Skewness:

$$m'(s)\Big|_{s=0} = e^{su + \sigma^2 s^2/2} (u + \sigma^2 s) = u$$

$$m''(s)\Big|_{s=0} = e^{su + \sigma^2 s^2/2} (\sigma^2) + (u + \sigma^2 s)^2 e^{su + \sigma^2 s^2/2}$$

$$= \sigma^2 + u^2$$

$$m'''(s)\Big|_{s=0} = \sigma^2 (u + \sigma^2 s) e^{su + \sigma^2 s^2/2}$$

$$+ 2(u + \sigma^2 s)(\sigma^2) e^{su + \sigma^2 s^2/2}$$

$$+ (u + \sigma^2 s)^3 e^{su + \sigma^2 s^2/2}$$

$$= u\sigma^2 + 2u\sigma^2 + u^3 = 3u\sigma^2 + u^3$$

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$$M^4(s)\Big|_{s=0} = \sigma^2(\mu+\sigma^2 s)^2 e^{s\mu+\sigma^2 s^2/2}$$
$$+\sigma^2(\sigma^2)e^{s\mu+\sigma^2 s^2/2} + 2(\sigma^2)(\sigma^2)e^{s\mu+\sigma^2 s^2/2}$$
$$+2\sigma^2(\mu+\sigma^2 s)^2 e^{s\mu+\sigma^2 s^2/2}$$
$$+(\mu+\sigma^2 s)^4 e^{s\mu+\sigma^2 s^2/2}$$
$$+3(\mu+\sigma^2 s)^2(\sigma^2)e^{s\mu+\sigma^2 s^2/2}$$

$$= 6\sigma^2\mu^2 + 3\sigma^4 + \mu^4$$

Skewness: $\alpha_3 = \dfrac{\mu_3}{(\mu_2)^{3/2}}$

$$\mu_3 = m_3 - 3m_1 m_2 + 2m_1^3$$
$$= \mu^3 + 3\mu\sigma^2 - 3\mu(\mu^2+\sigma^2) + 2\mu^3$$
$$= 3\mu^3 - 3\mu^3 + 3\mu\sigma^2 - 3\mu\sigma^2$$
$$= 0$$

$$\Rightarrow \boxed{\alpha_3 = 0}$$

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Kurtosis :

$$\mu_4 = m_4 - 4m_1 m_3 + 6m_1^2 m_2 - 4m_1^4 + m_1^4$$

$$= \mu^4 + 6\sigma^2\mu^2 + 3\sigma^4 - 4\mu(\mu^3 + 3\mu\sigma^2) + 6\mu^2(\mu^2+\sigma^2) - 3\mu^4$$

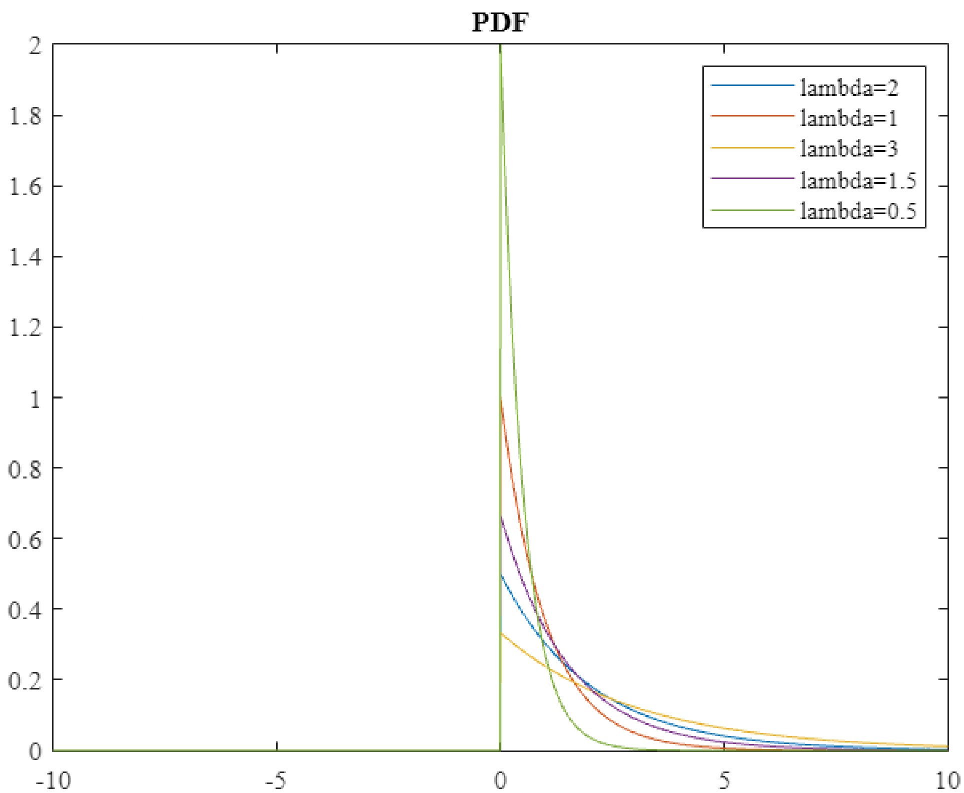
$$\Rightarrow \mu_4 = \mu^4 + 6\sigma^2\mu^2 + 3\sigma^4 - 4\mu^4 - 12\mu^2\sigma^2 + 6\mu^4 + 6\mu^2\sigma^2$$

