1

LAB - REPORT - 8

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Q1 (a) $\cos(\omega t) \cdot u(t)$ find $X(s)$

(b) $t \cos(\omega t)\, u(t)$

(c) $t \sin(\omega t)\, u(t)$

a) $$\cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$X(s) = \int_{-\infty}^{\infty} f(\omega t)\, e^{-st} dt$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} \left(e^{j\omega t} + e^{-j\omega t}\right) e^{-st}\, u(t)\, dt$$

$$= \frac{1}{2} \int_{0}^{\infty} \left[e^{(j\omega - s)t} + e^{(-j\omega - s)t}\right] dt$$

$$= \frac{1}{2} \frac{e^{(j\omega - s)t}}{(j\omega - s)}\Big|_0^{\infty} + \frac{1}{2} \frac{e^{(-j\omega - s)t}}{(-j\omega - s)}\Big|_0^{\infty}$$

2

$$= \frac{1}{2}\left[\frac{-1}{j\omega-s} + \frac{1}{(j\omega+s)}\right]$$

$$= \frac{j\omega+s + j\omega-s}{2(j^2\omega^2-s^2)}$$

$$= \frac{-2s}{2(-\omega^2-s^2)}$$

$$X(s) = \left(\frac{s}{\omega^2+s^2}\right)$$

b) $X(s) = \frac{1}{2}\int_{-\infty}^{\infty} t\,(e^{j\omega t} + e^{-j\omega t})\,u(t)\,e^{-st}\,dt$

$$= \frac{1}{2}\int_0^{\infty} t\,e^{(j\omega-s)t}\,dt$$

$$+ \frac{1}{2}\int_0^{\infty} t\,e^{(-j\omega-s)t}\,dt$$

Say $(j\omega - s) = k$

let $I = \int t e^{(j\omega - s)t} \, dt$

$$\Rightarrow I = \int t e^{kt} \, dt$$

$$= t \frac{e^{kt}}{k} - \int \frac{e^{kt}}{k} \, dt$$

$$= \left(\frac{t e^{kt}}{k} - \frac{e^{kt}}{k^2} \right)$$

$$\Rightarrow \int t e^{(j\omega - s)t} \, dt = \frac{t e^{(j\omega - s)t}}{(j\omega - s)} - \frac{e^{(j\omega - s)t}}{(j\omega - s)^2}$$

Similarly

$$\int t e^{(-j\omega - s)t} \, dt = \frac{t e^{(-j\omega - s)t}}{(-j\omega - s)} - \frac{e^{(-j\omega - s)t}}{(-j\omega - s)^2}$$

$$\cancel{X(s) = \frac{1}{2} \left[t e^{(j\omega - s)t} - e^{(j\omega - s)t} \right]_0^\infty + \frac{1}{2} \left[t e^{(-j\omega - s)t} - e^{(-j\omega - s)t} \right]}$$

(4)

$$X(s)=\frac{1}{2}\left[\frac{t\,e^{(j\omega-s)t}}{(j\omega-s)}-\frac{e^{(j\omega-s)t}}{(j\omega-s)^2}\right]_0^\infty+\frac{1}{2}\left[\frac{t\,e^{(-j\omega-s)t}}{(-j\omega-s)}-\frac{e^{(-j\omega-s)t}}{(-j\omega-s)^2}\right]_0^\infty$$

$$=\frac{1}{2}\left[0-\left(0-\frac{1}{(j\omega-s)^2}\right)\right]$$

$$+\frac{1}{2}\left[0-\left(0-\frac{1}{(-j\omega-s)^2}\right)\right]$$

$$=\frac{1}{2}\left[\frac{1}{(j\omega-s)^2}+\frac{1}{(j\omega+s)^2}\right]$$

c) $x(t)=t\sin(\omega t)\,u(t)$

$\cancel{X(s)}$ $\sin(\omega t)=\dfrac{e^{j\omega t}-e^{-j\omega t}}{2j}$

$$\Rightarrow X(s)=\int_0^\infty t\left(\frac{e^{j\omega t}-e^{-j\omega t}}{2j}\right)e^{-st}\,dt$$

From previous result

$$\int t e^{kt}\,dt=\frac{te^{kt}}{k}-\frac{e^{kt}}{k^2}$$

5

$$X(s) = \frac{1}{2j}\int_0^\infty t e^{(j\omega - s)t} dt - \frac{1}{2j}\int_0^\infty t e^{-(j\omega - s)t} dt$$

$$= \frac{1}{2j}\left[\frac{t e^{(j\omega-s)t}}{(j\omega-s)} - \frac{e^{(j\omega-s)t}}{(j\omega-s)^2}\right]\Bigg|_0^\infty$$

$$- \frac{1}{2j}\left[\frac{t e^{(-j\omega-s)t}}{(-j\omega-s)} - \frac{e^{(-j\omega-s)t}}{(-j\omega-s)^2}\right]\Bigg|_0^\infty$$

$$= \frac{1}{2j}\left[\frac{1}{(j\omega-s)^2} - \frac{1}{(j\omega+s)^2}\right]$$

Q2 $x(t) = e^{-at}\cos(\omega t)\, u(t)$, $X(s) = ?$

$$X(s) = \int_0^\infty e^{-at}\left(\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right) e^{-st} dt$$

$$= \frac{1}{2}\int_0^\infty \left[e^{(j\omega - s - a)t} + e^{(-j\omega - s - a)t}\right] dt$$

$$= \frac{1}{2}\, \frac{e^{(j\omega - a - s)t}}{(j\omega - a - s)}\Bigg|_0^\infty + \frac{1}{2}\, \frac{e^{(-j\omega - a - s)t}}{(-j\omega - a - s)}\Bigg|_0^\infty$$

⑥

$$= \frac{-1}{2(j\omega - a - s)} - \frac{1}{2(-j\omega - a - s)}$$

$$= \frac{-1}{2(j\omega - (a+s))} + \frac{1}{2(j\omega + (a+s))}$$

$$= \frac{1}{2}\left[\frac{-j\omega - a - s + j\omega - a - s}{(j\omega)^2 - (a+s)^2}\right]$$

$$X(s) = \frac{(a+s)}{\omega^2 + (a+s)^2}$$

Q3 $x(t) = u(t) - u(t-1)$, $X(s) = ?$

$$\Rightarrow x(t) = \begin{cases} 1 & 0 \le t < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} \, dt$$

$$= \int_0^1 1 \cdot e^{-st} \, dt + 0$$

$$= \frac{e^{-st}}{-s}\Big|_0^1 = \frac{e^{-s}-1}{-s} = \left(\frac{1-e^{-s}}{s}\right)$$

