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Q1(a) $S_{n+2} - 10S_{n+1} + 21S_n = 3n^2 - 2 \quad \forall n \geq 0$

$$S_{n+2} = 10S_{n+1} - 21S_n + 3n^2 - 2$$

Related linear homogenous recurrence relation

$$S_{n+2} = 10S_{n+1} - 21S_n$$

and let $f(n) = 3n^2 - 2$

for $S_{n+2} = 10S_{n+1} - 21S_n$,

$$C_1 = 10, \quad C_2 = -21$$

$$r^2 - C_1 r - C_2 = 0$$

$$r^2 - 10r + 21 = 0$$

$$(r-3)(r-7) = 0$$

$$\Rightarrow S_{n+2} = \alpha_1 (3)^{n+2} + \alpha_2 (7)^{n+2} = a_n^{(h)}$$

$\alpha_1, \alpha_2 \in \mathbb{R}$

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~~Q1 (b)~~ ~~$S = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$, orthogonal basis of S and $S^{\perp}$~~

$F(n) = 3n^2 - 2$

Probable soln $\Rightarrow a_n^{(P)} = (an^2 + bn + c)$

$(an^2 + bn + c) = 3(an^2 + bn + c)^2 - 2$

No base condition given?

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Q2 $A = \{1, 2, 3, 6, 12, 15, 36\}$

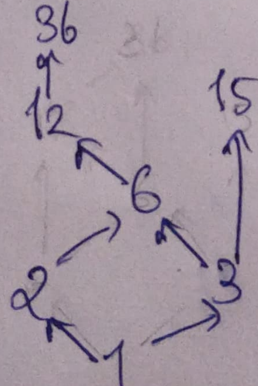
aRb if a divides b

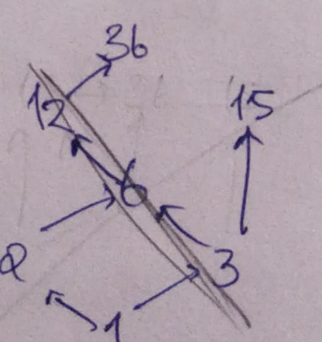
$\Rightarrow$ ~~R = {~~ $R = \{(1,2), (1,3), (1,6), (1,12), (1,15), (1,36),$
$(2,6), (2,12), (2,36), (3,6), (3,12), (3,15),$
$(6,12), (6,36), (3,36), (12,36), (1,1), (2,2), (3,3),$
$(6,6), (12,12), (15,15), (36,36)\}$

(a)

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 6 \\ 12 \\ 15 \\ 36 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 6 & 12 & 15 & 36 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

(b)





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~~(b)~~

(c) Maximal elements are 15 and 36

Minimal element is 1

There exists a least element, which is 1.
But there is no greatest element, since
there is no element divisible by every other
element.

Q3 $A = \{1,2,3,4,5,6,7,8,9,10\}$

xRy iff $(x-y)$ is a multiple of 4.

(a) for any $a \in A$,

$(a,a) \in R$ since $a-a=0$
is a multiple of 4

So, R is reflexive.

for any $a, b \in A$, $(a,b) \in R \Rightarrow a-b = 4k$ (say)

~~if $(a,b) \in R \Rightarrow$~~ here $k \in \mathbb{Z}$

So, $(b-a) = -4k$

Therefore, when ~~$(a,b) \in$~~ $(a,b) \in R$, $(b,a) \in R$

So, R is symmetric.

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for some $a, b, c \in A$

if $(a, b) \in R$, $a - b = 4k$ (let)

and if $(b, c) \in R$, $b - c = 4t$ (let)

~~here k and t~~
here $k, t \in \mathbb{Z}$

$$\begin{array}{rl} & a - b = 4k \\ + & b - c = 4t \\ \hline & a - c = 4(k+t) \end{array}$$

$\Rightarrow (a, c) \in R$

Therefore, when both $(a, b) \in R$ and $(b, c) \in R$,

$(a, c) \in R$

So, R is transitive.

R is an equivalence relation confirmed.

(b) $[1] = \{1, 5, 9\}$

$[2] = \{2, 6, 10\}$

$[3] = \{3, 7\}$

$[4] = \{4, 8\}$

$[5] = \{1, 5, 9\}$

$[6] = \{2, 6, 10\}$

$[7] = \{3, 7\}$

$[8] = \{4, 8\}$

$[9] = \{1, 5, 9\}$

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$[10] = \{2, 6, 10\}$

Partitions of A => {7}

$[1] = [5] = [9] = \{1, 5, 9\}$

$[2] = [6] = \cancel{[8]} = \{2, 6, \cancel{8}\ 10\}$
$[10]$

$[3] = [7] = \{3, 7\}$

$[4] = [8] = \{4, 8\}$

Q4 (a) $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & 4 \end{bmatrix}$

$R_2 \to 2R_2 - R_1$

$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & 9 \end{bmatrix}$

Pivot elements are x_1 and x_2 , free element x_3

$2x_1 + x_2 - x_3 = 0$

and $x_2 + 9x_3 = 0$

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Special solution

Set $x_3 = 1$,

$x_2 = -9$, $\cancel{x_1 =)}$

$2x_1 - 9 - 1 = 0$

$\Rightarrow x_1 = 5$

Solution $\rightarrow \begin{bmatrix} 5 \\ -9 \\ 1 \end{bmatrix}$

when $x_3 = 0$,

$x_2 = 0$ and $x_1 = 0$

~~Not considered~~

$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \\ -1 & 3 \end{bmatrix}$

find Projection matrix to project A onto ~~the~~ ~~orgonal~~ the ~~left~~ orthogonal complement of left null space of A.

Orthogonal complement of left null space = Column space

$\Rightarrow$ ~~To~~ Project A onto Column space