Optimisation et quantification d'incertitudes en dynamique non-linéaire des structures

Enora Denimal Goy

Inria, Saclay, France - Platon research team

enora.denimal@inria.fr

Collaborators

L. Renson (ICL), L. Salles (ICL), JJ. Sinou (ECL)

Main outlines

- Personal background
- Inria presentation
- Scientific seminar
  - Surrogate modelling for uncertainty quantification – Application to rotordynamics
  - Topology optimisation based on EGO algorithm for nonlinear resonances

Personal background

Master Engineering | PhD | Postdoc | Research scientist Inria – I4S Team | Research scientist Inria – Platon Team

2015 | 2018 | 2020 | 2024

Curriculum

- Master degree in mechanical engineering and structural dynamics
- PhD in structural dynamics for squeal noise
- Postdoc in structural dynamics for nonlinear resonance mitigation
- Researcher I4S team, Inria, Rennes
- Researcher PLATON team, Inria, Saclay

Research activity and topics

- Nonlinear structural dynamics (contact, friction, crack)
- Structural optimisation (topology, shape, parametric, model updating)
- Uncertainty propagation for large nonlinear dynamic systems
- Structural Health Monitoring

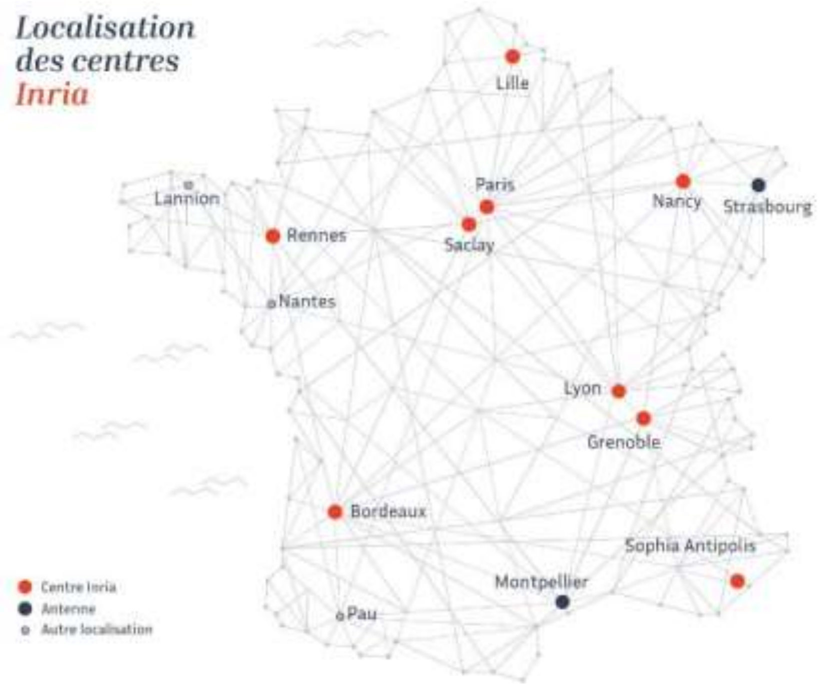
Inria - Institut National de la recherche en sciences du numérique

De nombreuses thématiques de recherche…

… pour de nombreux secteurs d'applications

Inria

Répartition en 9 centre de recherche sur le territoire



Structuration de l'institut

- 3900 scientifiques – 665 supports
  - Inria: 2700 personnes
  - Partenaires: 1700
- Structuration sous forme d'Equipe-Projet
  - 10 à 30 personnes
  - Feuille de route scientifique précise
  - Autonomie scientifique et financière
  - Évaluation internationale tous les 4 ans
  - Durée de vie maximale de 12 ans
  - Des équipes en coopération industrielle et internationale
- En 2020
  - 200 Equipes Projets
  - 80% en collaborations

Inria

- Partenariats industriels

- Des logiciels

- Laboratoires communs avec des PME

PLATON Uncertainty Quantification in Scientific Computing and Engineering

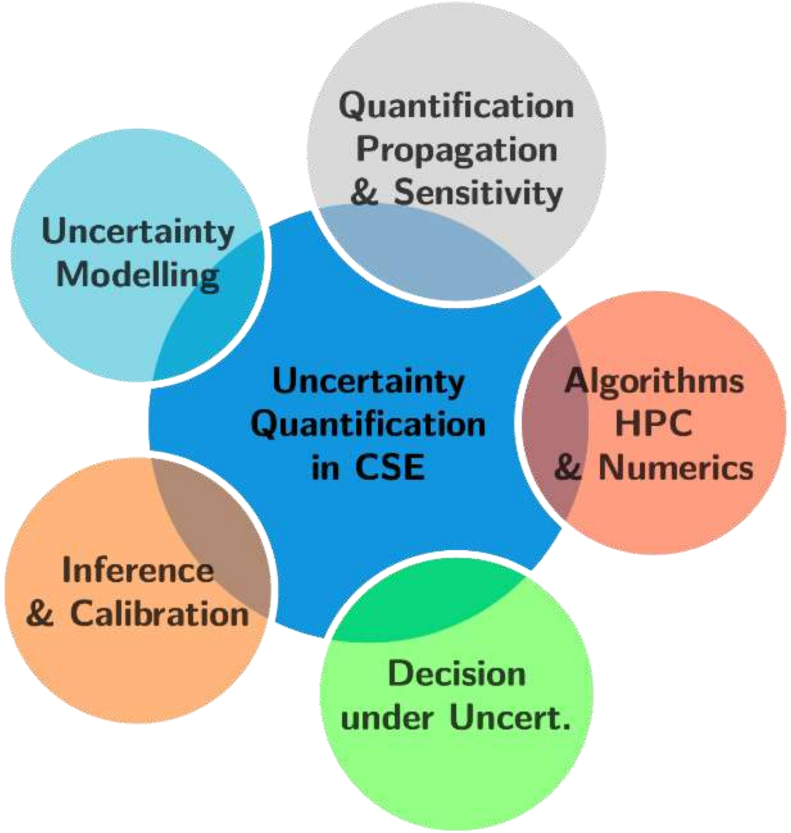
- Uncertainty Quantification in computation Science and Engineering

- What is PLATON ?
  - Join reserach group: Inria, CNRS and Ecole Polytechnique
  - Located in Saclay

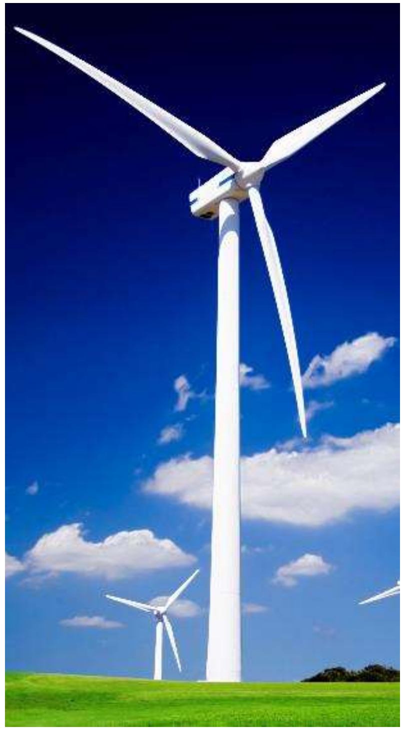
- PLATON's collaborations
  - Industries: ArianeGroup, Dassault systems, Framatome etc
  - EPICS: CEA, Onera, IFPEN etc
  - Academics: Stanford, Polytecnico di Milano, VKI, U-Grenoble, ICL etc

Key features of **PLATON**:

- Created in 2020
- Core activities on UQ
- Exhaustive in scope
- From methodology to implementation
- Applications with Partners
- Factorize developments
- Cross-fertilization between application domains



General context



Large industrial structures
- Civil engineering
- Energy production (rotor, wind turbines etc)
- Transport (aircraft, train, car etc)
- Spatial

Complex vibration behaviour
- Nonlinear vibrations
- Large and complex models
- Expensive solver

Current challenges
- Uncertainties
- Optimisation
- Design
- Etc…

General context

Nonlinear vibrations

joints, friction, contact,
large deformation, crack, etc…

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \boxed{\mathbf{F_{nl}}\,(\mathbf{x}, \dot{\mathbf{x}})} = \mathbf{F_e}$$

- Dedicated solvers: Harmonic Balance Method, Shooting method, etc
- Methods developed for linear structural not adapted anymore
    - Uncertainty Quantification
    - Optimisation
    - etc

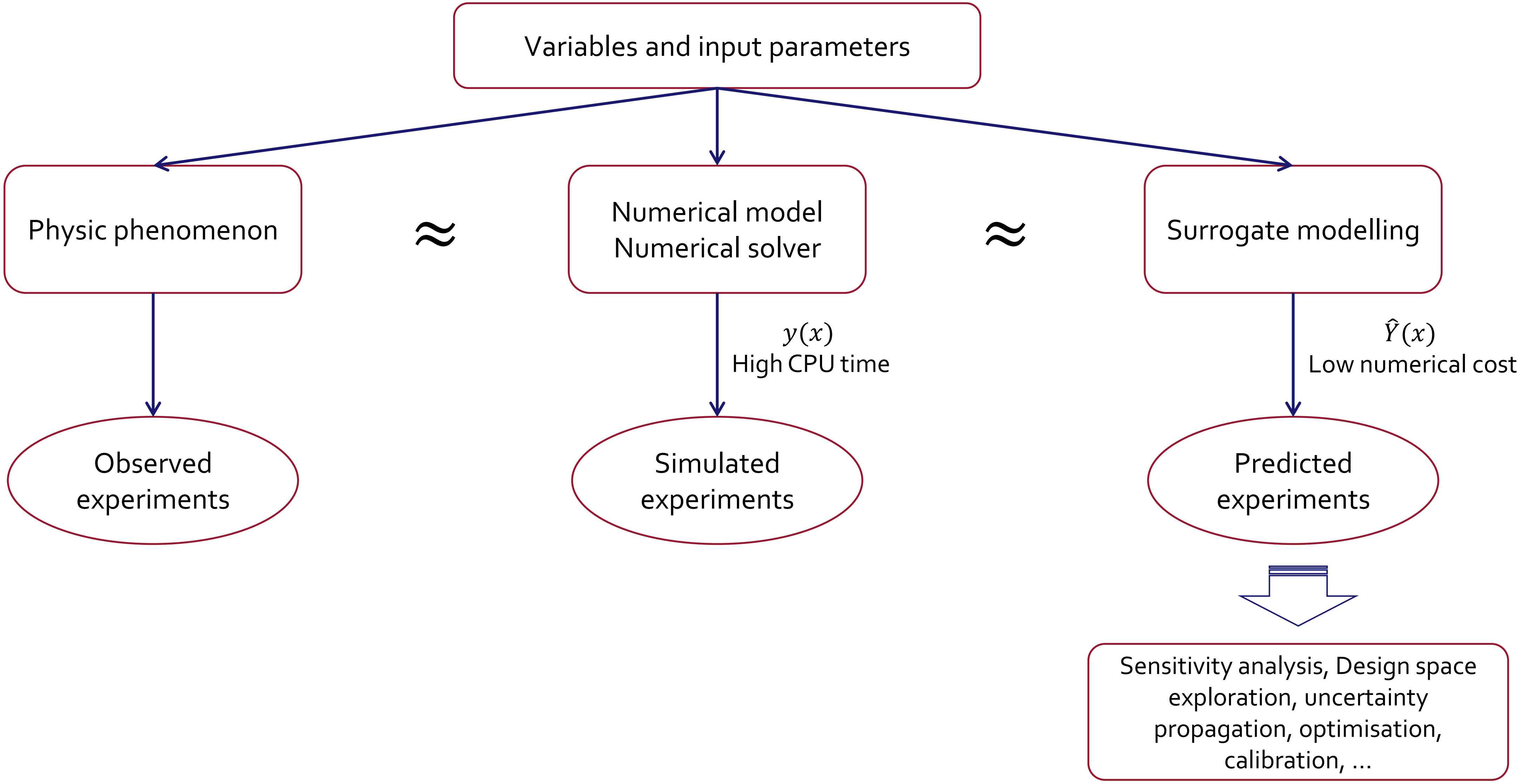
Limitations & current challenges

- Large computational time
    - Large models
    - Nonlinear behaviour
- Robust design of nonlinear structures
    - Uncertainties
    - Optimisation technics
    - Many parameters
    - New manufacturing technologies

Contributions

- Exploitation of surrogate modelling technics
    - Uncertainty quantification
    - Structural optimization
    - Robust Structural Health Monitoring

Surrogate modelling: why?



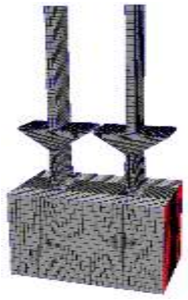
Surrogate modelling – general presentation

Input variables and parameters

$p_1, p_2, \dots, p_n$

- Sensitivity analysis
- Design space exploration
- Uncertainty propagation
- Optimisation
- …

Computational model



$+$ Nonlinear solver

High CPU time

Trained with N evaluations
$(\boldsymbol{p_1}, y_1)$
…
$(\boldsymbol{p_N}, y_N)$

Meta-modelling

cheap mathematical approximation

$\mathscr{M}(\boldsymbol{p})$

Negligible numerical cost

Output

y

Outlines

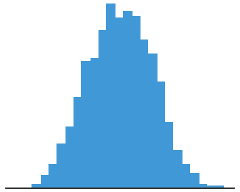
- Surrogate modelling for uncertainty quantification – Application to rotordynamics
  - Coupling PCE and kriging
  - Include physical knowledge in kriging
  - Prediction of dynamic features
  - Extension for rotor SHM in crack detection and localisation

- Topology optimisation based on EGO algorithm for nonlinear resonances
  - MMC framework
  - EGO algorithm
  - Application for friction ring dampers

Rotor modelling

- **Vibrations**
  - Critical speeds
  - Amplitudes

- **Real conditions**
  - Environmental variations
  - Manufacturing tolerances
  - Wear



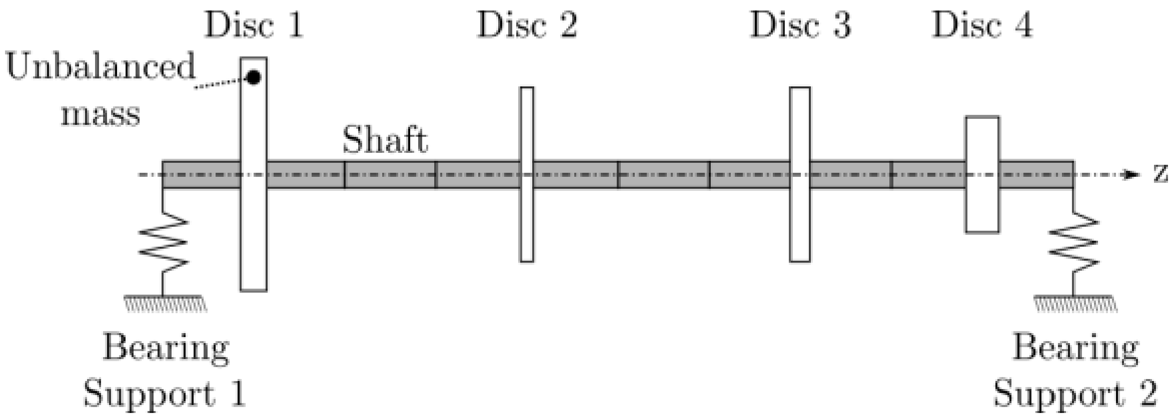
→ Numerous sources of uncertainties!

- **Design**
  - Few parameters
  - Robust design

Parametric studies

$p \in [p_{min}, p_{max}]$

Crucial to design robust rotors by taking into consideration uncertainties



Academic rotor under study

Shaft
- 10 Euler-beam finite elements
- mass, stiffness and gyroscopic matrices
- $\mathbf{C} = \alpha \mathbf{M_S} + \beta \mathbf{K_S}$ damping matrix

Bearings
- One on each side
- Stiffness in horizontal and vertical direction

Discs
- 4 discs at different locations
- Mass and gyroscopic matrices

Unbalanced mass on Disc 1

Rotor modelling

Equation of motion

$$\mathbf{M\ddot{x}} + (\mathbf{C} + \omega\mathbf{G})\mathbf{\dot{x}} + \mathbf{Kx} = \mathbf{f_1}$$

With $\mathbf{M} = \mathbf{M_S} + \mathbf{M_{d1}} + \mathbf{M_{d2}} + \mathbf{M_{d3}} + \mathbf{M_{d4}}$

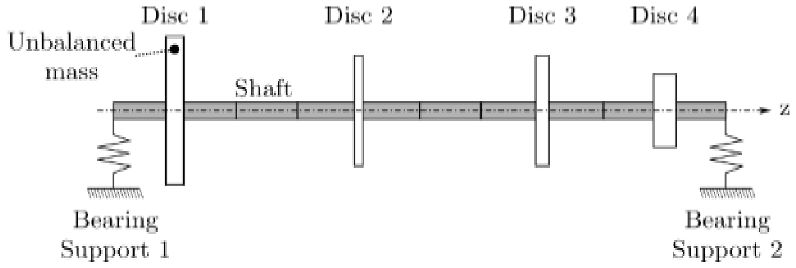
$\mathbf{G} = \mathbf{G_S} + \mathbf{G_{d1}} + \mathbf{G_{d2}} + \mathbf{G_{d3}} + \mathbf{G_{d4}}$

$\mathbf{K} = \mathbf{K_S} + \mathbf{K_{b1}} + \mathbf{K_{b2}}$

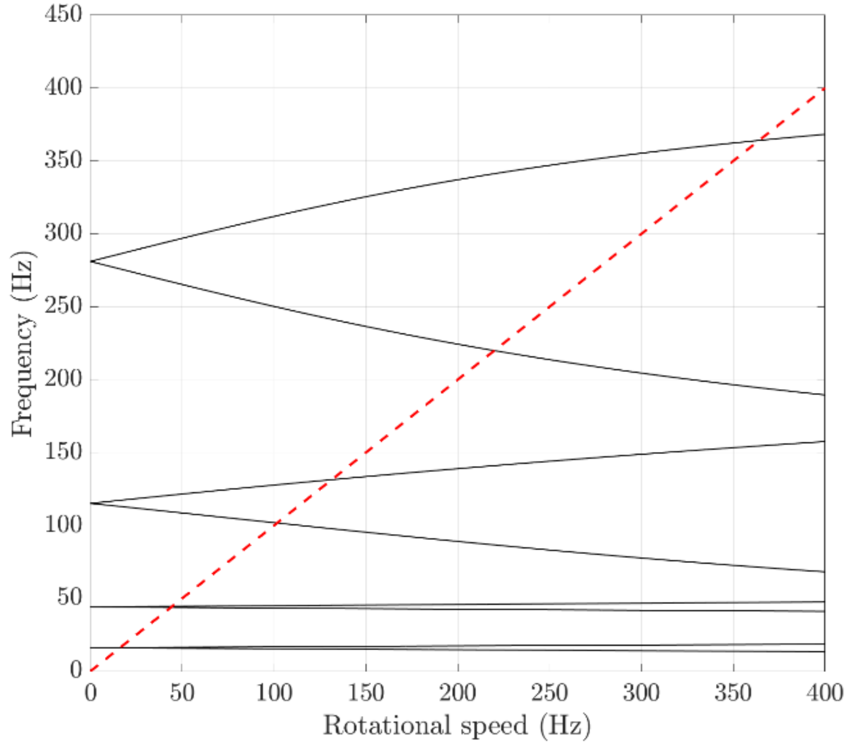
$\mathbf{x}$ dof displacements

- Eigenvalues and modeshapes depend on the rotational speed ω → Campbell diagram
- **Critical speeds**: crossing between engine order (red dotted line) and natural frequency → $\boxed{f^{(i)}}$
- **Vibration amplitude** at critical speed i → $\boxed{a^{(i)}}$

