



# **BIT MANIPULATION**

# **PROBLEM SOLVING 2**

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# Fortune Telling

$a_1 \quad a_2 \quad a_3$

$a_4 \quad a_5$

$x$

$$x = x + a_1$$

$$x = x \oplus a_2$$

$$x = x \oplus a_3$$

$$x = x \oplus a_4$$

$$x = x + a_5$$

$n+3$

$\xrightarrow{\text{Alice}}$   $x_1$

$\xrightarrow{\text{Bob}}$   $x_2$

$y$

Alice  $\rightarrow n$       Bob  $\rightarrow n+3$

Diagram showing Alice's and Bob's operations:

Alice's operation:  $5 + 3 = 6$  (circled 5 and 3, circled sum 6)

Bob's operation:  $5 + 3 = 8$  (circled 5 and 3, circled sum 8)

Result:  $6 + 6 = 12$  (circled 6 twice, circled sum 12)

Even check:  $6 + 6 = \underline{\quad}$  (circled 6 twice, circled sum, labeled "even")

$$\begin{array}{r} 101 \\ 011 \\ \hline 110 \\ \hline 5 \oplus \end{array}$$

$$\begin{array}{r} 6 + 3 = 5 \\ \hline 110 \\ 011 \\ \hline 101 \end{array} \quad | \quad 6 + 3 = 9$$

Similarity b/w XOR and AND operation →

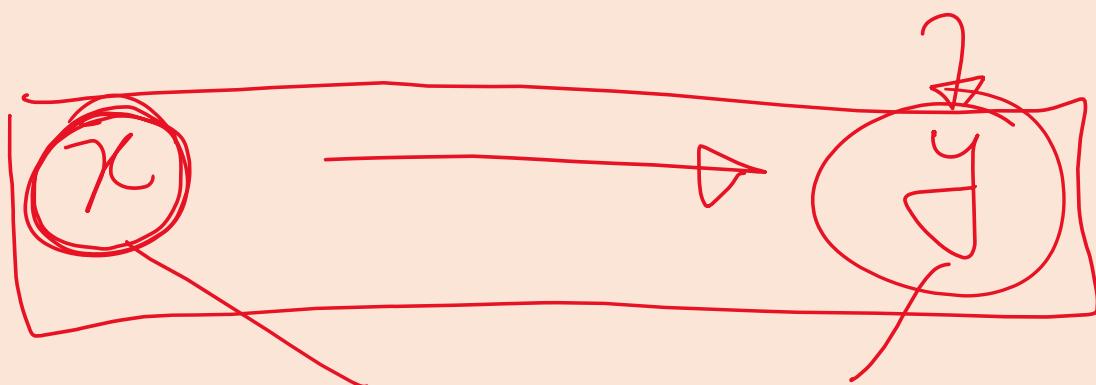
Number + odd → parity of numbers changed

