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GATE 2023-EC

EE23BTECH1205 - Avani Chouhan*

Question: 14

The value of the contour integral, $\oint_C \frac{z+2}{z^2+2z+2} dz$, where the contour C is $\{z : |z+1-\frac{3}{2}i|=1\}$, taken in the counter clockwise direction, is

- (A) $-\pi(1+j)$
- (B) $\pi(1 + j)$
- (C) $\pi(1-j)$
- (D) $-\pi(1-j)$

(GATE ST 2023)

Solution:

$$I = \oint_C \frac{z+2}{z^2+2z+2} dz; \quad C = \left| z+1 - \frac{3}{2}i \right| = 1 \quad (1)$$

The poles are given by $(z + 1)^2 + 1 = 0$

$$z+1=\pm\sqrt{-1} \tag{2}$$

$$z = -1 + j$$
, $z = -1 - j$

where -1 - i lies outside C and z = (-1, 1) lies inside C, by the Residue Theorem:

$$\oint_C f(z) dz = 2\pi i \operatorname{Res}(f(z), z = -1 + j)$$
 (3)

$$= 2\pi i \left(\frac{z+2}{2(z+1)} \right) \Big|_{z=-1+i}$$
 (4)

$$=2\pi i \left(\frac{-1+j+2}{2(-1+j+1)}\right) \tag{5}$$

$$=\pi(1+j). \tag{6}$$

Therefore, the correct answer is option (B).

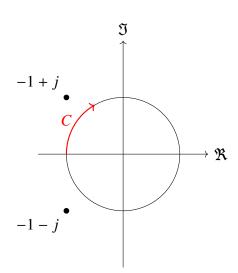


Fig. 0. Contour C and poles