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Group 3

Q1 a) $(p \wedge q) \rightarrow p$ = Tautology

p	q	$p \wedge q$	$(p \wedge q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Tautology

b) $p \rightarrow (p \vee q)$ = Tautology

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

c) $p \rightarrow (p \rightarrow q)$ = Contingency

p	q	$p \rightarrow q$	$p \rightarrow (p \rightarrow q)$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

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Q1 d) $(p \wedge q) \rightarrow (p \rightarrow q) \equiv$ Tautology

p	q	$p \wedge q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

e) $\neg(p \rightarrow q) \rightarrow \neg p \equiv$ Contingency

p	q	$\neg p$	$p \rightarrow q$	$\neg(p \rightarrow q)$	$\neg(p \rightarrow q) \rightarrow \neg p$
T	T	F	T	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	F	T

f) $\neg(p \rightarrow q) \rightarrow q \equiv$ Contingency

p	q	$p \rightarrow q$	$\neg(p \rightarrow q)$	$\neg(p \rightarrow q) \rightarrow q$
T	T	T	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	F	T

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Q1 g) $q \to (\neg p \to q) \equiv$ Tautology

p	$\neg p$	q	$\neg p \to q$	$q \to (\neg p \to q)$
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T

Q2 "If X sleeps, then either Y or Z will read. If Y reads, then X will not sleep. If W reads, then Z will not read.
X sleeps. Therefore, W will not read."

Say X: ~~X~~ sleeps

Y: Y reads

Z: Z reads

W: W reads

~~Given premises: $(X \to Y) \vee (X \to Z)$~~

~~$(Y \to \neg X)$~~

~~$(W \to \neg Z)$~~

~~X~~

~~Given Conclusion: $\neg W$~~

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Given premises: $X \rightarrow (Y \vee Z)$

$Y \rightarrow \neg X$

$W \rightarrow \neg Z$

X

Given conclusion: $\neg W$

1) $X \rightarrow (Y \vee Z)$ — premise
2) X — premise
3) $Y \vee Z$ — Modus ponens (1) and (2)
4) $Y \rightarrow \neg X$ — premise
5) $\neg Y$ — Modus tollens (4) and (2)
6) Z — from (3) and (5)
7) $W \rightarrow \neg Z$ — premise
8) $\neg W$ — Modus tollens (6) and (7)

Therefore, the given statement is true, demonstrated through rules of inference.

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Q3 All f, g and h from $Z^+_{+\{0\}}$ to $Z^+_{+\{0\}}$

$f(n) = n+1$, $g(n) = 3n$, $h(n) = \begin{cases} 0, & n \text{ is odd} \\ 1, & n \text{ is even} \end{cases}$

$(f \circ g) \circ h = ?$

$(f \circ g)(n) = f(g(n)) = f(3n) = 3n+1$

$[(f \circ g) \circ h](n) = f \circ g(h(n)) = \begin{cases} 3(0)+1, & n \text{ is odd} \\ 3(1)+1, & n \text{ is even} \end{cases}$

$\Rightarrow (f \circ g) \circ h\ (n) = \begin{cases} 1, & n \text{ is odd} \\ 4, & n \text{ is even} \end{cases}$

and $n \in Z^+ + \{0\}$

Q4 $\sum_{j=0}^{n} j(j+2) \equiv 0 + 1(3) + \cancel{2(4)} + 3(5) + \ldots$

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Q4 $\sum_{j=0}^{n} j(j+2) = \sum_{j=0}^{n}\left[(j)^2 + 2j\right]$

$= \cancel{\sum_{j=0}^{n} (j)^2 + 2\sum_{j=0}^{n} j}$

$= \sum_{j=0}^{n} (j)^2 + 2\sum_{j=0}^{n} j$

$= \frac{n(n+1)(2n+1)}{6} + \frac{\cancel{2}(n)(n+1)}{\cancel{2}}$

$\Rightarrow \boxed{\sum_{j=0}^{n} j(j+2) = \frac{n(n+1)(2n+1)}{6} + n(n+1)}$

$= n(n+1)\left(\frac{2n+1+6}{6}\right)$

$= \frac{n(n+1)(2n+7)}{6}$

So,

$\sum_{j=10}^{20} j(j+2) = \sum_{j=0}^{20} j(j+2) - \sum_{j=0}^{9} j(j+2)$

$= \frac{20(21)(41)}{6} + 20(21) - \frac{9(10)(19)}{6} - 9(10)$

$= \frac{\overset{10}{\cancel{20}}\,(\overset{7}{\cancel{21}})(47)}{\cancel{6}} - \frac{\overset{3}{\cancel{9}}(\overset{5}{\cancel{10}})(25)}{\cancel{6}}$

$= 3290 - 375 = \underline{2915}$

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Q5 TP → $2^n < (n+1)!$ $\forall\, n \geq 2$

Basis step: for $n=2$,

$(2)^2 = 4$, $(2+1)! = 6$

$\Rightarrow (2)^2 < (2+1)!$

True

Inductive step:

Say, statement is true for $n=k$

$(2)^k < (k+1)!$

TP → $(2)^{k+1} < (k+2)!$ (Statement true for $n=k+1$)

Since ~~(2)^k~~ $(2)^k < (k+1)!$

Multiplying both sides by k (Since $k \geq 2$)

$(2^k)k < (k+1)! \times k$

$(n! = n \times (n-1)!)$

$\Rightarrow (2^{k+1}) < (k+2)!$

Hence, the statement

$2^n < (n+1)!$ $\forall\, n \geq 2$ is proved.