# COEN 5830, Fall 2024
# Introduction to Robotics

## Lecture 8

## Path Planning Algorithms

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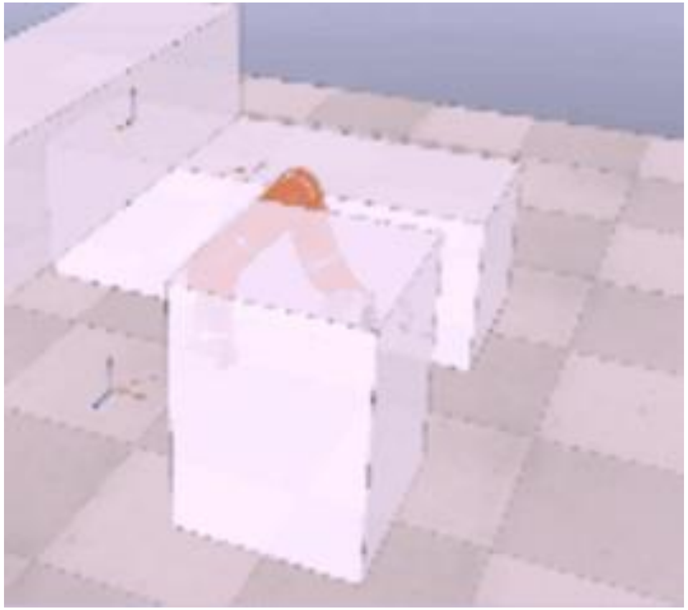
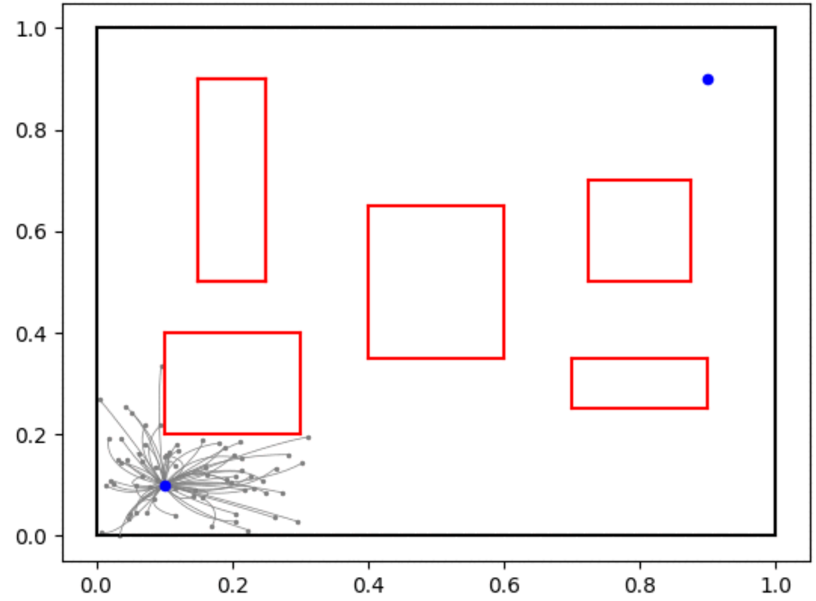
# Administrative

- HW1 will be graded by end of next week
- HW2 will be released this weekend

# What is path planning?

Path planning is the problem of finding a **set of robot states** from a **start state** to a **goal state** that **avoids obstacles** in the environment and satisfies **other constraints**, such as joint limits or degree of freedom limitations.





# Where do we explore next?

# Where do we explore next?

Answer: It depends on the planning algorithm.

