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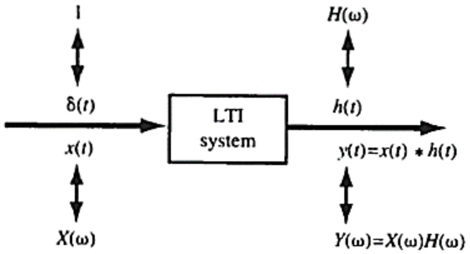
- Evaluate the response of the LTI system with impulse response

$$h(t) = e^{-at}u(t), \qquad a > 0$$

To the input signal $\quad x(t) = e^{-bt}u(t), \ b > 0$



From convolution theorem, we can write $\quad y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

(using convolution property) $\quad y(t) = x(t) * h(t) \xleftrightarrow{FT} \boldsymbol{X(\omega)H(\omega)}$

Fourier transform of $x(t)$ and $h(t)$ as, $\quad X(\omega) = \dfrac{1}{b+j\omega} \qquad \mathrm{H}(\omega) = \dfrac{1}{a+j\omega}$

$$Y(\omega) = X(\omega)H(\omega) = \frac{1}{(b+j\omega)(a+j\omega)} = \frac{A}{(a+j\omega)} + \frac{B}{(b+j\omega)}$$

Partial-fraction expansion

$$Y(\omega)=\frac{1}{(b+j\omega)(a+j\omega)}=\frac{A}{(a+j\omega)}+\frac{B}{(b+j\omega)}=\frac{A}{(a+s)}+\frac{B}{(b+s)}$$

$Let\ s=j\omega$

$$=\frac{1}{b-a}\frac{1}{(a+j\omega)}+\frac{1}{a-b}\frac{1}{(b+j\omega)}$$

$$A=[Y(\omega)(a+s)]_{s=-a}=\frac{1}{b-a}$$

$$B=[Y(\omega)(b+s)]_{s=-b}=\frac{1}{a-b}$$

$$=\frac{1}{a-b}\left[\frac{1}{b+j\omega}\right]-\frac{1}{a-b}\left[\frac{1}{a+j\omega}\right]$$

$$e^{-at}u(t)\overset{FT}{\leftrightarrow}\frac{1}{a+j\omega}$$

$$y(t)=\frac{1}{a-b}e^{-bt}u(t)-\frac{1}{a-b}e^{-at}u(t)$$

$$X(s)=\frac{1}{(s+1)(s+2)}=\frac{A}{s+1}+\frac{B}{s+2}.$$

we can evaluate the coefficients A and B by

$$A=[(s+1)X(s)]|_{s=-1}=1,$$

$$B=[(s+2)X(s)]|_{s=-2}=-1.$$

$$X(s)=\frac{1}{s+1}-\frac{1}{s+2}.$$

- The input output of a system is described by $y'(t) + 2y(t) = x(t) + x'(t)$

Compute the impulse response h(t) of the system

Taking Fourier-transform both side of the equation

$$y'(t) + 2y(t) = x(t) + x'(t)$$

$$\rightarrow \; j\omega Y(\omega) + 2\,Y(\omega) = X(\omega) + j\omega X(\omega)$$

$$\rightarrow (j\omega + 2)Y(\omega) = (1 + j\omega)X(\omega)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1+j\omega}{2+j\omega} = \frac{2+j\omega-1}{2+j\omega} = 1 - \frac{1}{2+j\omega}$$

$$Taking\ Inverse:\ \ h(t) = \delta(t) - e^{-2t}u(t)$$

Duality

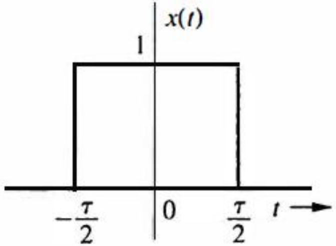
If $x(t) \xleftrightarrow{FT} X(\omega)$

$\boldsymbol{X(t)} \xleftrightarrow{FT} ?$

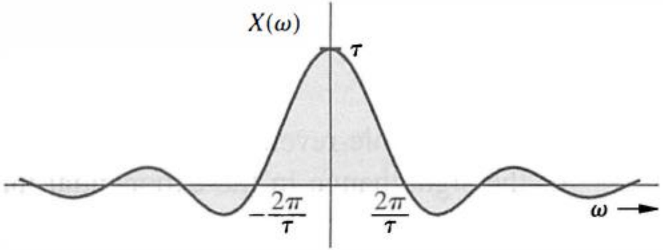
$$\int_{-\infty}^{\infty} X(t)e^{-j\omega t}dt = ?$$

