Bit Manipulation
Problem solving 1

--by Harsh Gupta

Maximum And

^, &

$1\,0\,1\,0\,0\,0\,0\,1\,1\,0$

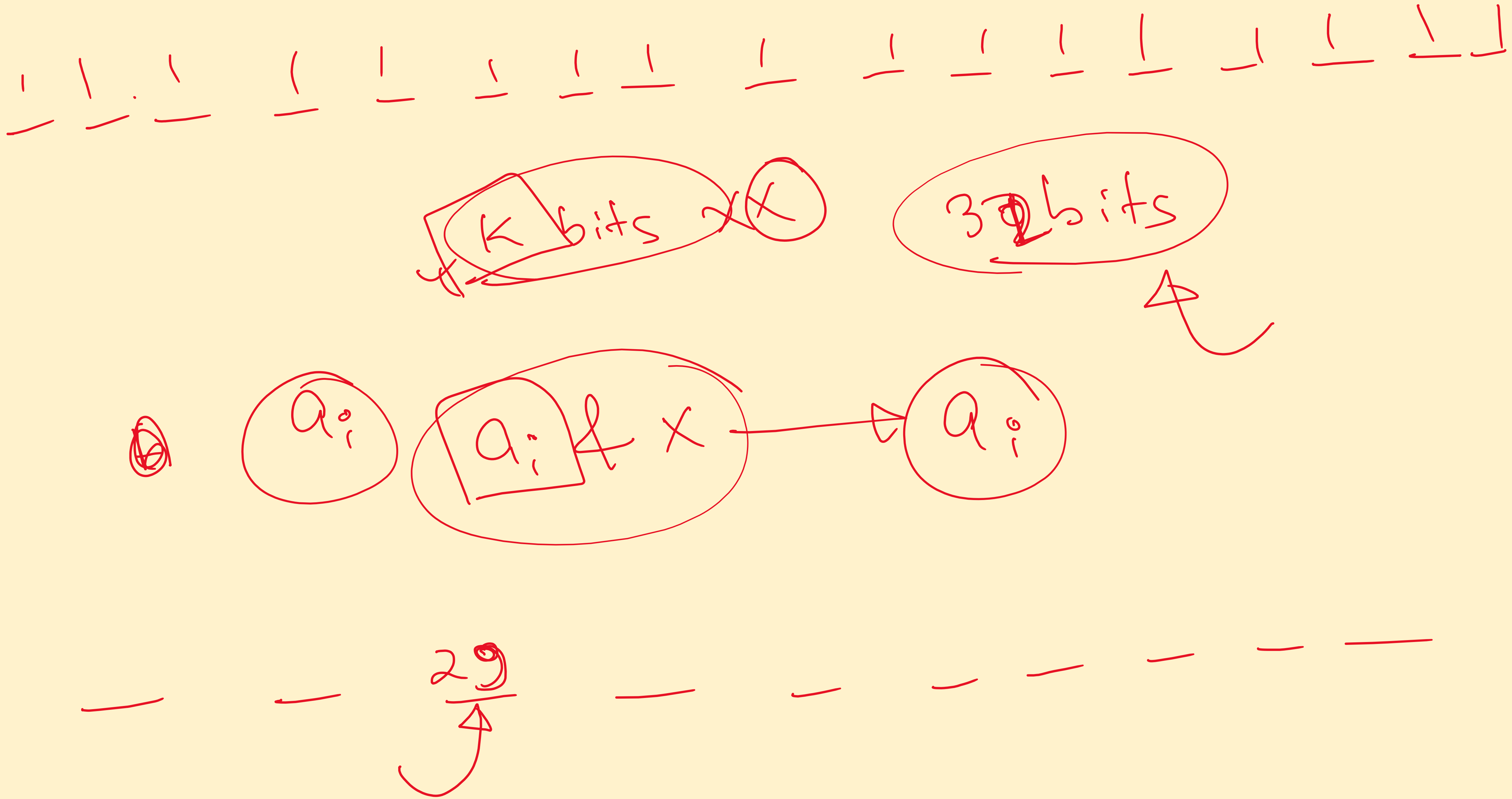
$a_1 \quad a_2 \quad a_3 \quad \cdots \quad a_n$

$X \longrightarrow$ exactly K bits are set

$$S = (x \,\&\, a_1) + (x \,\&\, a_2) + (x \,\&\, a_3) + \cdots + (x \,\&\, a_n)$$

$\downarrow$

maximize S, if same maximum value for multiple x, then o/p minimum X.


k bits
x
32 bits
0
a_i
a_i & x
a_i
29

$\boxed{29^{th}}$ bit is set in ~~x~~

S — bitwise — 29^{th} 28^{th} - . . .

