

# GATE 2022-IN

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## Question : 18

A signal  $x(t)$  is band-limited between 100 Hz and 200 Hz. A signal  $y(t)$  is related to  $x(t)$  as follows:

$$y(t) = x(2t - 5)$$

The statement that is always true is

- (A)  $y(t)$  is band-limited between 50 Hz and 100 Hz
- (B)  $y(t)$  is band-limited between 100 Hz and 200 Hz
- (C)  $y(t)$  is band-limited between 200 Hz and 400 Hz
- (D)  $y(t)$  is not band-limited

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## Solution:

$x(t)$  is band-limited to 100 Hz to 200 Hz

$$y(t) = x(2t - 5)$$

$$x(t) \rightleftharpoons X(\omega)$$

$$x(2t) \rightleftharpoons \frac{1}{2} X\left(\frac{\omega}{2}\right)$$

Time shifting will not change bandwidth,

So  $x(2t - 5)$  will be band-limited to 200 to 400.

**Derivation of Fourier transform of  $x(2t - 5)$ :**

$$x(t) \rightleftharpoons X(\omega)$$

$$x(at) \rightleftharpoons \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$$

$$x(2t) \rightleftharpoons \frac{1}{2} X\left(\frac{\omega}{2}\right)$$

$$x(t - t_0) \rightleftharpoons e^{-j\omega t_0} X(\omega)$$

$$x(2t - 5) \rightleftharpoons e^{-j5\omega} \cdot \frac{1}{2} X\left(\frac{\omega}{2}\right)$$

The operation  $x(2t - 5)$  compresses time by a factor of 2 and shifts 5 units rightward. This expands the frequency domain, doubling the bandwidth of  $x(t)$  from 100 Hz to 200 Hz to  $y(t)$  between 200 Hz and 400 Hz.

Hence, the correct answer is option (C).