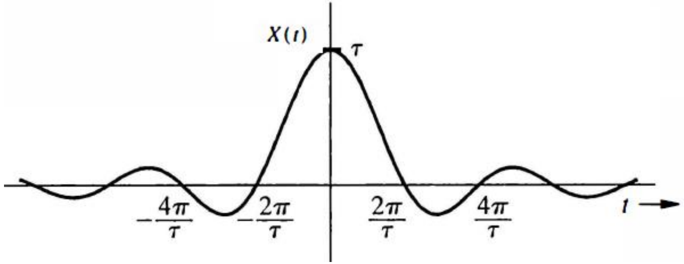
Called duality of time and frequency.

The duality principle may be compared with a photograph and its negative. A photograph can be obtained from its negative, and by using an identical procedure, a negative can be obtained from the photograph



$\Longleftrightarrow$





$\Longleftrightarrow$

?

Proof:

We know, $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$ ⇒ $= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\tau)e^{j\tau t}\,d\tau$

⇒ $2\pi\, x(t) = \int_{-\infty}^{\infty} X(\tau)e^{j\tau t}\,d\tau$

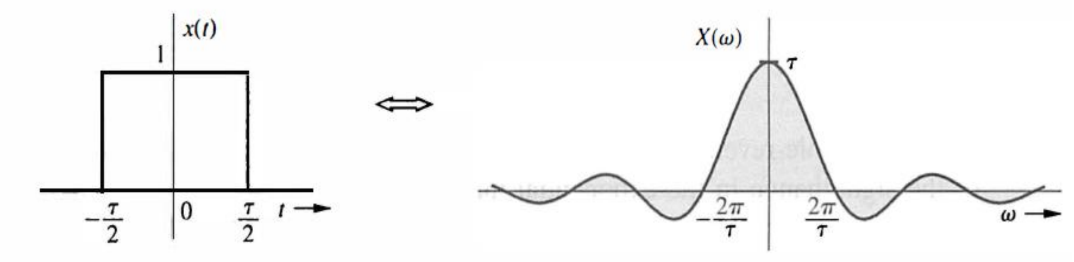
Putting, $t = -\omega$

⇒ $2\pi\, x(-\omega) = \int_{-\infty}^{\infty} X(\tau)e^{j\tau.-\omega}\,d\tau$

Putting, $\tau = \mathrm{t}$

$= \int_{-\infty}^{\infty} X(t)e^{-j\omega t}\,dt$

$= FT\{X(t)\}$



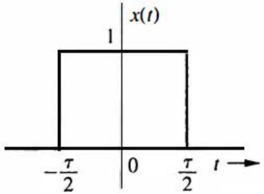
$$x(t) = \begin{cases} 1, |t| < \frac{\tau}{2} \\ 0, else \end{cases} = rect\left(\frac{t}{\tau}\right) \longleftrightarrow X(\omega) = \tau\frac{\sin\left(\omega\frac{\tau}{2}\right)}{\omega\frac{\tau}{2}} = \tau\,\mathrm{sinc}\left(\omega\frac{\tau}{2}\right)$$

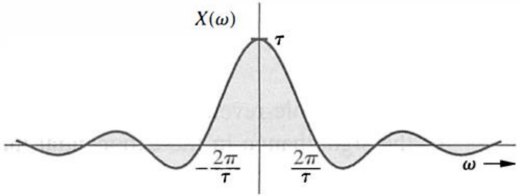
$$X(t) \xleftrightarrow{FT} 2\pi x(-\omega) \quad = x(-f)$$

