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ASSIGNMENT -2

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Q1 (a) $\phi \in \{\phi\}$

True, since any $a \in \{a\}$ is true.

(b) $\phi \in \{\phi, \{\phi\}\}$

True, since $a \in \{a, \{a\}\}$

(c) $\{\phi\} \in \{\phi\}$

False, a set cannot belong to itself, can only be a subset.

$\{a\} \notin \{a\}$

(d) $\{\phi\} \in \{\{\phi\}\}$

True, as if we represent $\{\phi\}$ as a, $a \in \{a\}$ is true for all sets/elements.

(e) $\{\phi\} \subseteq \{\phi, \{\phi\}\}$

True. Any $a \subseteq \{a, b\}$

(f) $\{\{\phi\}\} \subseteq \{\phi, \{\phi\}\}$

True. If a denotes $\{\phi\}$, b denotes ϕ, $\{a\} \subseteq \{b, a\}$ is true.

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(g) $\{\{\phi\}\} \subseteq \{\{\phi\}, \{\phi\}\}$

True. Similar to previous part, $\{a\} \subseteq \{a, a\}$ is true.

(h) $\phi \subseteq \{1,2,3\} \rightarrow \phi \not\subseteq \{(1,2), (2,3), (1,3)\}$

Null set ϕ is a subset for every set.

Therefore, $\phi \subseteq \{1,2,3\}$ is true,

$\phi \not\subseteq \{(1,2), (2,3), (1,3)\}$ is false,

$(T \rightarrow F)$ the ~~con~~ given statement is false.

(i) $A = \{1,2\} \rightarrow A \times \phi \neq \phi$ (x denotes cartesian product)

False, since Cartesian product of any set with the null set gives a null set.

(j) $\phi \times \phi \neq \phi$

False, since cartesian product of any set with ϕ gives ϕ

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Q2 Say a_n is the number of ternary strings of length n without two consecutive 0s or 1s or 2s.

(a) Say $(a_0)_n \Rightarrow$ String ends with 0
$(a_1)_n \Rightarrow$ String ends with 1
$(a_2)_n \Rightarrow$ String ends with 2

$$(a_0)_n = (a_1)_{n-1} + (a_2)_{n-1}$$

$$(a_1)_n = (a_0)_{n-1} + (a_2)_{n-1}$$

$$(a_2)_n = (a_0)_{n-1} + (a_1)_{n-1}$$

$$a_n = (a_0)_n + (a_1)_n + (a_2)_n$$

$$\Rightarrow a_n = (a_1)_{n-1} + (a_2)_{n-1} + (a_0)_{n-1} + (a_2)_{n-1} + (a_0)_{n-1} + (a_1)_{n-1}$$

$$a_n = 2\left[(a_0)_{n-1} + (a_1)_{n-1} + (a_2)_{n-1}\right]$$

$$\Rightarrow \boxed{a_n = 2a_{n-1}}$$

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(a) Total number of ternary strings $= 3^n$, $\boxed{S_0=0}$

(Strings of length=1) $a_1 = 3$ $(S_1 - S_0)$

(Strings of length=2) $a_2 = 3^2 - a_1$ $(S_2 - S_1)$

$= 6$

Initial conditions: $\underline{a_1 = 3}$, $\underline{a_2 = 6}$

(a) For consecutive 0s, 1s or 2s,

$b_n = 3^n - \cancel{b_{n-1}}\ a_n$ (Total $- a_n$)

$b_n = 3^n - 2[a_{n-1}] = 3^n - 2[3^{n-1} - b_{n-1}]$

$b_n = 3^n - 2(3^{n-1} - b_{n-1})$

$\underline{b_n} = 3^{n-1}(3-2) + 2b_{n-1} = \underline{3^{n-1} + 2b_{n-1}}$

(b) Initial $\Rightarrow$ $\underline{b_1 = 0}$

$\underline{b_2} = 3^1 + 2b_1 = \underline{3}$

(c) $b_3 = 3^2 + 2b_2 = 15$, $b_4 = 3^3 + 2b_3 = 57$,

$b_5 = 3^4 + 2b_4 = \underline{195}$

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$b_6 = 3^5 + 2b_5 = \boxed{633}$

(d) $b_n = 2b_{n-1} + 3^{n-1}$

Associated homogeneous part:

$$b_n = 2b_{n-1}$$

$$b_n^{(h)} = \alpha(2)^n$$

for $f(n) = 3^n$, probable solution $= 3^n = b_n^{(p)}$

$$b_n = b_n^{(h)} + b_n^{(p)} = 3^n + \alpha(2)^n$$

$b_1 = 0$, $n = 1$

$\Rightarrow 3 + 2\alpha = 0 \Rightarrow \boxed{\alpha = -3/2}$

$b_n = 3^n - \frac{3}{2}(2)^n \Rightarrow \boxed{b_n = 3^n - 3(2)^{n-1}}$

$b_6 = 3^6 - 3(2)^5 = 633$, same answer

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Q3 ~~Let~~ Consider $(3333+1) = 3334$ different positive integers consisting only of 1s:

(1, 11, 111, ...) where the last integer has (3334) 1s in its expansion.

When these integers are divided by 3333, there can be 3333 possible remainders (0 to 3332).

When we divide these 3334 integers by 3333, by pigeonhole principle, there have to be at least two integers (say, a and b) which have the same remainder.

Let $a = 3333(m) + r$
and $b = 3333(n) + r$
where m, n, r are positive integers, all distinct.