$$\Rightarrow \mu_4 = 3\sigma^4$$

$$\alpha_4 = \frac{\mu_4}{(\mu_2)^2} = \frac{3\sigma^4}{(\sigma^2)^2} = \boxed{3}$$

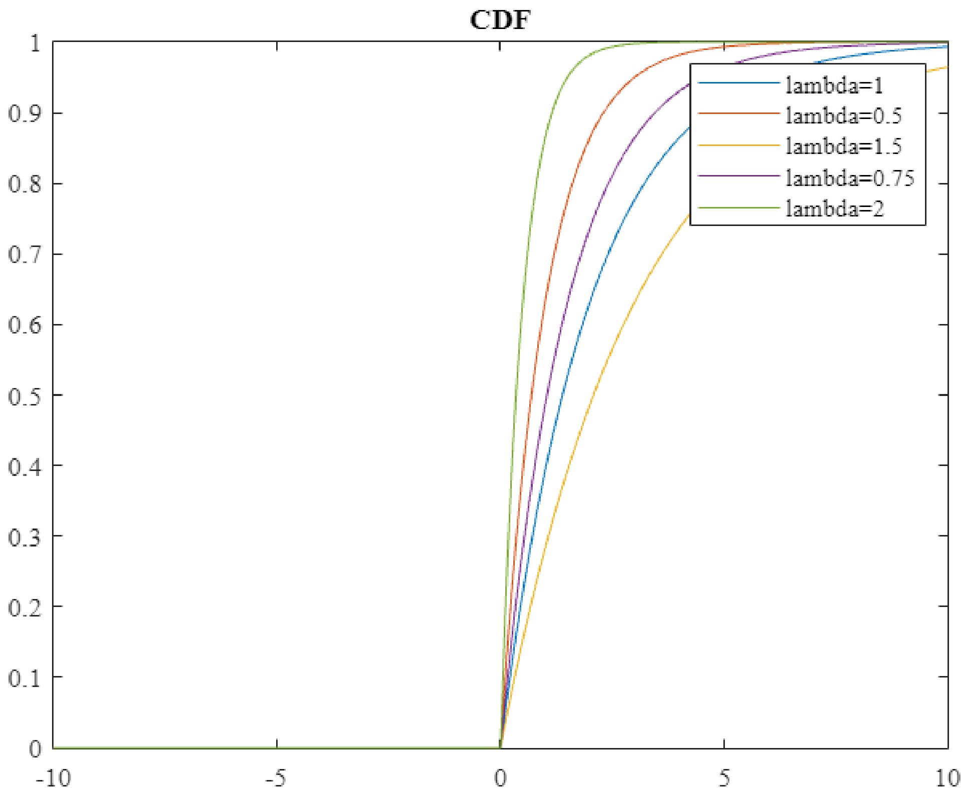
EXPONENTIAL FUNCTION

```
clear;
clc;
close all;
x=-10:0.01:10;
pdf=exppdf(x,2);
plot(x,pdf)
hold on;
pdf=exppdf(x,1);
plot(x,pdf)
hold on;
pdf=exppdf(x,3);
plot(x,pdf)
hold on;
pdf=exppdf(x,1.5);
plot(x,pdf)
hold on;
pdf=exppdf(x,0.5);
plot(x,pdf)
title("PDF");
hold off;
legend('lambda=2','lambda=1','lambda=3','lambda=1.5','lambda=0.5');
```




```
cdf=expcdf(x,2);
```