$X(t)$ is the same as $X(\omega)$ with ω replaced by t

$$x(t) = \begin{cases} 1, |t| < \frac{\tau}{2} \\ 0, else \end{cases}$$

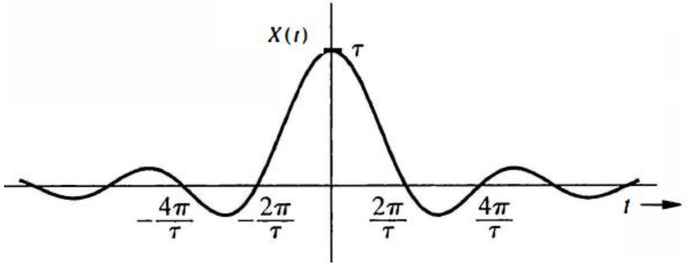
$$= rect(\frac{t}{\tau})$$





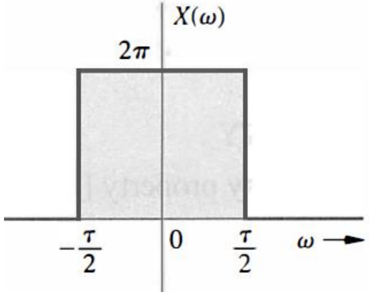
$$\boldsymbol{X(\omega)} = \tau \frac{\sin\left(\omega \frac{\tau}{2}\right)}{\omega \frac{\tau}{2}}$$

$$= \tau \sin c\left(\frac{\omega\tau}{2}\right)$$



$$2\pi x(-\omega)$$

$x(-\omega)$ is the same as $x\ (t)$ with t replaced by $-\omega$.



$$\underbrace{\tau \text{ sinc}\left(\frac{\tau t}{2}\right)}_{X(t)} \Longleftrightarrow \underbrace{2\pi \text{ rect}\left(\frac{-\omega}{\tau}\right)}_{2\pi x(-\omega)} = 2\pi \text{ rect}\left(\frac{\omega}{\tau}\right)$$

$$X(t) \xleftrightarrow{FT} 2\pi x(-\omega) = x(-f)$$

- Evaluate Fourier transform of the signal $x(t) = \dfrac{1}{1+t^2}$

$$y(t) = e^{-a|t|};\, a > 0$$

Using dual property: $z(t) \xleftrightarrow{FT} 2\pi z(-\omega)$

$$y(t) \xleftrightarrow{FT} Y(\omega) = \frac{2a}{a^2+\omega^2}$$

$$a = 1$$

$$y(t) = e^{-|t|} \xleftrightarrow{FT} Y(\omega) = \frac{2}{1+\omega^2}$$

$$Y(t) = \frac{2}{1+t^2} \xleftrightarrow{FT} 2\,\pi\, \boldsymbol{y(-\omega)}$$

$$\frac{1}{1+t^2} \xleftrightarrow{FT} \pi\ e^{-|-\omega|}$$

$$\frac{1}{1+t^2} \xleftrightarrow{FT} \pi\ e^{-|\omega|}$$

Given, $X(\omega)=\frac{1}{(1+\omega^2)}e^{-\frac{2\omega^2}{(1+\omega^2)}}$. **Determine the Fourier transform** of the following signals:

$(a)\ x(t-2)e^{jt}$

$(b)\ x(1-t)$

$(c)\ x\left(\frac{t}{2}-2\right)$

(a) Time shift property: $x(t-t_0)\overset{FT}{\leftrightarrow} e^{-j\omega t_0}X(\omega)$ ➡ $y(t)=x(t-2)\overset{FT}{\leftrightarrow} Y(\omega)=e^{-j\omega.2}X(\omega)$

Frequency shift property: $x(t)\ e^{j\omega_c t}\overset{FT}{\leftrightarrow} X(\omega-\omega_c)$ ➡ $y(t)e^{j.\omega_c=1.t}\overset{FT}{\leftrightarrow} Y(\omega-\omega_c)$

$y(t)\ e^{j.\omega_c=1.t}=x(t-2)e^{j\omega_c=1.t}\overset{FT}{\leftrightarrow} Y(\omega-\omega_c)=e^{-j2(\omega-\omega_c)}X(\omega-\omega_c)$

where $\omega_c=1$

(b) Scaling property: $x(at) \overset{FT}{\longleftrightarrow} \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$

$$\Rightarrow \quad x(-t) \overset{FT}{\longleftrightarrow} \frac{1}{|-1|} X\left(\frac{\omega}{-1}\right) = X(-\omega) \qquad \text{(for } a=-1\text{)} \quad \text{[folding]}$$

$$x(-t+1) \overset{FT}{\leftrightarrow} \int_{-\infty}^{\infty} x(-t+1)\, e^{-j\omega t} dt$$