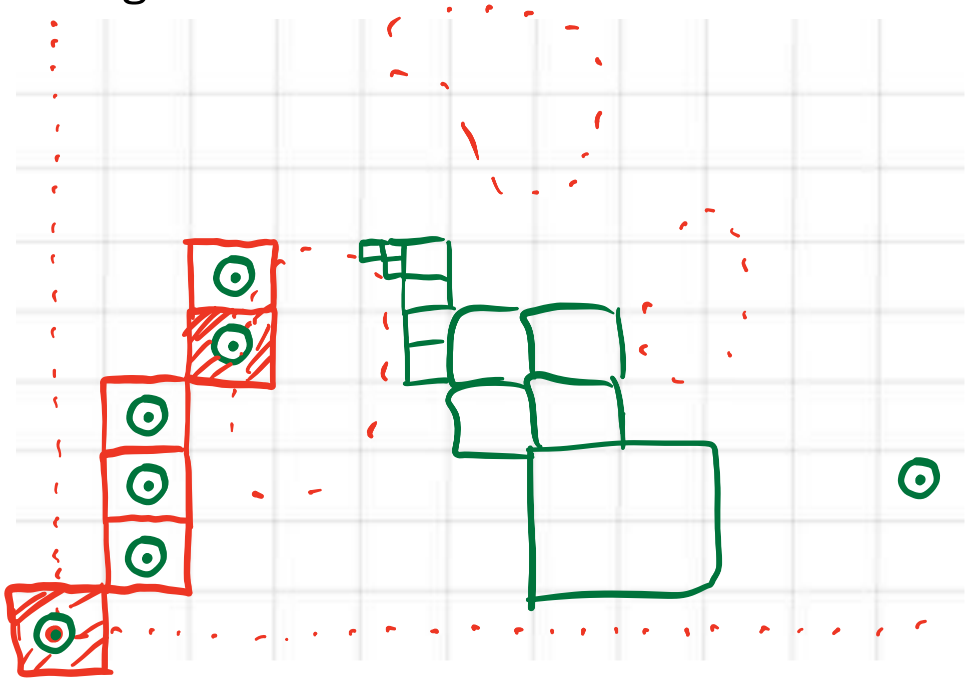
# Assumptions

- We are **not concerned** with the **equations of motion (EOM)** of the robot, ie. the robot can move in any direction (up/down, left/right, diagonally) in the grid of cells

# Discretization (for this class)

- Given:
  - List of **points** in configuration space that represent **obstacles**
  - Circular robot with specified radius, r

