# Numerical Method for visual computing and ML

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**Catastrophic cancellation** Of all the mathematical operations, one particular case is **especially dangerous**

$$a - b \quad (a \approx b)$$

The also happens for $a + b$ when $a \approx -b$ This can lead to catastrophic cancellation even if the substraction is done without error (this can happend not only if the there is an rounding error in the operation but maybe from before).

> *Example* For instance $3.140 = 2^1 \cdot 1.10010010$ and $-3.149 = +2^1 1.10010010$ When you sum both you get:
>
> $$= -2^{-7} 1.0_2$$

**Quantiying error**

- Absolute error: $|\text{true value} - \text{aprroximate value}|$
  - Example 2cm $\pm 0.1cm$
- Relative error: $\frac{\text{absolute error}}{|\text{true value}|}$
  - Example: 2cm $\pm 0.1\%$
  - Another common notation (true value) (1 $\pm$relative error)

**Forward ans Backward Error**

> *Example* For instance let us take
>
> $$\begin{aligned} f(x) &:= \sqrt{x} \\ y &= f(x) \\ \hat{\boldsymbol{y}} &= f_{IEE754}(x) \end{aligned}$$

Here is you wanted to compute the the backward error you would do it like this:

$$error = x - (f_{IEE754}(x))^2$$

**Conditioning of numerical problems**

# 1 Linear systems

**Geometric intrerpretation of matrix-vector product**

$$(Ax)_i = \sum_{k=1}^{n} A_{ik} x_k \text{ where } A = \begin{pmatrix} | & | & | \\ a^{(1)} & a^{(2)} & a^{(3)} \\ | & | & | \end{pmatrix}$$

**Basic matric transformation in 2D**

> *Scaling*
>
> $$\begin{pmatrix} v'_x \\ v'_y \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

> *Rotation*
>
> $$\begin{pmatrix} v'_x \\ v'_y \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

**Whyshould we care about linear function?**

- Solving linear problems is one of the (few) numerical problems that we know to solve **really well**
- When you can turn something into a linear system $\rightarrow$ good job, you are done.
- Nonlinear method are fragile

Almost everything boils down to a linear system

**Definition 1** *Matrix multiplication:*

$$(AB)_{ij} = \sum_{k=1}^{n} A_{ik} B_{kj}$$

The question however is why is it defined this way?
For instance if we take $f(x) = Ax, g(x) = Bx$ then you get that

$$g(f(x))BAx$$

**Pure versus numerical mathematics**

$$\begin{pmatrix} 1 & 0 \\ 1 & \varepsilon \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

**Effet Rebond**

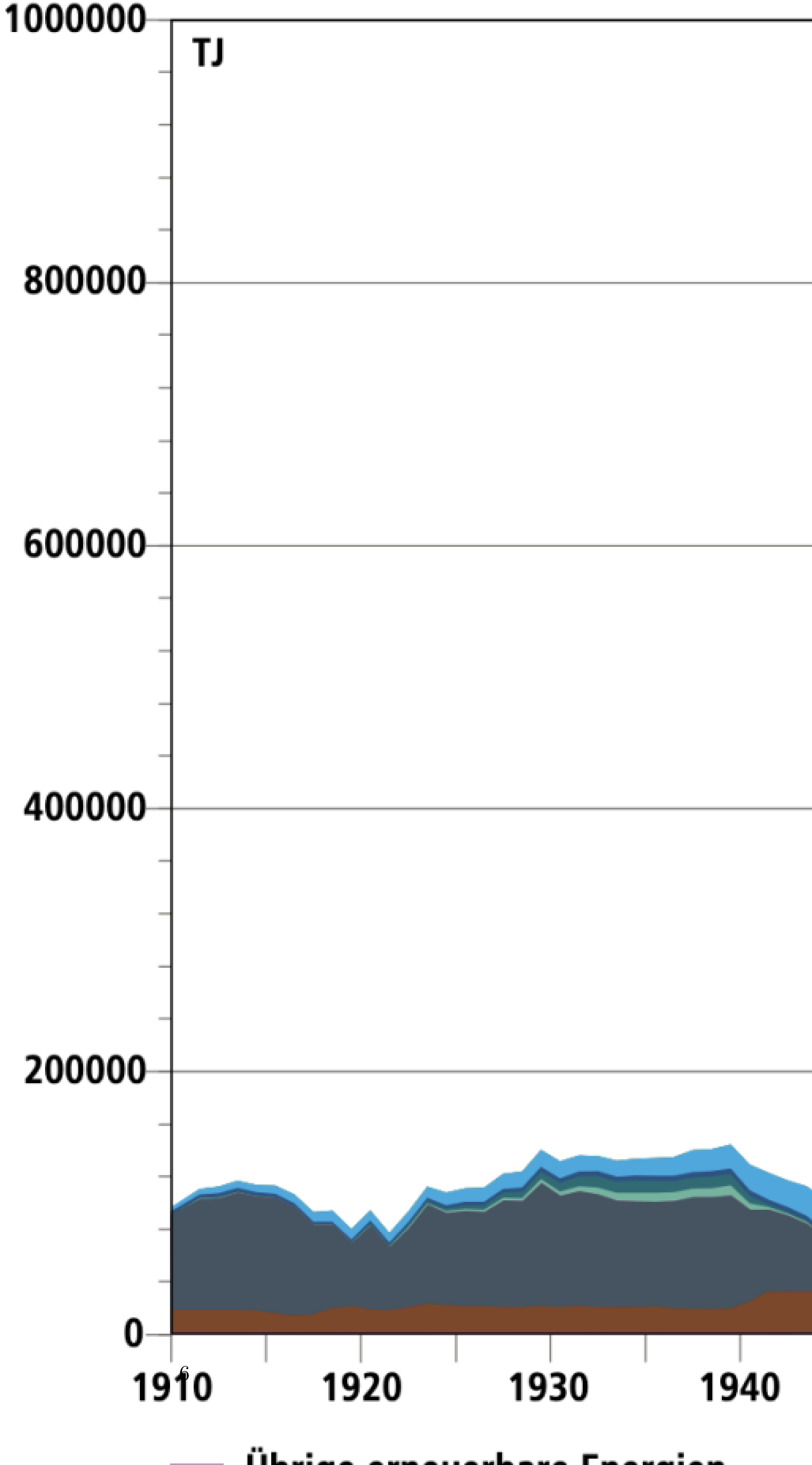
**Definition 2** *En trouvant une solution au problème la solution finalement empire ce problème*