Q4 $x(t) = \delta(t) - 3u(t) + 5e^{-2t}u(t)$

$$X(s) = \int_{-\infty}^{\infty} \delta(t)e^{-st}\,dt - 3\int_{-\infty}^{\infty} \cancel{e^{-st}u(t)\,dt}\; e^{-st}u(t)\,dt + 5\int_{-\infty}^{\infty} e^{-2t}e^{-st}u(t)\,dt$$

Since $\delta(t) = 1$ for $t=0$,

$$\int_{-\infty}^{\infty} \delta(t)e^{-st}\,dt = e^{-st}\Big|_{t=0} = 1$$

$$X(s) = 1 - 3\int_0^{\infty} e^{-st}\,dt + 5\int_0^{\infty} e^{(-2-s)t}\,dt$$

$$= 1 - 3\,\frac{e^{-st}}{-s}\Big|_0^{\infty} + 5\,\frac{e^{(-2-s)t}}{(-2-s)}\Big|_0^{\infty}$$

$$= 1 - \frac{3}{s} + \frac{5}{s+2}$$

Q5 $x(t) = \left(\cos 3t + e^{-5t}\right)u(t)$

$$X(s) = \int_0^\infty \cos(3t)\, e^{-st}\, dt + \int_0^\infty e^{-5t} e^{-st}\, dt$$

$$= \cancel{\int_0^\infty e^{(3j-s)t} + e^{(-3j-s)}}$$

$$= \int_0^\infty \left(\frac{e^{3jt} + e^{-3jt}}{2}\right) e^{-st}\, dt$$

$$+ \cancel{\int_0^\infty e^{-(5-s)}}$$

$$+ \int_0^\infty e^{(-5-s)t}\, dt$$

$$= \int_0^\infty \frac{e^{(3j-s)t}}{2}\, dt + \int_0^\infty \frac{e^{(-3j-s)t}}{2}\, dt$$

$$+ \left.\frac{e^{-5-s}}{(-5-s)}\right|_0^\infty$$

$$\cancel{= \frac{1}{2}\left[\frac{1}{(3j-s)} + \frac{1}{3j+s}\right.}$$

(9)

$$= \frac{1}{2}\left[\frac{e^{(3j-s)t}}{(3j-s)}\Big|_0^\infty + \frac{e^{(-3j-s)t}}{(-3j-s)}\Big|_0^\infty \right]$$

$$+ \frac{1}{s+5}$$

$$= \frac{1}{2}\cancel{\left[\frac{}{9j^2 -} \right.}$$

$$= \frac{1}{2}\left[\frac{-1}{(3j-s)} + \frac{1}{(3j+s)} \right] + \frac{1}{s+5}$$

$$X(s) = \frac{1}{2}\left[\frac{-2s}{-9-s^2} \right] + \frac{1}{s+5} = \left[\frac{1}{s+5} + \frac{s}{9+s^2} \right]$$

Q6 $x(t) = u(t) - u(t-2)$

$y(t) = x(t) * x(t)$, $Y(s) = ?$

$$Y(s) = X(s) \cdot X(s)$$

$$x(t) = \begin{cases} 1 & 0 \le t < 2 \\ 0 & \text{otherwise} \end{cases}$$

10

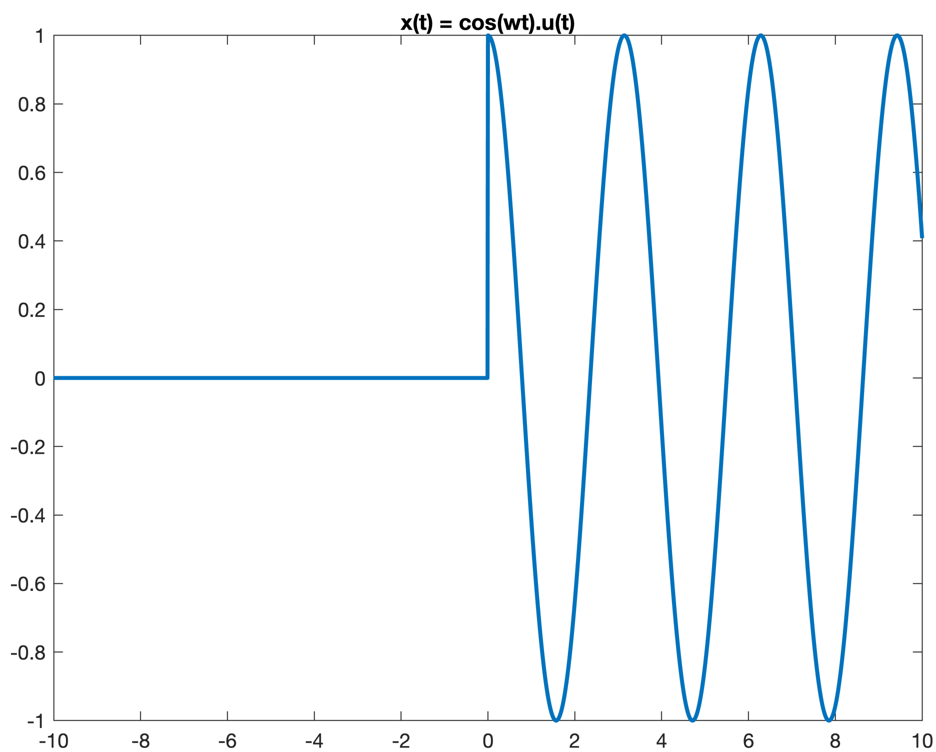
$$X(s) = \int_0^2 e^{-st}\,dt = \left.\frac{e^{-st}}{-s}\right|_0^2$$

