

~~$$Y_f = f \cdot 2000 \text{ ft} \rightarrow 2\pi m$$

$$f_s = 5 \quad b = 1 \quad t = 1 \quad m = f = \frac{1}{1000} \quad t = \boxed{100}$$

$$t_s \geq 2 \quad a + e^{-j\omega t} e^{j\omega t} + b e^{-j\omega t} \rightarrow a, c e^{-j\omega t}$$

$$(a_0 = -\frac{1}{2})$$~~

classmate

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Fourier Series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$a_k = \frac{1}{T} \int_0^T x(t) e^{-jk\omega_0 t} dt$$

Linearity :-  $Ax(t) + By(t) \xleftrightarrow{FS} Aa_k + Ba_k$

Time Shifting :-  $x(t-t_0) \xleftrightarrow{FS} e^{-jK\omega_0 t_0} a_k$

Time reversal :-  $x(-t) \xleftrightarrow{FS} a_{-k}$

Time Scaling :-  $\sum_{k=-\infty}^{\infty} a_k e^{jk(\alpha\omega_0)t} = x(\alpha t)$

Multiplication :-  $x(t) \xleftrightarrow{FS} a_k$   
 $y(t) \xleftrightarrow{FS} b_k$

$$x(t)y(t) \xleftrightarrow{FS} \sum_{l=-\infty}^{\infty} a_l b_{l-k}$$

Conjugation :-  $x^*(t) \xleftrightarrow{FS} a_k$   
 $x^*(t) \xleftrightarrow{FS} a_{-k}^*$

Parseval's :-  $\frac{1}{T} \int_T |a_k e^{jk\omega_0 t}|^2 dt = |a_k|^2$

Differentiation :-  $x(t) \xrightarrow{FS} ak$

$$\frac{dx(t)}{dt} \xrightarrow{FS} jk\omega_0 ak$$

Integration :-  $\int_{-\infty}^t x(t) dt \xrightarrow{j\omega_0} \frac{1}{j\omega_0} ak$

### Fourier Transform

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} dt$$

Linearity :-  $ax(t) + by(t) = aX(j\omega) + bY(j\omega)$

Time shifting :-  $x(t-t_0) \leftrightarrow e^{-j\omega t_0} X(j\omega)$

Conjugation :-  $x(t) \xleftrightarrow{F} X(j\omega)$   
 $x^*(t) \xleftrightarrow{F} X^*(j\omega)$

Differentiation :-  $\frac{dx(t)}{dt} \xleftrightarrow{F} j\omega X(j\omega)$

Integration :-  $\int_{-\infty}^t x(\tau) d\tau \xleftrightarrow{F} \frac{1}{j\omega} X(j\omega) + \pi X(0) \delta(\omega)$

Time & Frequency Scaling :-  $x(at) = \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$

Duality :-  $x(t) \longrightarrow x(j\omega)$   
 $x(t) \longrightarrow 2\pi x(-\omega)$

$$-jT x(t) \xleftarrow{F} \frac{d}{d\omega} x(j\omega)$$

$$e^{j\omega_0 t} x(t) \xleftarrow{F} x(j(\omega - \omega_0))$$

$$\frac{-1}{jt} x(t) + \pi x(0) \delta(t) \xleftarrow{F} \int_{-\infty}^{\infty} x(n) dn$$

Parseval's :-  $\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$

Convolution :-  $h(t) * x(t) \xleftarrow{F} H(j\omega) X(j\omega)$

Signal FT FS

$$\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_0) \quad a_k$$

$$e^{j\omega_0 t} \quad 2\pi \delta(\omega - \omega_0) \quad a_0 = 1 \quad a_k = 0$$

$$\cos \omega_0 t \quad \pi [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] \quad a_0 = a_{-1} = \frac{1}{2}, \quad a_k = 0$$

$$x(t) = 1 \quad 2\pi \delta(\omega) \quad a_0 = 1 \quad a_k = 0$$

$$\sin \omega_0 t \quad \frac{\pi}{j} [\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] \quad a_0 = a_{-1} = \frac{1}{2j}, \quad a_k = 0$$

$$\begin{cases} 1 & |t| < T_1 \\ 0 & T_1 < |t| < T_2 \end{cases} \quad \sum_{k=-\infty}^{\infty} \frac{2 \sin k\omega_0 T_1 \delta(\omega - k\omega_0)}{k} \quad \frac{\sin k\omega_0 T_1}{k\pi}$$

$$\sum_{n=-\infty}^{\infty} \delta(t-nT) \xrightarrow{T} \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$$

