* In asymptotic ignore the lower order term.

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Recursion Tree method for solving recurrences.

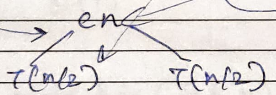
→ We consider the recursion tree and compute the total work done.

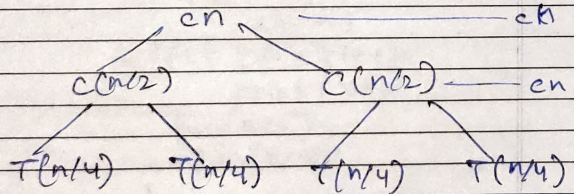
→ we write non-recursive part as root of the tree and write the recursive part as children.

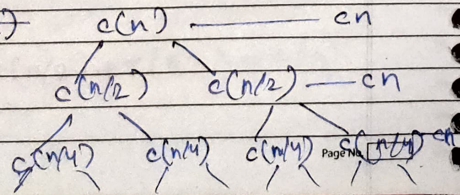
→ We keep expanding untill we see a pattern.

$T(n) = \underbrace{2T(n/2)}_{\text{recursive}} + \underbrace{cn}_{\text{non-recursive}}$

$T(1) = C$

① 

② 

③ ~~c(n/4)~~ 

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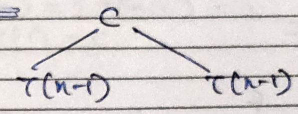
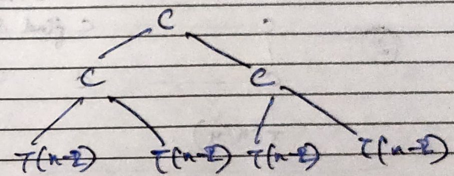
More example Recurrences

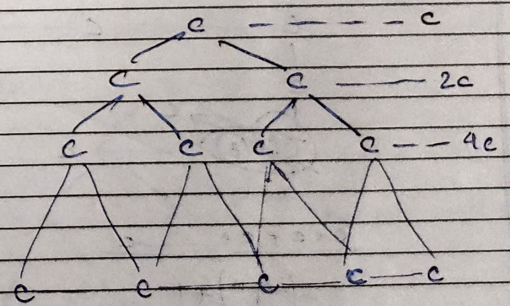
// Problem ①

$T(n) = 2T(n-1) + c$

$T(1) = c$

① 

② 

③ 

$c(1+2+4 \ldots)$ //AP

$\Rightarrow c\left(\frac{1\times(2^{n}-1)}{2-1}\right) \Rightarrow \cancel{c(2^{n-1})}\ c(2^n-1)$

$O(2^n)$

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// Problem 2

$T(n) = T(n/2) + c$

$T(1) = c$

① c
$T(n/2)$

*replacing $T(n/2)$ by c. doing untill find the pattern.

② c ——— c
c ——— c
$T(n/4)$

③ c ——— c
c ——— c
c
$T(n/8)$ ——— c

c+c+c+c+ ——— c

$(\log_2 n) + 1$

$\Theta(\log_2 n)$: Time Complexity

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// Problem 3

$T(n) = 2T(n/2) + c$

$T(1) = c$

① c ——— c
$T(n/2)$ $T(n/2)$

② c ——— c
c c ——— 2c
$T(n/4)$ $T(n/4)$ $T(n/4)$ $T(n/4)$

③ c ——— c
c c ——— 2c
c c c c ——— 4c
c c c c c

see example

c+2c+4c+ ———

$\Theta(\log_2 n)$

$\Theta\left(\frac{2^{\log_2 n} - 1}{2-1}\right) \Rightarrow \Theta(2^{\log_2 n}) \Rightarrow \Theta(n)$