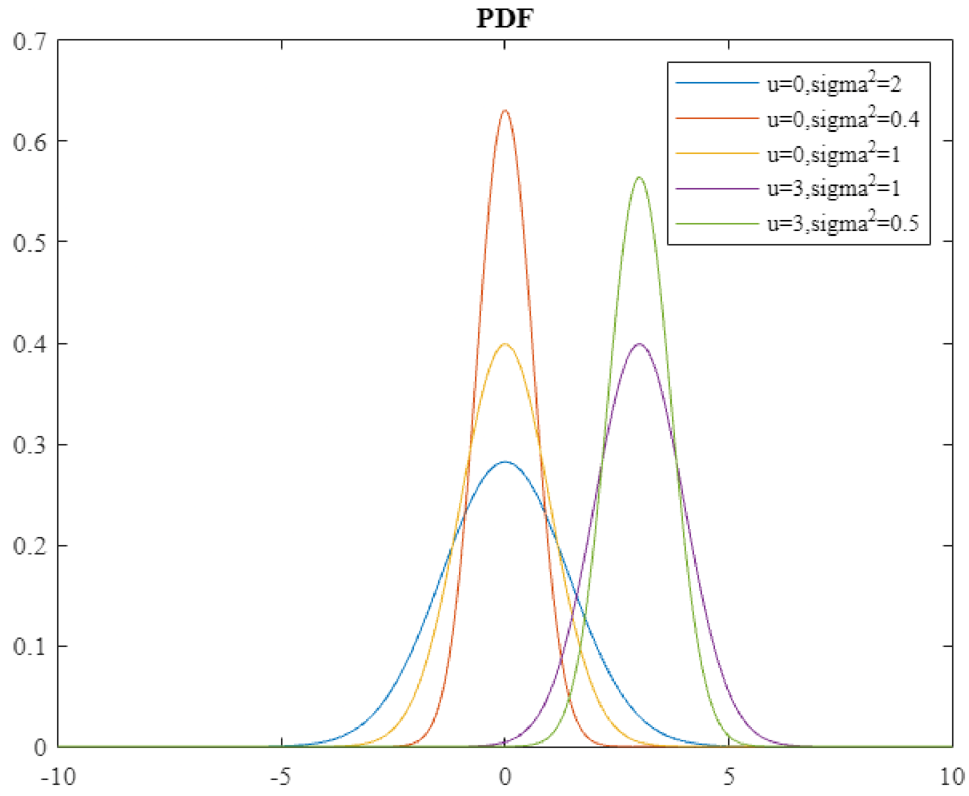
```
plot(x,cdf)
title("CDF");
hold on;
cdf=expcdf(x,1);
plot(x,cdf)
hold on;
cdf=expcdf(x,3);
plot(x,cdf)
hold on;
cdf=expcdf(x,1.5);
plot(x,cdf)
hold on;
cdf=expcdf(x,0.5);
plot(x,cdf)
hold off;
legend('lambda=1','lambda=0.5','lambda=1.5','lambda=0.75','lambda=2');
```