$$a_k = \frac{1}{T}$$

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & |t| \geq T_1 \end{cases} \quad \frac{2 \sin \omega T_1}{\omega}$$

$$\frac{\sin \omega t}{\pi t} \quad \begin{cases} 1 & |\omega| < \omega \\ 0 & |\omega| > \omega \end{cases}$$

$$\delta(t)$$

$$u(t)$$

$$\frac{1}{j\omega} + \pi \delta(\omega)$$

$$\delta(t-t_0)$$

$$e^{-j\omega t_0}$$

$$e^{-at} u(t)$$

$$\frac{1}{a+j\omega}$$

$$t e^{-at} u(t)$$

$$\frac{1}{(a+j\omega)^2}$$

$$\frac{t^{n-1}}{(n-1)!} e^{-at} u(t) \quad \frac{1}{(a+j\omega)^n}$$

Laplace

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(s) e^{st} ds$$

linearity :-  $a x_1(t) + b x_2(t) \longleftrightarrow a X_1(s) + b X_2(s)$

 $R_1 \cap R_2$ 

Time Shifting :-  $x(t-t_0) \longleftrightarrow e^{-st_0} X(s)$

Shift in Sdomain :-  $e^{s_0 t} x(t) \longleftrightarrow X(s-s_0)$

 $R + R \{s_0\}$ 

$$e^{j\omega t} x(t) \longleftrightarrow X(s-j\omega_0)$$

Time scaling :-  $x(at) \longleftrightarrow \frac{1}{|a|} X\left(\frac{s}{a}\right)$

$\kappa_1 = a\kappa$

Conjugation :-  $x^*(t) \longleftrightarrow X^*(s^*)$

Convolution :-  $x_1(t) * x_2(t) \longleftrightarrow X_1(s) X_2(s)$

 $R_1 \cap R_2$ 

Differentiation :-  $\frac{dx(t)}{dt} \longleftrightarrow s X(s)$

$$d -tx(t) \longleftrightarrow \frac{dX(s)}{ds}$$

Integration :-  $\int_{-\infty}^t x(\tau) d\tau \longleftrightarrow \frac{1}{s} X(s)$

 $R \cap \{Re\{s\} > 0\}$

Signal

LT

ROC

$$S(t)$$

$$1$$

All S

$$u(t)$$

$$\frac{1}{s}$$

$\operatorname{Re}\{s\} > 0$

$$-u(-t)$$

$$\frac{1}{s}$$

$\operatorname{Re}\{s\} < 0$

$$\frac{t^{n-1}}{(n-1)!} u(t)$$

$$\frac{1}{s^n}$$

$\operatorname{Re}\{s\} > 0$

$$-\frac{t^{n-1} u(t)}{(n-1)!}$$

$$\frac{1}{s^n}$$

$\operatorname{Re}\{s\} < 0$

$$e^{-\alpha t} u(t)$$

$$\frac{1}{s+\alpha}$$

$\operatorname{Re}\{s\} > -\alpha$

$$-e^{-\alpha t} u(t)$$

$$\frac{1}{s+\alpha}$$

$\operatorname{Re}\{s\} < -\alpha$

$$\frac{t^{n-1}}{(n-1)!} e^{-\alpha t} u(t)$$

$$\frac{1}{(s+\alpha)^n}$$

$\operatorname{Re}\{s\} > -\alpha$

$$-\frac{t^{n-1} e^{-\alpha t} u(t)}{(n-1)!}$$

$$\frac{1}{(s+\alpha)^n}$$

$\operatorname{Re}\{s\} < -\alpha$

$$S(t-T)$$

$$e^{-sT}$$

All S

$$(\cos \omega t) u(t)$$

$$\frac{s}{s^2 + \omega^2}$$

$\operatorname{Re}\{s\} > 0$

$$(\sin \omega t) u(t)$$

$$\frac{\omega}{s^2 + \omega^2}$$

$\operatorname{Re}\{s\} > 0$

$$(e^{-\alpha t} \cos \omega t) u(t) \quad \frac{s + \alpha}{(s + \alpha)^2 + \omega^2} \quad \operatorname{Re}\{s\} > -\alpha$$

$$(e^{-\alpha t} \sin \omega t) u(t) \quad \frac{\omega}{(s + \alpha)^2 + \omega^2} \quad \operatorname{Re}\{s\} > -\alpha$$

$$u_n(t) = \frac{d^n s(t)}{dt^n} \quad s^n \quad \text{all } s$$

$$u_{-n}(t) = u(t) * \dots * u(t) \quad \frac{1}{s^n} \quad \operatorname{Re}\{s\} > 0$$