$$= \frac{e^{-2s} - 1}{-s}$$

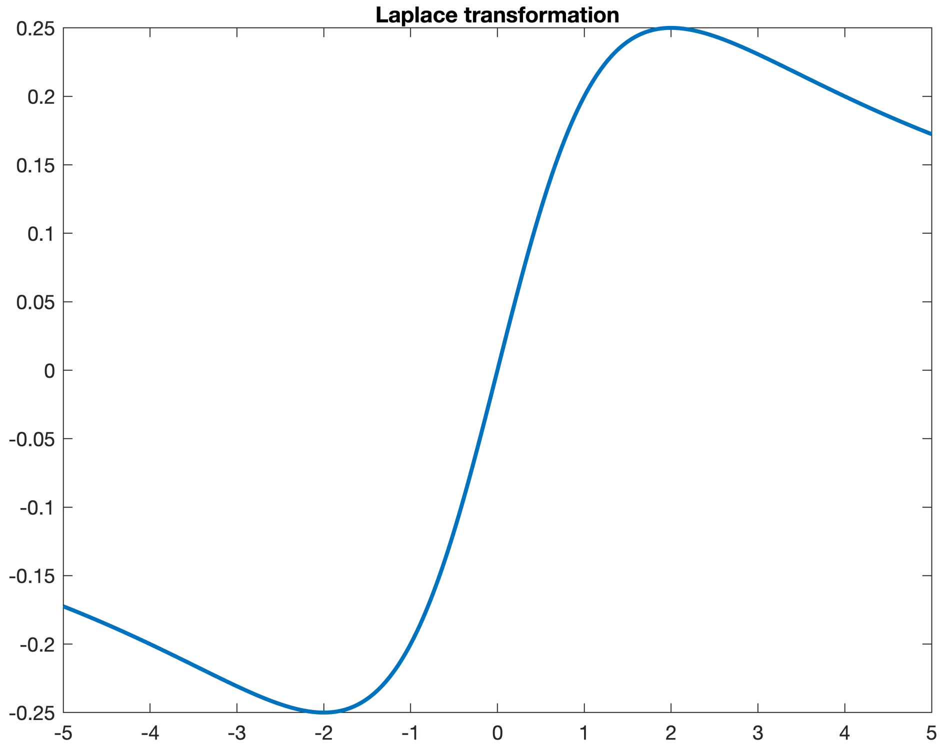
$$= \frac{1 - e^{-2s}}{s}$$

$$\Rightarrow Y(s) = \left(\frac{1 - e^{-2s}}{s}\right)^2$$

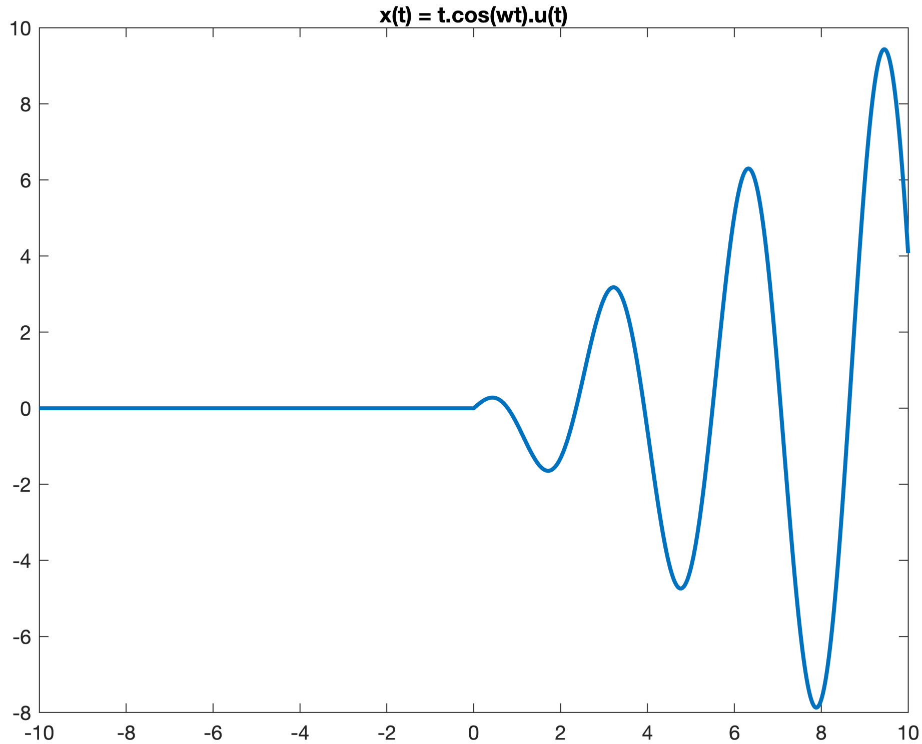
```
clear;
close all;
clc;
warning("off");
t = -10:0.01:10;
w = 2;
x = [];
count = 1;
for k = t
    if k >= 0
        x(count) = cos(w*k);
    else
        x(count) = 0;
    end
    count = count + 1;
end
plot(t, x, 'LineWidth', 2);
title("x(t) = cos(wt).u(t)");
```



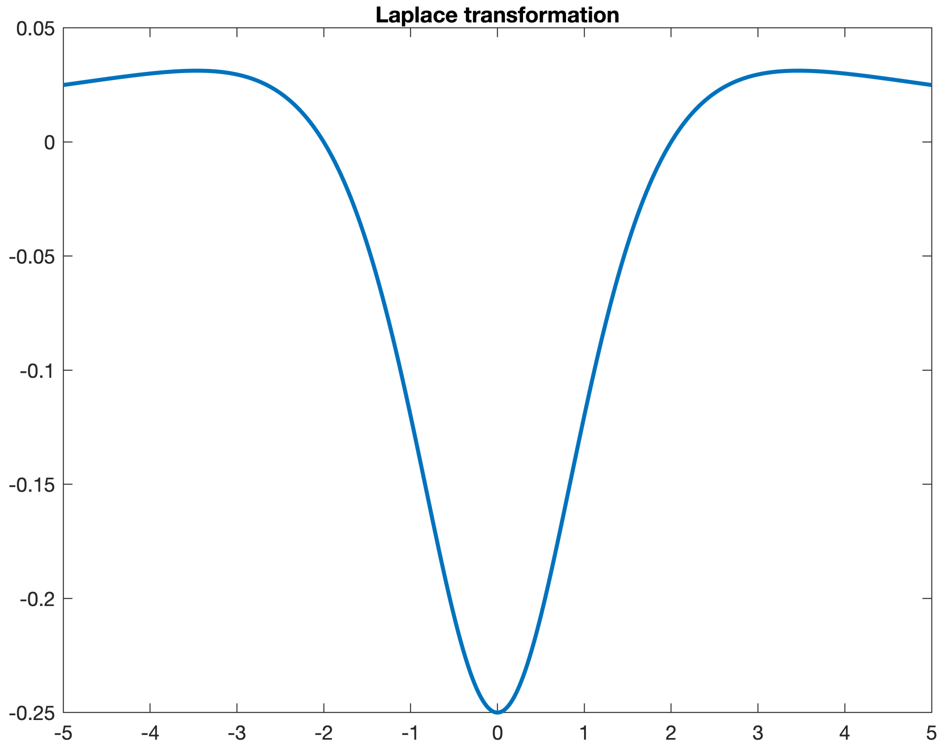

```
Xs = [];
s = -5:0.01:5;
Xs = s./(w.^2 + s.^2);
plot(s, Xs, 'LineWidth', 2);
title("Laplace transformation");
```