Parameters to predict



Academic rotor under study



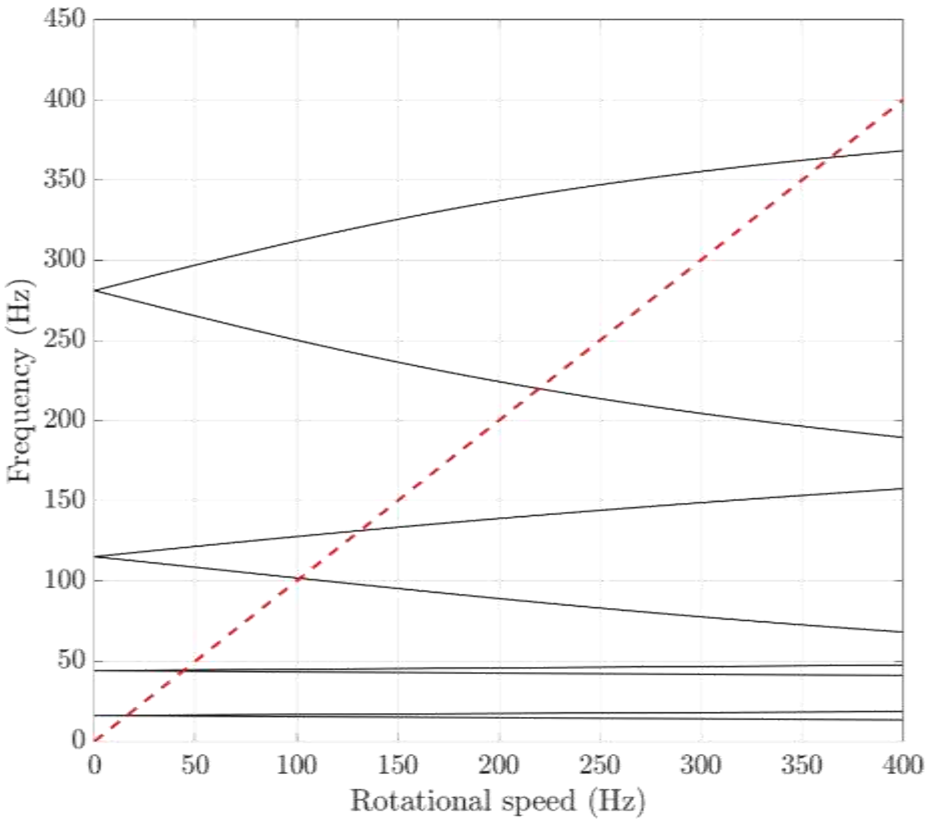
Campbell diagram of the rotor

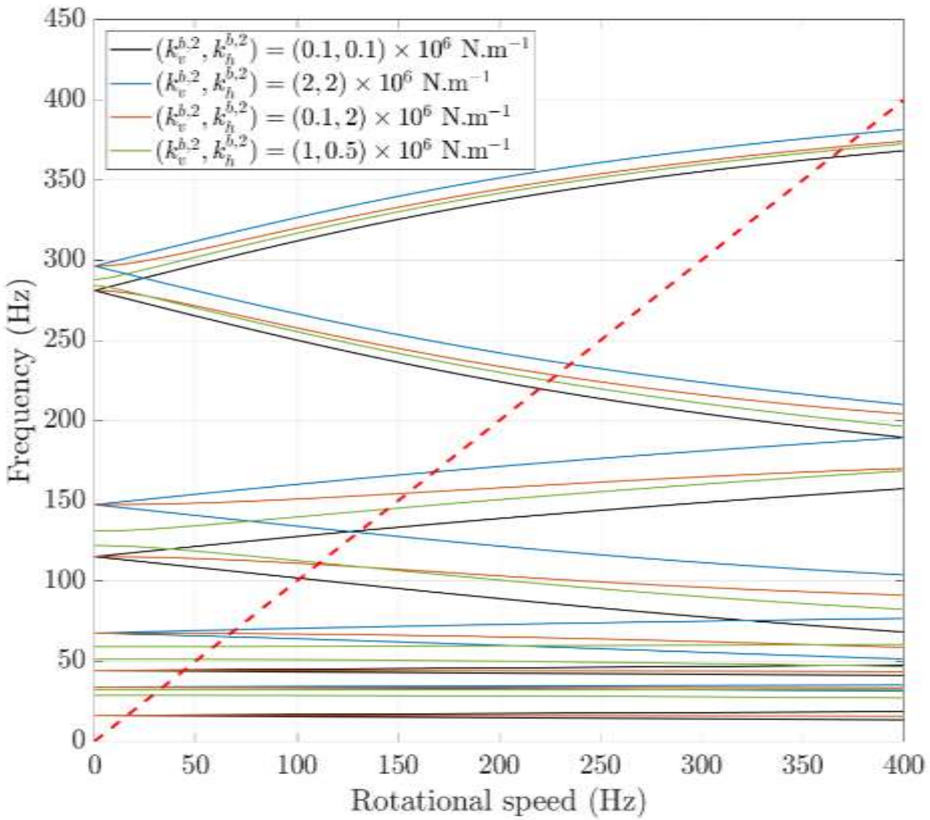
Rotor modelling

- **Critical speeds**: crossing between engine order (red dotted line) and natural frequency → $\boxed{f^{(i)}}$

Parameters to predict

- **Vibration amplitude** at critical speed i → $\boxed{a^{(i)}}$





Campbell diagram of the rotor – impact of bearing stiffness

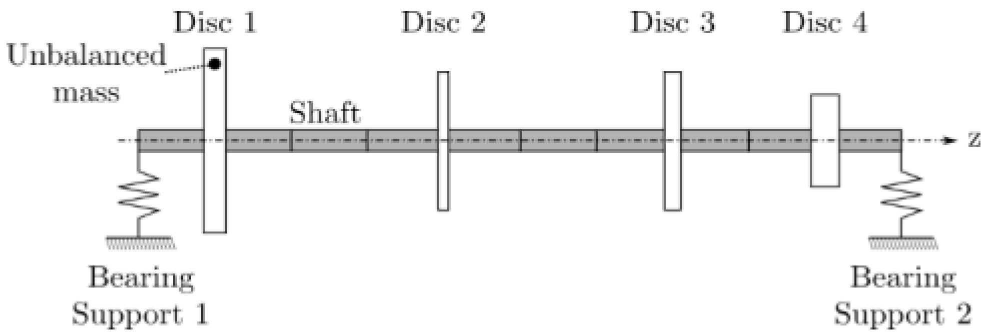
Uncertain parameters in the model

Uncertain parameters
- Manufacturing, environment, wear etc
- Modelled as random (**PDF**)
- Rotor response = stochastic
- Indicators = stochastic
- 7 uncertain parameters with different contributions

Design parameters
- 2 design parameters
- $k_{x,2}$ and $k_{y,2} \in [0.1; 2]10^6 N/m$.
- Scanning

Problem
Scanning + Monte Carlo = too expensive !

Hybrid formulation
- PCE for stochastic
- Kriging for scanning



Academic rotor under study

Parameter	Notation	% variation	Law	Contribution
Young modulus – shaft	E	$\pm 5\%$	Uniform	$\mathbf{K_s}$
Thickness - disc 1	t_1	$\pm 10\%$	Uniform	$\mathbf{G_{d1}}, \mathbf{M_{d1}}$
Thickness - disc 2	t_2	$\pm 10\%$	Uniform	$\mathbf{G_{d2}}, \mathbf{M_{d2}}$
Thickness – disc 3	t_3	$\pm 10\%$	Uniform	$\mathbf{G_{d3}}, \mathbf{M_{d3}}$
Thickness – disc 4	t_4	$\pm 10\%$	Uniform	$\mathbf{G_{d4}}, \mathbf{M_{d4}}$
Stiffness – right support – vert.	$k_{x,1}$	$\pm 10\%$	Uniform	$\mathbf{K_{b2}}$
Stiffness – right support – hor.	$k_{y,1}$	$\pm 10\%$	Uniform	$\mathbf{K_{b2}}$

Uncertain parameters

Polynomial Chaos – Mathematical background

Approximation:

$$\lambda(\xi) \approx \sum_{i=0}^{m} a_i \Psi_i(\xi)$$

With

ξ Random parameter

$f(\xi)$ Probability density function

$(\Psi_i)_{i\in 1,\dots,m}$ Basis of othogonal polynomials of degree i (m is the chaos order)

Inner Product

$$\langle \Psi_j, \Psi_k \rangle = \int_D \Psi_j(x)\Psi_k(x) f(x)dx$$

Given by the Askey scheme

$\hat{\mathbf{a}} = (a_i)_{i=0,\dots,m}$ Unknown deterministic coefficients

To be determined

Regression method:

- Determination of an experimental design of N samples
  $$\mathbf{\Xi} = \left\{\xi^{(1)},\dots,\xi^{(N)}\right\}$$
- Computation of their evaluations
  $$\mathbf{Y} = \left\{\lambda(\xi^{(1)}),\dots,\lambda(\xi^{(N)})\right\}$$
- Least squares minimization
  $$\hat{\mathbf{a}} = (\mathbf{W}^{\mathrm{T}}\mathbf{W})^{-1}\mathbf{W}^{\mathrm{T}}\mathbf{Y}$$

with $W_{ij} = \Psi_j(\xi^{(i)})$

Post processing of the coefficients

- Coefficients $(a_i), i \in [0, m]$ directly related to the stochastic properties of λ

$$E[\lambda] = a_0$$

$$\sigma_\lambda^2 = \sum_{k=1}^{m} a_k^2 ||\Psi_k||$$

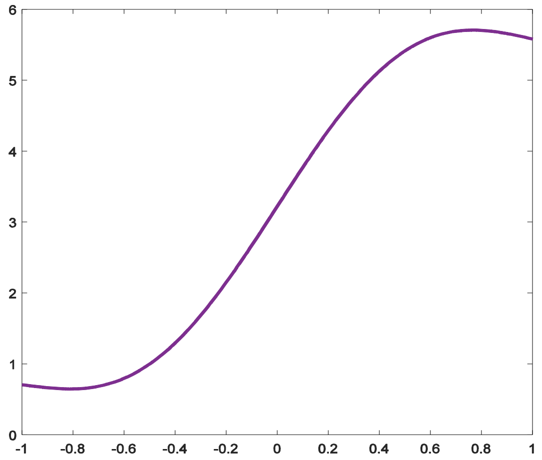
- Directly related to the Sobol indices

$$S_i = \frac{V_i}{\sigma_\lambda^2}$$

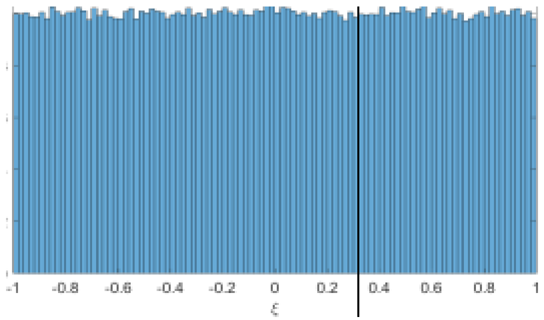
With $V_i = \sum_{j\in v_i} a_j^2 ||\Psi_k||$

Polynomial Chaos - Example

Function f



ξ a uniform random parameter



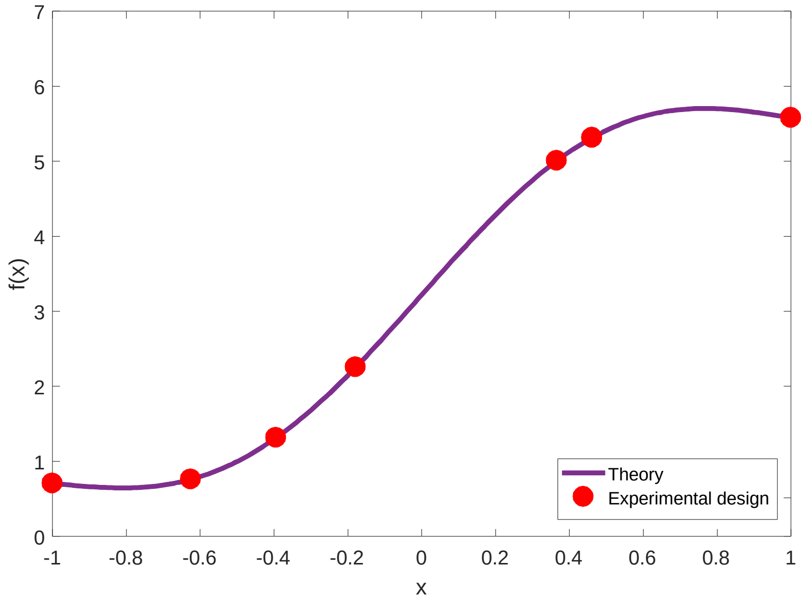
Experimental Design

7 samples

$$(x_1, \ldots, x_7)$$

and their evaluations

$$\mathbf{Y} = (f(x_1), \ldots, f(x_7))$$



Creation of the basis Chaos of order 2

Legendre polynomial

$$\Psi_0(\xi) = 1$$

$$\Psi_1(\xi) = \xi$$

$$\Psi_2(\xi) = (3\xi^2 - 1)/2$$

Computation of the coordinates in the bases

Computation of $\mathbf{W}$

$$\mathbf{W}_{ij} = \begin{pmatrix} \Psi_0(x_1) & \Psi_1(x_1) & \Psi_2(x_1) \\ \vdots & \ddots & \vdots \\ \Psi_0(x_7) & \Psi_1(x_7) & \Psi_2(x_7) \end{pmatrix}$$

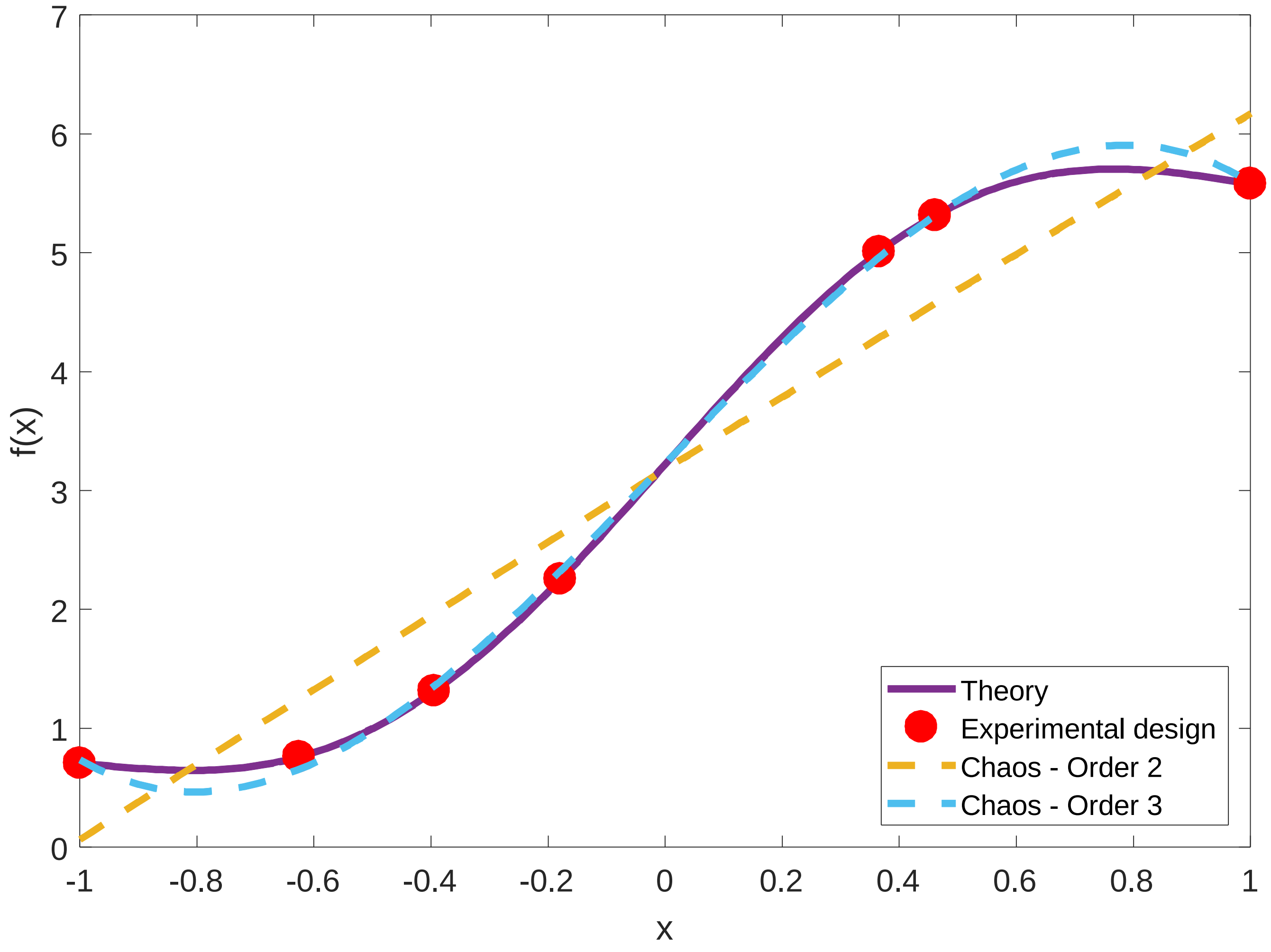
Computation of $\mathbf{a}$

$$\hat{\mathbf{a}} = (\mathbf{W}^T\mathbf{W})^{-1}\mathbf{W}^T\mathbf{Y}$$

$$\hat{\mathbf{a}} = (a_0, a_1, a_2)$$

Polynomial Chaos - Example

Reconstruction of the function with polynomial chaos



Kriging – Mathematical background

Approximation:

$$\lambda(\mathbf{p}) = \sum_{i=1}^{m} \beta_i f_i(\mathbf{p}) + Z(\mathbf{p})$$

with

$\mathbf{p} \in \mathbf{R}^k$ parameters vector

N samples $\mathbf{S} = \left(\mathbf{s}^{(1)}, \dots, \mathbf{s}^{(N)}\right)$

and their evaluations

$\mathbf{Y}_S = \left(\lambda(\mathbf{s}^{(1)}), \dots, \lambda(\mathbf{s}^{(N)})\right)$

$(f_i)_{i=1,\dots,m}$ Regression functions

m polynomial basis

$(\beta_i)_{i=1,\dots,m}$ Regression coefficients

solution of a least square problem

Global behaviour

$Z(.)$ Zero-mean Gaussian process

Covariance $\mathrm{E}\left[Z(\mathbf{s}), Z(\mathbf{x})\right] = \sigma^2 \mathrm{R}\,(\boldsymbol{\theta}, \mathbf{s}, \mathbf{x})$

Process variance σ^2

Correlation function $\mathrm{R}\,(\boldsymbol{\theta}, \mathbf{s}, \mathbf{x}) \in [0,1]$

Depends on the distance between $\boldsymbol{s}$ and $\boldsymbol{x}$

Zero distance → Correlation = 1

Infinite distance → Correlation = 0

$\boldsymbol{\theta}$ characterizes the distance of influence

Solution of a maximum likelihood problem

Local behaviour

Characteristics to choose

- **Regression function**
  - Polynomials
- **Correlation function**
  - Exponential, Gaussian, linear, spherical, matern…
  - Depends of the regularity of the phenomenon
- **Experimental Design**
  - Input/output set
  - Learning points
  - Smallest as possible