(replacing $-t+1 = \tau,\ dt = -d\tau$)

$$\overset{FT}{\leftrightarrow} \int_{+\infty}^{-\infty} x(\tau)\, e^{-j\omega(-\tau+1)} \,.\, -d\tau$$

$$\overset{FT}{\leftrightarrow} e^{-j\omega} \int_{+\infty}^{-\infty} x(\tau)\, e^{j\omega\tau} \,.\, -d\tau$$

$$\overset{FT}{\leftrightarrow} e^{-j\omega} \int_{-\infty}^{\infty} x(\tau)\, e^{j\omega\tau}\, d\tau = e^{-j\omega} \int_{-\infty}^{\infty} x(\tau)\, e^{-j.(-\omega).\tau}\, d\tau = e^{-j\omega} X(-\omega)$$

Using Time shift property (delay):

$$x(t-t_0) \overset{FT}{\leftrightarrow} e^{-j\omega t_0} X(\omega)$$

$$x(-t+1) \overset{FT}{\leftrightarrow} e^{-j\omega.1} X(-\omega)$$

(c) Using Time shift property (delay): $x(t-t_0) \overset{FT}{\leftrightarrow} e^{-j\omega t_0} X(\omega)$

$$y(t) = x(t-2) \overset{FT}{\leftrightarrow} Y(\omega) = e^{-j\omega.2} X(\omega)$$ [shifting]

Using scaling property $x(at) \overset{FT}{\longleftrightarrow} \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$ [Scaling]

$$y\left(\frac{t}{2}\right) = x\left(\frac{t}{2}-2\right) \overset{FT}{\leftrightarrow} \frac{1}{1/2} Y\left(\frac{\omega}{\frac{1}{2}}\right) = 2.Y(2\omega) = 2e^{-j.2\omega.2} X(2\omega)$$

Solve the problems

Q1. Evaluate the Fourier transform of the following signals:

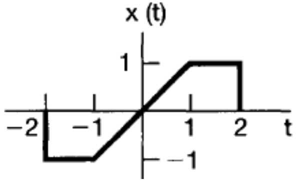
$(a)\ e^{-\alpha t}\ cos\omega_0 t\ u(t), \alpha > 0$

$(b)\ e^{-3|t|}\sin 2t$

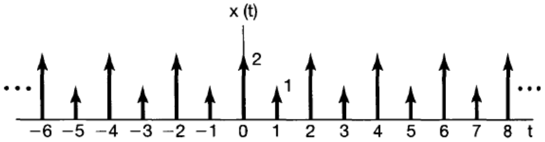
$(c)\ x(t) = \begin{cases} 1+\cos\pi t, & |t| \le 1 \\ 0, & |t| > 1 \end{cases}$

(d) $\sum_{k=0}^{\infty} \alpha^k \delta(t-kt),\ |\alpha| < 1$

(h)



(i)



$(e)\ te^{-2t}\sin 4t\ u(t)$

(f) $x(t) = \begin{cases} 1-t^2, 0 < t < 1 \\ 0,\ else \end{cases}$

(g) $x(t) = \sum_{n=-\infty}^{\infty} e^{-|t-2n|}$

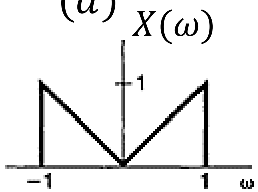
Q2. Determine the signal $x(t)$ from the Fourier transform of the signals,

$(a)\ X(\omega) = \dfrac{2\sin[3\,(\omega-2\pi)]}{(\omega-2\pi)}$

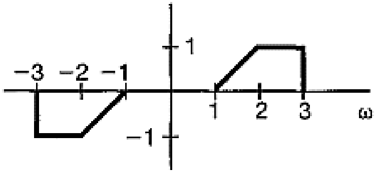
$(b)\ X(\omega) = \cos(4\omega + \frac{\pi}{3})$

$(c)\ X(\omega) = 2[\delta(\omega-1) - \delta(\omega+1)] + 3\,[\delta(\omega-2\pi)] + \delta(\omega+2\pi)$

(d)



(e)



Thank you