Number  $\oplus$  odd → parity of result opposite to that of numbers

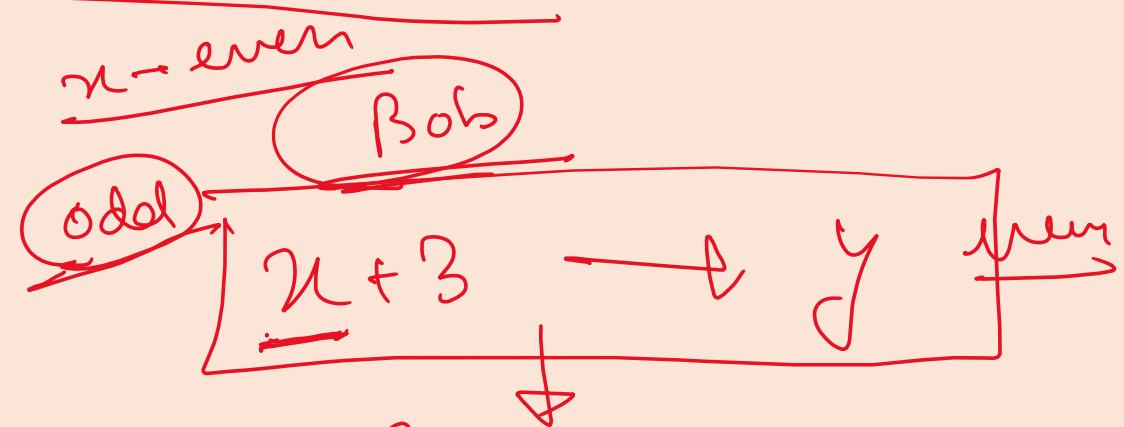
Parity → odd/even

array  $a$

~~$\frac{+a_i}{+a_i}$~~



Parity of  $x$  and  $y$   
should be different



Parity of  $n$  and  $y$   
should be same

$\chi$  → even

1st odd → odd

2<sup>nd</sup> odd → even

$3^{\text{rd}}$  odd  $\Rightarrow$  odd

left odd → even

5<sup>th</sup> odd → odd

$n^3 \rightarrow$  odd

even

cnt of odd # is odd

x → y

n+3 → y  
n+3 and y  
n and y

if (x & y are of same parity){

Alice  
+

Bob

}

else {

Alice

3

cnt of odd # is even

If (x & y are of

same parity){

else {

Alice

Bob

3

Intuition

$x$

even

$x + 3$

odd

$\oplus$

$\rightarrow$

different parity

$$x \begin{array}{c} \oplus \\ + \end{array}$$

$x \oplus/+ \text{ even} \rightarrow \text{Parity same}$

$x \oplus/+ \text{ odd} \rightarrow \text{Parity changes}$

odd  $\oplus/+ \text{ even} \rightarrow \boxed{\text{odd}}$

odd  $\oplus/+ \boxed{\text{odd}} \rightarrow \boxed{\text{even}}$

~~even~~  
~~odd~~

~~odd~~

even  
~~even~~  
odd

~~even~~  
~~even~~  
even

odd  
odd  
odd

odd  
even  
even

# GARDENER AND THE ARRAY

$\{ \dots \}$  =  $\{ \_ \_ \_ \}$

*i<sup>th</sup> bit*

$\boxed{a_1 \ a_2 \ a_3 \ a_n \ a_5 \dots a_n}$

$$x_1 = \{ a_2, a_n, a_6 \} \quad x_2 = \{ a_1, a_2 \}$$

$$f(x_1) = f(x_2)$$
$$(a_2 | a_n | a_6) = (a_1 | a_2) \rightarrow \boxed{\text{Yes}} \text{ else } \boxed{\text{No}}$$

a<sub>i</sub>

0 0 0 1 0 1 0 0 1 0 0

7<sup>th</sup> bit    5<sup>th</sup> bit    2<sup>nd</sup> bit

a    b

32 bits

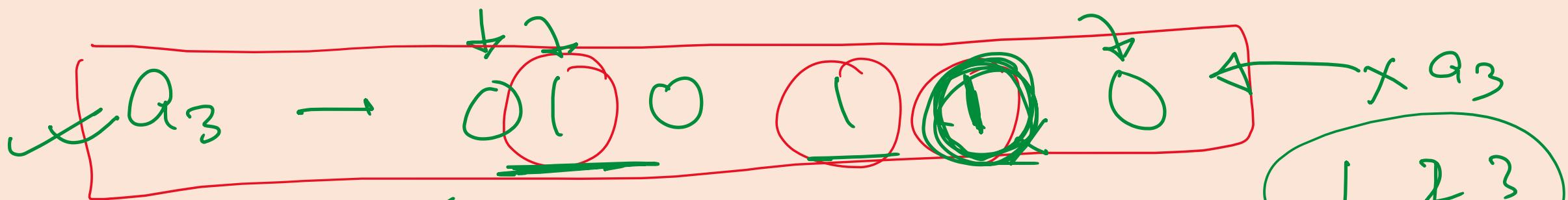
$\rightarrow (a_1, a_2, a_3, a_n \dots a_n)$