NORMAL DISTRIBUTION

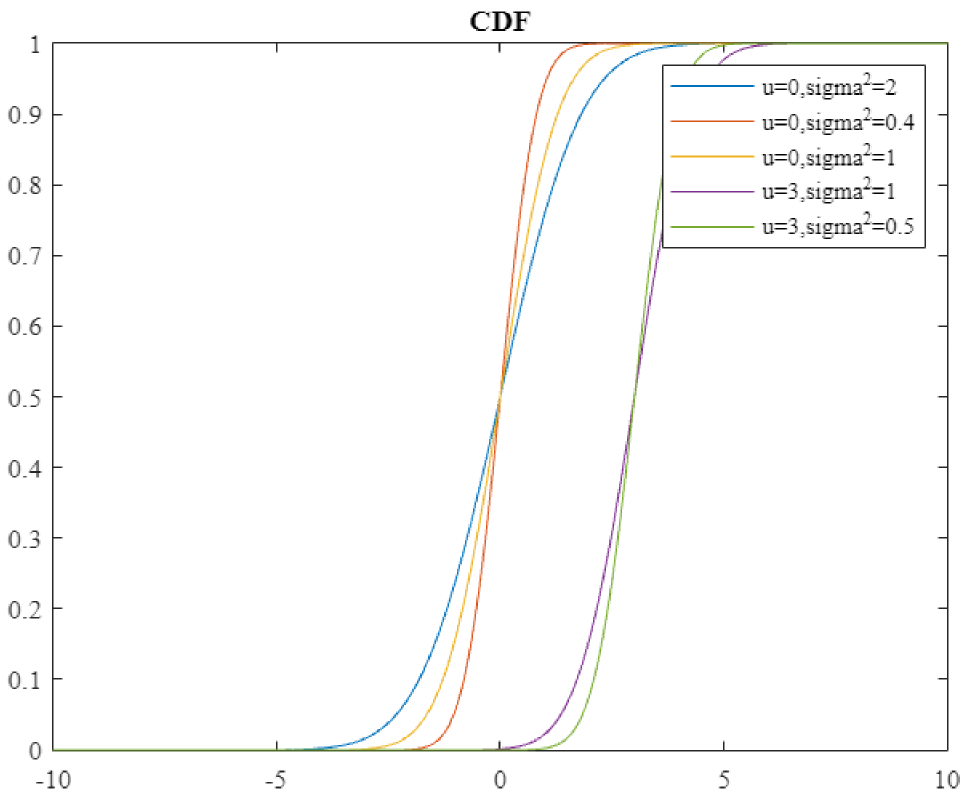
```
clear;
x=-10:0.01:10;
pdf=normpdf(x,0,sqrt(2));
plot(x,pdf)
title("PDF");
hold on;
```

```
pdf=normpdf(x,0,sqrt(0.4));
plot(x,pdf)
hold on;
pdf=normpdf(x,0,sqrt(1));
plot(x,pdf)
hold on;
pdf=normpdf(x,3,sqrt(1));
plot(x,pdf)
hold on;
pdf=normpdf(x,3,sqrt(0.5));
plot(x,pdf)
hold off;
legend('u=0,sigma^2=2','u=0,sigma^2=0.4','u=0,sigma^2=1','u=3,sigma^2=1','u=3,sigma^2=0
```




```
cdf=normcdf(x,0,sqrt(2));
plot(x,cdf)
title("CDF");
hold on;
pdf=normcdf(x,0,sqrt(0.4));
plot(x,pdf)
hold on;
pdf=normcdf(x,0,sqrt(1));
plot(x,pdf)
hold on;
pdf=normcdf(x,3,sqrt(1));
plot(x,pdf)
```

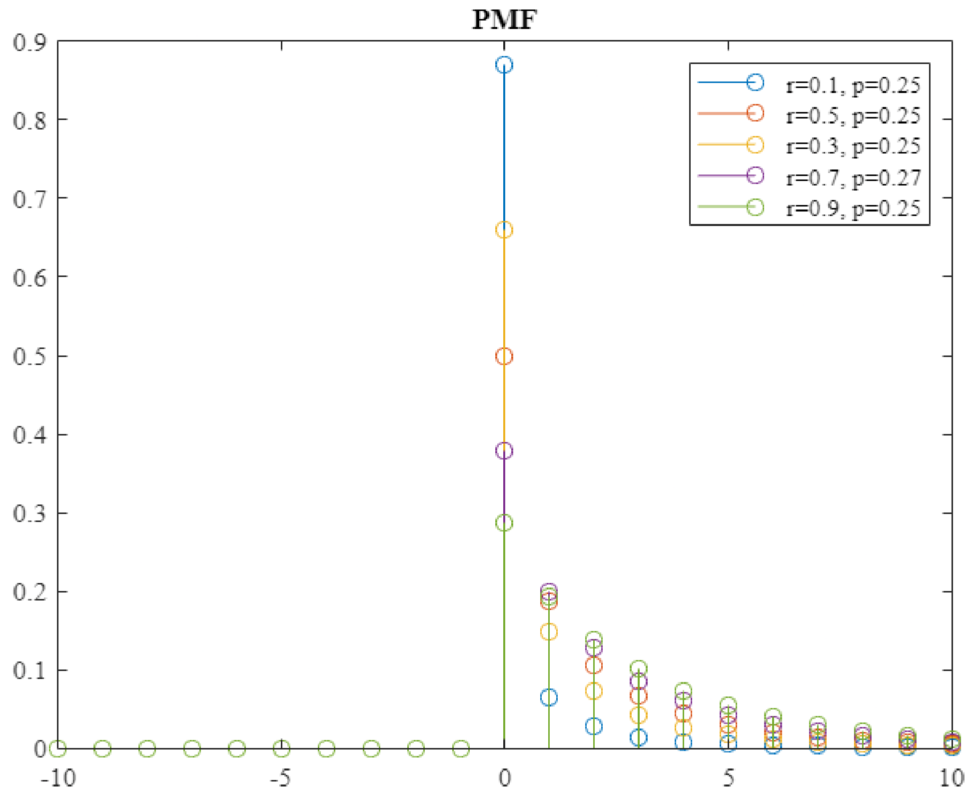
```
hold on;
pdf=normcdf(x,3,sqrt(0.5));
plot(x,pdf)
hold off;
legend('u=0,sigma^2=2','u=0,sigma^2=0.4','u=0,sigma^2=1','u=3,sigma^2=1','u=3,sigma^2=0
```