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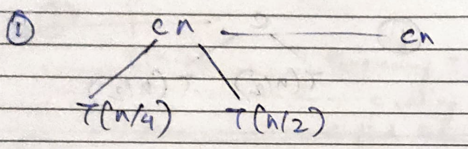
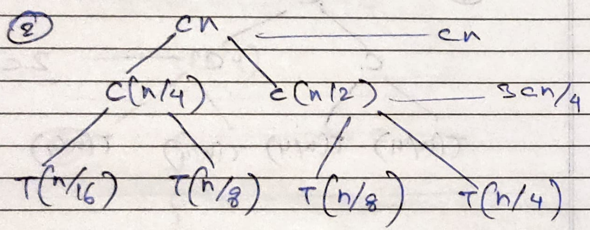
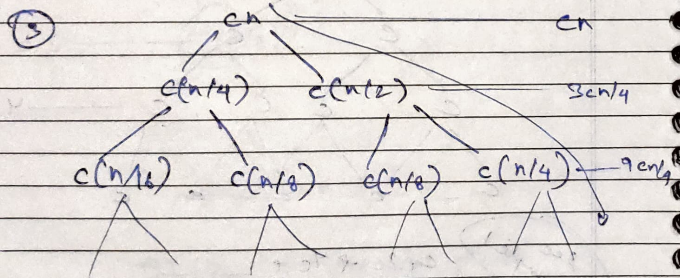
Upper Bounds using Recursion Tree method

$T(n) = T(n/4) + T(n/2) + cn$

$T(1) = c$







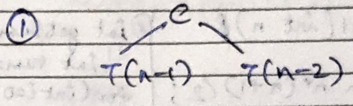
$cn + 3cn/4 + 9cn/4 + \ldots$

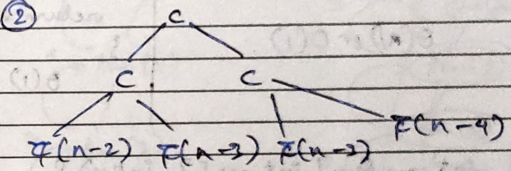
$O(\log n) \Rightarrow O\left(\frac{cn}{1-3/4}\right) \Rightarrow O(n)$

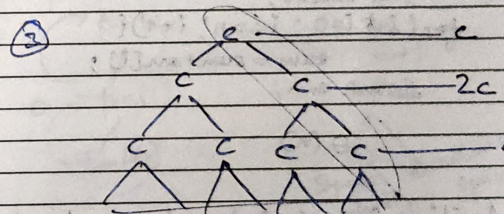
Problem 2 $T(n) = T(n-1) + T(n-2) + c$

$T(1) = c$

$T(0) = c$







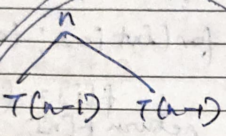
$O(\underbrace{c + 2c + 4c + \ldots}_{n \text{ terms}})$

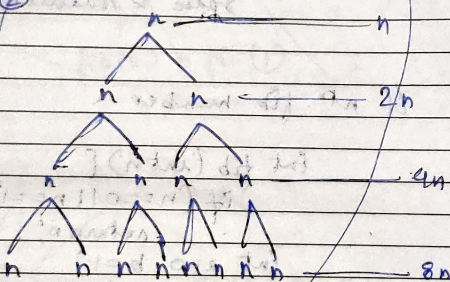
$O(2^n)$

Quiz 1 (Problems)

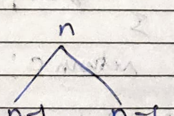
$T(1) = 1$

$T(n) = 2T(n-1) + n \;,\; n \geq 2$

① 

② 

$n(1+2+4+8+ \ldots)$

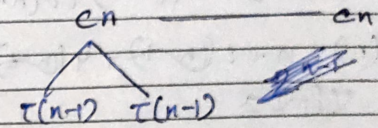


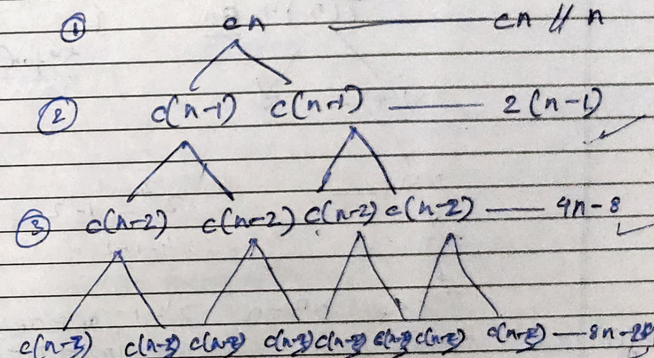
$\Theta(n)$

$\log n$

$T(1) = c$

$T(n) = 2T(n-1) + cn$

① 



$n-(n-1)$ ✓ $n + 2(n-1) + 4(n-2) + 8(n-3) + \ldots$

$2^0(n-0) + 2^1(n-1) + 2^2(n-2) + 2^3(n-3) + \ldots$

2^{n-1}

$2^n\left(n - \frac{n \times (n+1)}{2}\right)$

$2^n\left(\frac{2n - n(n+1)}{2}\right) \Rightarrow 2^n\left(\frac{2n - n^2 - n}{2}\right)$