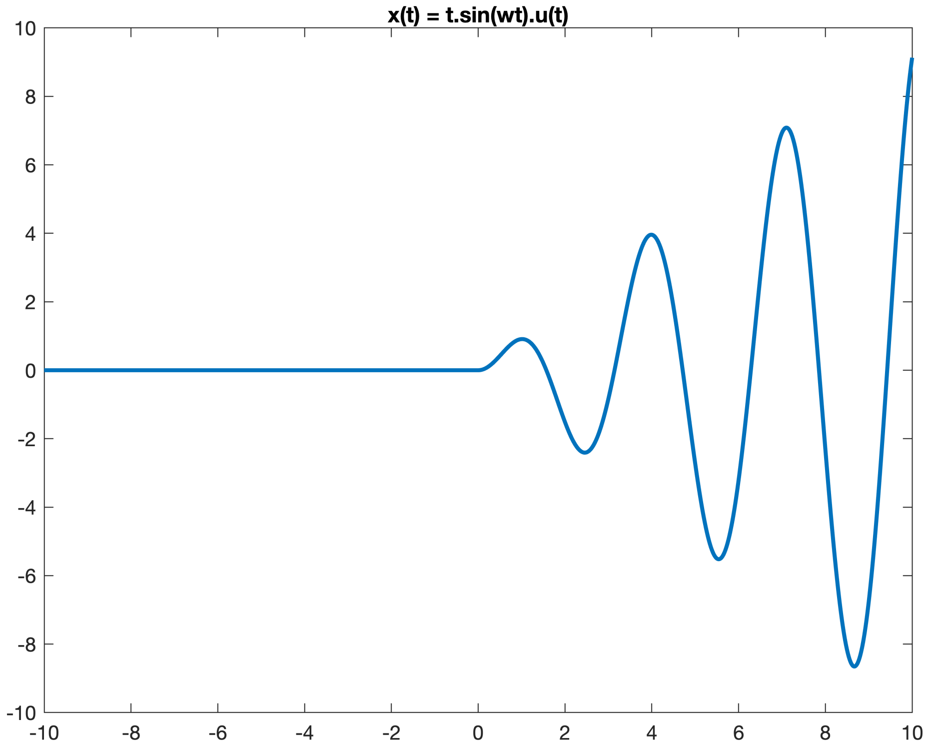

```
clear x;
clear count;
clear Xs;
count = 1;
x = [];
for k = t
    if k >= 0
        x(count) = k * cos(w*k);
    else
        x(count) = 0;
    end
    count = count + 1;
end
plot(t, x, 'LineWidth', 2);
title("x(t) = t.cos(wt).u(t)");
```



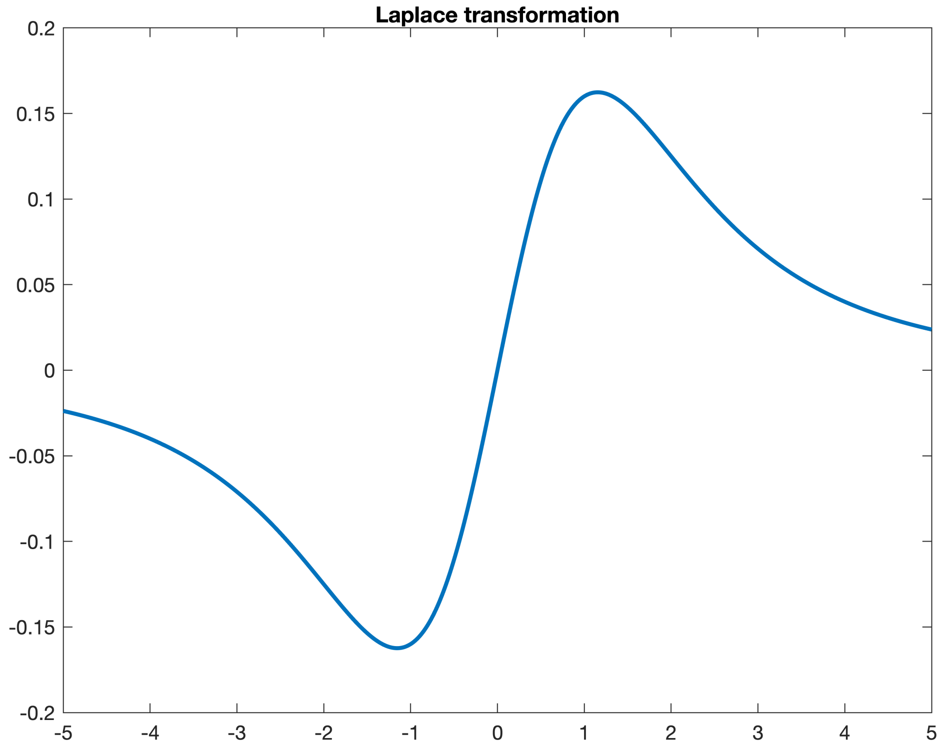

```
clear Xs;
Xs = [];
s = -5:0.01:5;
Xs = 0.5.* (1j.*w - s).^-2 + 0.5.*(s + 1j*w).^-2;
plot(s, Xs, 'LineWidth', 2);
title("Laplace transformation");
```



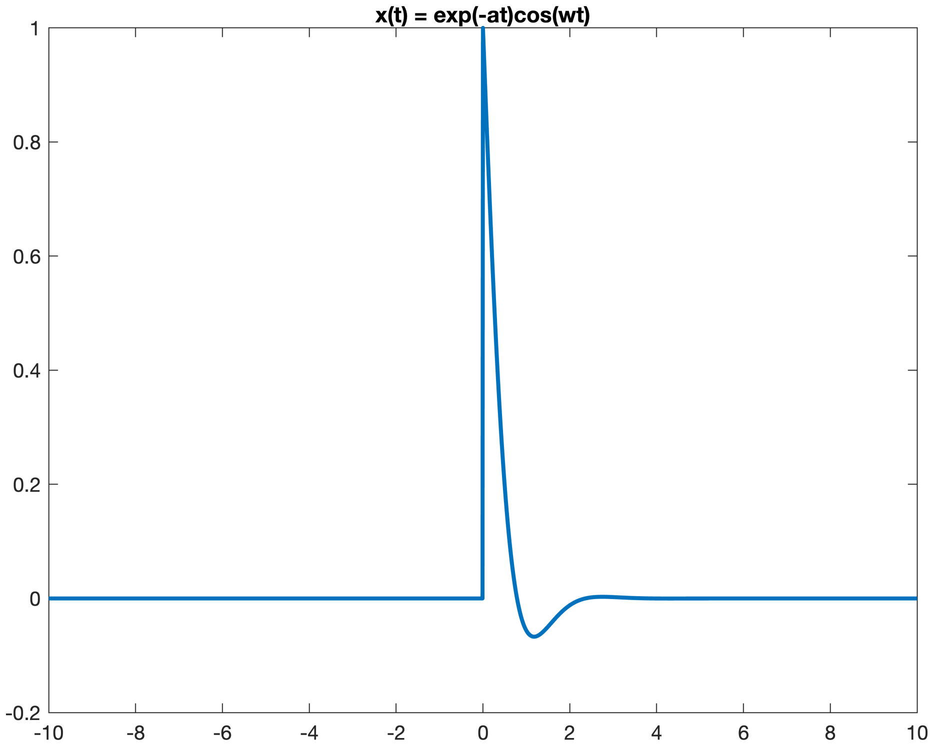

```
clear x;
clear count;
clear Xs;
count = 1;
x = [];
for k = t
    if k >= 0
        x(count) = k * sin(w*k);
    else
        x(count) = 0;
    end
    count = count + 1;
end
plot(t, x, 'LineWidth', 2);
title("x(t) = t.sin(wt).u(t)");
```