Hybrid formulation

Formulation

Main idea: express the PCE coefficients as a function of the design parameters

$$\lambda(\xi,p) = \sum_{k=0}^{m} a_k(p)\Psi(\xi) = \sum_{k=0}^{m} \mathcal{M}^{(k)}(p)\Psi(\xi)$$

Construction

Function $\lambda(\xi,p)$

Random parameter ξ
DoE $\boldsymbol{\Xi} = (\xi_1, \dots, \xi_Q)$

Interval parameter p
DoE $\mathbf{P} = (\mathrm{p}_1, \dots, \mathrm{p}_\mathrm{N})$

For each p_k: construction of a PCE → N values of each chaos coefficient $a_j \to \left(a_j^{(1)}, \dots, a_j^{(N)}\right)$

For each a_j: construction of a kriging surrogate model $\mathcal{M}^{(j)}$ → $a_j(p) = \mathcal{M}^{(j)}(p)$

Hybrid Model:

$$\lambda(\xi,p) = \sum_{j=0}^{P-1} \mathcal{M}^{(j)}(p)\Psi(\xi)$$

Advanced DoE for the kriging

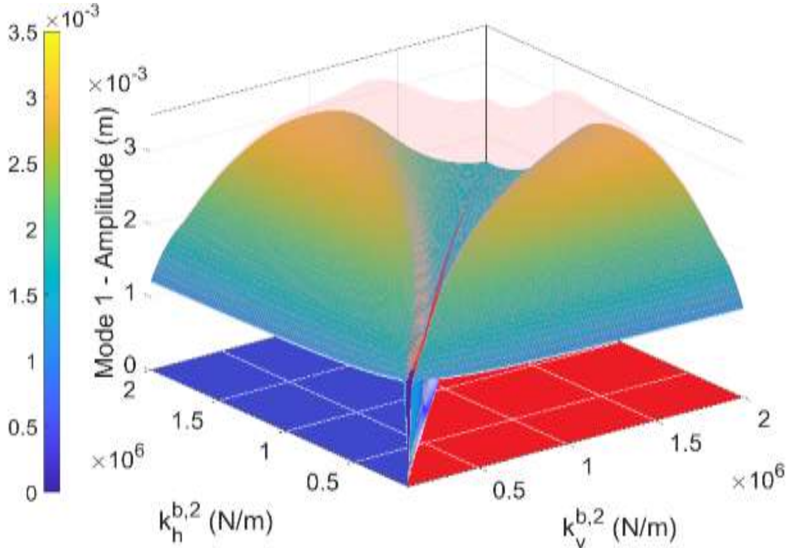


Fig.: Vib. amplitude at first critical speed versus the two design parameters – Average values (coloured surface) – Average ± standard deviation (red and blue surfaces)

- **Problem knowledge**: symmetry w.r.t. $k_v^{(b,2)} = k_h^{(b,2)}$ (design variables)

- **3 strategies to integrate this knowledge**
  - *Classic strategy*: regression g, correlation $\mathcal{R}$, no sampling strategy
  - *Half-design space*: kriging construction only on $k_v^{(b,2)} \leq k_h^{(b,2)}$, reconstruction by symmetry

    Symmetry in the DoE
  - *Symmetric regression*

$$\mathbf{g_s}(x_1, x_2) = \begin{cases} \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2 & \text{if } x_1 \leq x_2 \\ \mathbf{g_s}(x_2, x_1) & \text{otherwise} \end{cases}$$

    Symmetry in the Kriging regression

- **Evaluation of the performances**
  - Different DoE size pour kriging: [20,40,60,80]
  - 4000 validation points

DoE strategies comparison

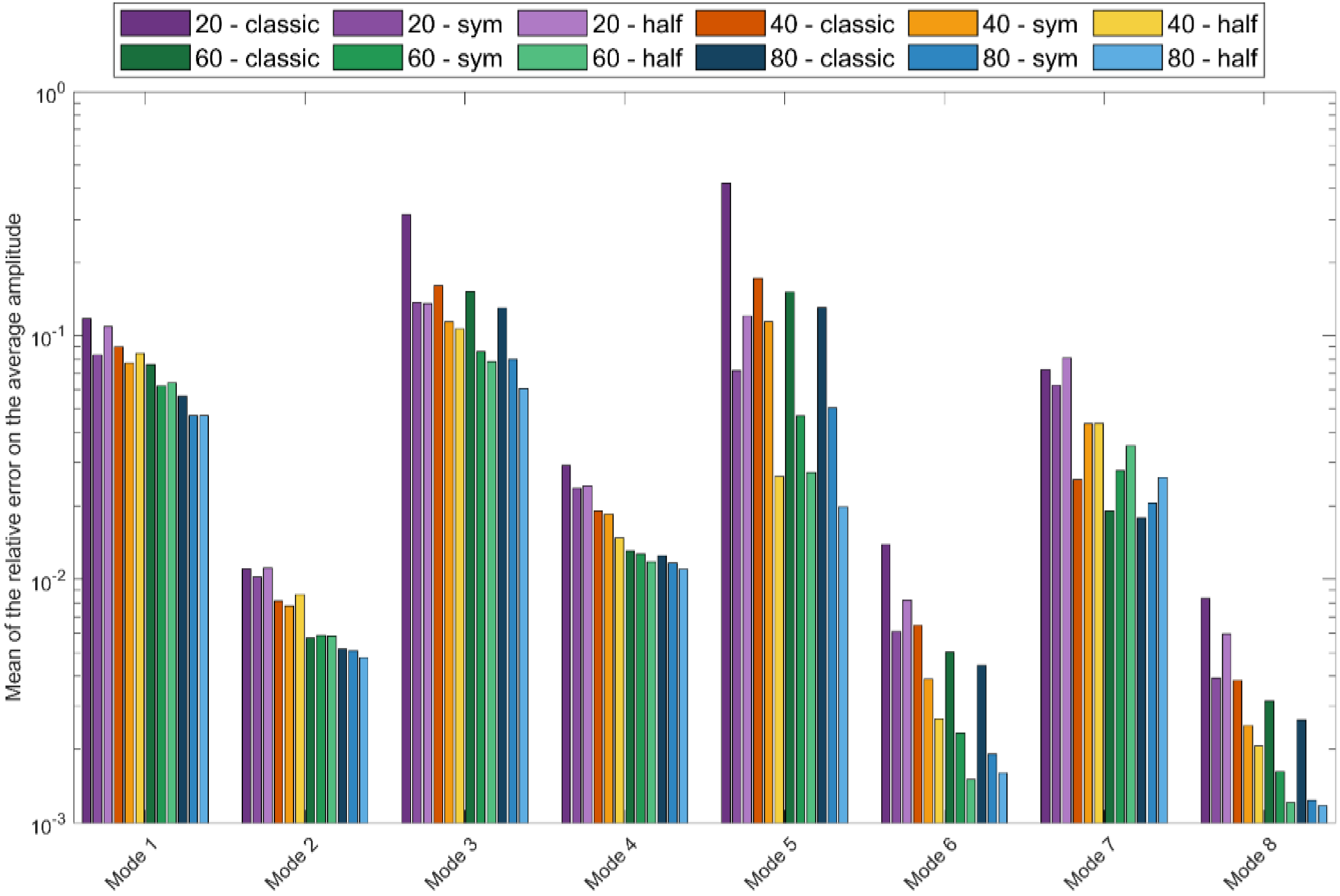


Fig.: Mean on the relative error on the average amplitude

- Error lower for forward modes (even number) than backward modes (odd number)
- When DoE size increases, error decreases
- Classic kriging: worst strategy
- Half: best strategy

Integrating the symmetry property in the DoE is the best strategy here

Evolution of the critical speeds and vibration amplitudes

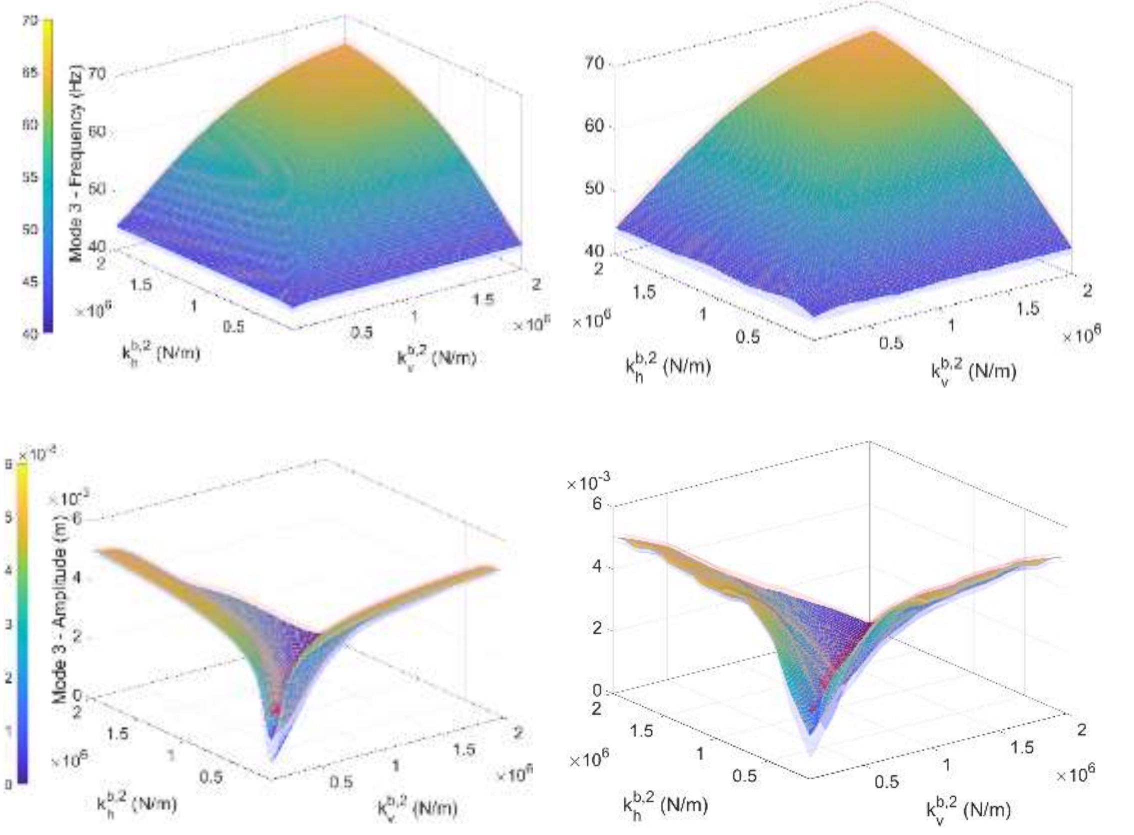
MCS + Scanning	VS	Hybrid
10^7	VS	7500

Freq. [Hz]

Ampl. [m]

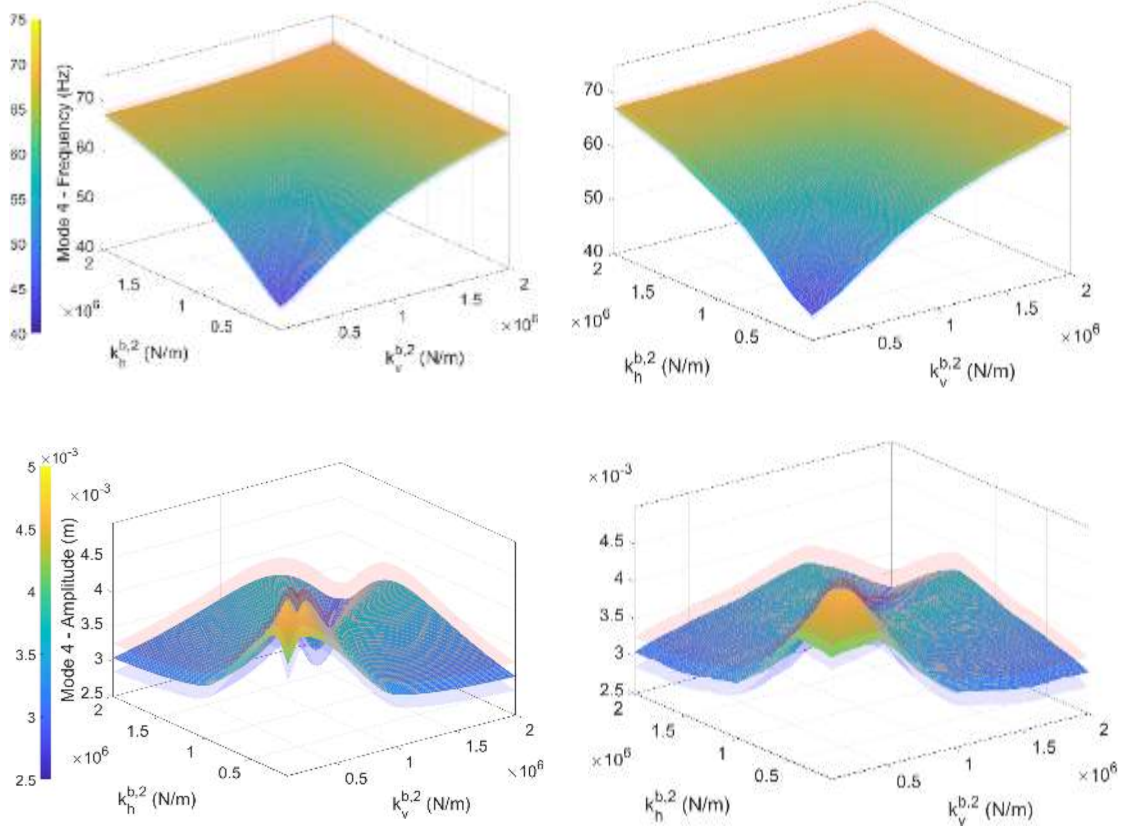
Mode 3

Reference | Prediction



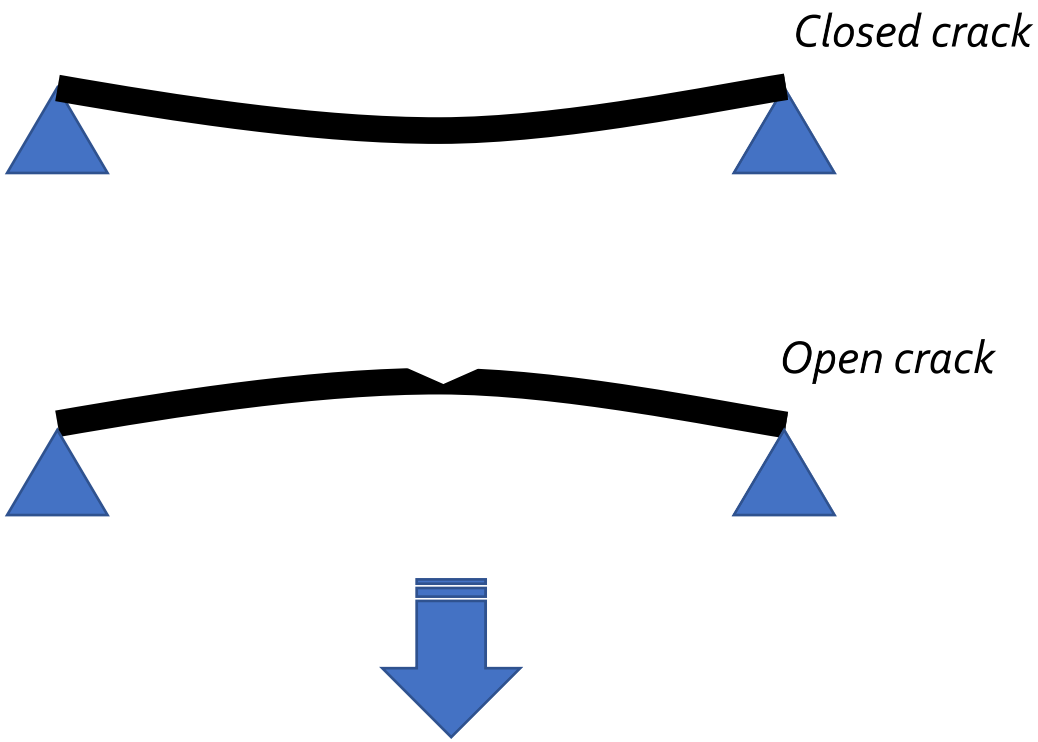
Mode 4

Reference | Prediction



Application for the SHM of cracked rotor

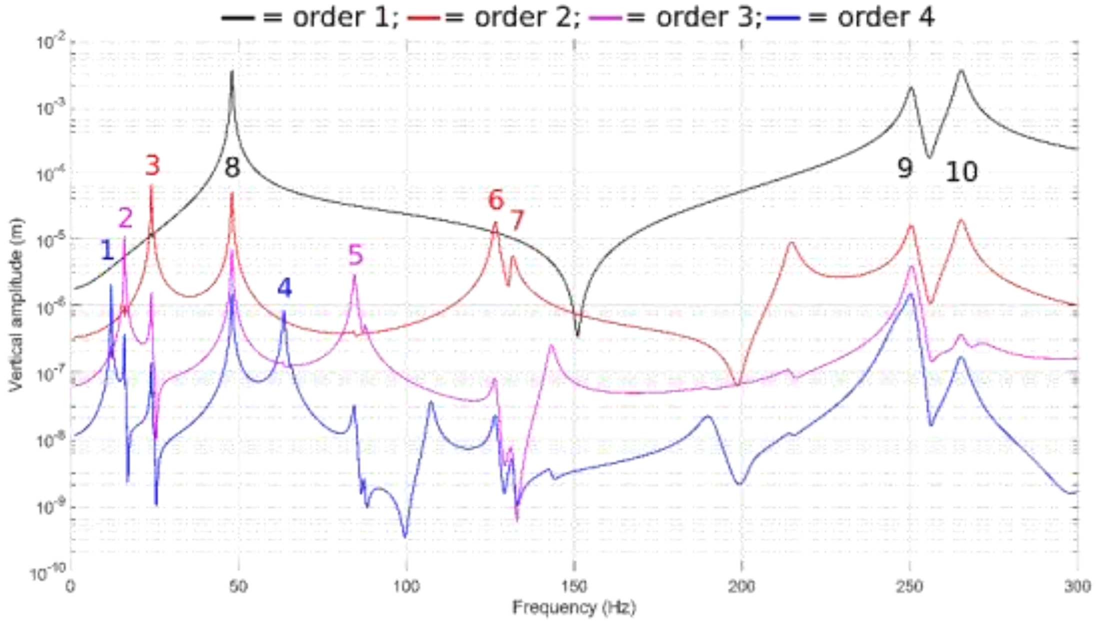
Deformation of the shaft during rotation



Breathing mechanism of the crack

Nonlinear dynamic response

Nonlinear FRF

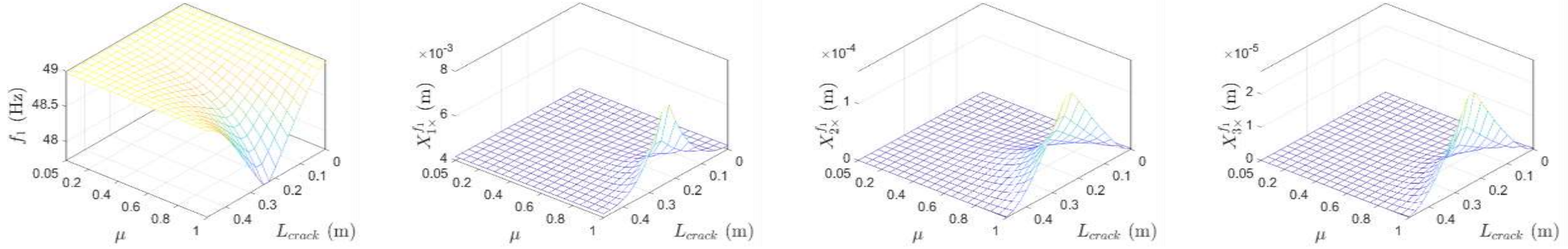


Objectif

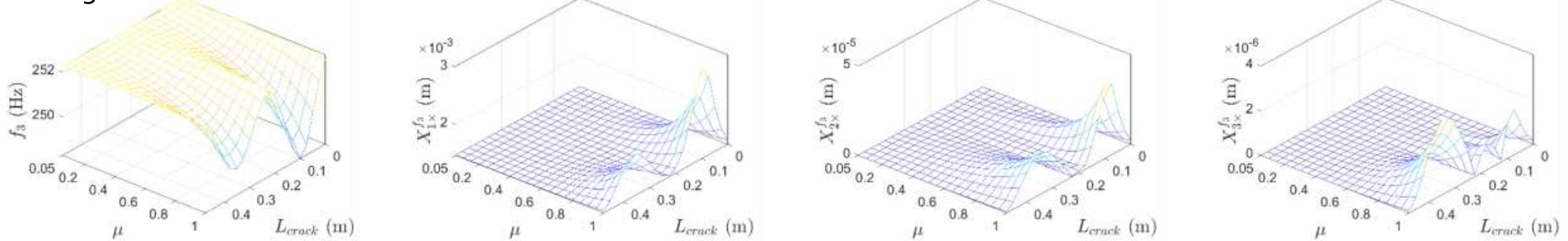
Prediction of sub-critical and critical speeds and corresponding vibration amplitudes

Impact of the crack on the deterministic response

Mode 1

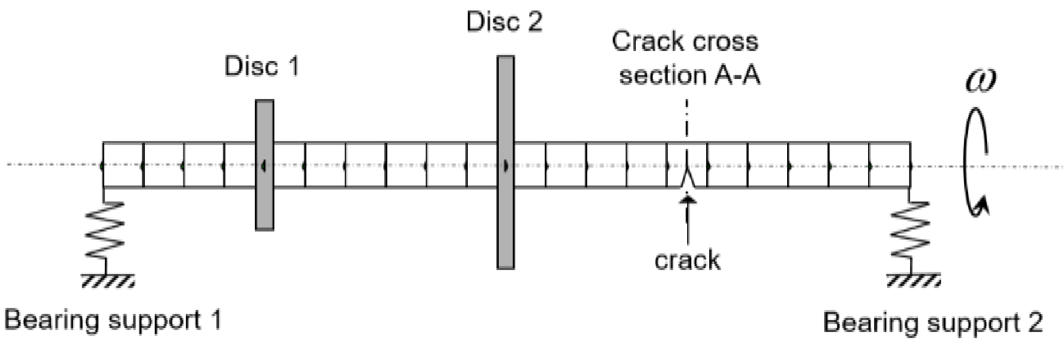


Mode 3



But what happens when the system is uncertain ?

