LAB REPORT-6

Submitted by -
S2021001 0027
Anushthan Saxena

Dtd: 7/6/22

Q1 $x(t) = \begin{cases} 1 & |t| \leq T \\ 0 & \text{otherwise} \end{cases}$

given $T=2$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$\Rightarrow X(j\omega) = \int_{-2}^{2} 1 \cdot e^{-j\omega t}\, dt$$

$$= \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-2}^{2}$$

$$\Rightarrow X(j\omega) = \frac{e^{2j\omega} - e^{-2j\omega}}{j\omega}$$

Q2 $x(t) = e^{-a|t|} = \begin{cases} e^{-at} & t > 0 \\ e^{at} & t < 0 \end{cases}$

$$X(j\omega) = \int_{-\infty}^{0} e^{at} e^{-j\omega t} dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t} dt$$

$$= \int_{-\infty}^{0} e^{(a-j\omega)t} dt + \int_{0}^{\infty} e^{-(a+j\omega)t} dt$$

$$= \cancel{\left(\frac{1}{a-j\omega}\right)}$$

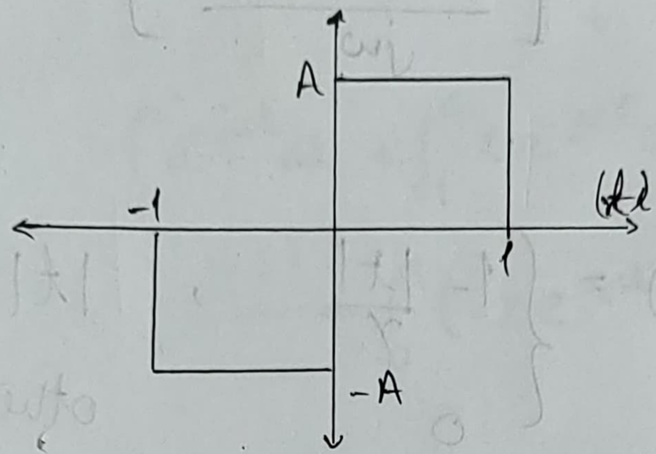
$$= \left.\frac{e^{a-j\omega}}{a-j\omega}\right|_{-\infty}^{0} + \left.\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right|_{0}^{\infty}$$

$$= \left(\frac{1}{a-j\omega}\right) + \left(\frac{1}{a+j\omega}\right)$$

$$= \frac{a+j\omega+a-j\omega}{a^2+\omega^2}$$

$$= \frac{2a}{a^2+\omega^2}$$

Q3



$X(j\omega) = ?$

given $A = 2$

$$\Rightarrow x(t) = \begin{cases} A & 0 < t \leq 1 \\ -A & -1 \leq t < 0 \end{cases}$$

where $A = 2$

$$\Rightarrow X(j\omega) = -2\int_{-1}^{0} e^{-j\omega t}\,dt + 2\int_{0}^{1} e^{-j\omega t}\,dt$$

$$= 2\left[\frac{e^{-j\omega t}}{j\omega}\Big|_{-1}^{0} + \frac{e^{-j\omega t}}{-j\omega}\Big|_{0}^{1} \right]$$

$$= 2\left[\frac{1 - e^{j\omega}}{j\omega} - \frac{(e^{-j\omega} - 1)}{j\omega} \right]$$

$$\Rightarrow X(j\omega) = 2\left[\frac{2 - e^{j\omega} - e^{-j\omega}}{j\omega}\right]$$

Q4 $x(t) = \begin{cases} 1 - \frac{|t|}{\gamma}, & |t| \leq \gamma \\ 0 & \text{otherwise} \end{cases}$

given $\gamma = 1$

Since $\gamma = 1$,

$$x(t) = \begin{cases} 1 + \frac{t}{\gamma} & \text{for } -\gamma \leq t < 0 \\ 1 - \frac{t}{\gamma} & \text{for } 0 < t \leq \gamma \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow x(t) = \begin{cases} 1 + t & -1 \leq t < 0 \\ 1 - t & 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