$(a-b) = 3333(m-n)$
Since $(m-n) \neq 0$, $(a-b)$ is divisible by 3333.

$(a-b)$ will only consist of 0s and 1s,
and hence we proved that there exists at least one number consisting of 0s and 1s which is divisible by 3333.

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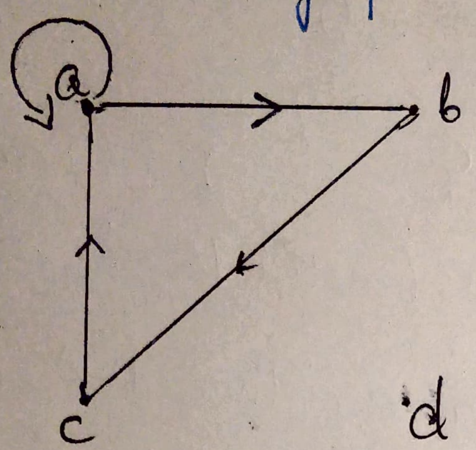
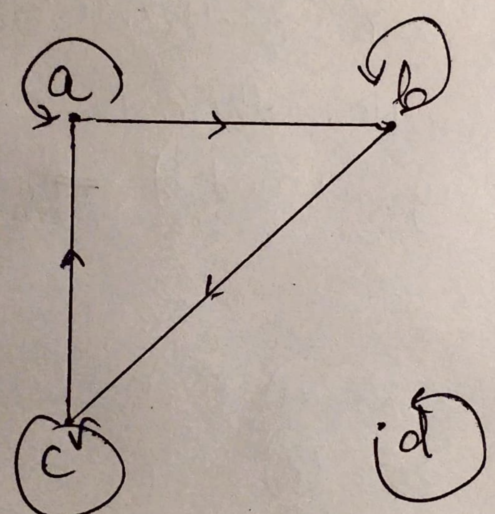
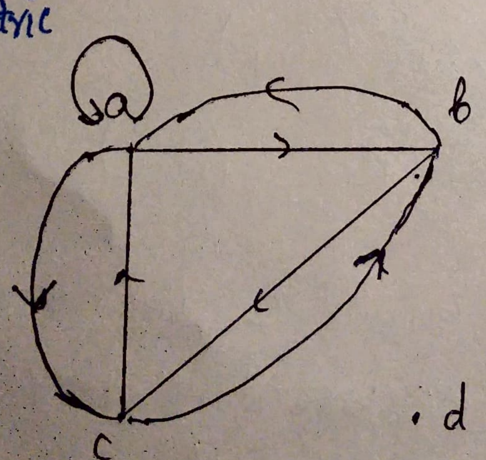
Q4 Given directed graph:



The reflexive closure of this relation will be:



The ~~transitive~~ symmetric closure will be:



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Q5 $R = \{ (0,1), (0,2), ~~(2,1)~~ (1,1), (1,3), (2,2), (3,0) \}$
defined on $A = \{0,1,2,3\}$

Transitive closure of $R \Rightarrow \{(0,1), (0,2), (1,1), (1,3), (2,2),$
$(3,0), (0,3), (1,0), (3,1), (3,2), (3,3),$
$(1,2), (0,0)\}$

Zero-one matrix form =
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Q6 Population at the end of 2020 = 8 000 000
No. of immigrants at the end of year "n" = 25000n

Rate of increment of population = 8% p.a.

To find: Population of city at the end of 2030 (using recurrence relation)

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Let a_n be the population of city ~~at~~ after the end of year n, then

$$a_n = 108\% \text{ of } a_{n-1} + 25000n$$

$$\Rightarrow a_n = 1.08a_{n-1} + 25000n$$

Population at the end of 2020 = 8 000 000

$$= a_0$$

Associated homogenous recurrence relation $\Rightarrow$

$$a_n = 1.08a_{n-1}$$

Solution $a_n^{(h)} = \alpha(1.08)^n$

If $F(n) = 25000n$, say $p_n = cn+d$

where p_n denotes a trial solution, c and d are constants.

So,

$$cn+d = 1.08\left(c(n-1)+d\right) + 25000n$$

$$cn+d = 1.08cn - 1.08c + 1.08d + 25000n$$

$$-0.08cn - 0.08d + 1.08c = 25000n$$

$$-1.08c + 25000n + 0.08cn + 0.08d = 0$$

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$(25000 + 0.08c)n + (0.08d - 1.08c) = 0$

$\Rightarrow$ Since $(cn+d)$ is a trial solution,

$25000 + 0.08c = 0$ and $0.08d - 1.08c = 0$

$\underline{c = -312500}$

$0.08d = 1.08c$

$\Rightarrow d = -4218750$

So, $a_n^{(p)} = -312500n - 4218750$

$a_n = a_n^{(p)} + a_n^{(h)}$

$= -312500n - 4218750 + \alpha(1.08)^n$

$a_0 = 8000000, n = 0$

$\Rightarrow$

$8000000 = -4218750 + \alpha$

$\Rightarrow \alpha = 12218750$

So, $a_n = -312500n - 4218750 + 12218750(1.08$

For the year 2030, $\underline{n = 10}$

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$$a_{10} = -312\,500\,(10) - 4218\,750 + 12\,218\,750\,(1.08)^{10}$$

$$a_{10} = 19\,035\,614.81$$

$\Rightarrow$ Population of city at the end of 2030 = 19035614