Uncertain parameters in the model



Parameter	Notation	% variation	Law	Contribution
Young modulus – shaft	E	$\pm 5\%$	Uniform	$\mathbf{K_s}$
Density – shaft	ρ	$\pm 5\%$	Uniform	$\mathbf{M_s}$ and $\mathbf{q}$
Thickness - disc 1	t_1	$\pm 10\%$	Uniform	$\mathbf{G_{d1}}, \mathbf{M_{d1}}$ and $\mathbf{q}$
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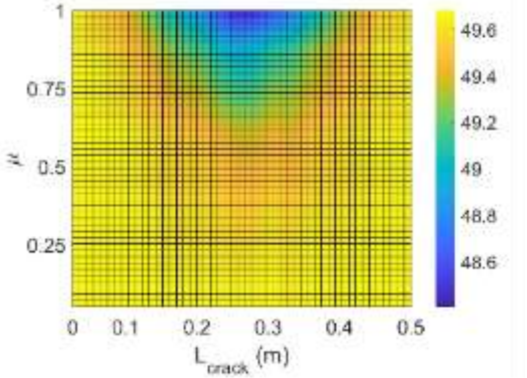
Stiffness – right support – vert.	$k_{x,2}$	$\pm 10\%$	Uniform	$\mathbf{K_{b2}}$
Stiffness – right support – hor.	$k_{y,2}$	$\pm 10\%$	Uniform	$\mathbf{K_{b2}}$

f

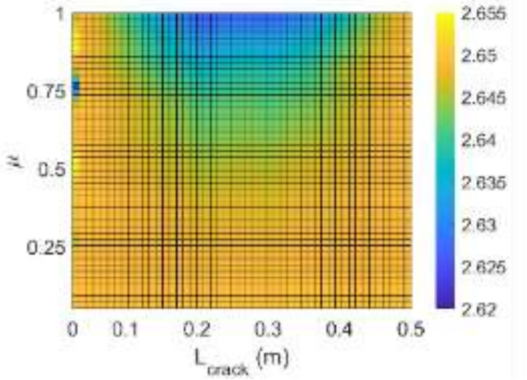
PCE + kriging

Prediction of the resonance frequencies and amplitudes

$E[f_1]$



Var$[f_1]$



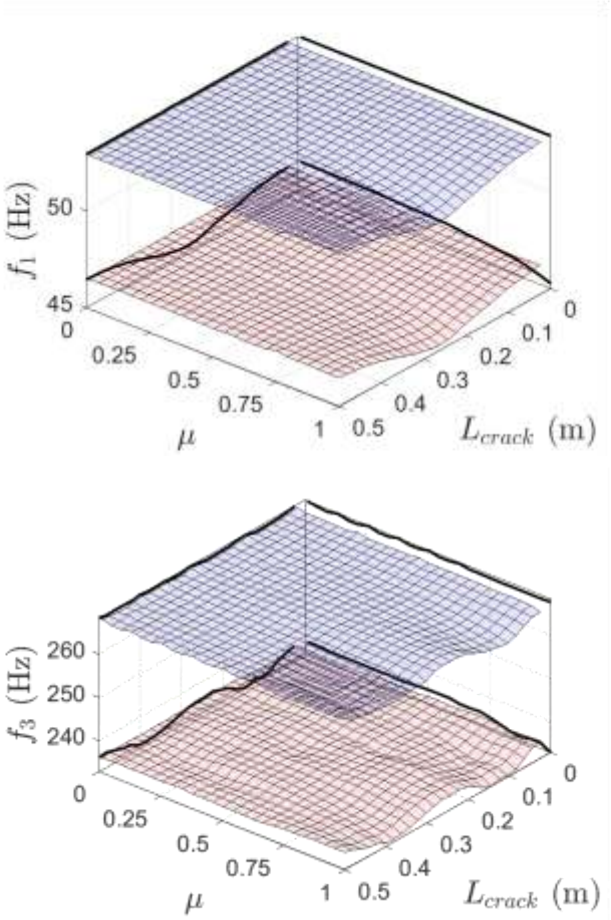
Analysis of the 95% confidence interval

- Determination of the 95% confidence interval over the map:
  - $CI_a(\mu, L_c)$: Confidence interval of the parameter a.

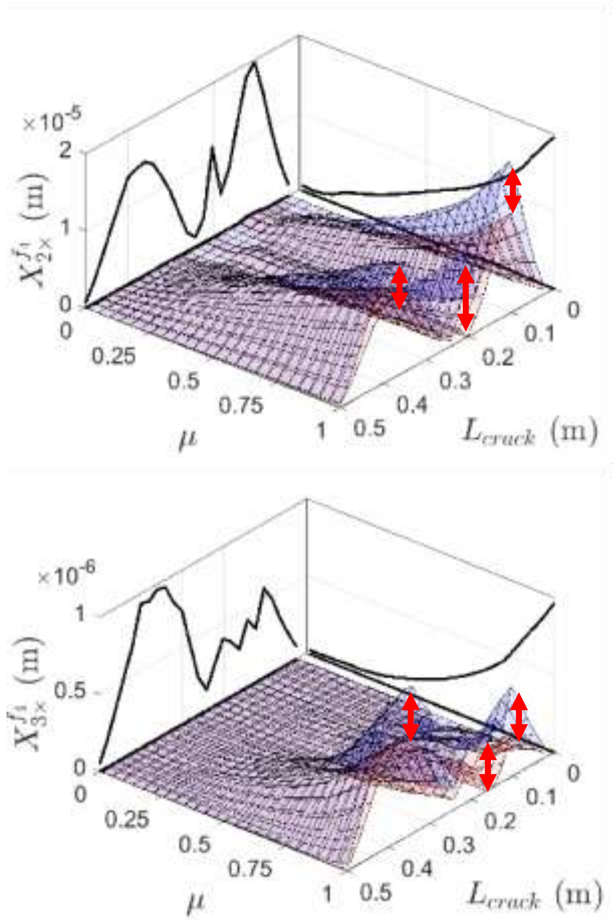
- Three categories:
  - Category 1: the variation of the minimum and maximum bounds are too small compared to the width of the CI→ not an indicator
  - Category 2: the variation of the minimum and the maximum bounds are predominant compared to the width of the CI, but for small variations of the crack parameters the bounds remain in the interval due to uncertainties → partial indicators
  - Category 3: impact of uncertainties is negligible → robust indicators

Illustration of the indicators

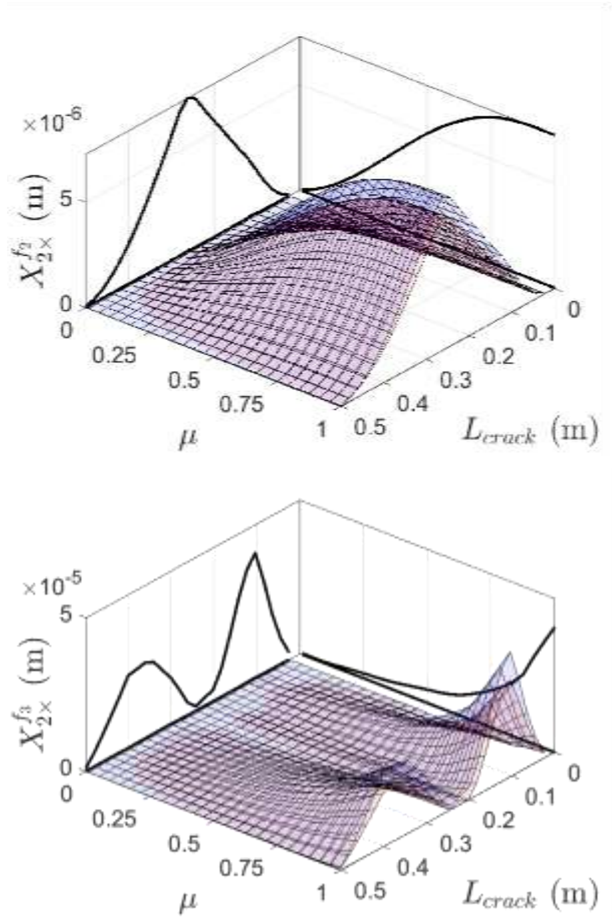
Caterogy 1



Caterogy 2



Caterogy 3



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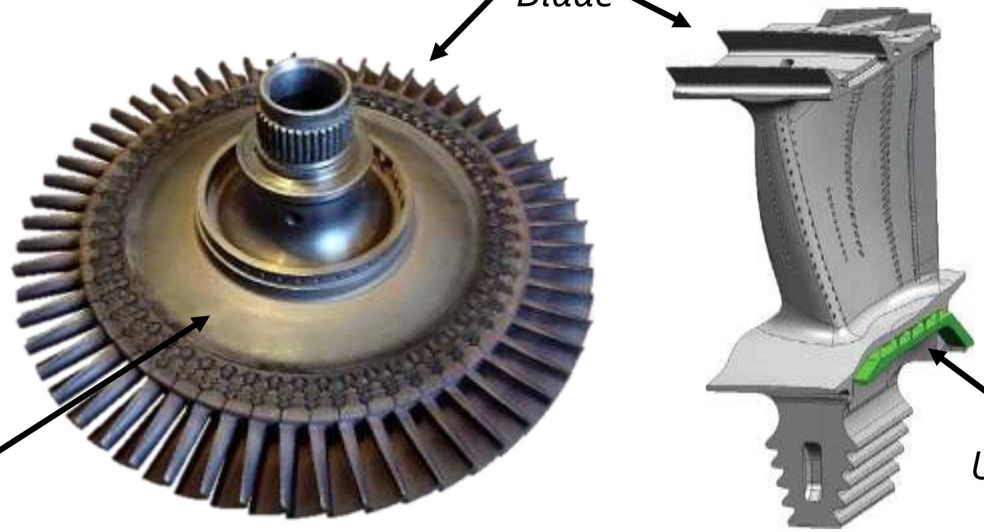
Bladed discs and friction dampers

Technology 1: Bladed disc



Technology 2: Integrally bladed disc (blisk)

Control of vibrations

- Introduction of damping
- Dampers are small metal pieces
- Use of friction to dissipate vibration energy

Dynamic equation of the system

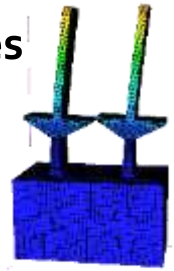
$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} + \boxed{\mathbf{F}_{\mathbf{nl}}(\mathbf{U})} = \mathbf{F}(t)$$

Non-linear equation solved with specific solvers
→ Numerically expensive !

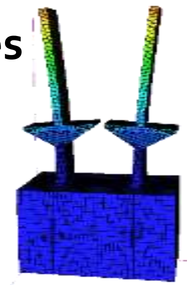
Full nonlinear dynamic response highly sensitive to the contact interface

Impact of damper geometry

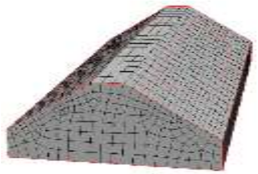
In-phase mode of the blades

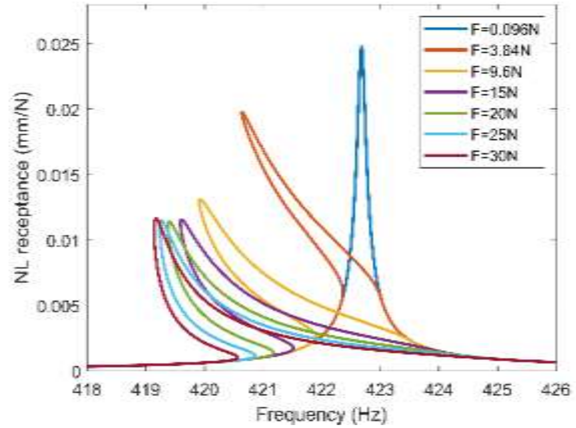


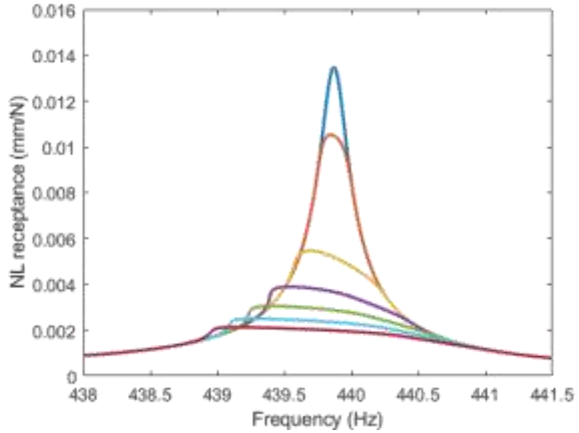
Out-of-phase mode of the blades



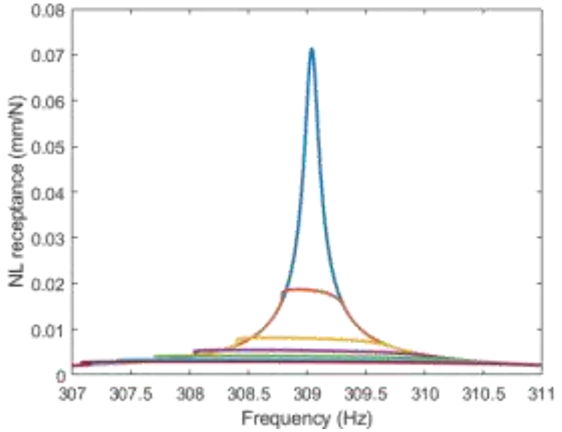
Wedge damper

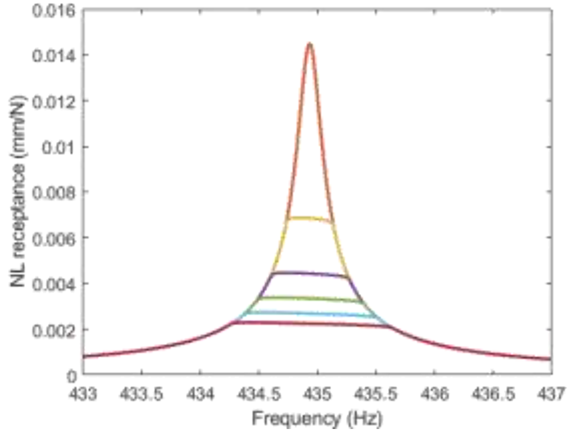






Conical damper





Damper shape highly influent: optimisation of the topology

Topology optimisation – general presentation

- Objective: over a design domain Ω, find the best material domain Ω_m to minimise the objective function $f_{obj}(\Omega_m)$ under the constraints $g(\Omega_m) \leq 0$

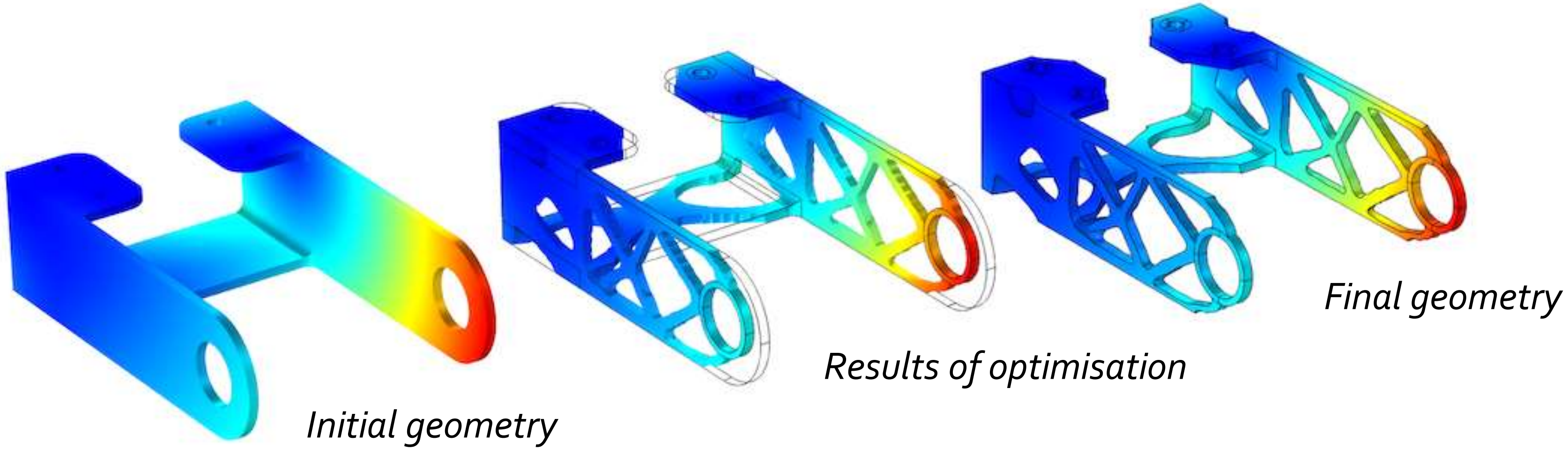


Fig.: Illustration of the topological optimisation process (Comsol website)