$2^n\left(\frac{n - n^2}{2}\right)$

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$T(n) = n + 2(n-1) + 4(n-2) + 8(n-3) + 2^{n-1}(n-(n-1))$

$T(n) = n + 2(n-1) + 4(n-2) + 8(n-3) + 2^{n-1}(1)$

$2T(n) = 2n + 4(n-1) + 8(n-2) + 16(n-3) + 2^n$

$T(n) = n + 2\cancel{(n-1)} + 4\cancel{(n-2)} + 8\cancel{(n-3)} + 2^n - 2^{n-1}$

$\cancel{T(n) =}$

fun(on)
{
$2 * fun(n-1)$

$T(5) + 5n$

$2 \times T(4) +$

$T(3) +$

$T(2) + 1$

$T(1) +$

$2^n - n$ $\quad 2^1 - 1 =$ $\quad a_n = a_{n-1} + a_{n-2}$

$2^2 - 1 - 2$

$4 - 1 - 2$

$\boxed{r^2 = r -} = 0$

$4 - 2$

$f(n) = \alpha(2)^n - n$

$1 = \alpha \times 2 - 1$

$1 = 2\alpha - 1$

$2 = 2\alpha$ $\boxed{T(n) = 2T(n-1)} + n$

$\alpha = 1$

$\alpha'(x)^n$

$T(1) = 1$

Homo $\qquad$ Func.

$-n$

$f(n) = 0$

$r - 2 = 0$

$\boxed{r = 2}$

$2 \boxed{n} = 0$

$2 \neq 0$ X

$c = -1 \quad d = 0$

$\cancel{r\,c} : (kn + c) \times \alpha^1$

$\boxed{cn + d}$

$a_n = \alpha(2)^n$

$cn + d = 2cn + 2d + n.$

$\cancel{c + d = 2c + 2d + n}$

$-n + 0$

$n(c+1) + d = 0$

$\cancel{\alpha c}$ $\quad d = cn + 2d + n$ $\qquad cn + d + n = 0$

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$T(n) = 2T(n-1) + n.$

$T(n+1) = 2T(n+1-1) + n + 1$

$= 2T(n) + n + 1$

$T(n+1) = 2T(n) + n + 1$

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$T(n) = 2T(n-1) + n.$

$T(n+1) = 2T(n) + n + 1$

$T(n+2) = 2T(n+1) + n + 2$

$f(n) = n + 2$

$T(n+2) - 2T(n+1) = 0$

$r^2 - 2r = 0$ $\qquad f(n) = n + 2 \neq 0$

$r(r-2) = 0$

$r = 0 \quad r = 2$ $\qquad cn + d = 0$

Homo → $\alpha_1(0)^n + \alpha_2(2)^n$ $\qquad f(n) = cn + d$

$\to \cancel{\alpha_1} + \alpha_2(2)^n$

$\to \alpha_2 2^n.$

$c(n+2) + d - 2(c(n+1) + d) = 0$

$cn + 2c + d - 2cn - 2c - d = 0$

$-cn$

$c(n+2) + d = 2(c(n+1) + d) + n + 2$

$cn + \cancel{2c} + \cancel{d} = 2cn + \cancel{2c} + \cancel{2d} + n + 2.$

$cn + d + n + 2 = 0$

$n(c+1) + (d+2) = 0$

$c = -1 \quad d = -2$

$-n - 2.$ $\qquad \alpha 2^n - n - 2 = f(n).$

2 $\qquad 2\alpha - 1 - 2 = 1.$

$2\alpha - 3 = 1$

$\alpha = 2$

$2\alpha 2^n - n - 2$ $\qquad 2^{n+1} - n - 2$

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