DDA 5002 Homework # 3 Fall 2025

Due: Thursday, October 30 at 11:59pm

Please note: Problems 2, 4, and 5 ask you to solve a model in Python. Put your .py file you create and the solution output in a single zip file and submit it via Blackboard. Please name your file `Lastname_StudentID.zip`. Also, give meaningful names to your decision variables and constraints, and add comments to your code liberally. Both the problem set and the code are due at end of the day at 11:59pm on Thursday, October 30th.

1. Consider the following linear programs:

$$\begin{aligned} \min_{x} \quad & 5x_1 + x_2 - 4x_3 \\ \text{s.t.} \quad & x_1 + x_2 + x_3 + x_4 \geq 19 \\ & 4x_2 + 8x_4 \leq 55 \\ & x_1 + 6x_2 - x_3 = 7 \\ & x_1 \text{ free}, x_2 \geq 0, x_3 \geq 0, x_4 \leq 0, \end{aligned}$$

and

$$\begin{aligned} \max_{x} \quad & 2x_1 - 7x_2 + 6x_3 + 5x_4 \\ \text{s.t.} \quad & 2x_1 - 3x_2 - 5x_3 - 4x_4 \leq 20 \\ & 7x_1 + 2x_2 + 6x_3 - 2x_4 = 35 \\ & 4x_1 + 5x_2 - 3x_3 - 2x_4 \geq 15 \\ & 0 \leq x_1 \leq 10, 0 \leq x_2 \leq 5, x_3 \geq 2, x_4 \geq 0. \end{aligned}$$

(a) Write the dual of each model.

(b) Write the KKT optimality conditions (primal feasibility, dual feasibility, and complementary slackness conditions) for each model. Since you have written down the dual for part (a), you don't have to repeat dual feasibility.

2. LGUAir must decide how to partition one of its planes for a Shenzhen $\leftrightarrow$ Beijing route. The plane has space for 200 coach-fare seats. It is also possible to create first-class and business-class sections, but each first-class seat takes up the space of 2 coach-fare seats and each business-fare seat takes up the space of 1.5 coach-fare seats. The profit on a first-class ticket is 3 times the profit of a coach-fare ticket and the profit on a business-class ticket is 2 times the profit of a coach-fare ticket. Once the plane has been partitioned, it cannot be changed. LGUAir has estimated that there are three scenarios that will occur in a typical week, each with the same frequency (i.e., there are the same number of flights under each scenario): (i) weekday morning and evening traffic, (ii) weekend traffic, and (iii) weekday midday traffic. For each flight demands are estimated as follows. Under scenario (i): they can sell up to 20 first-class tickets, 50 business-class tickets, and 200 coach-class tickets. Under scenario (ii): they can sell up to 10 first-class tickets, 25 business-class tickets, and 175 coach-class tickets. Under scenario (iii): they can sell up to 5 first-class tickets, 10 business-class tickets, and 150 coach-class tickets. Assume that LGUAir will sell no more tickets than there are seats in each section.

(a) Formulate a linear program that, when solved, will yield a partitioning of the seats that maximizes profit. Formulate your model in a **data-independent** manner. (Even though it should really be an integer program, please formulate and solve it as an LP.)

(b) Implement and solve your linear program using Python with COPT. Report your solution "by hand" with clear explanations (i.e., don't just point to the Python output). Your solution should involve all of your decision variables, even if some are just constructs to compute what you want.

For the following questions, you may use Python to help you answer them. If you choose to do so, please submit the codes of steps to obtain the results along with the optimization modeling codes for part (a) and (b).

(c) What is the shadow price associated with the constraint that says the plane can hold at most $b = 200$ coach-size seats?

(d) Use your answer from part (c) to predict the optimal value, z^*, if we change the plane's capacity from $b = 200$ to $b = 201$ coach-size seats.

(e) Resolve the linear program with $b = 201$. How does your prediction compare with the optimal value, z^*?

(f) Another constraint in the linear program said, "don't sell more tickets than the demand" for each fare class and for each scenario. What are the shadow prices for these constraints? Use a specific example in this problem to explain why some of the shadow prices are zero.

3. Consider the following linear program:

$$\begin{aligned} \max_{x} \quad & 4x_1 + 2x_2 + 3x_3 \\ \text{s.t.} \quad & 2x_1 + 3x_2 + x_3 \leq 12 \\ & x_1 + 4x_2 + 2x_3 \leq 10 \\ & 3x_1 + x_2 + x_3 \leq 10 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

(a) Form the dual of the given linear program.

(b) Use complementary slackness (for both the primal and dual) to derive an optimal solution to the dual, given that an optimal solution to the primal is $(x_1^*, x_2^*, x_3^*) = (2, 0, 4)$.

(c) Confirm that strong duality holds; i.e., verify that the objective function value of the primal is the same as the objective function value of the dual for their respective solutions.

4. In the manufacturing of printed circuit boards, holes need to be drilled on the boards through which chips and other components are later wired. It is required that these holes be drilled as quickly as possible; hence, the problem of the most efficient order to drill the holes is to find a shortest possible route that visits each hole exactly once and return to the starting hole. In the table below are the distances (in millimeters) between any pair of hole locations. Hence, minimizing the total time is equivalent to finding the minimum total distance traveled between the hole locations.

Hole Locations	1	2	3	4	5
1	-	13	8	15	7
2	13	-	5	7	14
3	8	5	-	15	17
4	15	7	15	-	8
5	7	14	17	8	-

(a) Formulate a **data-specific** integer program to determine the minimum distance required to drill all the needed holes. Define your index sets, parameters, and decision variables clearly.

(b) Solve your integer program in Python. Report your solution, and the optimal distance, in a fashion that can be understood by the manufacturing engineer (i.e., do not just point to Python output).

Hint: check the traveling salesperson problem.

5. To graduate from FastTrack University with a major in operations research, a student must complete at least two math courses, at least two OR courses, and at least two computer courses. Some courses can be used to fulfill more than one requirement (shown by “X”), and some courses are prerequisites for others. This information is given in the following table.

Courses	Fulfill requirements for Math	OR	Computer	Prerequisite
1. Calculus	X			
2. Operations Research	X	X		
3. Data Structures	X		X	Intro to Programming
4. Business Statistics	X	X		Calculus
5. Computer Simulation		X	X	Intro to Programming
6. Intro to Programming			X	
7. Forecasting	X	X		Business Statistics

(a) Formulate and write down a linear integer programming that minimizes the number of courses needed to satisfy the major requirements.

(b) Implement and solve your model in Python. Report your solution “by hand” in a way that someone without optimization background can understand.