- Interests
  - Lighter and more efficient structures
  - Identify new geometries
  - 3D printing: open the possibilities

Topological optimisation – general presentation

- Objective: over a design domain Ω, find the best material domain Ω_m to minimise the objective function $f_{obj}(\Omega_m)$ under the constraints $g(\Omega_m) \leq 0$

- Classical approaches: density-based and Level-Set Method
  - Compliance, deflection, natural frequencies, stress, mass etc
  - <u>But:</u> require the sensitivities of the objective function w.r.t to the densities or shape function

    → Not adapted for nonlinear vibrations

- Global optimisation
  - No need for gradient
  - Well adapted to non-convex optimisation problems
  - <u>But:</u> require a parametrisation of the problem

Presentation of the mechanical system

- **2D-geometry**
  - Reduction computational time
  - Reduction of the number of optimization parameters
  - Problem can be approximated as 2D

- **Friction contact elements**
  - 3 status: stick, slip, separated
  - Uniform contact pressure

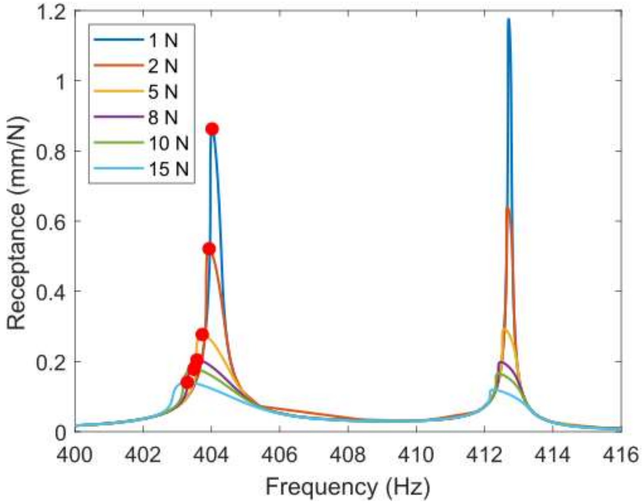


Fig.: Receptances FRF for different excitation amplitudes

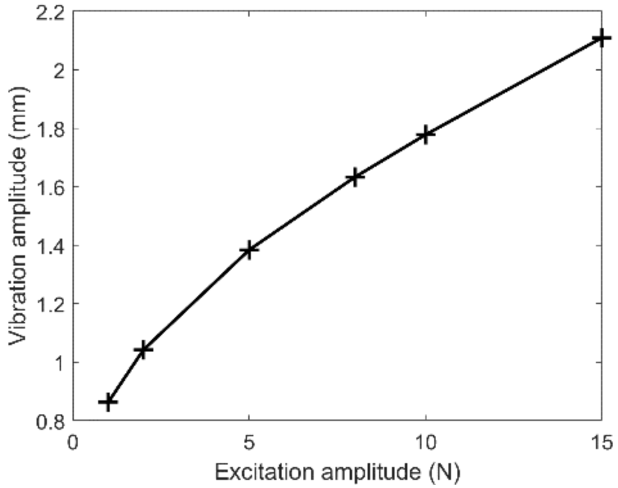


Fig.: Evolution of the amplitude at resonance for different excitation amplitudes

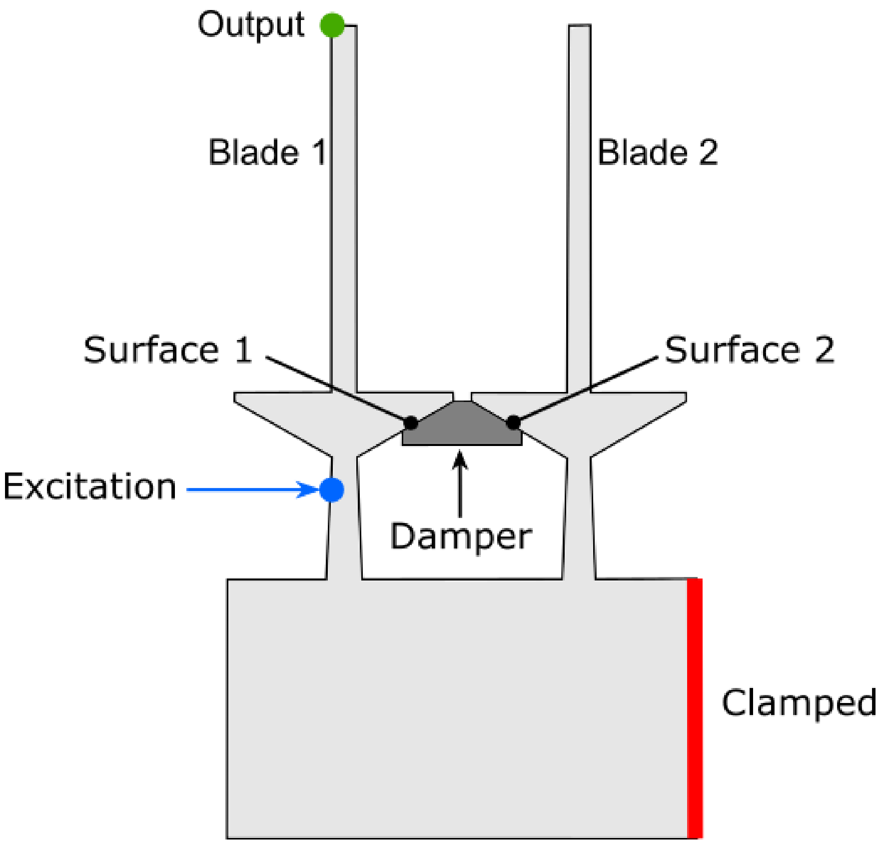
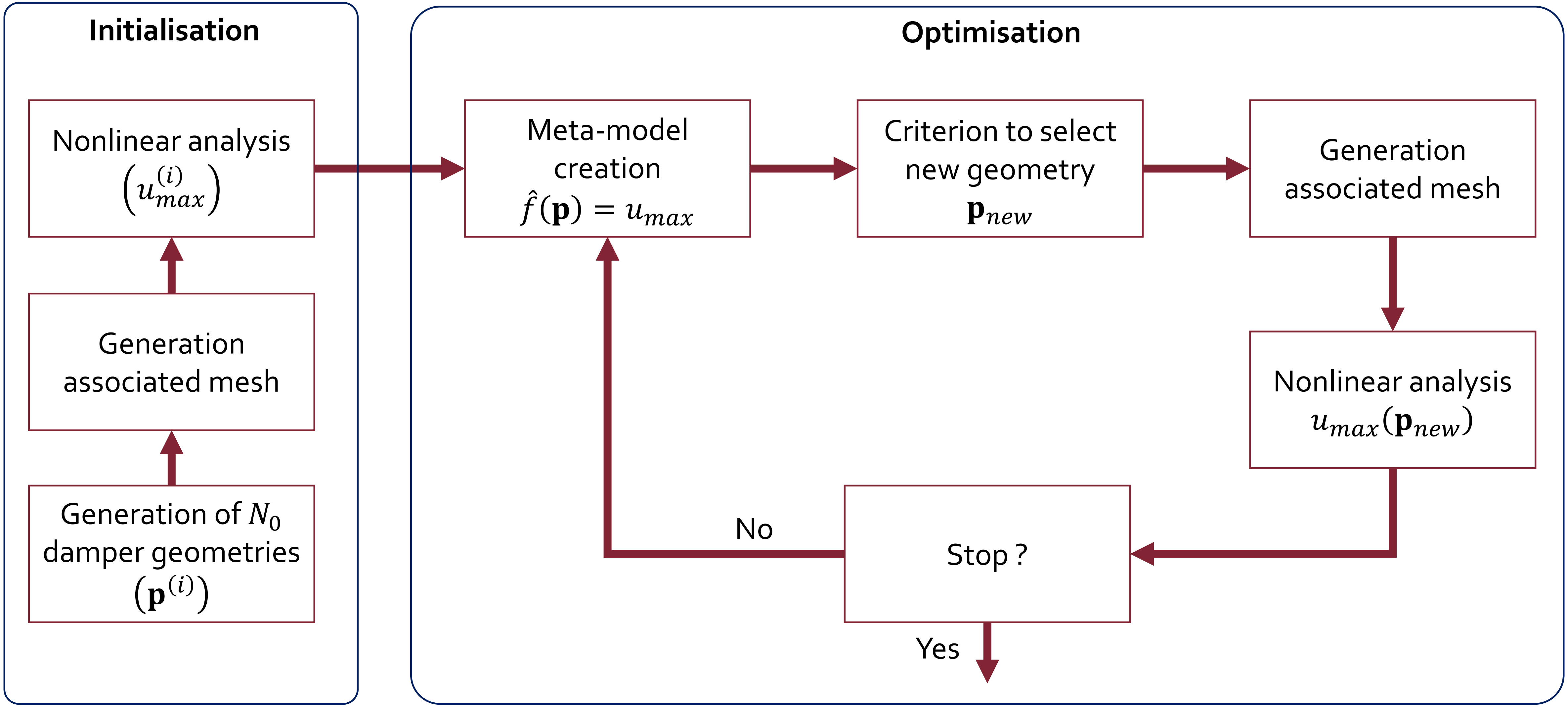
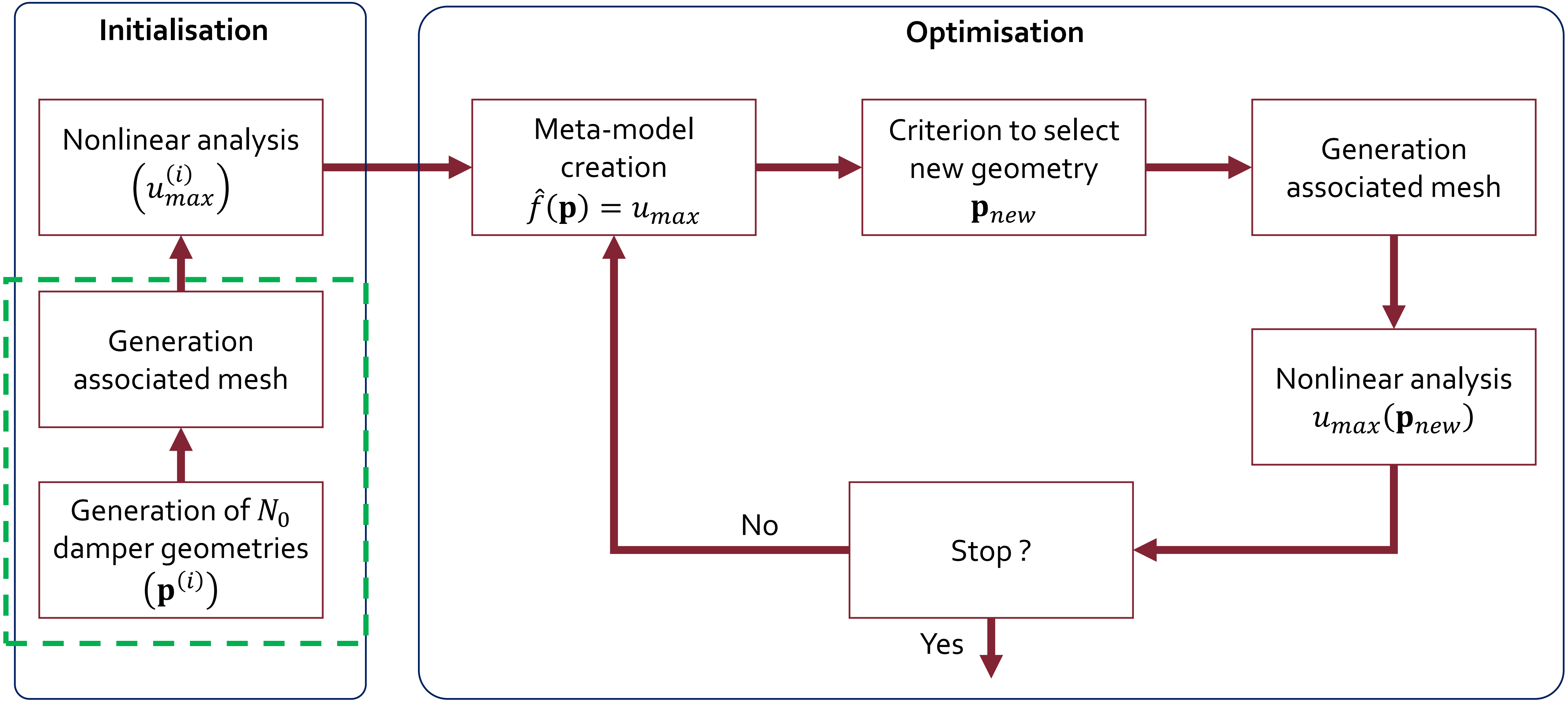


Fig.: Studied mechanical system

Surrogate modelling – optimisation process



Surrogate modelling – optimisation process



Damper parametrisation

Moving Morphable Components (MMC) framework :

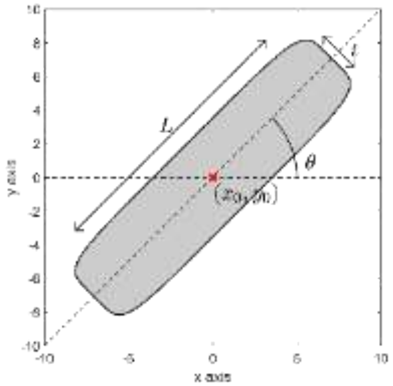
- Structure = assembly of elementary components
  - 1 component = 5 parameters
  - location (x_0, y_0), inclination θ, length L and thickness t
- Moving, shrinking, expanding components => complex topologies
- Translated into a Level-Set Function (LSF) $\Psi(\boldsymbol{u})$:

$$\begin{cases} \Psi(\boldsymbol{u}) > 0, \boldsymbol{u} \text{ is in the material domain} \\ \Psi(\boldsymbol{u}) < 0, \boldsymbol{u} \text{ is in the void domain} \\ \Psi(\boldsymbol{u}) = 0, \boldsymbol{u} \text{ is at the boundary} \end{cases}$$

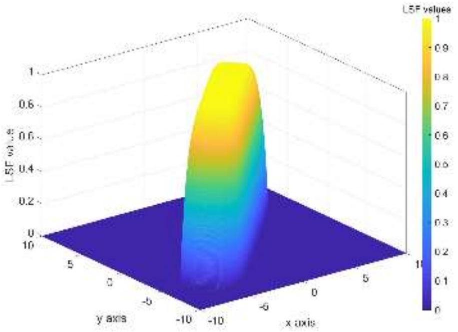
- 1 component = 1 unitary LSF Ψ_i
- Combination of the elementary LSF => global LSF

$$\Psi(\boldsymbol{u}) = \max_i \Psi_i(\boldsymbol{u})$$

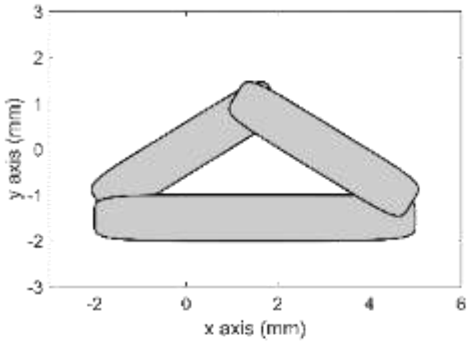
Illustration:



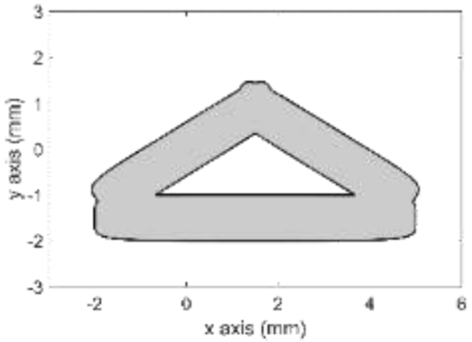
(a) Unitary component



(b) LSF of a unitary component



(c) Assembly of different components



(c) Final geometry

Fig.: Illustration of the MMC framework
Grey = material domain

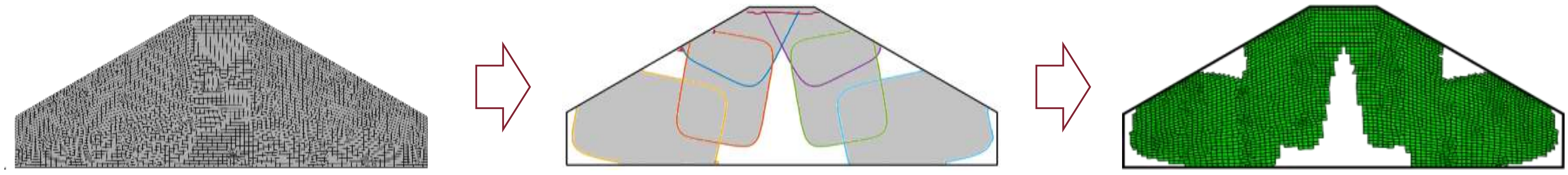
Damper parametrisation – mesh creation

Mesh mapping

- Initial damper mesh
- Discretisation of the LSF in the center of each element
- Void elements removed

Geometrical constraints

- 1 component set to the top: seal the platforms
- Damper symmetric
- 1 center fixed to the contact line



Initial mesh

LSF computation
(color line = component limits, grey =material domain)

Selection of elements to keep

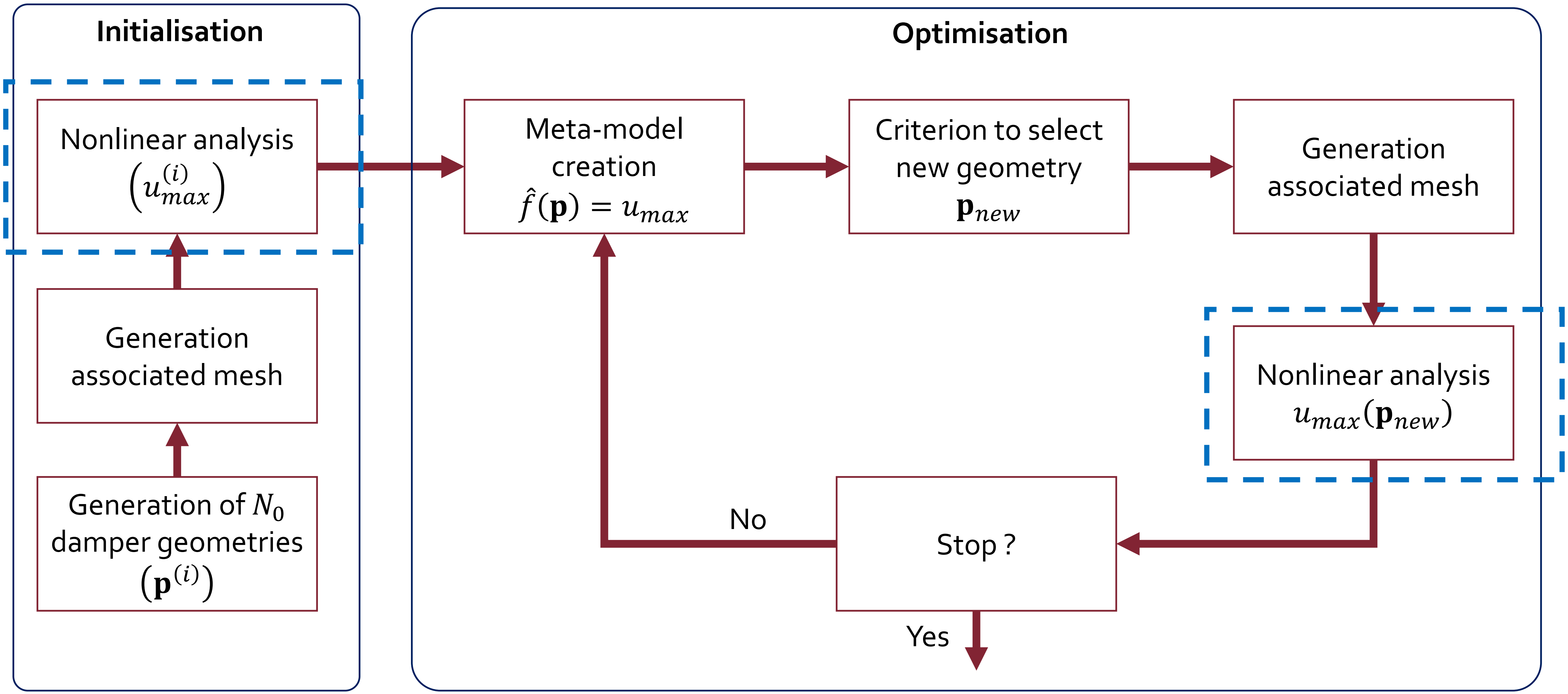
Fig.: General process for damper mesh creation

