Mean & Variance of Poisson Distribution -

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \dots \infty$$

$$\text{mean} = E(x) = \sum_x \cancel{p(x)}\, x P(x)$$

$$= \sum_x x \cdot \frac{\lambda^x e^{-\lambda}}{x!} = \sum_x \frac{\cancel{x} \lambda^x e^{-\lambda}}{\cancel{x}(x-1)!}$$

$$= e^{-\lambda} \sum_x \frac{\lambda \cdot \lambda^{x-1}}{(x-1)!} = e^{-\lambda} \lambda \sum_x \frac{\lambda^{x-1}}{(x-1)!} = e^{-\lambda} \lambda e^{\lambda}$$

$$= \lambda$$

$$\therefore \boxed{\text{Mean} = \lambda}$$

Variance -

$$E(x(x-1)) = \sum_x x(x-1) P(x) = \sum_x x(x-1) \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= \sum_x x(x-1) \frac{\lambda^x e^{-\lambda}}{x(x-1)(x-2)!} = \sum_x \cancel{x(x-1)} \frac{\lambda^2 \cdot \lambda^{x-2} e^{-\lambda}}{\cancel{x(x-1)}(x-2)!}$$

$$= \lambda^2 e^{-\lambda} \sum_x \frac{\lambda^{x-2}}{(x-2)!} = \lambda^2 e^{-\lambda} \cdot e^{\lambda} = \lambda^2$$

~~So, Variance : $E(x(x-1)) = \lambda^2$~~ $E(x(x-1)) = \lambda^2$

$$E(x(x-1)) = E(x^2) - E(x)$$

$$\Rightarrow E(x^2) = E(x(x-1)) + E(x)$$

$$= \cancel{E(x^2) - E(x)} \; \lambda^2 + \lambda$$

$$\text{Var} = E(x^2) - (E(x))^2 = \lambda^2 + \lambda - \lambda^2 = \lambda$$

i.e $\sigma^2 = \lambda$

$\sigma = \sqrt{\lambda}$

* Mean & variance of Poisson distribution is same.

Q) P.T $P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$ is probability mass function.

Also find MGF, CF & PGF of Poisson distribution.

Sol^n: For PMF,

① $P(x) \geq 0$

② $\sum_x P(x) = 1$

$$\sum_x P(x) = \sum_x \frac{\lambda^x e^{-\lambda}}{x!} = e^{-\lambda} \sum_x \frac{\lambda^x}{x!}$$

$$= e^{-\lambda} e^{\lambda} = 1$$

Moment generating f^n:-

$$M_x(t) = E(e^{xt}) = \sum_x e^{xt} P(x) = \sum_x e^{xt} \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= e^{-\lambda} \sum_x \frac{(e^t \lambda)^x}{x!} = e^{-\lambda} e^{\lambda e^t}$$

$$= e^{-\lambda + \lambda e^t} = e^{\lambda(e^t - 1)}$$

Characteristic f^n:

$$\phi_x(t) = E(e^{ixt}) = e^{\lambda(e^{it} - 1)}$$

Probability generating f^n -

$$Z_x(t) = E(z^x) = e^{\lambda(z-1)}$$

Mean & Variance by MGF -

$$E(x) = \frac{d(M_x(t))}{dx} = \frac{d(e^{-\lambda} e^{\lambda e^t})}{dx}\Big|_{t=0}$$

$$= e^{-\lambda}(e^{\lambda e^t} \lambda e^t)_{t=0} = e^{-\lambda} e^{\lambda} \lambda$$

$$\boxed{E(x) = \lambda}$$

$$E(x^2) = \frac{d^2(M_x(t))}{dt^2}\Big|_{t=0} = \frac{d^2(e^{-\lambda} e^{\lambda e^t})}{dt^2}\Big|_{t=0} = e^{-\lambda} \frac{d(e^{\lambda e^t} \lambda e^t)}{dt}\Big|_{t=0}$$

$$= e^{-\lambda}\left[e^{\lambda e^t}(\lambda e^t)^2 + e^{\lambda e^t} \lambda e^t\right]_{t=0}$$

$$= e^{-\lambda}\left[e^{\lambda}\lambda^2 + e^{\lambda} \cdot \lambda\right] = \lambda^2 + \lambda$$

$$\therefore E(x^2) = \lambda^2 + \lambda$$

$$\text{Var.} = E(x^2) - (E(x))^2 = \lambda^2 + \lambda - \lambda^2 = \lambda$$

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Q) Given that 2% of the fuses manufactured by a firm are defective. Find probability that a box containing 200 fuses has