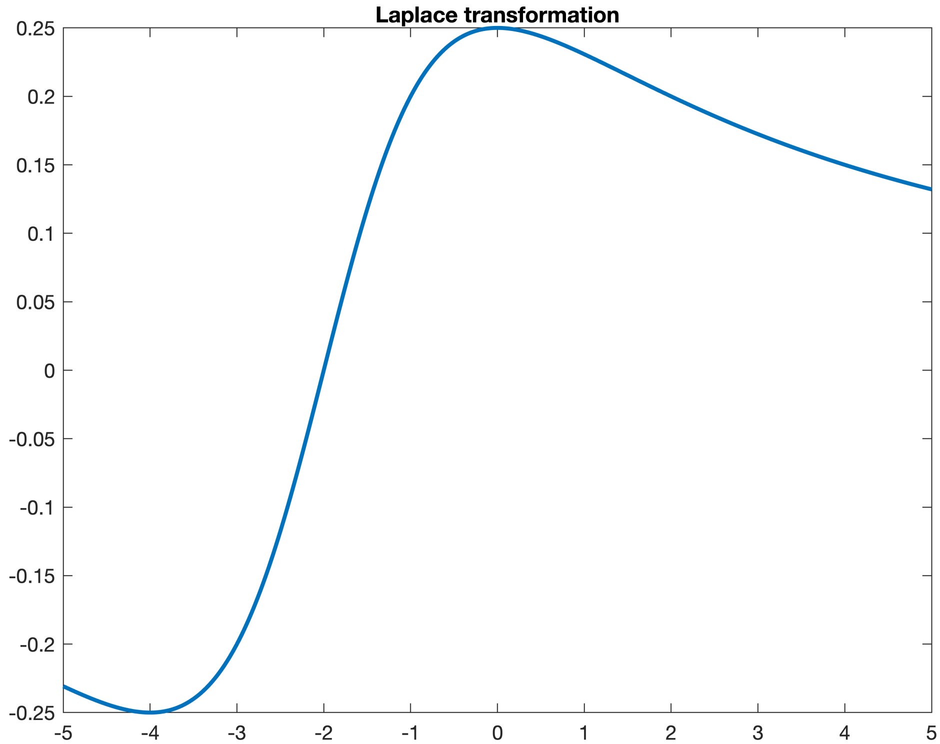

```
clear Xs;
Xs = [];
s = -5:0.01:5;
Xs = (0.5/1j).* (1j.*w - s).^-2 - (0.5/1j).*(s + 1j*w).^-2;
plot(s, Xs, 'LineWidth', 2);
title("Laplace transformation");
```



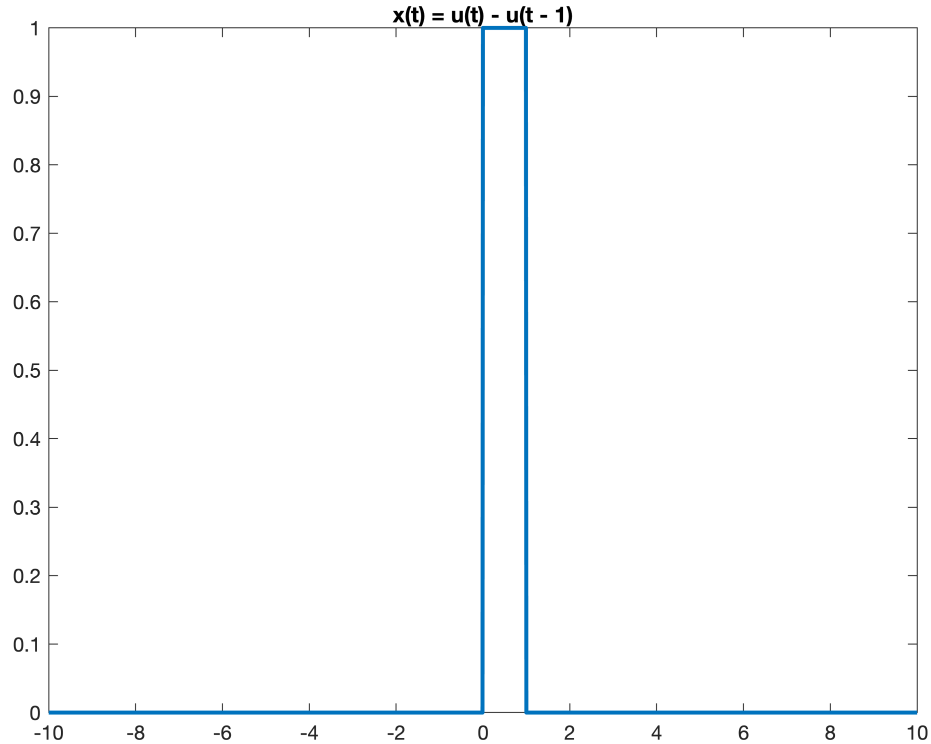

```
clear x;
clear count;
clear Xs;
x = [];
a = 2;
count = 1;
for k = t
    if k >= 0
        x(count) = exp(-1*a*k)*cos(w*k);
    else
        x(count) = 0;
    end
    count = count + 1;
end
plot(t, x, 'LineWidth', 2);
title("x(t) = exp(-at)cos(wt)");
```