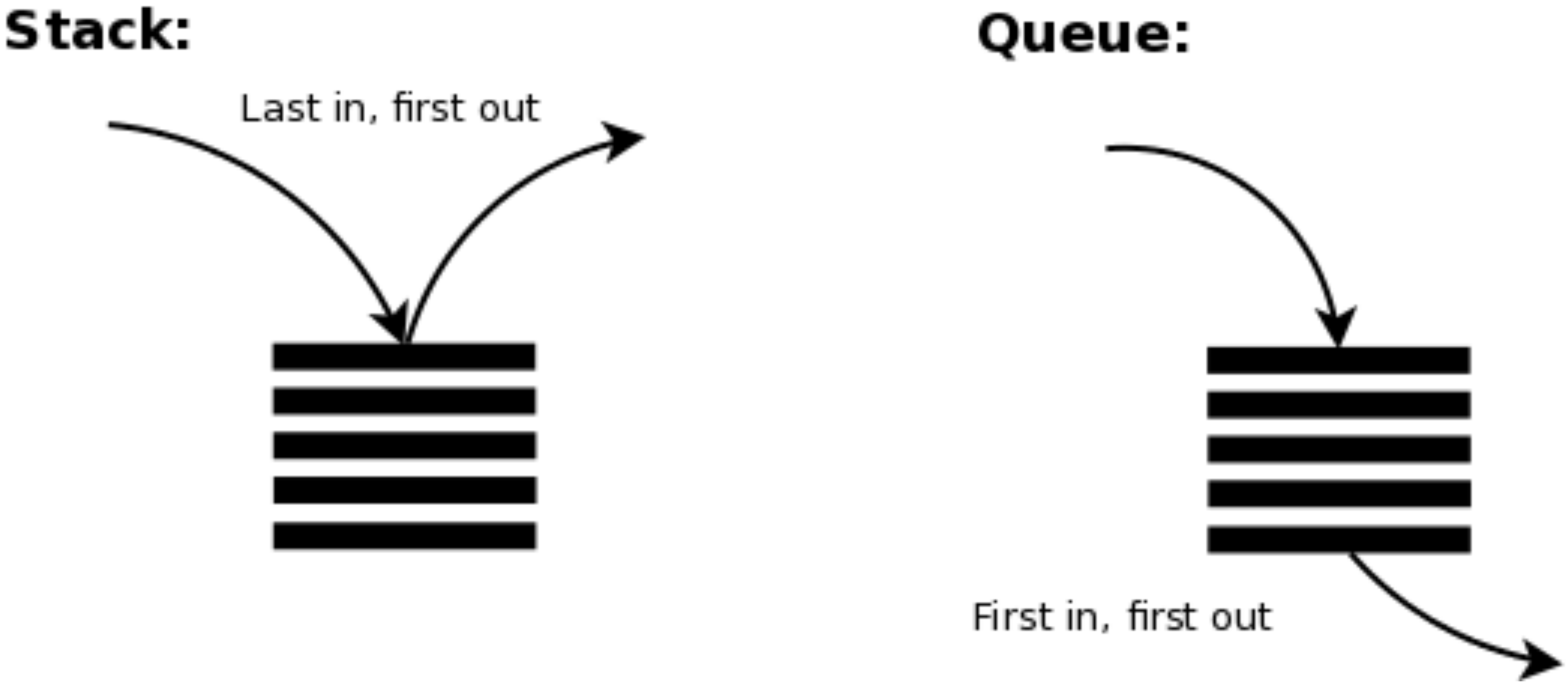

- Task:
  - Create discretization grid of traversable areas



# Graphs and Trees

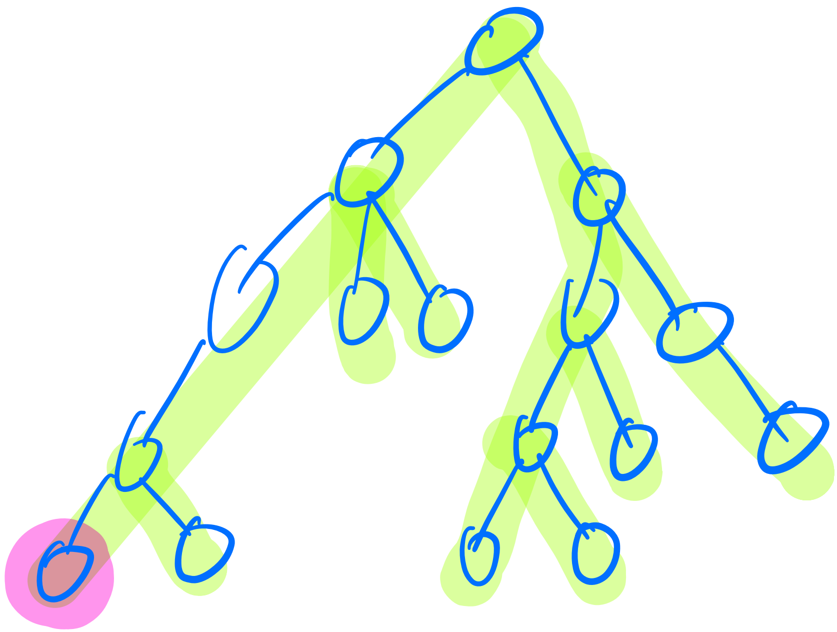
- **Search-based** planners represent the **C-space as a graph** through discretization
- A graph consists of a collection of **nodes** and **edges**
- Edges **connect** two nodes.
- **Nodes** typically represent **robot states**, while edges indicate the ability **to move between nodes without collision**
- Edges are often **weighted** by the **cost** to **move** from one node to another
- A **tree** is a **directed** graph (edges can only move in one direction) with **no cycles** and each node has **at most one parent**