1. At least 1 defective fuses.
2. 3 or more defective fuses.
3. No defective fuses.

Sol^n: $n = 200$ $\quad p = \frac{2}{100} = 0.02$

$\lambda = np = 4$

$P(x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{4^x \cdot e^{-4}}{x!}$

1. $P(x \geq 1)$

$= 1 - P(x<1) = 1 - P(0) = 1 - \frac{4^0 \cdot e^{-4}}{0!} = 1 - e^{-4}$

2. $P(x \geq 3)$

$= 1 - P(x<3) = 1 - P(0) - P(1) - P(2)$

$= 1 - e^{-4} - 4e^{-4} - 8e^{-4}$

$= 1 - e^{-4}(1+4+8) = 1 - 13e^{-4}$

3. $P(0)$

$= e^{-4}$

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Q2. In certain factory turning out blades there is small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Using poisson distribution find approximate no. of packets containing

1. No. of defective blades
2. One defective blade in consignment of 10000 packets.

Sol^n: (1) $p = 0.002$ $\quad n = 10$

$\lambda = np = 0.02$

$P(x) = \frac{e^{-0.02}(0.02)^x}{x!}$

$P(0) = e^{-0.02} = 0.9802$

No. of 0 defective blades in 10000 consignment $= 10000(0.9802)$
$= 9802$ blades.

(2) ~~$n = 10000$ $\quad p = 0.002$~~

~~$\lambda = 20$~~

~~$P(x) = \frac{e^{-20}(20)^x}{x!}$~~

~~$P(1) = e^{-20} \cdot 20$~~

(2) $\lambda = 0.02$

$P(x) = \frac{e^{-0.02}(0.02)^x}{x!}$

$P(1) = \frac{e^{-0.02} \cdot 0.02}{1} = 0.02(0.9802)$

No. of 1 defective blades in consignment of 10000 $= 10000(0.02 \times 0.9802)$

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Q) If probability of a bad reaction from a certain infection is 0.01. Find the chance that out of 200 individuals ~~morth~~ more than 2 will get bad reaction.

Solⁿ: $p = 0.01 \quad n = 200$

$\lambda = np = 2$

$P(x) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-2} 2^x}{x!}$

$P(x > 2)$

$= 1 - P(x \le 2)$

$= 1 - P(0) - P(1) - P(2)$

$= 1 - e^{-2} - 2e^{-2} - 2e^{-2}$

$= 1 - 5e^{-2}$

Fitting of Poison distribution :-

Q) A skilled typist on routine work kept a record of mistakes made per day during 300 working days

Mistakes/day	0	1	2	3	4	5	6
No. of days	143	90	42	12	9	3	1

$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$

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Solⁿ:

Mistakes/day	f	xf	P(x)	300 P(x)
0	143	0	0.411	≈ 123
1	90	90	0.365	≈ 110
2	42	84	0.163	≈ 49
3	12	36	0.048	≈ 14
4	9	36	0.011	≈ 3
5	3	15	0.002	≈ 1
6	1	6	0.003	≈ 0
	300	265		300

$\bar{x} = \frac{\Sigma xf}{\Sigma x} = \frac{265}{300} = 0.89$

Exponential distribution –

* A continuous R.V x which has the following pdf $\boxed{f(x) = \lambda e^{-\lambda x}}$; $\lambda > 0 \quad 0 < x < \infty$ is called exponential distribution

Mean ~~&~~ & Variance of Exponential distribution –

$E(x) = \int_{-\infty}^{\infty} x \cdot f(x) = \int_0^{\infty} x \lambda e^{-\lambda x} \cdot dx = \lambda \int_0^{\infty} x e^{-\lambda x} \cdot dx$

$= 1/\lambda$

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$\boxed{\therefore \text{Mean } (E(x)) = \frac{1}{\lambda}}$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x) \cdot dx = \lambda \int_0^{\infty} x^2 \cdot e^{-\lambda x} \cdot dx = \frac{2}{\lambda^2}$$

~~Variance~~ $E(x^2) = \frac{2}{\lambda^2}$

$$\text{Variance} = E(x^2) - (E(x))^2$$
$$= \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

mgf of Exponential distribution –

$$M_x(t) = E(e^{xt}) = \int_{-\infty}^{\infty} e^{xt} f(x) \cdot dx$$

$$= \int_0^{\infty} e^{xt} \lambda \cdot e^{-\lambda x} \cdot dx = \lambda \int_0^{\infty} e^{-x(\lambda - t)} \cdot dx$$

$$= \lambda \left(\frac{e^{-x(\lambda - t)}}{\lambda - t} \right)_0^{\infty} = \lambda \left(0 - \frac{1}{\lambda - t} \right)$$

$$M_x(t) = \frac{\lambda}{t - \lambda}$$

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Characteristic function –

$$\phi_x(t) = E(e^{ixt}) = \frac{\lambda}{it - x}$$

Q) The length of telephone conversation is an exponential variable with mean = 3 min. Find probability that call

① ends in less than 3 min

② takes b/w 3 – 5 min.