Computation of the new damper characteristics

- Structural matrices $\mathbf{M}_{\mathrm{damper}}, \mathbf{C}_{\mathrm{damper}}, \mathbf{K}_{\mathrm{damper}}$ from Craig-Bampton reduction
- Contact parameters updating (normal pressure, contact stiffness)

Intermediary conclusion – Damper geometry parametrized: depends on a few parameters $\mathbf{p}$

Surrogate modelling – optimisation process



Computation of the non-linear response

Equation of motion

$$\mathbf{M\ddot{X}} + \mathbf{C\dot{X}} + \mathbf{KX} + \mathbf{F_{nl}}(\mathbf{X}, \mathbf{\dot{X}}) = \mathbf{F_e}$$

$\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the global mass, damping and stiffness matrices

Harmonic Balance Formulation

$$\mathbf{J_1}(\mathbf{Q}, \omega) = \mathbf{Z}(\omega)\mathbf{Q} + \tilde{\mathbf{F}}_{\mathrm{nl}}(\mathbf{Q}) - \tilde{\mathbf{F}}_{\mathrm{ex}} = \mathbf{0}$$

$\mathbf{Q}$ vector of sine and cosine coefficients

Phase quadrature

$$\mathrm{J}_2(\mathbf{Q}, \omega) = \phi - \frac{\pi}{2} = 0$$

ϕ phase between excitation and response

→ directly on the resonance peak

Finding Q and ω so that $[\mathbf{J_1}(\mathbf{Q}, \omega); \mathrm{J}_2(\mathbf{Q}, \omega)] = \mathbf{0}$

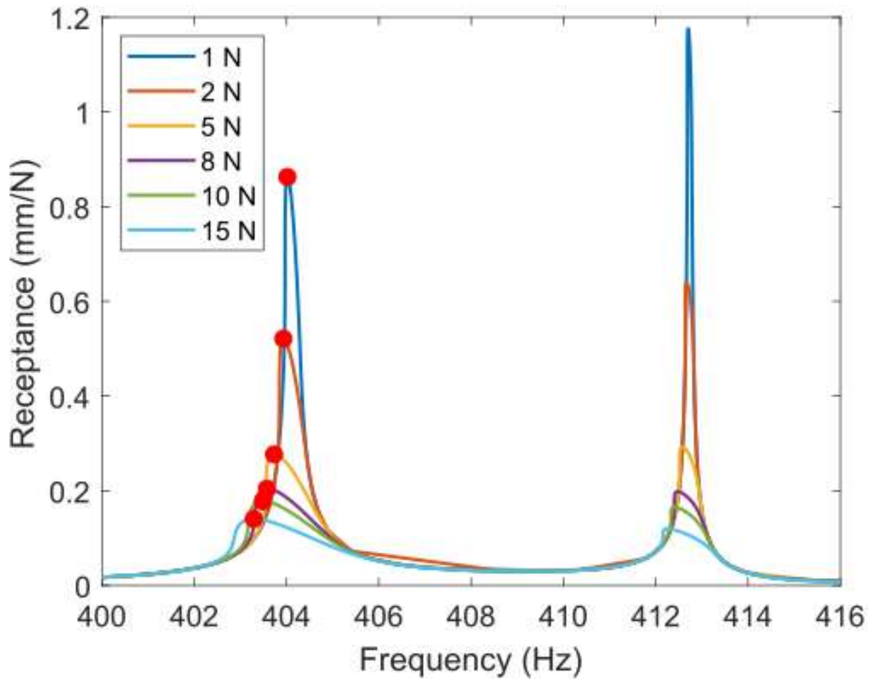
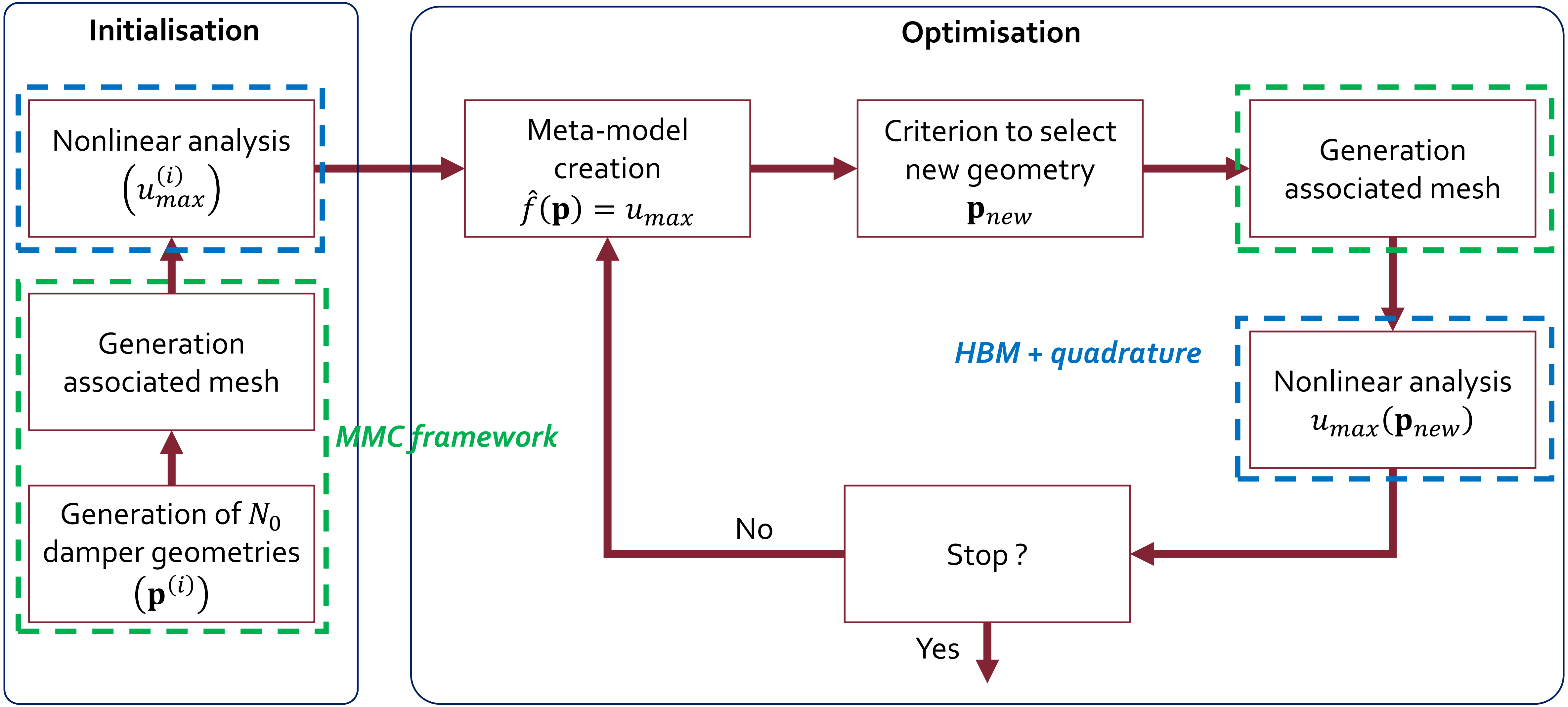
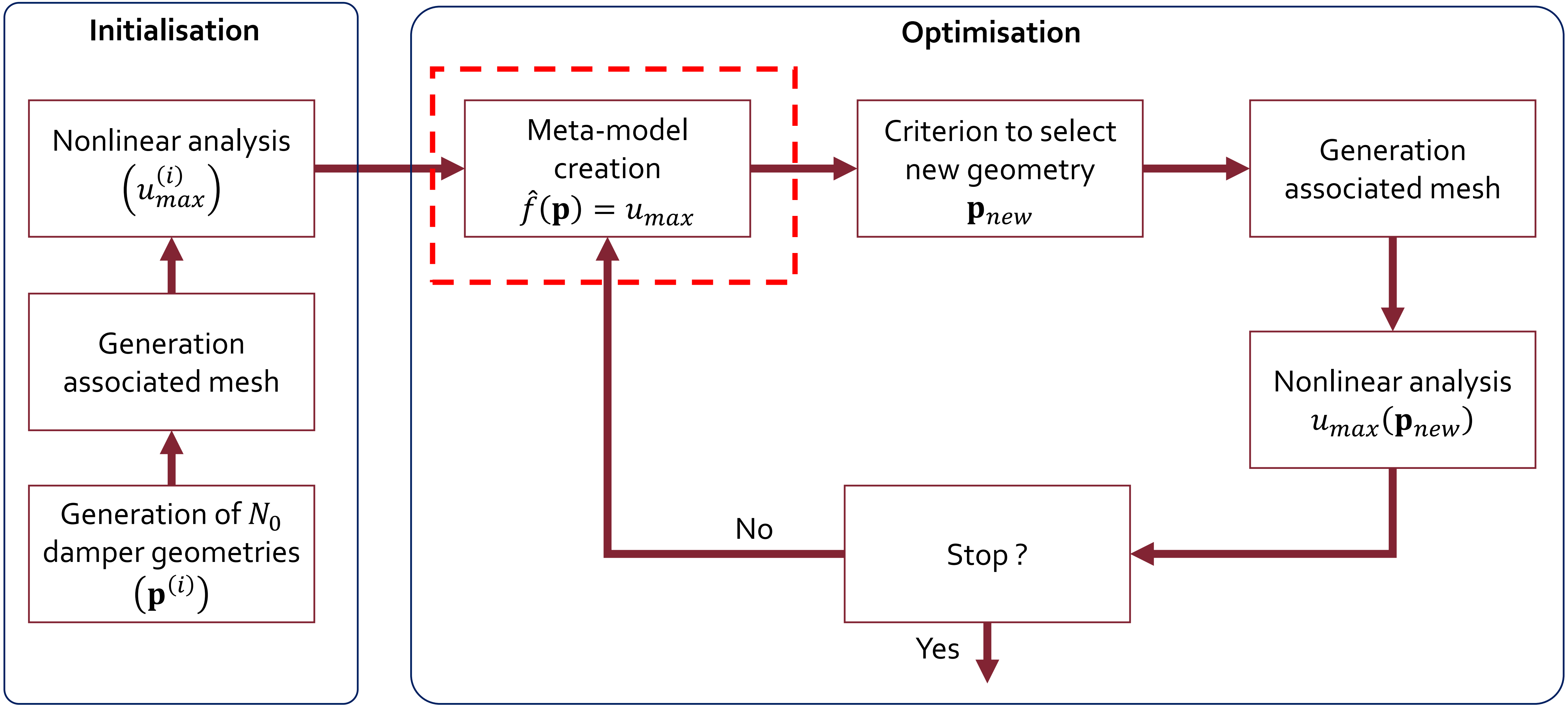


Fig.: Full FRF versus Phase quadrature criterion
Vibration at tip blade
(●): phase quadradure solution

Surrogate modelling – optimisation process



Surrogate modelling – optimisation process



Kriging – general formulation

Approximation :

$$f(\mathbf{p}) \sim \hat{f}(\mathbf{p}) = \boxed{\sum_{i=1}^{m} \beta_i g_i(\mathbf{p})} + \boxed{Z(\mathbf{p})}$$

Regressive part
Global behaviour

Gaussian process
Local behaviour

with

$\mathbf{p} \in \mathbf{R}^k$ Parameters vector (input)

N samples $\mathbf{P} = \left(\mathbf{p}^{(1)}, \dots \mathbf{p}^{(N)}\right)$
and their evaluations $\mathbf{F} = \left(f\left(\mathbf{p}^{(1)}\right), \dots, f\left(\mathbf{p}^{(N)}\right)\right)$

Called Design of Experiments (DoE)

$(g_i)_{i=1,\dots,m}$ Regression functions

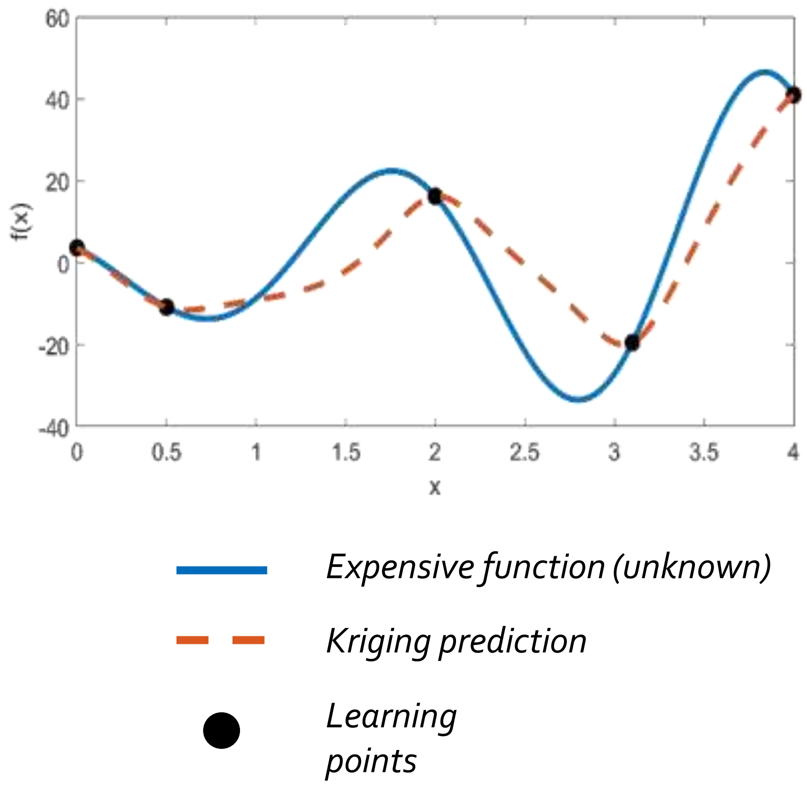
$(\beta_i)_{i=1,\dots,m}$ Regression coefficients

$Z(.)$ Zero-mean Gaussian process

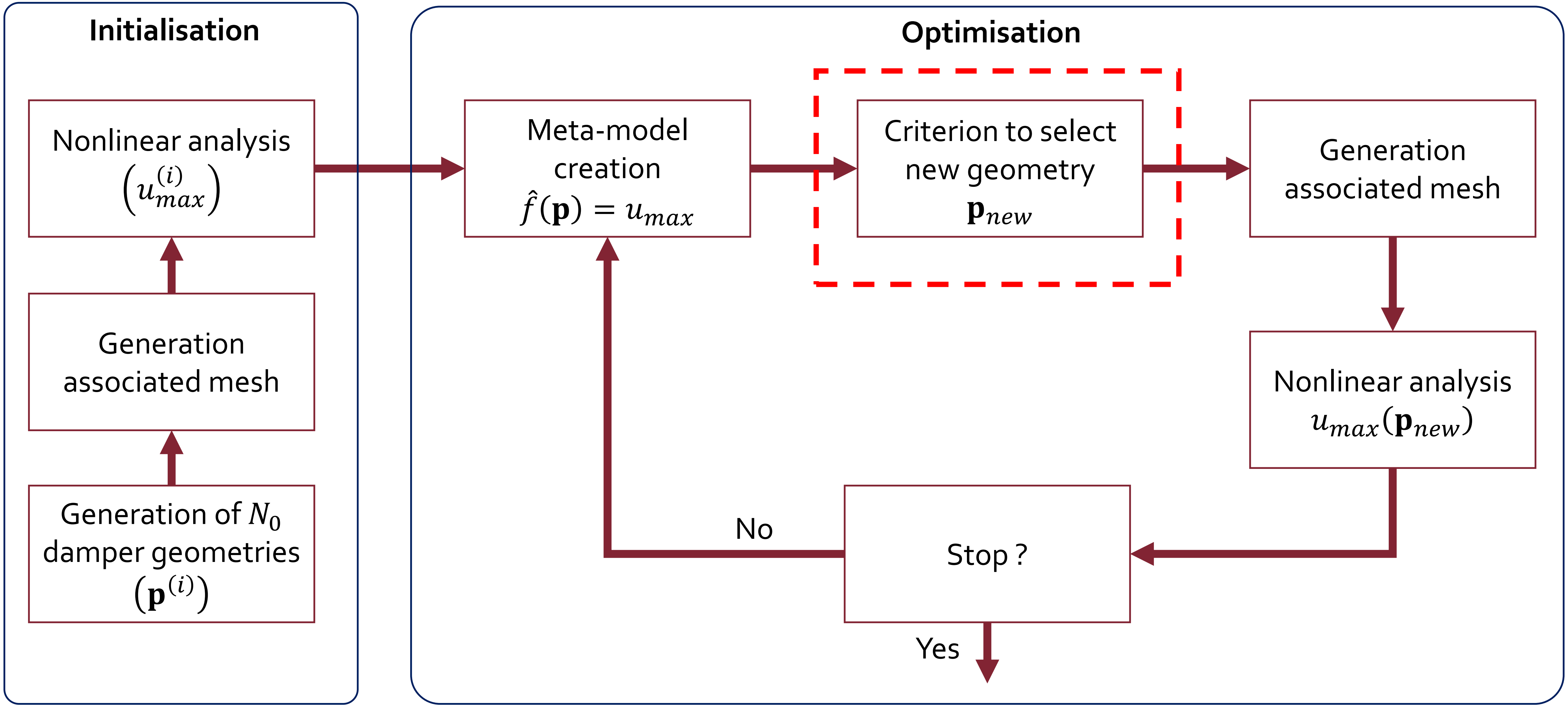
Covariance $\mathbb{E}[Z(\mathbf{s}), Z(\mathbf{x})] = \sigma^2 \mathcal{R}(\boldsymbol{\theta}, \mathbf{s}, \mathbf{x})$

Correlation function $\mathcal{R}(\boldsymbol{\theta}, \mathbf{s}, \mathbf{x})$

Illustration – 1d case :



Surrogate modelling – optimisation process



Expected improvement criterion

Modified Expected Improvement criterion:

$$EI(\mathbf{p}) = \begin{cases} \left(f_{min} - \hat{f}(\mathbf{p})\right)\Phi\left(\frac{f_{min} - \hat{f}(\mathbf{p})}{\hat{s}(\mathbf{p})}\right) + \hat{s}(\mathbf{p})\phi\left(\frac{f_{min} - \hat{f}(\mathbf{p})}{\hat{s}(\mathbf{p})}\right) \\ 0 \\ Penalty < 0 \quad \text{if infeasible (disconnected)} \end{cases}$$

New point

$$\mathbf{p}_{new} = \underset{\mathbf{p} \in \mathrm{R}^p}{\operatorname{argmax}}\, \mathrm{EI}(\mathbf{p})$$

Resolution with genetic algorithm

where

- $\hat{f}(\mathbf{p})$ kriging prediction at $\mathbf{p}$
- $\hat{\boldsymbol{s}}(\mathbf{p})$ standard error of the kriging at $\mathbf{p}$ (known)
- f_{min} current minimum observed so far
- Φ cumulative distribution function of the normal law
- ϕ probability density function of the normal law