# Stack vs Queue

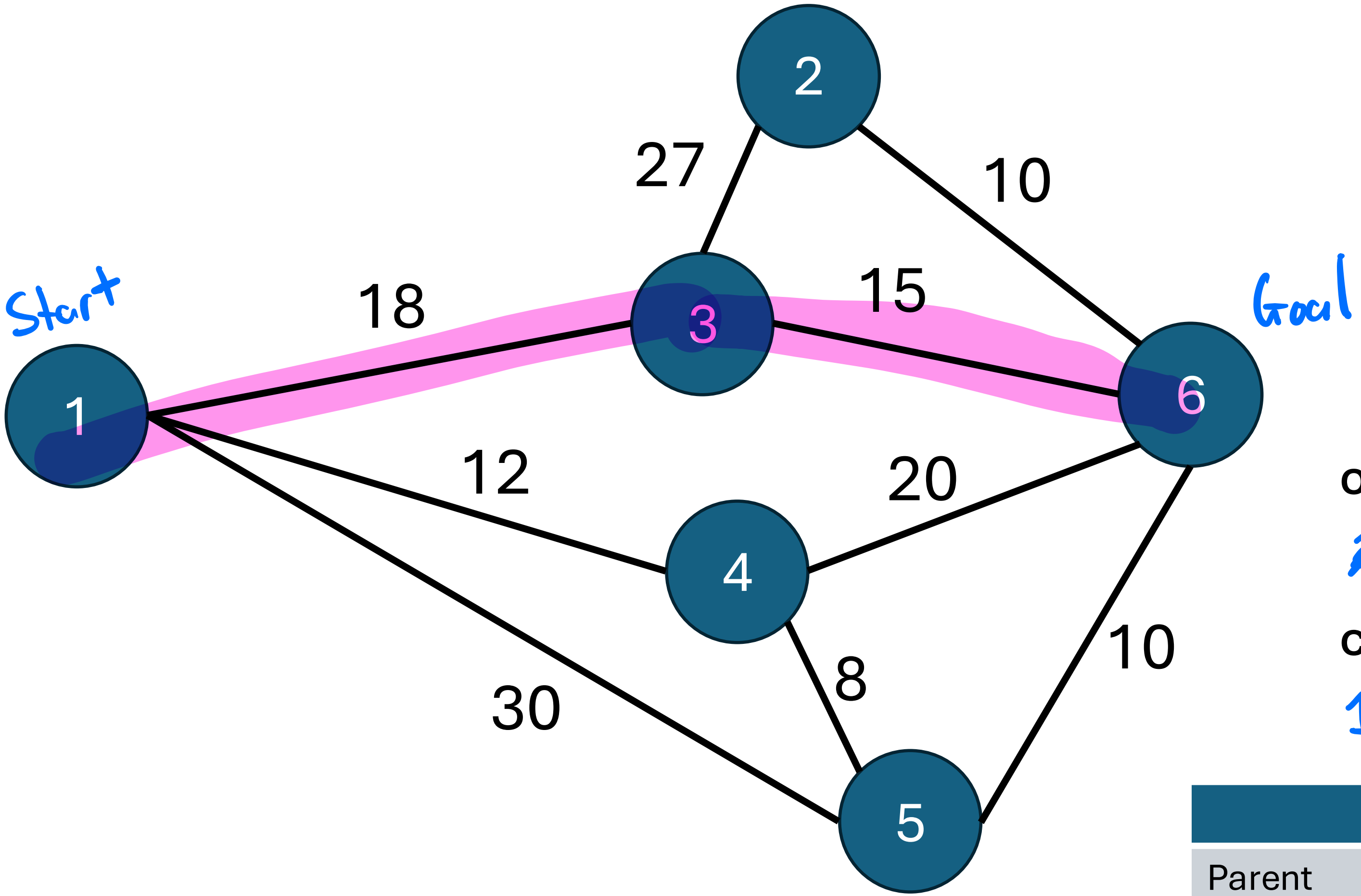


# Depth-First Search

- Search **as far down one tree branch** as possible before backtracking and searching the next available branch.
- Uses a stack structure where the **newest unexplored node** is explored



# Depth-First Search Example (Generic Graph)




**OPEN**

~~1~~ 5 4 ~~3~~ 6 ~~2~~

**CLOSED**

1 5 4 3 6 2

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Parent Node | - | 3 | 1 | 1 | 1 | 3 |

# ~~Depth-First Search Pseudocode~~

```
open_set, closed_set = dict(), dict()
open_set[self.calc_index(start_node)] = start_node

while True:
    if len(open_set) == 0:
        break
    current_node = open_set[-1]

    if current_node = goal_node:
        goal_node.parent = current_node.parent
        goal_node.cost = current_node.cost
        break

    for motion in allowed_motions:
        node = create_node(motion)
        if node in closed_set:
            continue
        if node not valid:
            continue
        if node not in open_set:
            open_set += node
        else:
            if node.cost < open_set[node].cost:
                open_set[node] = node

    del open_set[current_node]
    closed_set += current_node

calculate_final_path()
```