*Exemple*

Par exemple, vouloir manger moins de calorie en mangant de la glace basse en calorie mais ducoup en manger plus car moins calorique → plus de calorie au final.

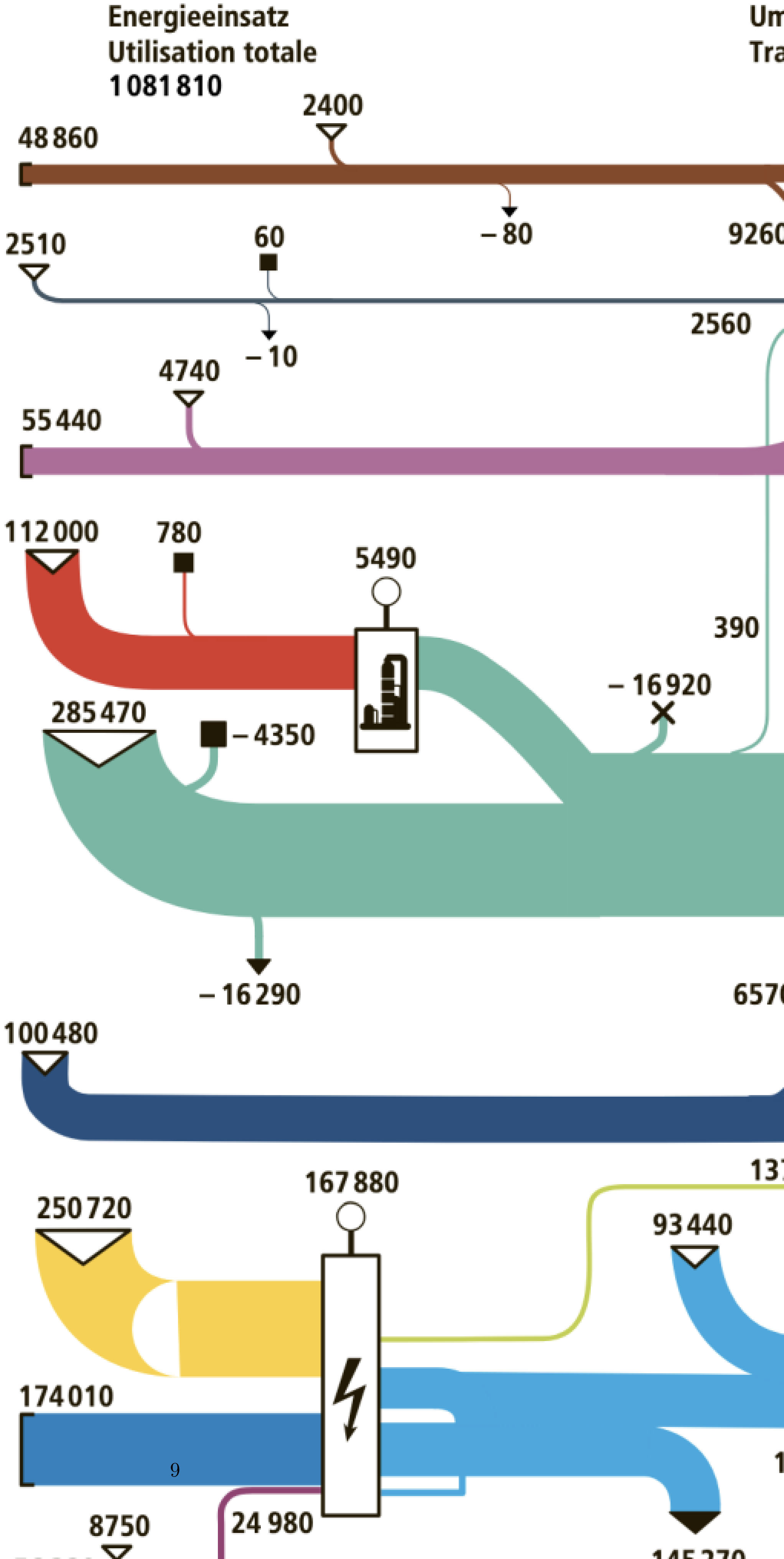
**Evolution de la consommation finale d'energie**

**Fig. 1 Endenergieverbrauch 1910–2024 na**
**Consommation finale 1910–2024 sel**



sur le terrain Suisse) néanmoins beaucoup de la production est délocalisé de nos jours ce qui expliquer pourquoi la production peut baisser.

**Energie primaire, secondaire, finale, utile**


Energieeinsatz
Utilisation totale
1 081 810
2400
48 860
– 80
2510
60
– 10
2560
4740
55 440
112 000
780
5490
390
– 16 920
285 470
– 4350
– 16 290
100 480
167 880
250 720
93 440
174 010
8750
24 980