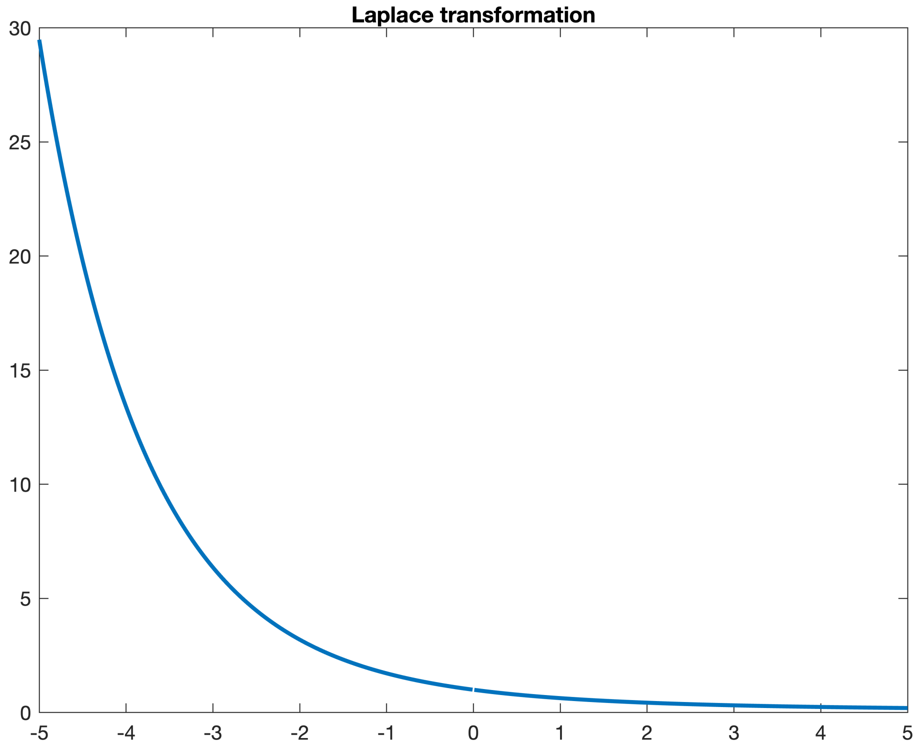

```
Xs = [];
s = -5:0.01:5;
Xs = (a + s)./(w.^2 + (a + s).^2);
plot(s, Xs, 'LineWidth', 2);
title("Laplace transformation");
```



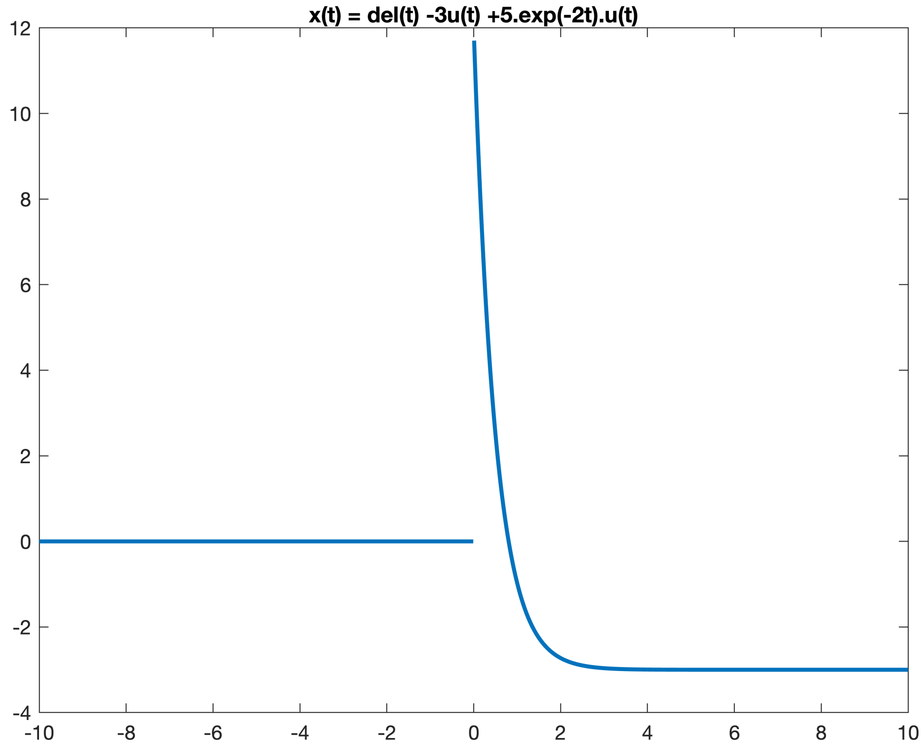

```
clear x;
clear count;
clear Xs;
count = 1;
x = [];
for k = t
    if k >= 0 && k < 1
        x(count) = 1;
    else
        x(count) = 0;
    end
    count = count + 1;
end
plot(t, x, 'LineWidth', 2);
title("x(t) = u(t) - u(t - 1)");
```



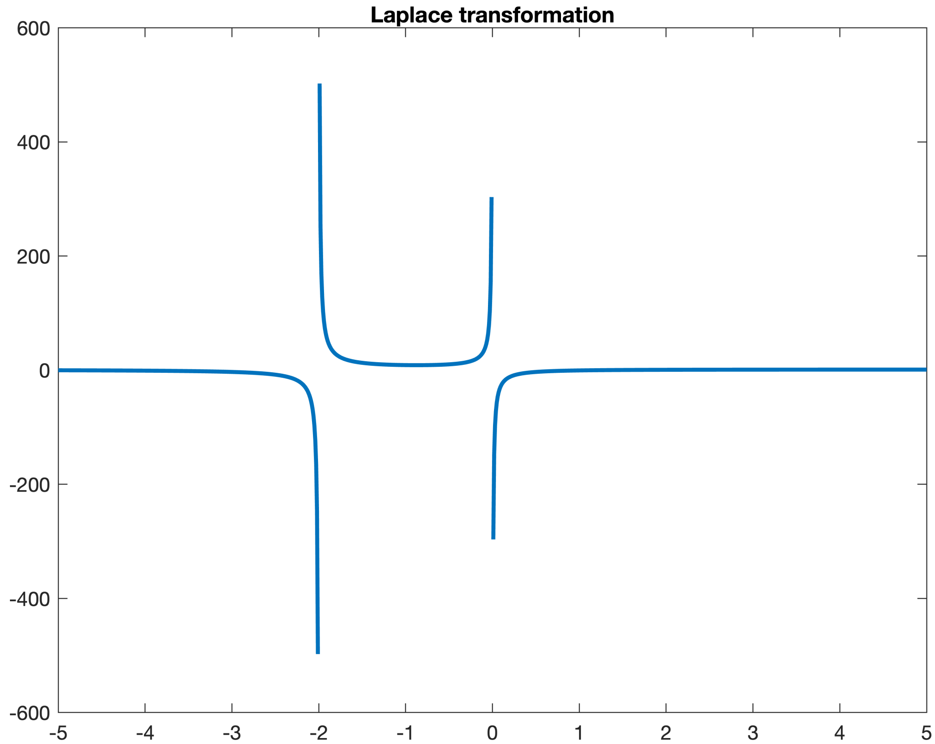

```
Xs = [];
s = -5:0.01:5;
Xs = (1 - exp(-s))./s;
plot(s, Xs, 'LineWidth', 2);
title("Laplace transformation");
```



