Differential and partial differential equation:

Differential Equations

- ***A differential equation is a mathematical equation that relates some function with its derivatives.***
- An example of a linear differential equation with constant coefficients to describe the **wheel suspension system of an automobile** can be given as

$$M\ddot{x} + D\dot{x} + Kx = KF(t)$$

The dependent variable x appears with its first and second order derivative $\dot{x}$ & $\ddot{x}$ and the term involving these quantities are multiplied by constant coefficient and added. The quantity $F(t)$ is an input to the system depending upon the independent variable t.

Differential and partial differential equation:

- A dependent variable is a variable whose value depends upon independent variables.
- The dependent variable is what is being measured in an experiment or evaluated in a mathematical equation. The dependent variable is sometimes called "*the outcome variable*." In a simple mathematical equation, for example:

$$a = b/c$$

the dependent variable, a , is determined by the values of b and c .

Differential and partial differential equation:

- If the dependent variable or any of its derivate appears in any other form such as being raised to a power or are combined in any other way for e.g. by being multiplied together, the differential equation is said to be **non-linear**.
- **When more than one independent variable occurs in a differential equation, the differential equation is said to be partial differential equation.**
- It involves the derivative of the same dependent variable with respect to each of the independent variable.

Differential and partial differential equation:

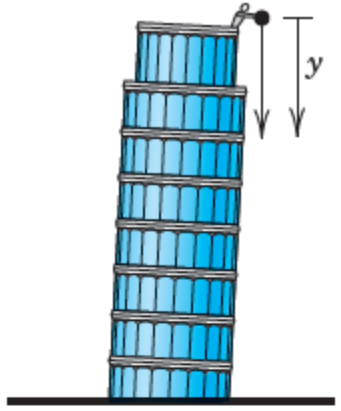
- ***A linear equation is always a polynomial of degree 1 (for example x+2y+3=0).***
- ***Every other equation is nonlinear. Higher degree polynomials are nonlinear ($x^2+3x+2=0$).***

E.g. Equation of heat flowing through 3-D body i.e.

$$\frac{\partial u}{\partial t} - \alpha\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = 0$$

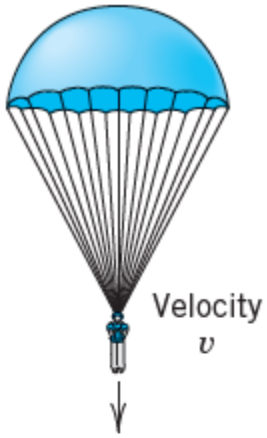
Here the dependent variable is u and independent variable is x, y, z and t.

Some applications of differential equations



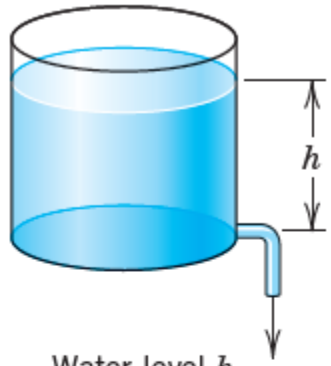
Falling stone

$y'' = g = const.$



Parachutist

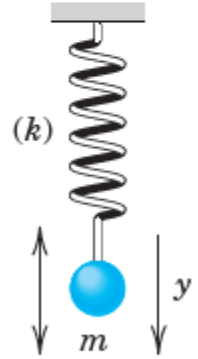
$mv' = mg - bv^2$



Water level h

Outflowing water

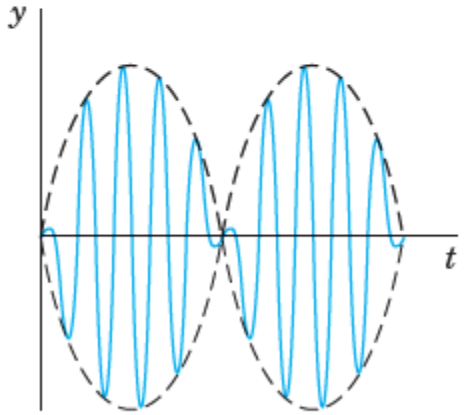
$h' = -k\sqrt{h}$



Displacement y

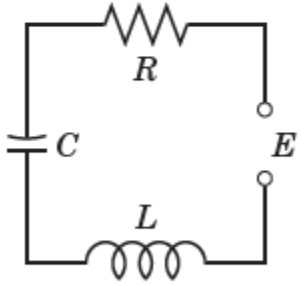
Vibrating mass on a spring

$my'' + ky = 0$



Beats of a vibrating system

$y'' + \omega_0^2 y = \cos\ \omega t, \quad \omega_0 \approx \omega$



Current I in an RLC circuit

$LI'' + RI' + \frac{1}{C}I = E'$