$\boxed{2^{29}}(1+0+1$

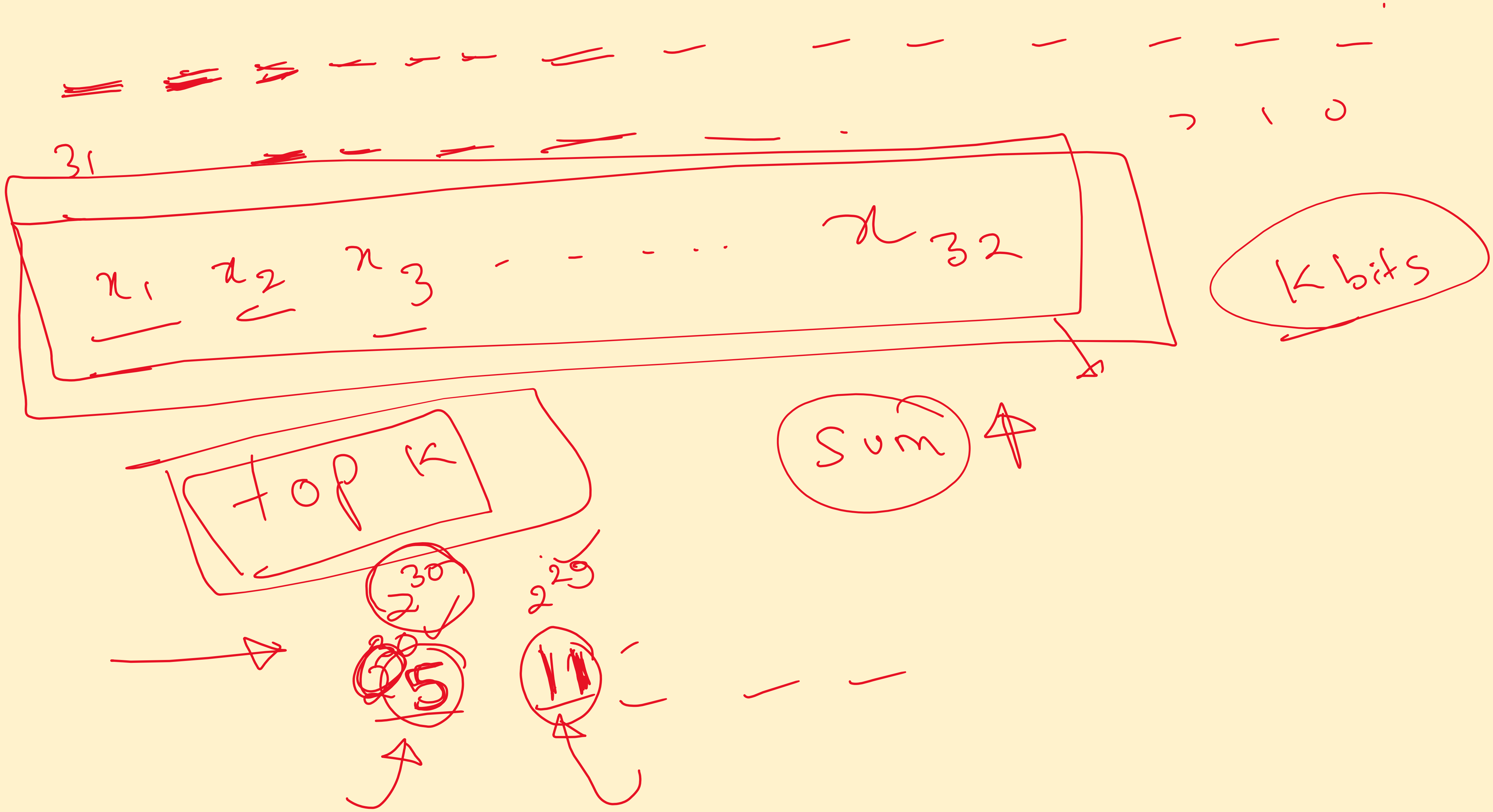
$\boxed{2 \times 2^{29}}$

29^{th}

x	a_1	0	0	1	0	0	1	0	0 x
	a_2	0	0	0	0	0	1	0	0
	a_3	0	0	1	0	0	0	0	

$2^{29}+0+2^{29}$

$\boxed{2^{29}(2)}$


31
2 1 0
x1 x2 x3 - - - . . . x32
K bits
Sum
top K
2^30
2^29
25
11

2^5

x — 5th bit

$\cancel{x}$ & a_i

32 possible

k option

$S = \left[2^5 \times \left(\text{No. of numbers in array for which this bit is set} \right) \right]$