Solⁿ: ~~$x = 3$ min~~ Mean $= \frac{1}{\lambda}$

so, $\frac{1}{\lambda} = 3$ i.e $\lambda = \frac{1}{3}$

$$f(x) = \lambda \cdot e^{-\lambda x} = \frac{1}{3} e^{-\frac{1}{3}x}$$

① $P(x < 3)$

$$\int_0^3 f(x) \cdot dx = \int_0^3 \frac{1}{3} e^{-\frac{1}{3}x} \cdot dx = \frac{1}{3} \left. \frac{e^{-\frac{1}{3}x}}{-\frac{1}{3}} \right|_0^3$$

$$= -e^{-\frac{1}{3}x} \Big|_0^3 = -\left[e^{-1} - e^{0}\right] = -(e^{-1} - 1) = 1 - \frac{1}{e}$$

② $$\int_3^5 f(x) \cdot dx = -e^{-\frac{1}{3}x} \Big|_3^5 = -\left[e^{-\frac{5}{3}} - e^{-1}\right]$$

$$= e^{-1} - e^{-\frac{5}{3}}$$

Moments :-

Moment about origin.	Moment about mean	Moment about any other point.
$\mu_r' = E(x-0)^r$	$\mu_r = E(x-\bar{x})^r$	$\mu_r'' = E(x-A)^r$
		$\mu_1'' = E(x-A)$
$\mu_0' = E(x-0)^0 = 1$	$\mu_0 = E(x-\bar{x})^0 = 1$	$\mu_2'' = E(x-A)^2$
$\boxed{\mu_1' = E(x) = mean}$		$\mu_3'' = E(x-A)^3$
	$\mu_1 = E(x-\bar{x}) = E(x) - E(\bar{x})$	$\mu_4'' = E(x-A)^4$
$\mu_2' = E(x^2)$	$= \bar{x} - \bar{x} = 0$	
	i.e. $\boxed{\mu_1 = 0}$	
$\mu_3' = E(x^3)$		
$\mu_4' = E(x^4)$		

$$\mu_2 = E(x-\bar{x})^2 = E(x^2 - 2x\bar{x} + \bar{x}^2)$$
$$= E(x^2) - 2\bar{x}E(x) + \bar{x}^2$$
$$= E(x^2) - 2E(x)E(x) + (E(x))^2$$
$$= E(x^2) - 2(E(x))^2 + (E(x))^2$$

$$\boxed{Var(x) = \sigma^2 = \mu_2 = E(x^2) - (E(x))^2}$$

$$\mu_2 = \mu_2' - (\mu_1')^2$$

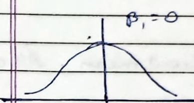
$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

Skewness & Kurtosis :-

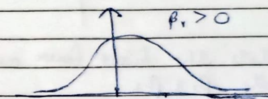
Skewness : mean lack of symmetry.

$$\boxed{\beta_1 = \frac{\mu_3^2}{\mu_2^3}}$$



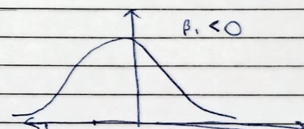
Mean = median = Mode

Symmetrical



mode median mean

Positively skewed.

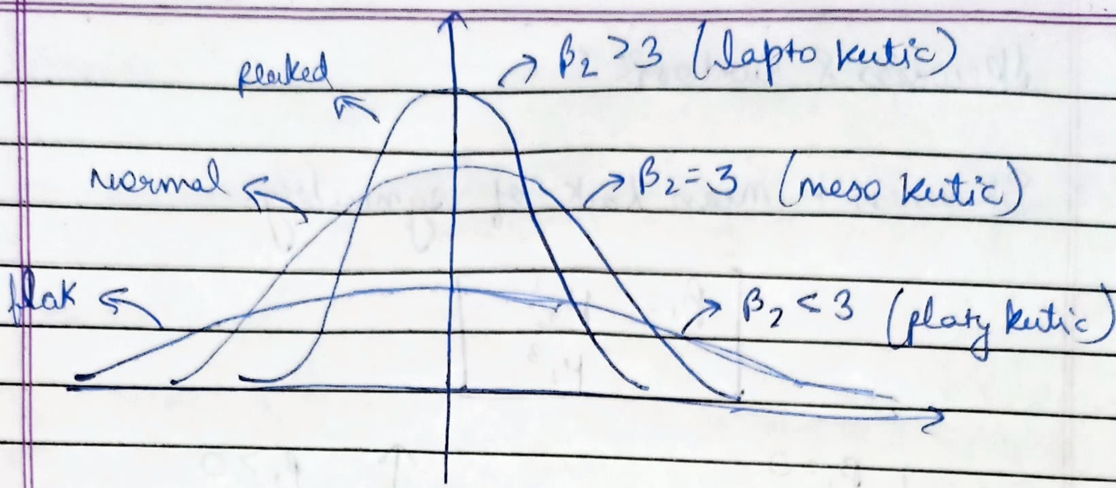


mean Median mode

Negatively skewed.

* Kurtosis : The extent to which a distribution is peaked or flat is called kurtosis.

$$\boxed{\beta_2 = \frac{\mu_4}{\mu_2^2}}$$



Q) Calculate the first four moment about mean. Also calculate β_1 & β_2.

x_i	1	2	3	4	5	6	7	8	9
y_i	1	6	13	25	30	22	9	5	2