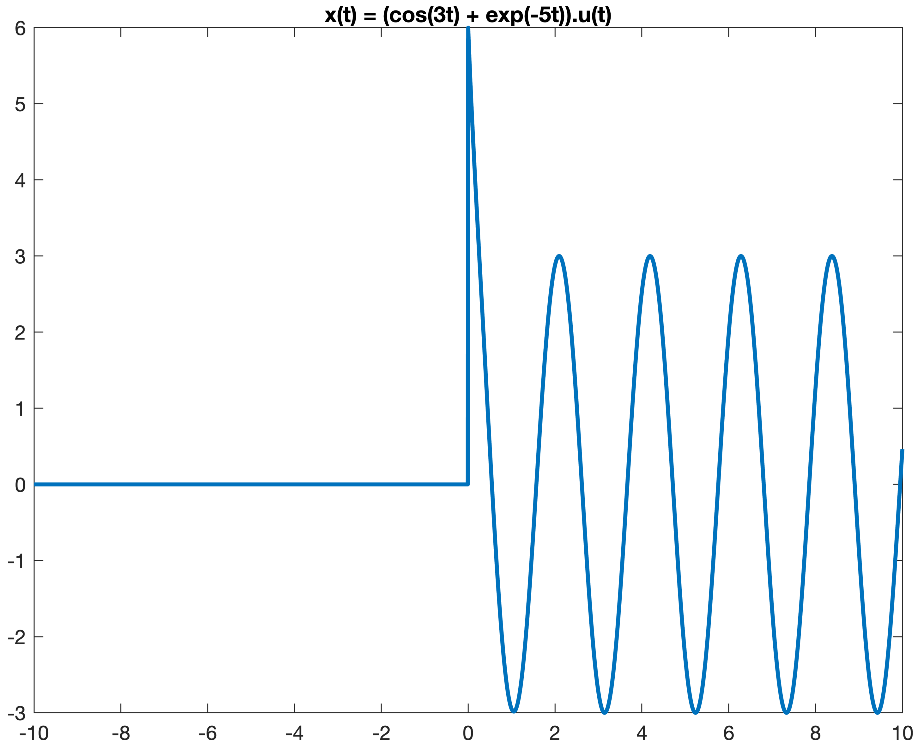

```
clear x;
clear count;
clear Xs;
count = 1;
x = [];
del = dirac(t);
u = [];
u2 = [];
for k = t
    if k >= 0
        u(count) = 1;
    else
        u(count) = 0;
    end
    count = count + 1;
end
u = 3.*u;
u2 = 5*exp(-2.*t).*u;
x = del - u + u2;
plot(t, x, 'linewidth', 2);
title("x(t) = del(t) -3u(t) +5.exp(-2t).u(t)");
```



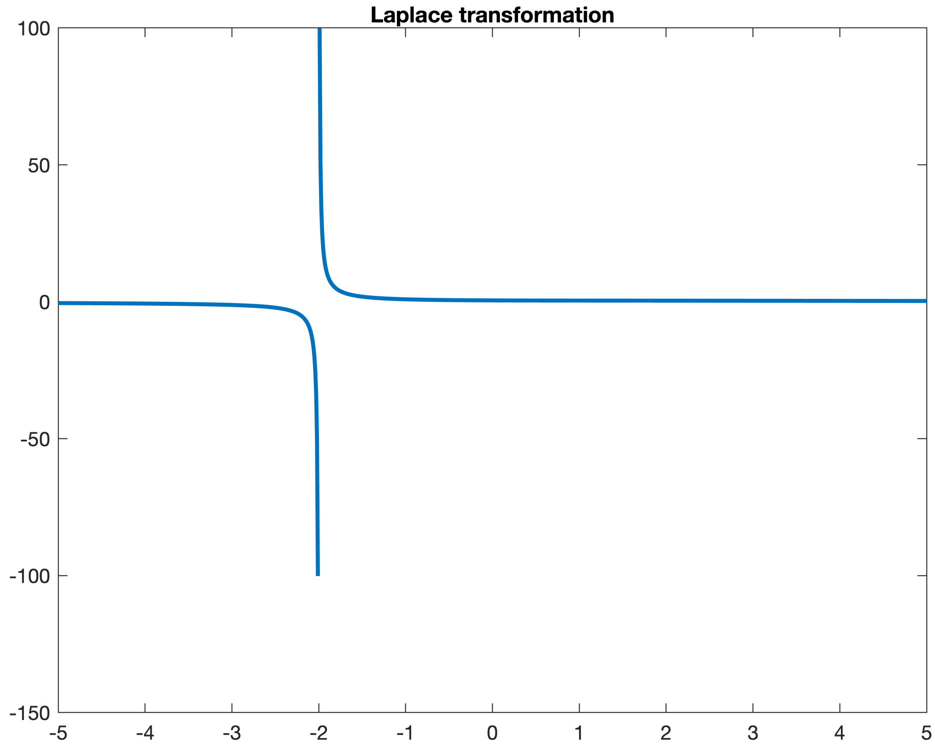

```
Xs = 1 - 3./s + 5./(s+2);
plot(s, Xs, 'LineWidth', 2);
title("Laplace transformation");
```