No solutio

→ NB

Goal met

Adding new nodes

→ closed set += node

This is wrong

See Below

↓

```
open_set, closed_set = dict(), dict()
open_set[self.calc_index(start_node)] = start_node

while True:
    if len(open_set) == 0:
        break
    current_node = open_set.pop([-1])

    if current_node = goal_node:
        goal_node.parent = current_node.parent
        goal_node.cost = current_node.cost
        break

    for motion in allowed_motions:
        node = create_node(motion)

        if node not valid:
            continue

        if node not in closed_set:
            open_set += node
            closed_set += node

calculate_final_path()
```

Initialization

No path possible (No nodes left to explore)

Select last node added as current node (remove from OPEN list)

End condition (Path found)

Add new nodes by considering all motions from current node

create new node based on current node + motion

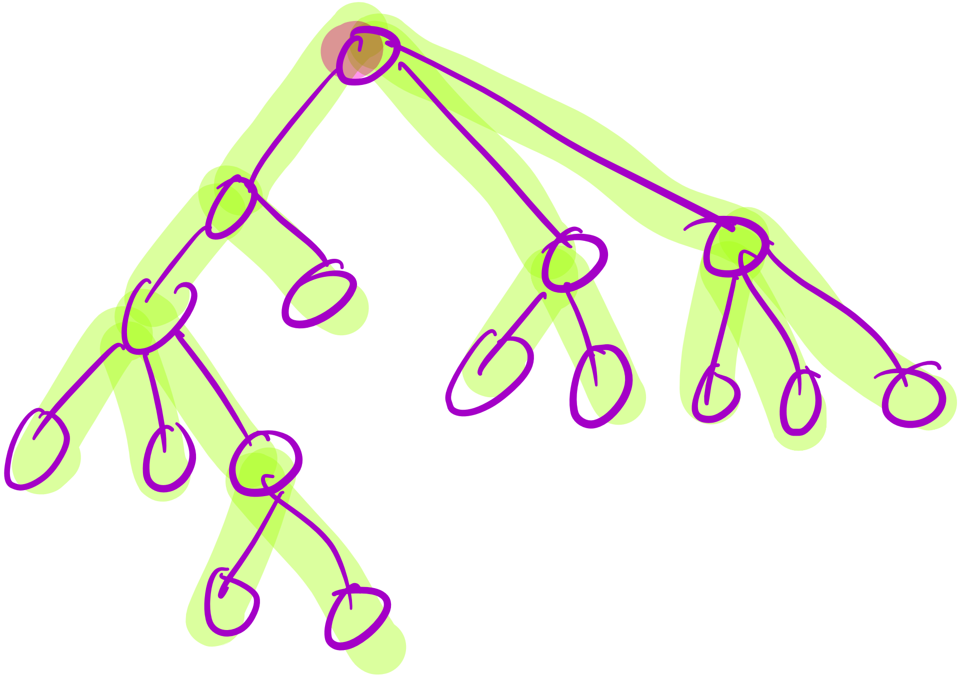
Are we in collision?

Add node to OPEN and CLOSED if not already closed.