$S = \boxed{} \rightarrow$

$\cdots \to 6 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1 \quad 0$

$a_1 = 1\ 0\ 0\ 1\ 0\ 0\ 1$

$a_2 = 1\ 1\ 0\ 0\ 1\ 0\ 0$

$a_3 = 0\ 1\ 1\ 1\ 1\ 0\ 1$

$a_4 = 1\ 0\ 0\ 0\ 0\ 0\ 1$

$a_5 = 0\ 1\ 0\ 1\ 0\ 1\ 0$

$\boxed{S \uparrow}$

$\boxed{k=2}$

$\boxed{2 \text{ bits}} = k$

(value, bit)

X

S → decimal

$\boxed{(1 \ll 6) \times 3}$

$30 \to 2^{30} \times (0) = 0$

$29, 28, 27 \ldots 7 = 0$

descending

k bits

6^{th} bit $\to \boxed{(2^6 + 2^6 + 2^6)} = \boxed{3 \times 2^6}$

5^{th} bit $\to \boxed{3 \times 2^5}$

4^{th} bit $\to \boxed{1 \times 2^4}$

3^{rd} bit $\to \boxed{3 \times 2^3}$

2^{nd} bit $\to \boxed{2 \times 2^2}$

1^{st} bit 1×2^1

0^{th} bit $\to 3 \times 2^0$

$X \to 0\ 1\ 0\ 1\ 0\ 0\ L$

0

2 bits

$2^{\text{bit pos}}$ 2^2 2^1 2^0

S 4

bit

0 0 0

0 0 0 2 0

0 0 0 2 5

0 0 0 6 5

1 1 0

$(0\times10^0)+(5\times10^0)+(5\times10^0)$

$= 10^0(10) = \boxed{10}$

$(2\times10^1)+(2\times10^1)+(6\times10^1)$

$= 10^1(6+2+2)$

$= \boxed{100}$

s

sum

x_{min}

4^{th} bit

2nd bit in x

4

40

40

32

$= 32$

$2^3 \times 4$

$2^4 \times 2$

2^2

4 3 2 1 0

2

4

sorted

↓

sum (descending)

↓

bit values (descending)

↓

ascending

~~pair~~ <OB, {P1, P2}

V → { { }, { }, { } }

desending on the basis of P1
ascending " " " of P2

true P1, P2
false P2, P1

```
class comparator {
  bool ()( pair<ll, ll> &P1, pair<ll, ll> &P2)
    if (P1.first != P2.first) { return P1.first > P2.first }
    return P1.second < P2.second;
}
```

