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Parseval's theorem (continuous-time periodic signals)

- The average power (i.e., energy per unit time) in one period of the periodic signal $x(t)$ is

$$P = \frac{1}{T}\int_T |x(t)|^2\, dt = \sum_{k=-\propto}^{\propto} |a_k|^2$$

Proof: $|x(t)|^2 = x(t)\,x^*(t) \quad = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \left(\sum_{L=-\infty}^{\infty} a_L^* e^{-jL\omega_0 t} \right)$

$$P = \frac{1}{T}\int_T |x(t)|^2\, dt = \frac{1}{T}\int_T \left\{ \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \left(\sum_{L=-\infty}^{\infty} a_L^* e^{-jL\omega_0 t} \right) \right\} dt$$

$$\int_T e^{j(k-L)\omega_0 t}\, dt = T, \qquad if\ k = L$$
$$= 0, \qquad k \neq L$$

a_k^*

$k=L$

$$= \frac{1}{T}\left[\sum_{k=-\infty}^{\infty} a_k \sum_{L=-\infty}^{\infty} a_L^* \left\{ \int_T e^{j(k-L)\omega_0 t}\, dt \right\} \right] = \frac{1}{T}.T \sum_{k=-\propto}^{\propto} |a_k|^2 \quad = \sum_{k=-\propto}^{\propto} |a_k|^2$$

- Evaluate the complex-exponential Fourier-series expansion of the signal

$x(t) = 2 + 3\cos 2\pi t + 4\sin 3\pi t$ and then verify the Parseval's theorem.

By the definition of synthesis equation, we can write

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

ω_0 is *unknown* i.e. we have to *first determine* the T

$2 \rightarrow$ *periodic* any value of T

$\cos 2\pi t \rightarrow T_1 = 1$

$\sin 3\pi t \rightarrow T_2 = \frac{2}{3}$

$$T = \text{Least} - \text{common multiplier}\left(1, \frac{2}{3}\right) = 2$$

$$\omega_0 = 2\pi F_0 = 2\pi . \frac{1}{2} = \pi$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk.\pi.t}$$

$$= \ \ldots + a_{-2}e^{-2j\pi t} + a_{-1}e^{-j\pi t} + a_0 + a_1 e^{j\pi t} + a_2 e^{2j\pi t} + \ \ldots.$$

Using Euler relation, we can expand the following equation $x(t) = 2 + 3\cos 2\pi t + 4\sin 3\pi t$

$$x(t) = 2 + 3.\frac{1}{2}\left[e^{j2\pi t} + e^{-j2\pi t}\right] + 4.\frac{1}{2j}\left[e^{j3\pi t} - e^{-j3\pi t}\right]$$

$$= 2 + \frac{3}{2}e^{-j2\pi t} + \frac{3}{2}e^{-j2\pi t} - \frac{4}{2j}e^{-j3\pi t} + \frac{4}{2j}e^{j3\pi t}$$

Comparing two equations

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0} = \ldots + a_{-2}e^{-2j\pi t} + a_{-1}e^{-j\pi t} + a_0 + a_1 e^{j\pi t} + a_2 e^{2j\pi t} + \ldots$$

Fourier-series expansion

$$a_{-2} = \frac{3}{2} = a_2$$
$$a_{-1} = 0 = a_1$$
$$a_0 = 2$$
$$a_3 = \frac{4}{2j}$$
$$a_{-3} = -\frac{4}{2j}$$

To verify Parseval's theorem: $x(t)$ has *period* 2

$$a_{-2} = \frac{3}{2} = a_2$$
$$a_{-1} = 0 = a_1$$
$$a_0 = 2$$

$$a_3 = \frac{4}{2j}$$
$$a_{-3} = -\frac{4}{2j}$$

As per definition of power of a signal, we can write:

$$P = \frac{1}{T}\int_0^T |x(t)|^2 dt \qquad = \frac{1}{T}\int_0^T |2 + 3\cos 2\pi t + 4\sin 3\pi t|^2\, dt \quad =?$$

From Parseval's theorem: $$\sum_{k=-\infty}^{\infty} |a_k|^2 = \sum_{k=-3}^{3} |a_k|^2 \quad = |a_{-3}|^2 + |a_{-2}|^2 + |a_{-1}|^2 + |a_0|^2 + |a_1|^2 + |a_2|^2 + |a_3|^2$$

$$= 2^2 + \left(\frac{3}{2}\right)^2 + 0^2 + 2^2 + 0^2 + \left(\frac{3}{2}\right)^2 + 2^2$$

$$= \frac{33}{2}$$

$$= 16.5$$

Conjugate: As per synthesis equation, we can write

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\left(\frac{2\pi}{T}\right)t} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$x^*(t) = \left(\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}\right)^* = \sum_{k=-\infty}^{\infty} a_k^* e^{-jk\omega_0 t} = \sum_{k=-\infty}^{\infty} a_{-k}^* e^{jk\omega_0 t}$$

(using time-reversal property)

If $x(t)$ is real valued → $\overset{*}{x}(t) = x(t)$

$$\rightarrow \overset{*}{x}(t) = \sum_{k=-\infty}^{\infty} a_{-k}^* e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$\rightarrow a_{-k}^* = a_k$$

Try yourself for if $x(t)$ is pure imaginary

A periodic signal $x(t)$ with fundamental period T_0 has complex-exponential Fourier- Series coefficients a_k. Express the following signal in terms of a_k.

(a) $x^*(t)$

By the definition of Fourier-series for $given\ x(t)$ and a_k, we can write

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\left(\frac{2\pi}{T}\right)t} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

Let $y(t) = x^*(t)$

Now $$y(t) = x^*(t) = \left(\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}\right)^*$$

$$= \sum_{k=-\infty}^{\infty} a_k^* e^{-jk\omega_0 t}$$

(using conjugate property)

Solve problems

Q1. Determine the complex exponential Fourier series representation for each of the following signals:

$(a)\ x(t) = \cos\omega_0\, t$

$(c)\ x(t) = \cos(2t + \frac{\pi}{4})$

$(d)\ x(t) = \cos 4\,t + \sin 6\,t$

$(e)\ x(t) = \sin^2(t)$

$(f)\ x(t) = 2 + \cos(\frac{2\pi}{3}t) + 4\sin(\frac{5\pi}{3}t)$

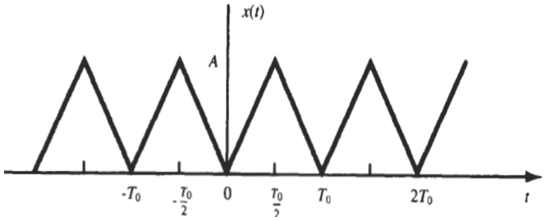
$(g)\ x(t) = \begin{cases} 1.5; 0 \le t < 1 \\ -1.5; 1 \le t < 2 \end{cases}$

Q2. Let $x_1(t)$ be a continuous-time periodic signal with fundamental frequency ω_1 and Fourier coefficients a_k. Given that

$$x_2(t) = x_1(t-1) + x_1(1-t)$$

Determine the relation of fundamental frequency ω_2 of $x_2(t)$ related to ω_1?
Evaluate a relationship between the Fourier-series coefficients b_k of $x_2(t)$ and a_k.

Q3. Consider the triangular wave $x(t)$ shown in Fig. Using the differentiation technique,
Evaluate (a) the complex exponential Fourier series of $x(t)$,
and (b) the trigonometric Fourier series of $x(t)$.



Thank you