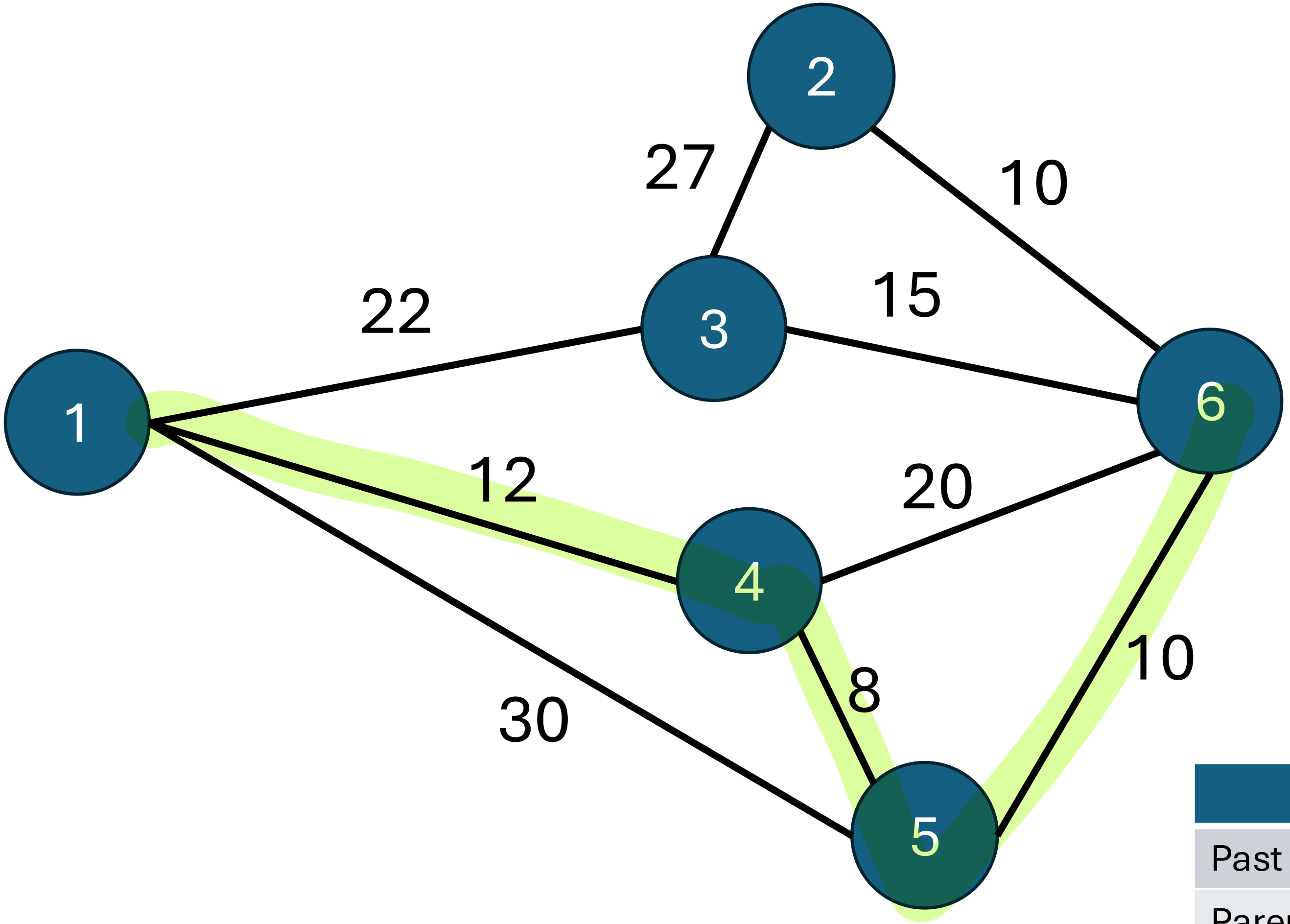
Prevents creation of nodes with multiple parents.

# Dijkstra’s Algorithm

- Explore branches connected to the node with the **lowest total cost**. Keep exploring until the next node to be explored is the goal node.
- Considered a **variant** of a **breadth-first search**



# Dijkstra Example (Generic Graph)




**OPEN**

2(49)

~~1(0)~~ ~~3(22)~~ ~~4(12)~~ ~~5(20)~~ 6(30)

**CLOSED**

1(0) 4(12) 5(20) 3(22)

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Past Cost | 0 | 49 | 22 | 12 | 20 | 30 |
| Parent Node | – | 3 | 1 | 1 | 4 | 5 |

# Dijkstra Pseudocode

```
open_set, closed_set = dict(), dict()
open_set[self.calc_index(start_node)] = start_node

while True:
    if len(open_set) == 0:
        break
    current_node = min(open_set.cost)
    if current_node = goal_node:
        goal_node.parent = current_node.parent
        goal_node.cost = current_node.cost
        break

    for motion in allowed_motions:
        node = create_node(motion)
        if node in closed_set:
            continue
        if node not valid:
            continue
        if node not in open_set:
            open_set += node
        else:
            if node.cost < open_set[node].cost:
                open_set[node] = node

    del open_set[current_node]
    closed_set += current_node

calculate_final_path()
```

Initialization

No solution

Solution found

Exploring nodes

Improving cost

# A* Search

- Operates in the same way as Dikstra's algorithm, but with an **added heuristic measure**
- The **heuristic** is an **estimate** of how far away the goal node is from any particular node.
- There are 2 requirements for the heuristic function:
  - **Always optimistic** (estimated remaining path length is less than actual path length). This estimate serves as a lower bound on the cost to go.
  - Simple and easy to evaluate