$f(a) = f(b)$

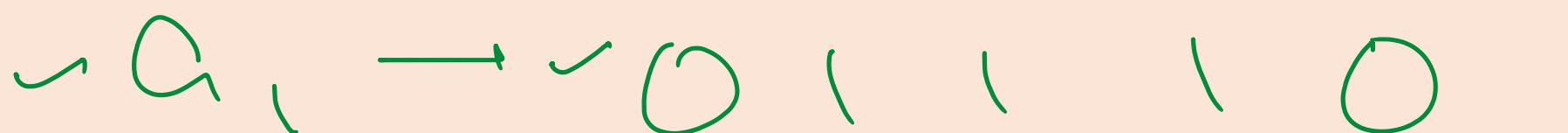
b

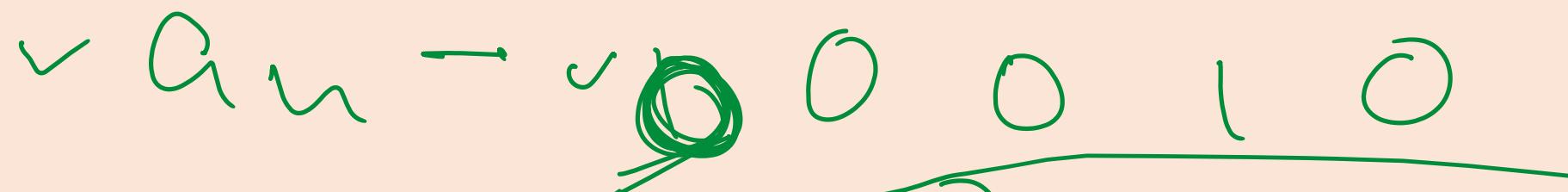
$(a_1, a_2, a_n \dots \underline{a_n})$

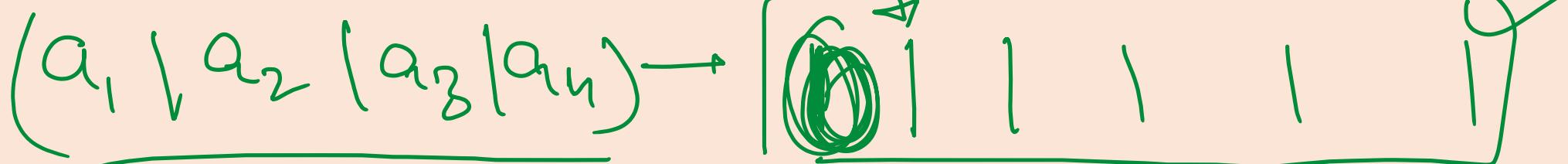
skipped a<sub>3</sub>

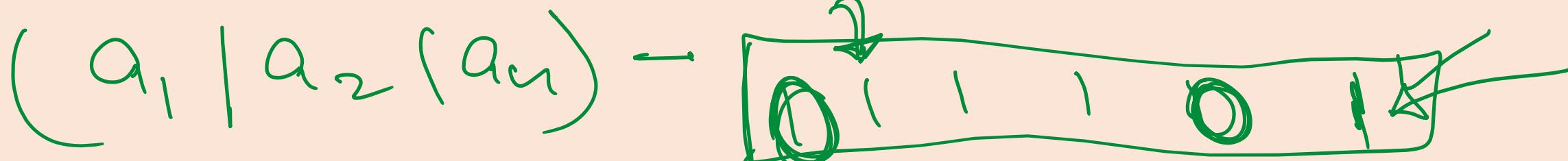
~~$\exists Q_3 \rightarrow$~~   $\exists Q_3 \rightarrow$  

$\checkmark a_2 \rightarrow$  

$\neg a_1 \rightarrow$  

$\checkmark a_n \rightarrow$  

$(a_1 | a_2 | a_3 | a_n) \rightarrow$  

$(a_1 | a_2 | a_n) \rightarrow$  

can I exclude a number  
from entire array?

(  ,   ,   ,   ,   )

iter bit → # that have iter bit is set





cnt → 3

$$x_1 = (a_1 | a_2 | a_3 | a_4 | a_5) \times \text{snumber}$$

1

$$x_2 = (a_1 | \underline{a_2} | \underline{a_3} | \underline{a_4} | \underline{a_5}) \times$$

$a_2 \rightarrow$

$a_2$

— — — — — — — — — —  
1 —  
X