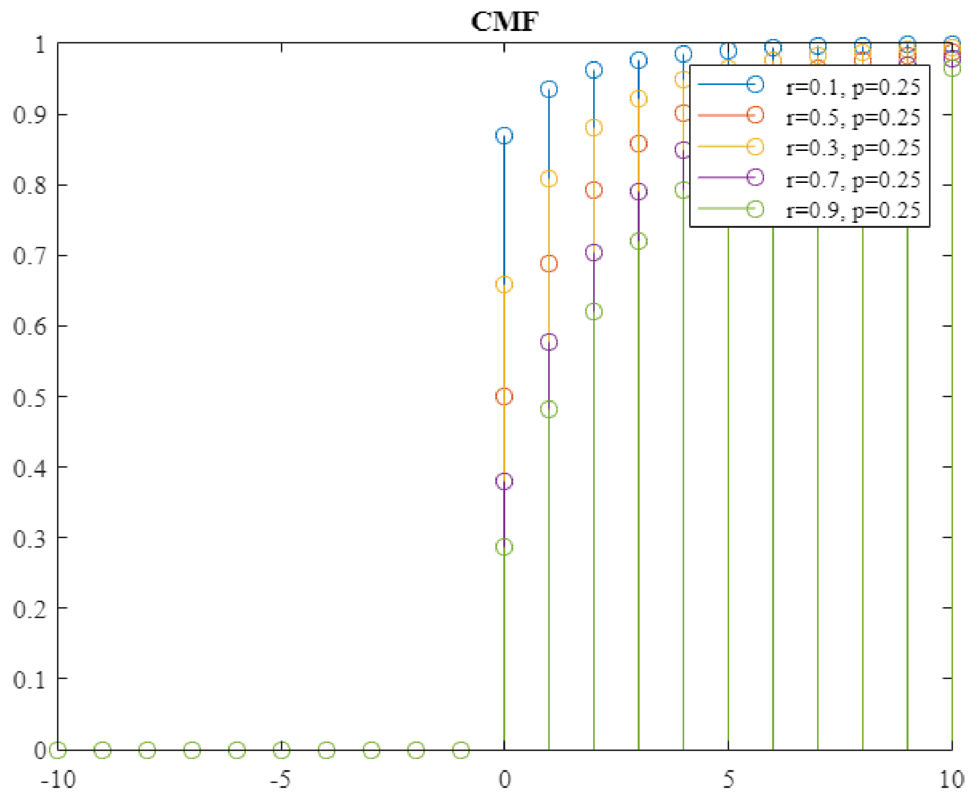
NEGATIVE BINOMIAL

```
clear;
x=-10:10;
pdf=nbinpdf(x,0.1,0.25);
stem(x,pdf)
title("PMF");
hold on;
pdf=nbinpdf(x,0.5,0.25);
stem(x,pdf);
hold on;
pdf=nbinpdf(x,0.3,0.25);
stem(x,pdf);
hold on;
pdf=nbinpdf(x,0.7,0.25);
stem(x,pdf);
hold on;
pdf=nbinpdf(x,0.9,0.25);
stem(x,pdf);
```

```
hold off;
legend('r=0.1, p=0.25','r=0.5, p=0.25','r=0.3, p=0.25','r=0.7, p=0.27','r=0.9, p=0.25'
```




```
cdf=nbincdf(x,0.1,0.25);
stem(x,cdf)
title("CMF");
hold on;
cdf=nbincdf(x,0.5,0.25);
stem(x,cdf);
hold on;
cdf=nbincdf(x,0.3,0.25);
stem(x,cdf);
hold on;
cdf=nbincdf(x,0.7,0.25);
stem(x,cdf);
hold on;
cdf=nbincdf(x,0.9,0.25);
stem(x,cdf);
hold off;
legend('r=0.1, p=0.25','r=0.5, p=0.25','r=0.3, p=0.25','r=0.7, p=0.25','r=0.9, p=0.25'
```


CMF
r=0.1, p=0.25
r=0.5, p=0.25
r=0.3, p=0.25
r=0.7, p=0.25
r=0.9, p=0.25
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-10
-5
0
5
10