# A* Pseudocode

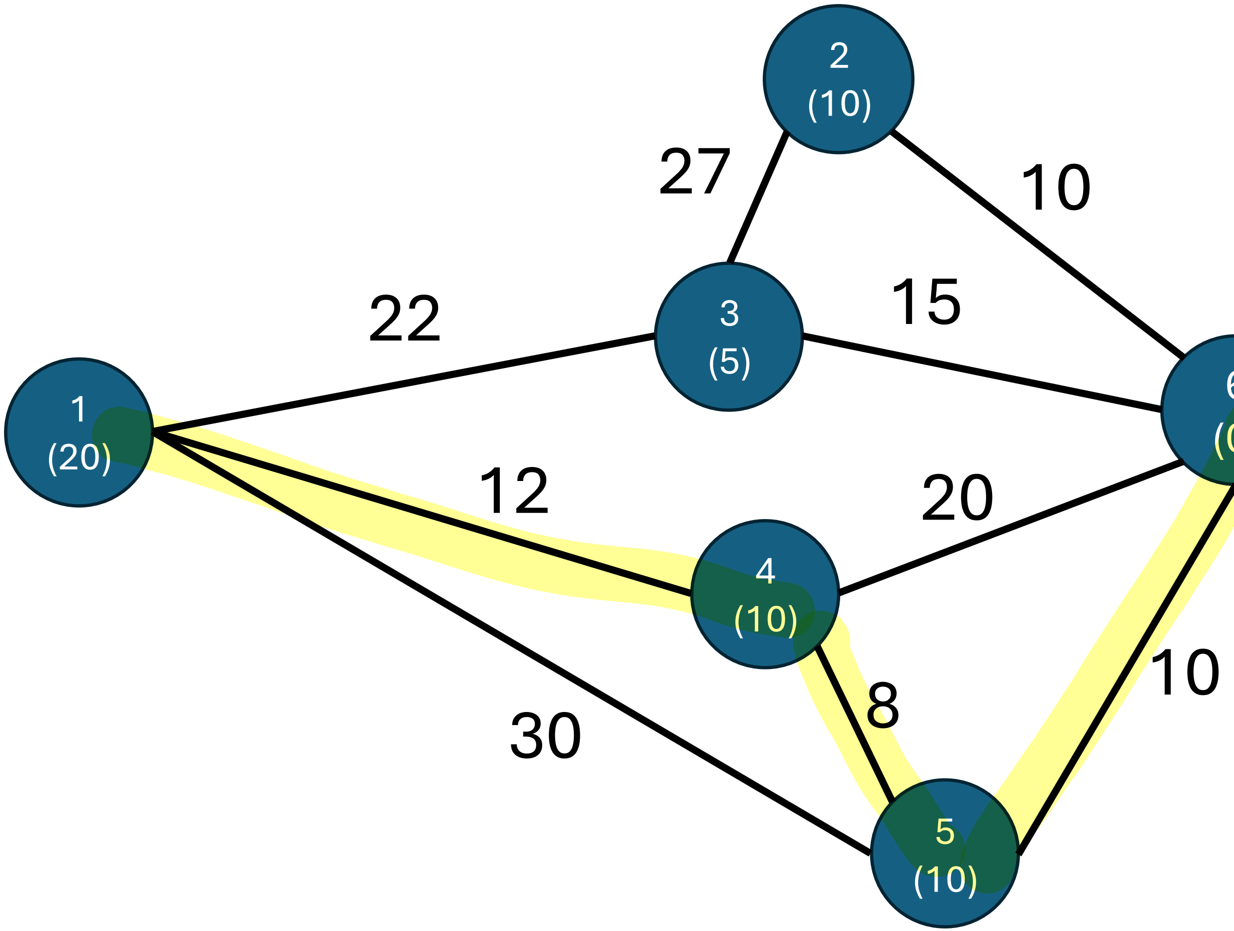
Dijkstra

```
open_set, closed_set = dict(), dict()
open_set[self.calc_index(start_node)] = start_node
while True:
    if len(open_set) == 0:
        break
    current_node = min(open_set.cost + calc_heuristic(open_set))
    if current_node = goal_node:
        goal_node.parent = current_node.parent
        goal_node.cost = current_node.cost
        break

    for motion in allowed_motions:
        node = create_node(motion)
        if node in closed_set:
            continue
        if node not valid:
            continue
        if node not in open_set:
            open_set += node
        else:
            if node.cost < open_set[node].cost:
                open_set[node] = node
    del open_set[current_node]
    closed_set += current_node

calculate_final_path()
```

