

GATE 2023-EC

EE23BTECH1205 - Avani Chouhan*

Question : 14

The value of the contour integral, $\oint_C \frac{z+2}{z^2+2z+2} dz$, where the contour C is $\{z : |z+1-\frac{3}{2}i| = 1\}$, taken in the counter clockwise direction, is

- (A) $-\pi(1+j)$
- (B) $\pi(1+j)$
- (C) $\pi(1-j)$
- (D) $-\pi(1-j)$

(GATE ST 2023)

Solution:

$$I = \oint_C \frac{z+2}{z^2+2z+2} dz; \quad C = \left| z+1-\frac{3}{2}i \right| = 1 \quad (1)$$

The poles are given by $(z+1)^2 + 1 = 0$

$$z+1 = \pm \sqrt{-1} \quad (2)$$

$$z = -1+j, z = -1-j$$

where $-1-i$ lies outside C and $z = (-1, 1)$ lies inside C , by the Residue Theorem:

$$\oint_C f(z) dz = 2\pi i \text{Res}(f(z), z = -1+j) \quad (3)$$

$$= 2\pi i \left(\frac{z+2}{2(z+1)} \right) \Big|_{z=-1+i} \quad (4)$$

$$= 2\pi i \left(\frac{-1+j+2}{2(-1+j+1)} \right) \quad (5)$$

$$= \pi(1+j). \quad (6)$$

Therefore, the correct answer is option (B).

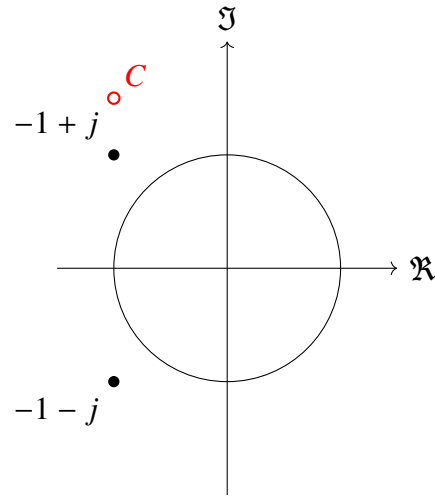


Fig. 0. Contour C and poles