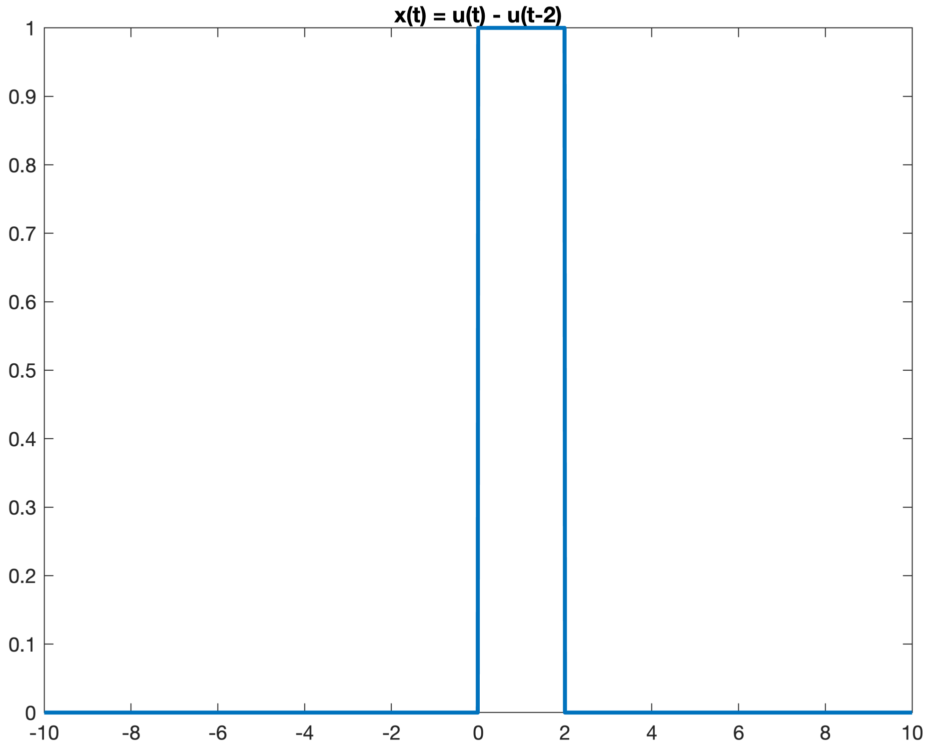

```
clear x;
clear Xs;
x = (cos(3.*t) + exp(-5*t)).*u;
plot(t, x, 'LineWidth', 2);
title("x(t) = (cos(3t) + exp(-5t)).u(t)");
```



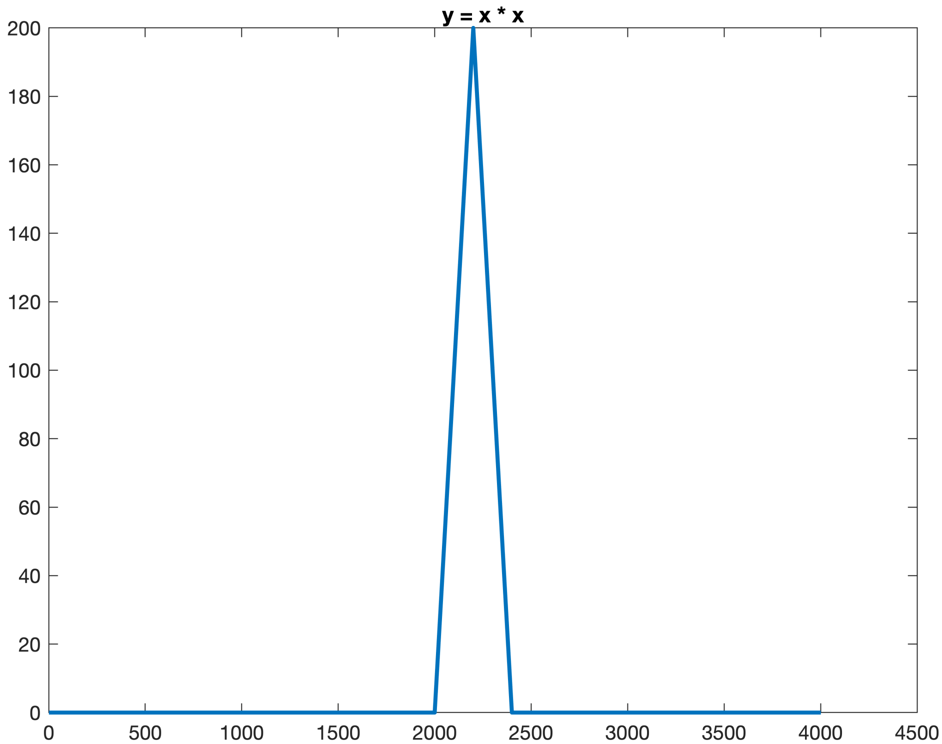

```
Xs = s./(9 + s.^2) + 1./(2 + s);
plot(s, Xs, 'LineWidth', 2);
title("Laplace transformation");
```



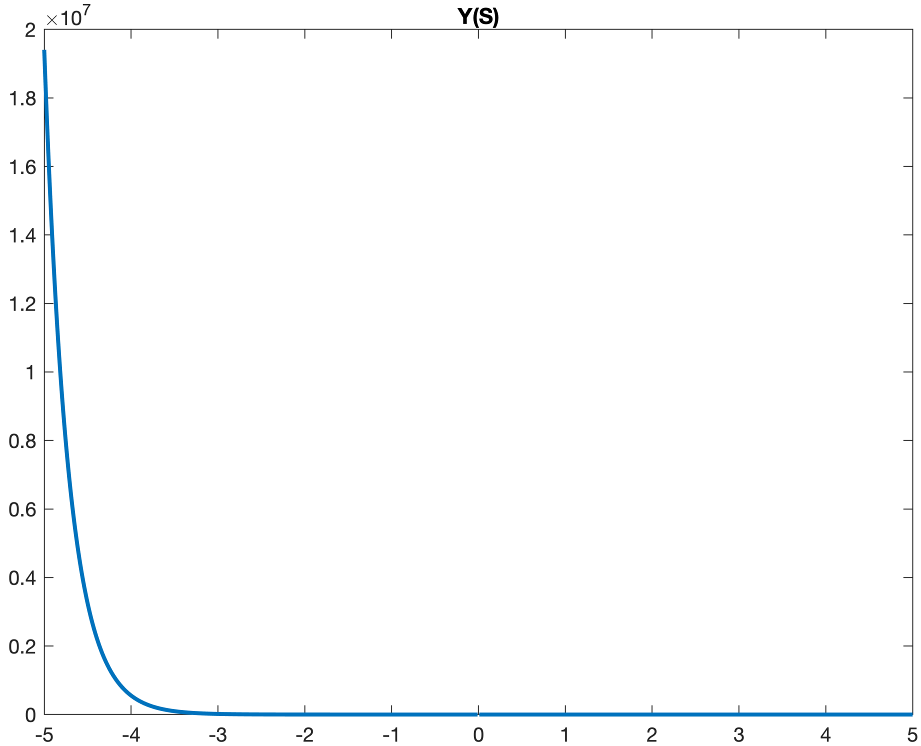

```
clear x;
clear Xs;
clear count;
count = 1;
x = [];
for k = t
    if k >= 0 && k < 2
        x(count) = 1;
    else
        x(count) = 0;
    end
    count = count + 1;
end
plot(t, x, 'LineWidth', 2);
title("x(t) = u(t) - u(t-2)")
```




```
y = conv(x, x);
plot(y, 'LineWidth',2);
title("y = x * x");
```




```
Xs = ((1- exp(-2.*s))./s).^2;
plot(s, Xs, 'Linewidth', 2);
title("Y(S)");
```


Y(S)
×10^7
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-5
-4
-3
-2
-1
0
1
2
3
4
5