# A* Example (Generic Graph)




**OPEN**

6(30) 2(59)

~~1(20)~~ ~~3(27)~~ ~~4(22)~~ ~~5(30)~~

**CLOSED**

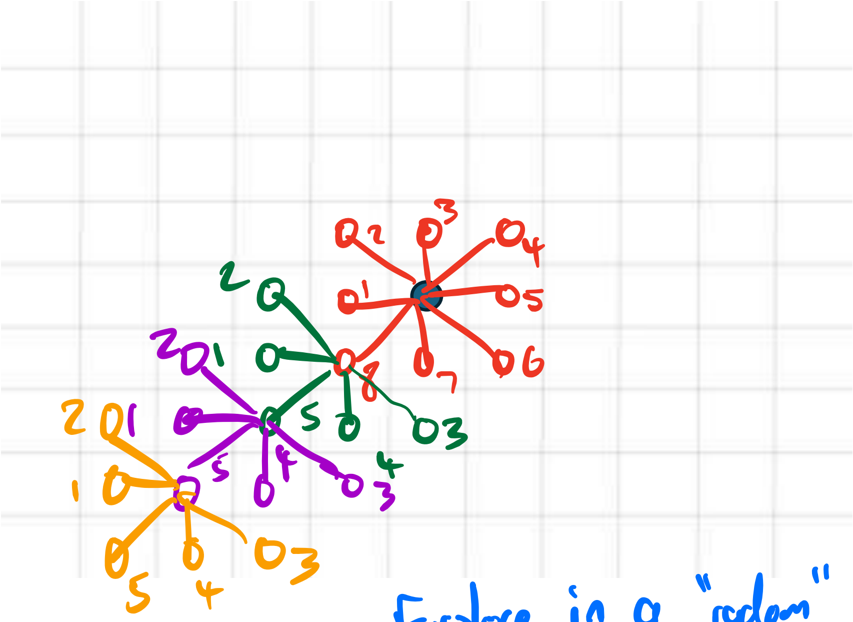
1 4 3 5

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Past Cost | 0 | 49 | 22 | 12 | 20 | 30 |
| Heuristic | 20 | 10 | 5 | 10 | 10 | 0 |
| Est Total Cost | 20 | 59 | 27 | 22 | 30 | 30 |
| Parent Node | -1 | 3 | 1 | 1 | 4 | 5 |

# DFS vs Dijkstra (and A*)

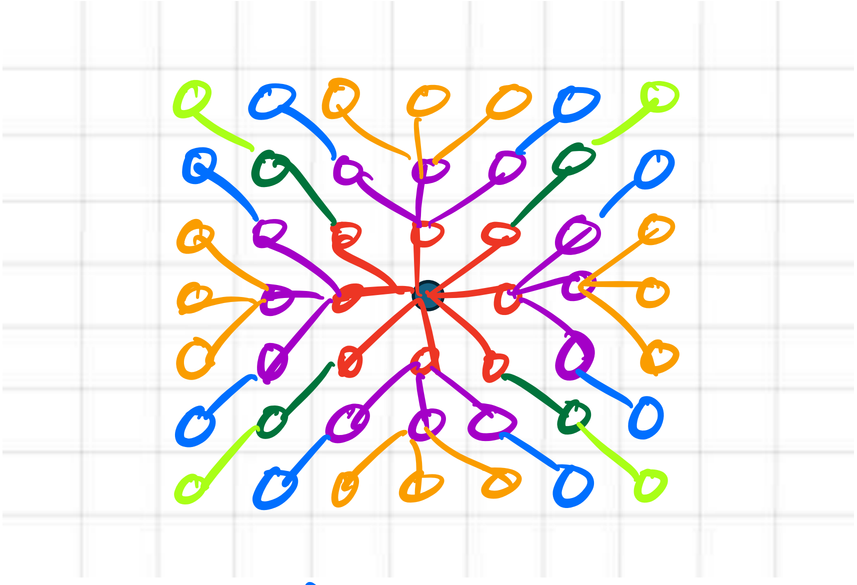
## Exploration patterns

DFS



Explore in a "random" single direction

Dijkstra (and A*)



Uniform exploration