* $X(j\omega) = \int_{-1}^{0} (1+t)\, e^{-j\omega t}\, dt + \int_{0}^{1} (1-t)\, e^{-j\omega t}\, dt + 0$

$= \int_{-1}^{0} e^{-j\omega t}\, dt + \int_{-1}^{0} t\, e^{-j\omega t}\, dt + \int_{0}^{1} e^{-j\omega t}\, dt$

$+ \int_{0}^{1} (-t\, e^{-j\omega t})\, dt$

$$\int t\, e^{-j\omega t}\, dt = t\, \frac{e^{-j\omega t}}{-j\omega} + \int \frac{d(t)}{dt} \left(\frac{e^{-j\omega t}}{+j\omega}\right) dt$$

$$= \frac{t\, e^{-j\omega t}}{-j\omega} \times \frac{j\omega}{j\omega} + \frac{e^{-j\omega t}}{-j^2\omega^2}$$

$$= \frac{j\omega t\, e^{-j\omega t} + e^{-j\omega t}}{\omega^2}$$

$$\int t\, e^{-j\omega t}\, dt = \frac{e^{-j\omega t}\,(1+j\omega t)}{\omega^2}$$

$\Rightarrow \cancel{X(j\omega) = \frac{e^{-j\omega t}}{-j\omega}\Big|_{-1}^{0}} + \frac{e^{-j\omega t}}{-j\omega}\Big|_{0}^{1}$

$$X(j\omega) = \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-1}^{0} + \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{0}^{1} + \left.\frac{e^{-j\omega t}(1+j\omega t)}{\omega^2}\right|_{-1}^{0}$$

$$- \left.\frac{e^{-j\omega t}(1+j\omega t)}{\omega^2}\right|_{0}^{1}$$

$$= \frac{1-e^{j\omega}}{-j\omega} + \frac{e^{-j\omega}-1}{-j\omega}$$

$$+ \frac{1-e^{j\omega}(1-j\omega)}{\omega^2} - \cancel{e^{-j\omega t}}$$

$$- \frac{e^{-j\omega}(1+j\omega)-1}{\omega^2}$$

$$\cancel{= \frac{e^{-j\omega}-e^{j\omega}}{-j\omega}} + \cancel{\frac{2(}{\omega^2}}$$

$$X(j\omega) = \frac{j\omega}{j\omega}\left(\frac{e^{-j\omega} - e^{j\omega}}{(-j\omega)}\right) + \frac{1 - e^{j\omega} + j\omega e^{j\omega}}{\omega^2}$$

$$- \frac{e^{-j\omega} + j\omega e^{-j\omega} - 1}{\omega^2}$$

$$= \frac{(e^{-j\omega} - e^{j\omega})j\omega}{\omega^2} + \frac{2 - e^{-j\omega} - \cancel{e^{j\omega} + j\omega(e^{j\omega} - e^{-j\omega})}}{\cancel{\omega^2}}$$

$$+ \frac{2 - e^{-j\omega} - e^{j\omega} + j\omega(e^{j\omega} - e^{-j\omega})}{\omega^2}$$

$$= \frac{2 - e^{j\omega} - e^{-j\omega}}{\omega^2}$$

Q5 $x(t) = u(t-2) - e^{-2t}u(t)$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{2}^{\infty} e^{-j\omega t} dt - \int_{0}^{\infty} e^{-2t} e^{-j\omega t} dt$$

$$= \frac{e^{-j\omega t}}{-j\omega}\Big|_{2}^{\infty} - \int_{0}^{\infty} e^{-(2+j\omega)t} dt$$

$$= \frac{e^{-j\omega(2)}}{j\omega} + \left(\frac{e^{-(2+j\omega)t}}{-(2+j\omega)} \right) \Bigg|_0^{\infty}$$

$$= \frac{e^{-2j\omega}}{j\omega} - \frac{1}{2+j\omega}$$