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$$A = \begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix}$$

Eigen values : ~~det(A)~~

$$\det(A - \lambda I) = 0$$

$A - \lambda I =$ we subtract λ on diagonal

$$\begin{bmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{bmatrix}$$

Since $\det(A - \lambda I) = 0$

On a 4x4 M_{11} we hide Row 1 and col 1 to Remain with a 3x3

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$M_{11} = d$

$[(-9-\lambda)\det$

$[(-2)\ de$

$(-4)\ de$

Each

* $\det[(5$

$= (5-\lambda$

$= \lambda^2$

* $\det[$

$= 10$

$=$

* $\det$

$=$

$=$

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$$M_{11} = \det \begin{bmatrix} -9-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ 13 & -14 & -13-\lambda \end{bmatrix}$$

$$\left[(-9-\lambda)\det(5-\lambda, -10; -14, -13-\lambda)\right] - \left[(-2)\det(10, -10; -13, -13-\lambda)\right] + \left[(-4)\det(10, 5-\lambda; -13, -14)\right]$$

Each det

* $\det[(5-\lambda, -10; -14, -13-\lambda)$
$= (5-\lambda)(-13-\lambda) - (-10)(-14)$
$= \lambda^2 + 8\lambda - 205$

* $\det[10, -10; -13, -13-\lambda]$
$= 10(-13-\lambda) - (-10)(-13)$
$= -260 - 10\lambda$

* $\det[10, 5-\lambda; -13, -14]$
$= 10(-14) - (5-\lambda)(-13)$
$= -75 - 13\lambda$

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$M_{11} = (-9-\lambda)(\lambda^2+8\lambda-205) + 2(-260-10\lambda) - 4(-75-13\lambda)$

$= (-9-\lambda)(\lambda^2+8\lambda-205) - 520 - 20\lambda + 300 + 52\lambda$

$= (-9-\lambda)(\lambda^2+8\lambda-205) + 32\lambda - 220$

$= -\lambda^3 - 17\lambda^2 + 165\lambda + 1625$

$M_{21} \neq 0$ Remove Row 2 Col 1

$$M_{21} = \det \begin{bmatrix} 8 & -1 & -2 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{bmatrix}$$

$\det [5-\lambda, -10 ; -14, -10]$

$= (5-\lambda)(-10) - (-10)(-14)$

$= -50 + 10\lambda - 140$

$= 10\lambda - 190$

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$\det \begin{bmatrix} 10 & -13-\lambda \\ \end{bmatrix}$;

$\det \begin{bmatrix} 10 & -10 \\ -13 & -13-\lambda \end{bmatrix}$

$= 10(-13-\lambda) - (-10)(-13)$

$= -130 - 10\lambda - 130$

$= -260 - 10\lambda$

$\det \begin{bmatrix} 10 & 5-\lambda \\ -13 & -14 \end{bmatrix}$

$= 10(-14) - (5-\lambda)(-13)$

$= -140 - (-65 + 13\lambda)$

$= -140 + 65 + 13\lambda$

$= -75 + 13\lambda$

$M_{21} = 8(10\lambda - 190) - (5-\lambda)(-260 - 10\lambda)$
$+ (-13-\lambda)(-75 + 13\lambda)$

$= 8\lambda^2 + 80\lambda - 1750$

$=$

13
140
65
75

13
x 5
65

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M_{41} → Remove Row 4, Col 1

$$M_{41} = \det\begin{bmatrix} 8 & -1 & -2 \\ -9-\lambda & 2 & -4 \\ 10 & 5-\lambda & -10 \end{bmatrix}$$

$\det[-2, -4 ; 5-\lambda, -10]$

$= (-2)(5-\lambda) - (-4)(-10)$

$= -10 - 2\lambda - 40$

$= -50 - 2\lambda$

$\det[-9-\lambda, 10 ; -4, -10]$

$= (-9-\lambda)(-4) - (10)(-10)$

$= 36 - 4\lambda + 100$

$= -4\lambda + 64$

$\det[(-9-\lambda)(10) ; -2, 5-\lambda]$

$= (-9-\lambda)(-9-\lambda)(-2) - (10)(5-\lambda)$

$= (18+2\lambda) - 50 + 10\lambda$

$= 12\lambda - 32$

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$M_{41} = -2\lambda$

Th

det (λ -

$(4-\lambda)(-\lambda^3$

$2(8\lambda^2 +$

$(-2\lambda^2 +$

wi

I got

=

col 1

-2

-4

$-\lambda \quad -10$

$0)$ $\quad$ 9

36

64

18

32

$-10)$

$\lambda)$

$(10)(5-\lambda)$

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$M_{41} = -2\lambda^2 + 50\lambda + 1060$

The polynomial

$\det(A - \lambda I) = 0$

$\Downarrow$

$$(4-\lambda)(-\lambda^3 - 17\lambda^2 + 165\lambda + 1625) + 2(8\lambda^2 + 80\lambda - 1750) + (-2\lambda^2 + 50\lambda + 1060) = 0$$

with the use of a calculator

I got

$$= \boxed{\lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3100 = 0}$$

The polynomial