Homework

The Lost Array

Make Almost Equal With Mod

1 1 0 0 $\boxed{}$

$2^0 \; (12 \% 1 = 0) \longrightarrow 0$

$2^1 \; (12 \% 2) \longrightarrow 0$

$2^2 \; (12 \% 4) \longrightarrow 0 \; 0$

$2^3 \; (12 \% 8) \longrightarrow 1 \; 0 \; 0$

$2^4 \; (12 \% 16) \longrightarrow 1 \; 1 \; 0 \; 0$

12

2^5 ______

2^6 ______

______

$k = 2^n$

0 1

$\boxed{2^2}$

2^1

a_1 1 0

a_2 0 0

a_3 1 0

a_4 1 0

a_5 1 0

a_6 0 0

a_7 0 0

2^1 2^1

2^0

$\boxed{0}$

$\boxed{2^2}$

exactly
2 remainders
possible

2^2 4th 3rd

2^3

1 1 0

0 1 0

0 1 1
1 1 0
0 1 0
1 0 1 1 0
0 1 1 0

0

1/0

1 1 0
1 1 6
1 1 6
0 1 6
0 1 6
0 1 6
1 1 0

1<<k

00

2^{d} → d bits

k2

$1 << (3+1)$

bit 3

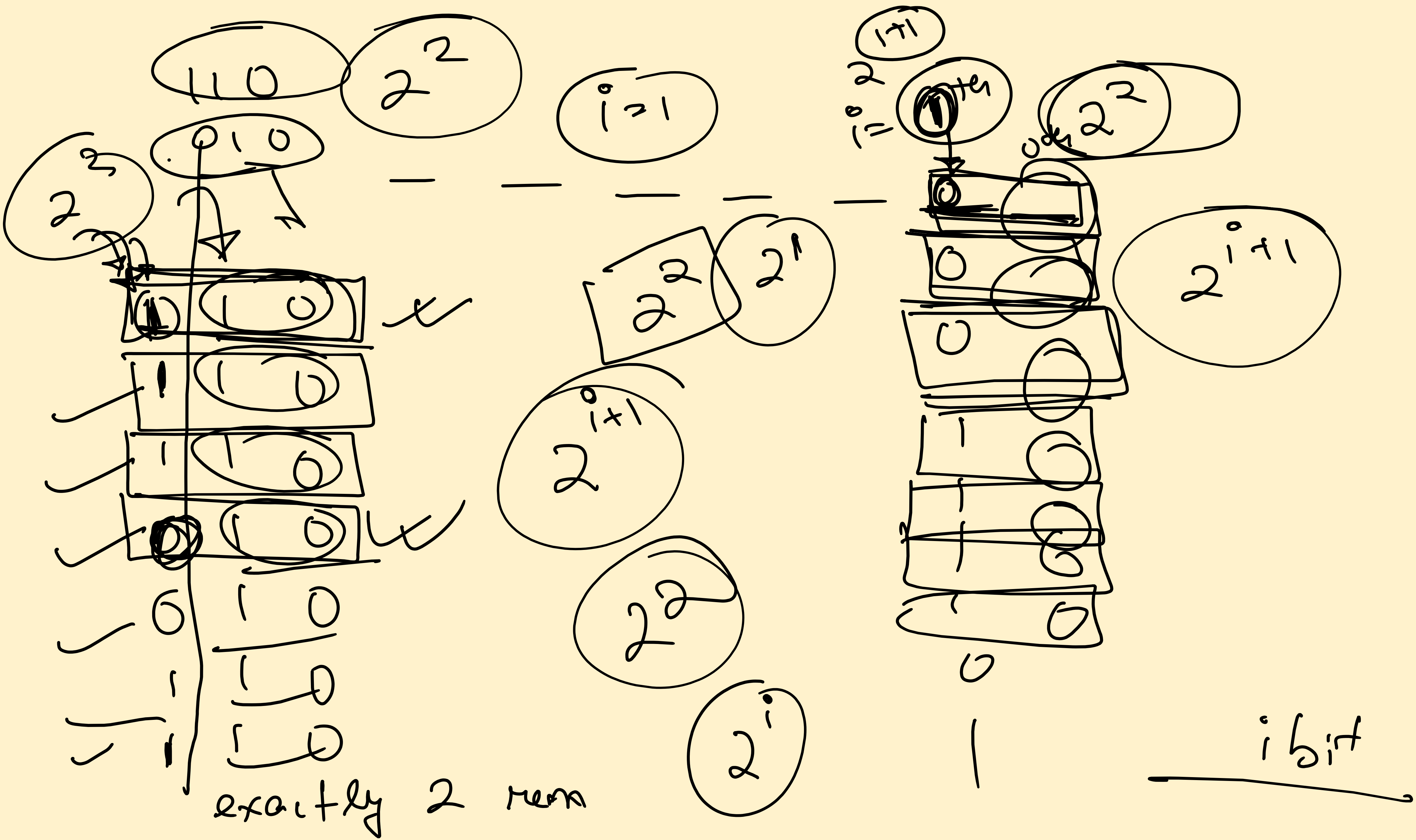
2^{4}

3^{4}

2^{4}

3	2	1	0
1	_	_	_

Dividing a number with 2^j gives you the last j bits of the number.



110
2^2
010
2^3
i=1
2^2
2^1
2^(i+1)
2^2
2^i
exactly 2 rem
2^(i+1)
i bit

2^1

0^{th} bit

1 0 1 $\quad 2^{0+1} \quad$ 2

odd

1 1 0

even

1 1 1

1 0 0

0 1 1

$i \to 63$

$\boxed{2^{63}}$ x $\quad$ ((1LL) << i)

exactly 2

the

$6 \rightarrow 0\ 1\ 1\ 0$

$8 \rightarrow 1\ 0\ 0\ 0$

$4 \rightarrow 0\ 1\ 0\ 0$

$2^2 = 4$

1

0

$6 \% 4 = 2$ ✓

$8 \% 4 = 0$ ✓

$4 \% 4 = 0$ ✓

i^{th} bi..