Good balance between global and local search:

- Exploration of the space
  - area where no information is available
  - far from learning points
- Improvement of the minimum found
  - area where the probability to find the minimum of the objective function is high

Illustration of the process on a 1d case

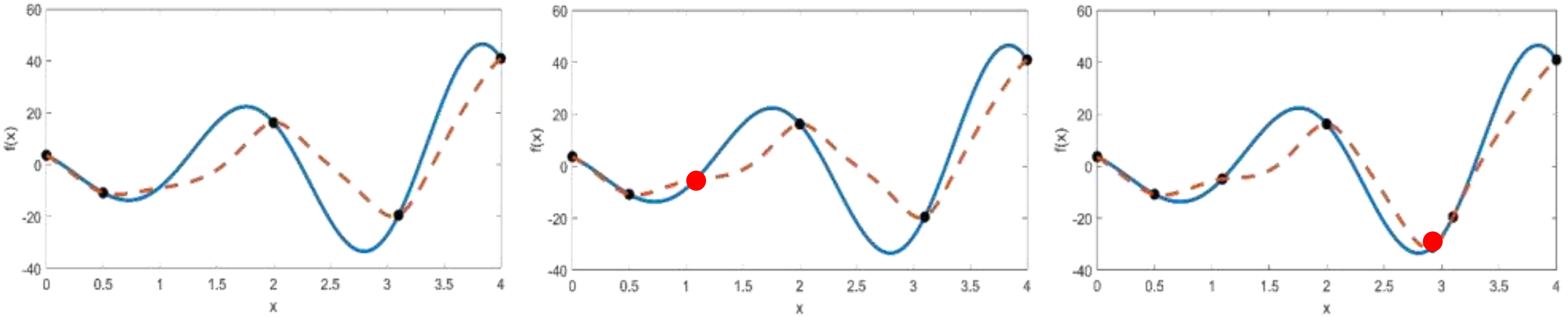
Function | **Step 1** | **Step 2** | **Step 3**

Function

Kriging prediction

Learning points

Last point added



Expected Improvement

Expected Improvement

Max EI

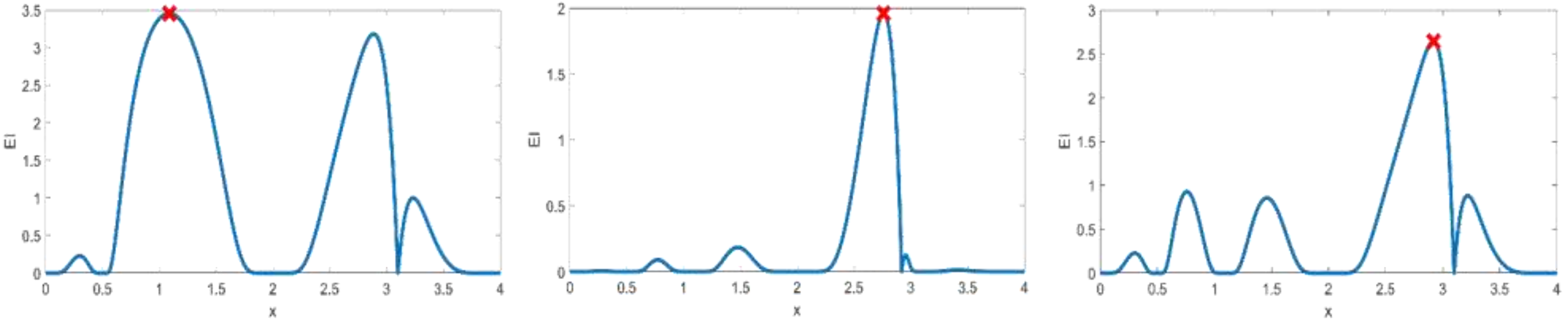
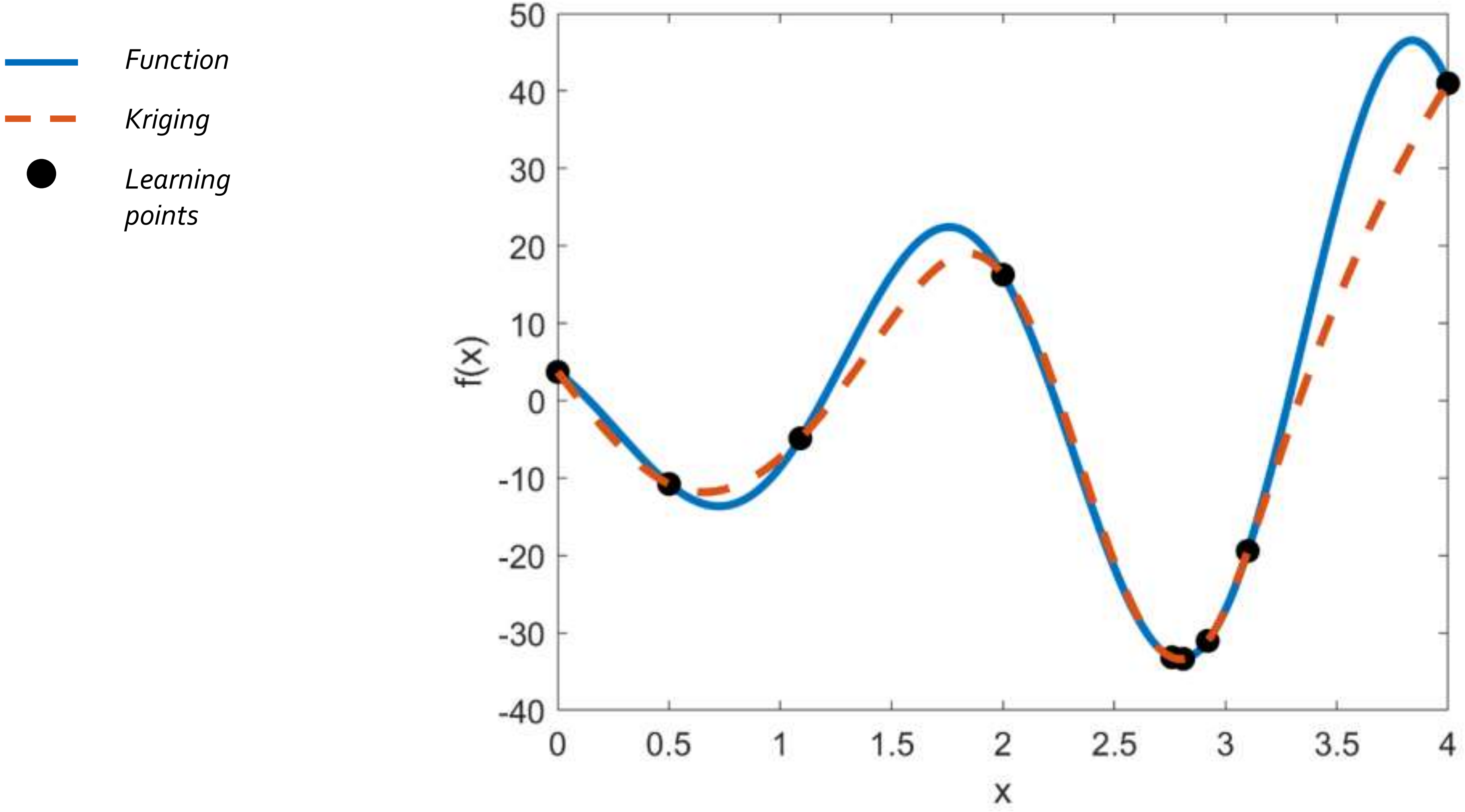
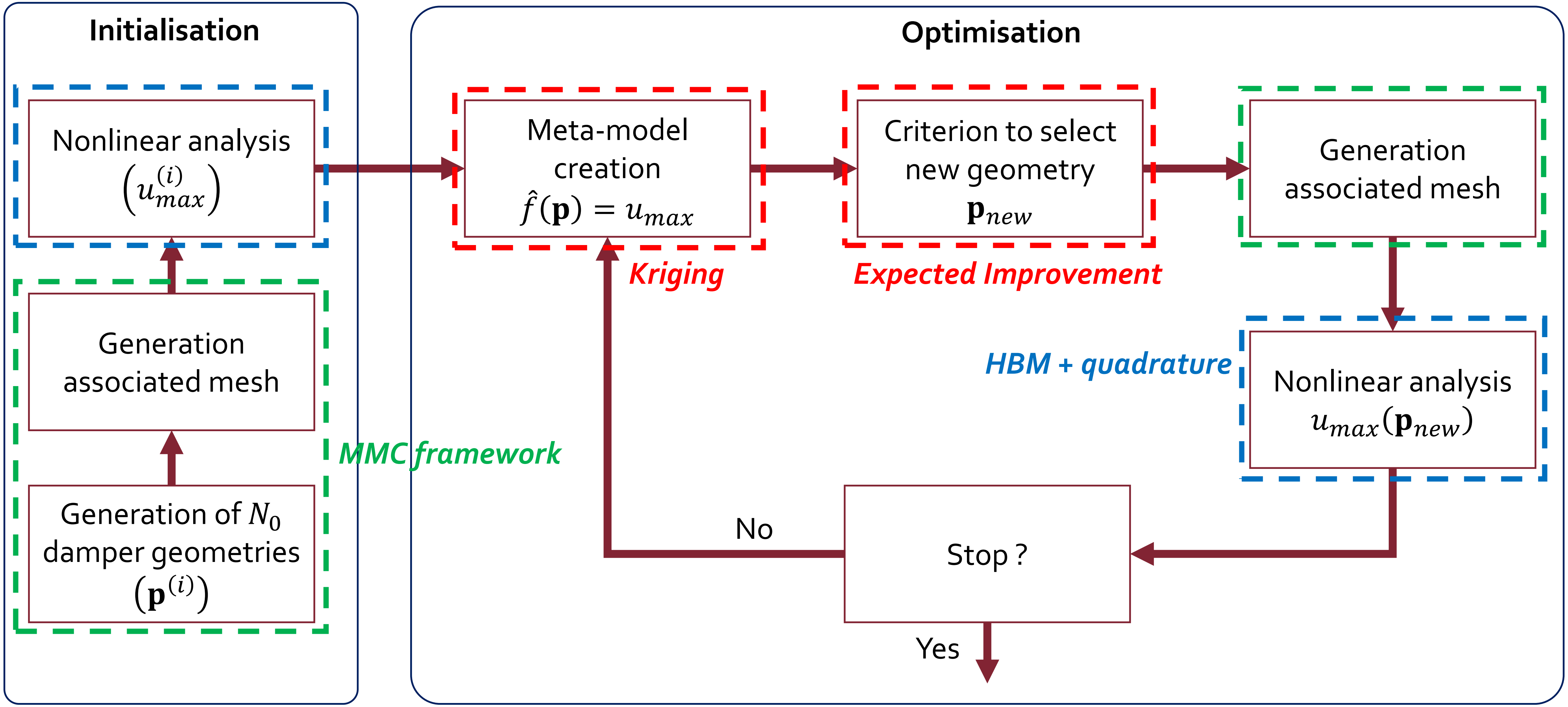


Illustration of the process on a 1d case

Step 5



Surrogate modelling – optimisation process



Results – general overview

Optimisation parameters:

- Optimisation of $-1/u_{max}$ instead of u_{max}
- 9 optimisation parameters
- 55 initial points
- 200 iterations

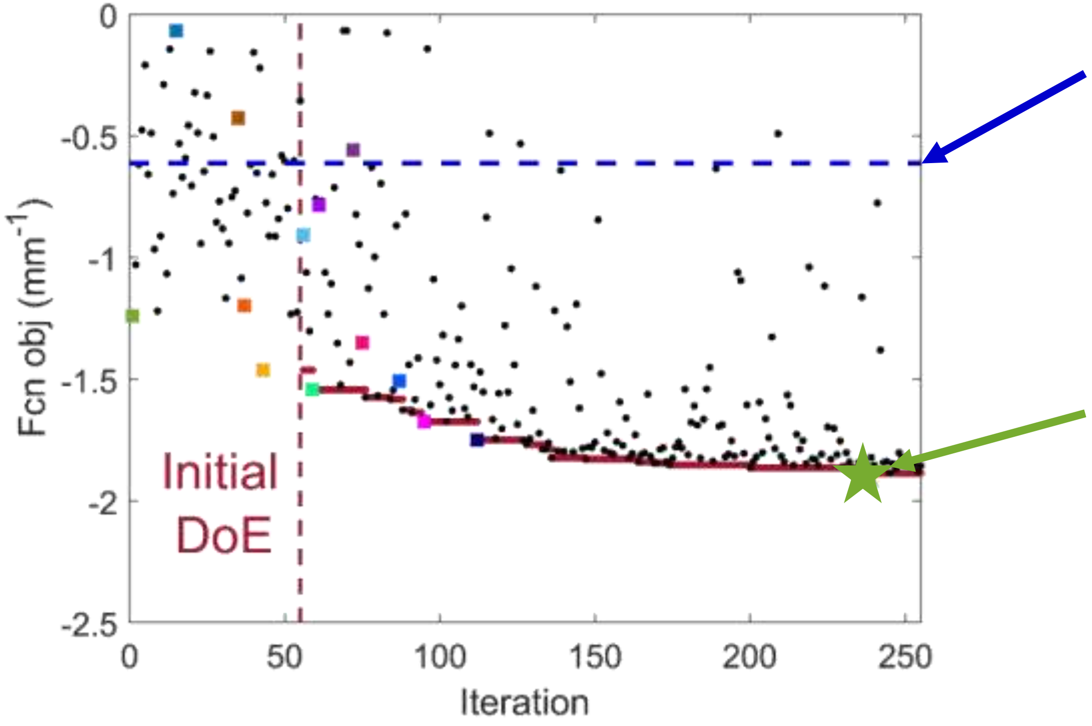


Fig.1: Evolution of the objective function vs iteration number
( ) : current minimum

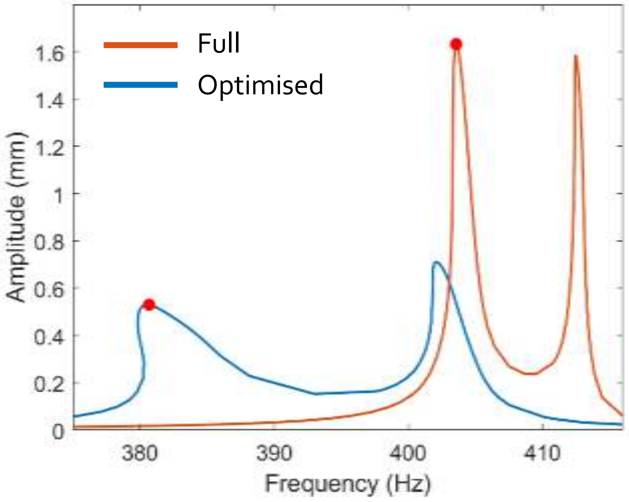
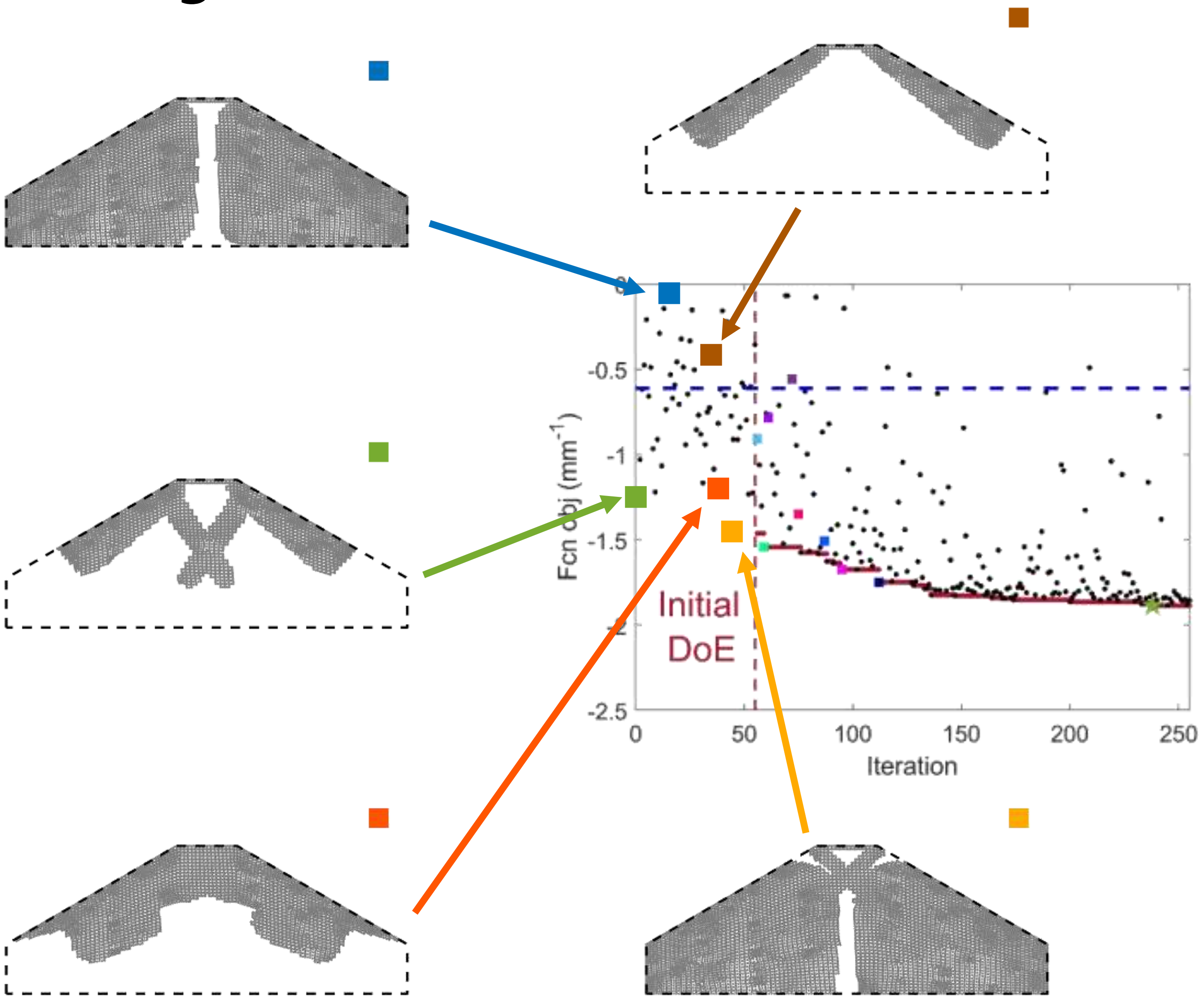


Fig.2: FRFs of the optimised (blue) and full damper (orange) dampers

Comparison:

- Vibrations divided by 3
- Frequency shift of 20 Hz
- Mass reduced by 87%

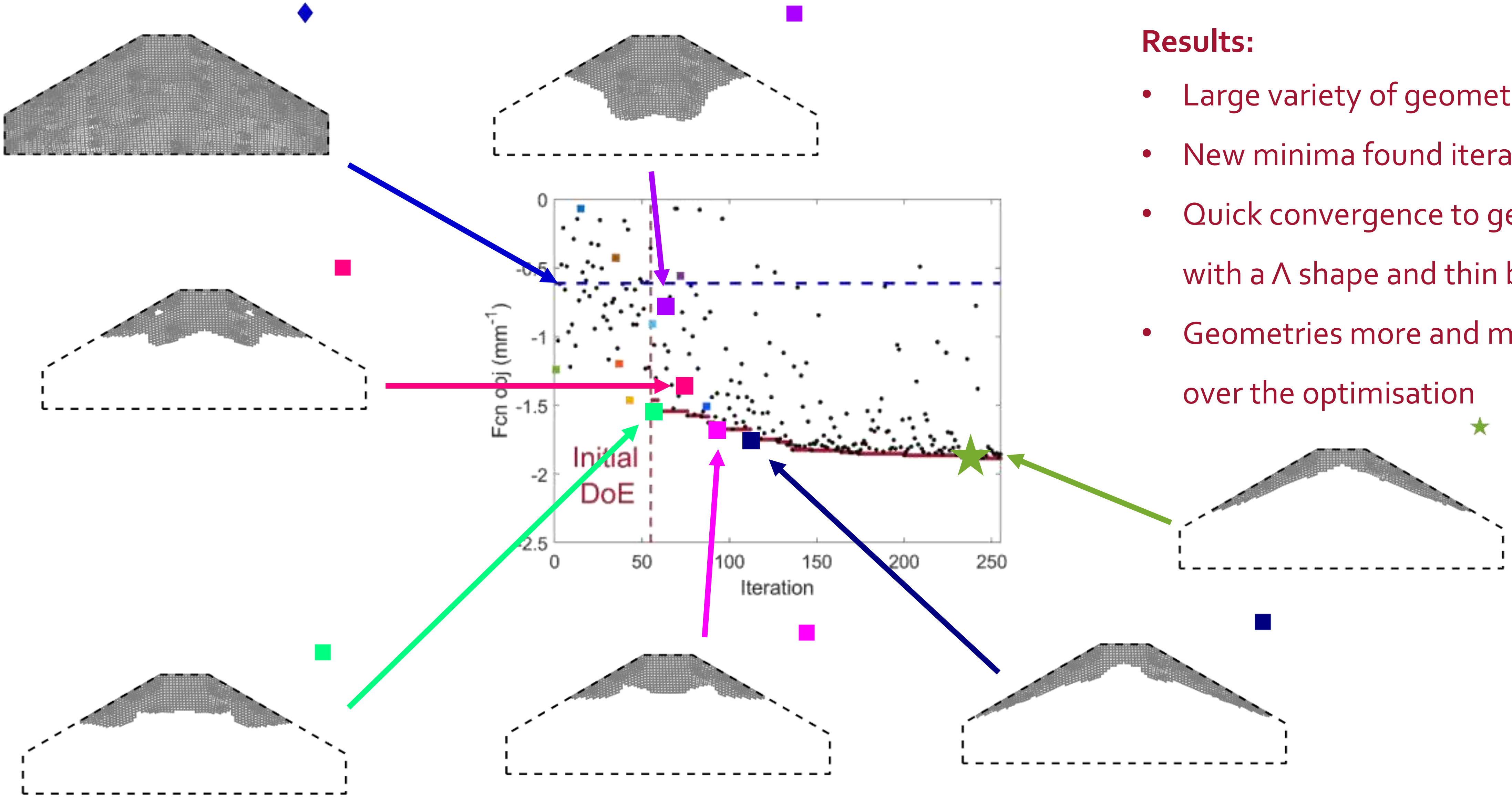
Results – general overview



Results:

- Large variety of geometries
- New minima found iteratively
- Quick convergence to geometry with a Λ shape and thin branches
- Geometries more and more similar over the optimisation

Results – general overview



Results:

- Large variety of geometries
- New minima found iteratively
- Quick convergence to geometry with a Λ shape and thin branches
- Geometries more and more similar over the optimisation

Results – general properties of the tested geometries

(●) : initial points – (★) : best geometry – (♦) : full damper

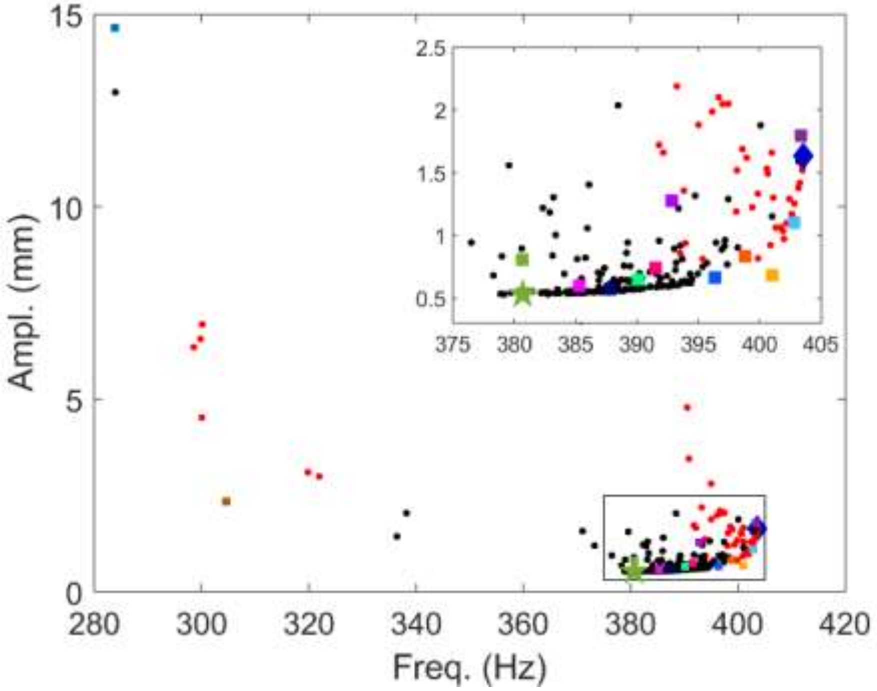


Fig.1: amplitude vs resonance frequency

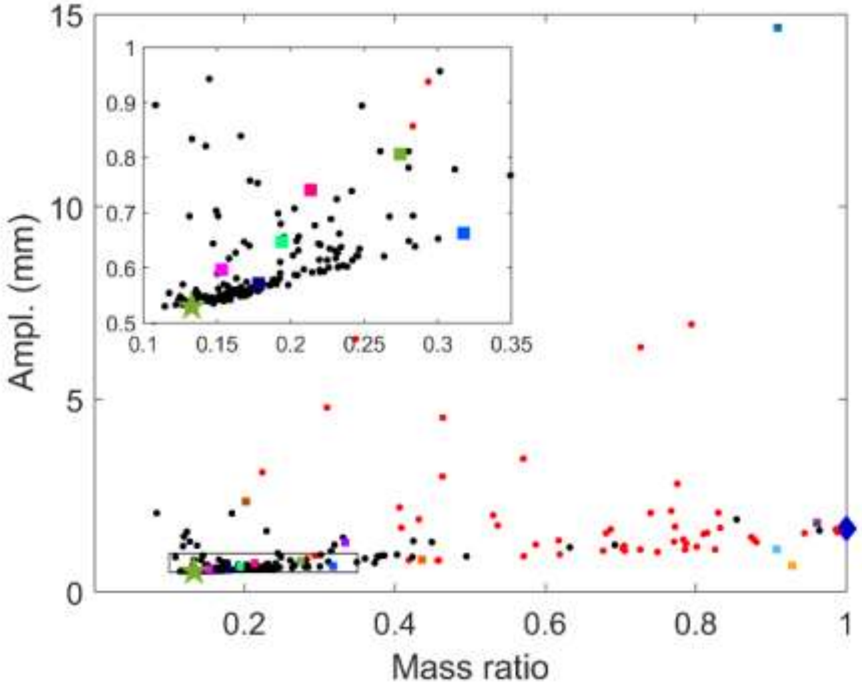


Fig.2: amplitude vs mass ratio

Results:

- Reduction of amplitude => reduction of resonance frequency
- Reduction of mass => reduction the vibration amplitude

Results – geometry clustering

Idea: group similar geometries to get general properties related to the geometry

Distance definition $d(\mathbf{p}^{(1)}, \mathbf{p}^{(2)})$:
comparison of elements in common in the geometry $\mathbf{p}^{(1)}$ and in the geometry $\mathbf{p}^{(2)}$

- $d = 1$: all elements are the same (void and material at the same location)
- $d = 0$: all elements are different

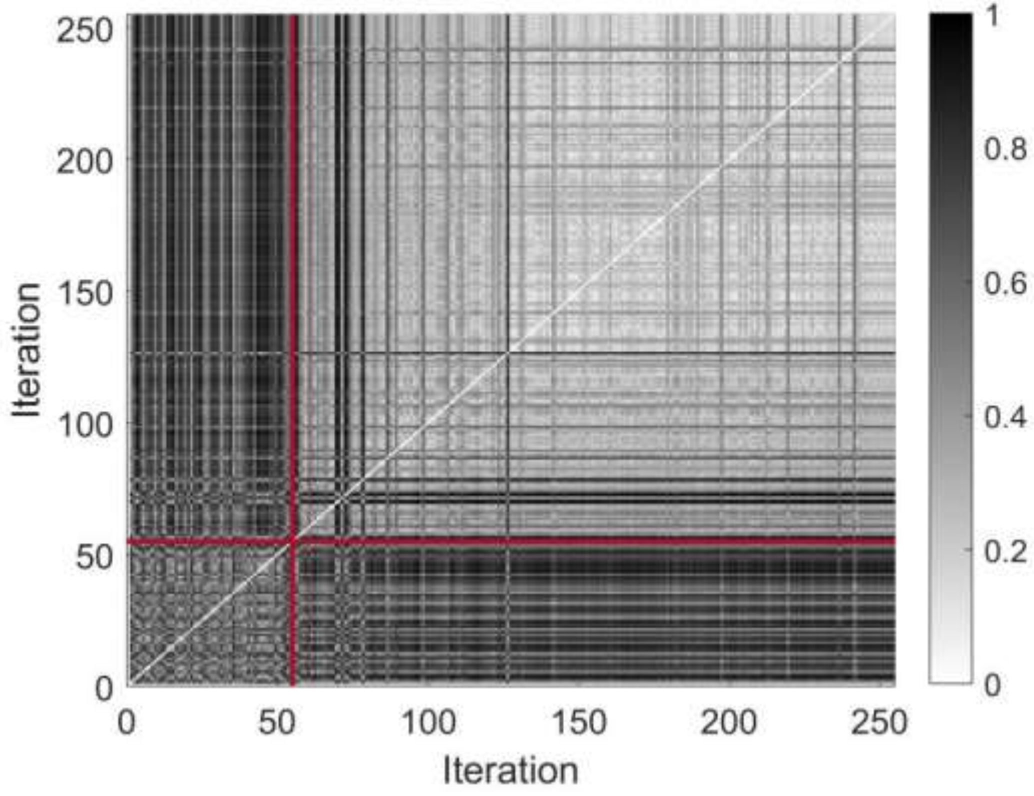
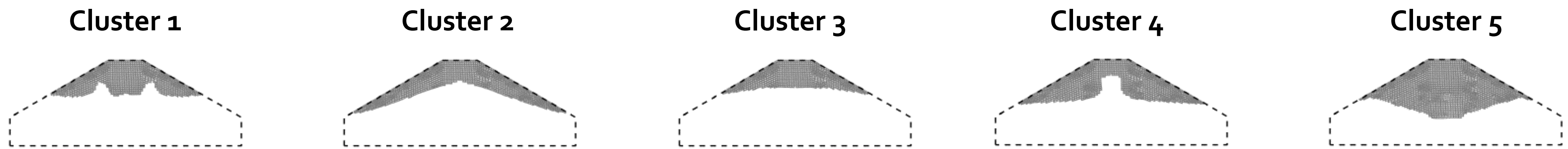


Fig.: matrix of the distances $d_{i,j} = d(\boldsymbol{p}^{(1)}, \boldsymbol{p}^{(2)})$

Results:
- Geometries different from each other in the initial set
- Geometries more and more similar over the optimisation

Results – geometry clustering

Idea: group similar geometries to get general properties related to the geometry



Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5
Bumps on side	*Λ shape*	*Flat*	*Central hole*	*Central bump*
33 geom.	*123 geom.*	*15 geom.*	*15 geom.*	*5 geom.*

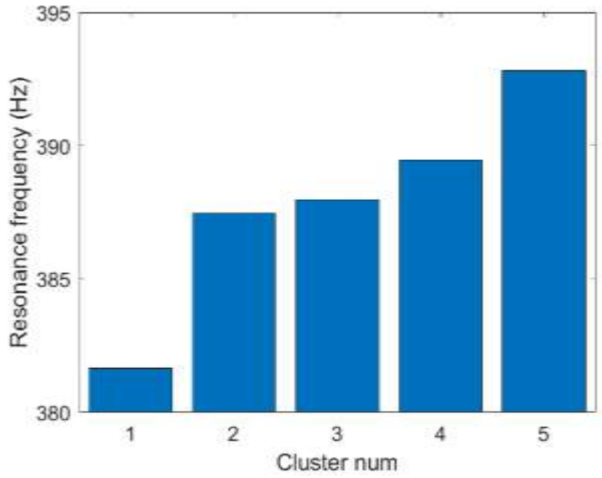


Fig.: Resonance frequency on average

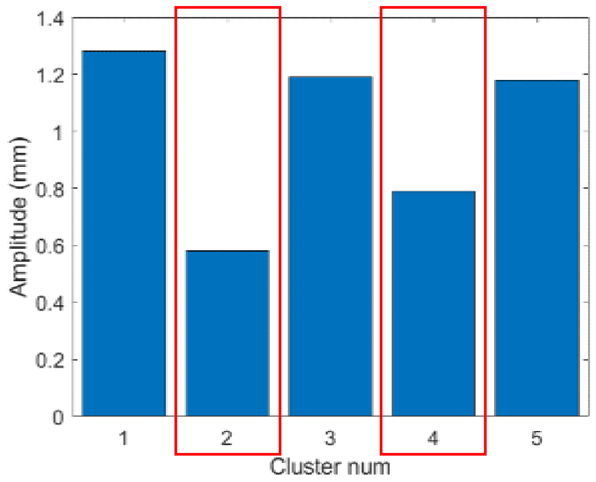


Fig.: Vib. amplitude on average

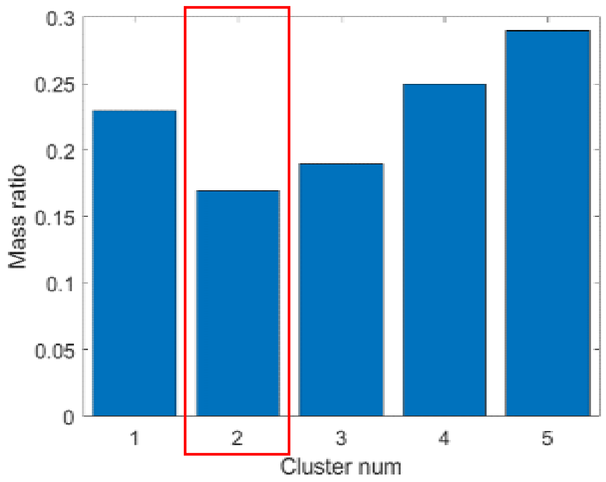
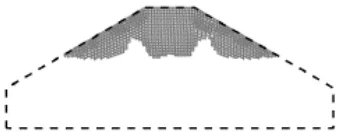


Fig.: Mass ratio on average

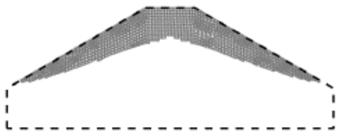
Results – geometry clustering

Cluster 1



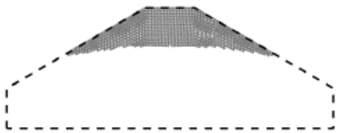
Bumps on side
33 geom.

Cluster 2



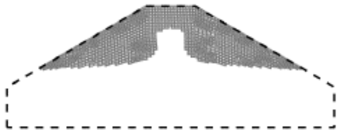
Λ shape
123 geom.

Cluster 3



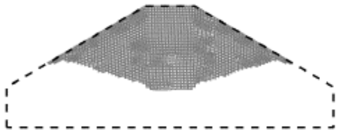
Flat
15 geom.

Cluster 4



Central hole
15 geom.

Cluster 5



Central bump
5 geom.

N = constant*m/A

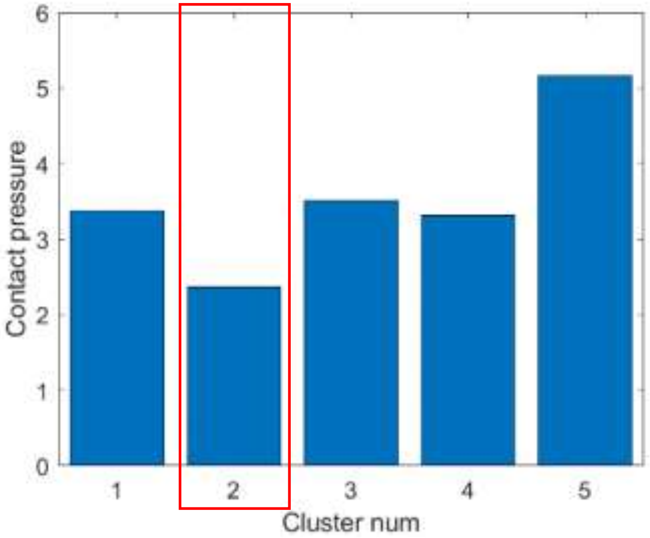


Fig.: Initial contact pressure on average

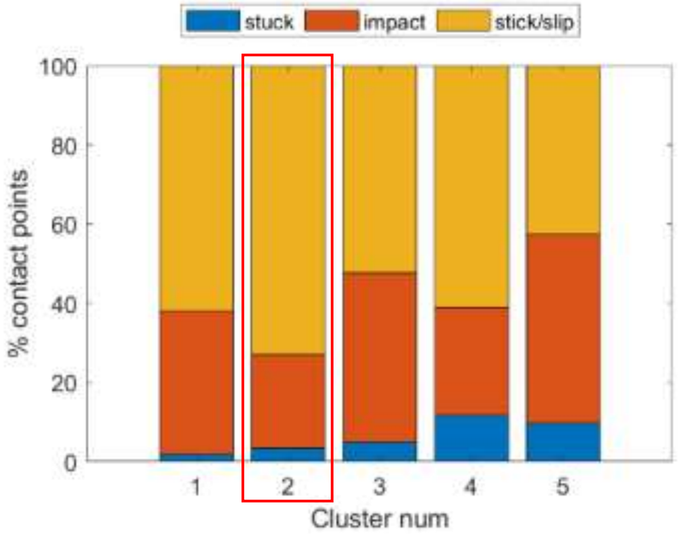


Fig.: Contact satus distribution on average

Best geometries tend to minimise the initial contact pressure to minimise stuck points and maximise stick/slip
→ low mass and large contact area

Conclusion and perspectives

- Design of nonlinear dynamic systems = challenging
- Limitations due to computational time and realistic modelling of manufacturing, operating condition etc

- Surrogate modelling = promising tool
  - Physical properties integrated in DoE or regression
  - Prediction of QoI at reduced numerical cost
  - Deep understanding in the dynamic behaviour of complex systems
  - Topology ans structural optimisation for nonlinear resonance mitigation

- Perspectives
  - Optimisation: robustness, 3D, experimental validation
  - Include more advanced nonlinear dynamic features (bifurcation)

Thanks!

Enora Denimal Goy

Inria, Saclay, France - Platon research team

enora.denimal@inria.fr

Related publications

- E. Denimal, J-J. Sinou, *Advanced kriging-based surrogate modelling and sensitivity analysis for rotordynamics with uncertainties*, European Journal of Mechanics - A/Solids, 2021, 90:1-20 [DOI] [HAL].
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- E. Denimal, L. Renson, C. Wong, L. Salles, *Topology optimisation of friction under-platform dampers using Moving Morphable Components and the Efficient Global Optimization algorithm*, Structural and Multidisciplinary Optimization, 65, 56 (2022), [DOI].