Part B-1

1 *Excellence* #@REL-EVA-RE@#

1.1 Quality and pertinence of the project's research and innovation objectives

Proof assistants — also called *interactive theorem provers* (ITPs) — are software tools used to rigorously verify formal modelling and reasoning. Contemporary systems such as *Rocq*, *Lean*, and *Isabelle*[1] offer powerful frameworks for constructing formal specifications and proofs: they have been used successfully in various applications, ranging from the verification of advanced theorems in mathematics to the certification of complex software artifacts such as programming language compilers, operating system kernels and cryptographic protocols[2]. Yet they remain notoriously difficult to learn and use, limiting broader adoption in various settings where they could bring transformative societal impact, such as:

- **mathematics education**, where they could act as a unifying medium for interactive exploration and understanding of mathematical concepts, fostering closer collaboration amongst students and offloading some teaching burden (e.g. grading) through automation[3];
- **formal verification**, where they could bring higher standards of quality assurance (QA) to businesses that rely on complex hardware and software systems, especially in safety-critical industries such as healthcare, transportation and energy;
- **artificial intelligence (AI) safety**, where the uncertainty inherent to current technologies based on probabilistic techniques such as large language models (LLMs) could be mitigated by the exact logical reasoning capabilities of ITPs, an approach sometimes termed *neurosymbolic AI*.

In view of this large potential for applications, it is natural to ask what exactly limits adoption of the current generation of ITPs. The recent surge of interest arising both in academia and industry — in great part due to the popularity of the Lean language and its Mathlib library — suggests that social factors such as public communication, community building, and vast amounts of expository content and learning resources all play an important role in the widespread appropriation of this complex technology. Even more recently, the promise of a new kind of generative AI free from so-called "hallucinations" that could aid in accelerating scientific discoveries has been a powerful narrative attracting much attention and funding[4].

However, many researchers in the field agree that current ITPs suffer from more *foundational* issues that affect directly their accessibility and ease of use, as well as their ability to scale to larger developments. While these issues are quite diverse in nature, a recurring theme since the birth of the technology in the 60s is the overwhelming **bureaucracy** involved in formalization efforts: formal proofs require a level of care and detail that is far superior and much more time consuming than what is expected from informal paper proofs. This has been measured by the so-called *DeBruijn factor* comparing the size of formal and informal proofs, often reaching a value of 4 on average[5].

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[1] Team, The Rocq Development. The Rocq Prover. Apr. 2025. https://doi.org/10.5281/zenodo.15149629; Moura, Leonardo de, and Sebastian Ullrich. *Automated Deduction – CADE 28.* Edited by André Platzer and Geoff Sutcliffe, 2021, pp. 625–35; Tobias Nipkow, et al., *Isabelle/HOL: a proof assistant for higher-order logic*, vol. 2283 (Springer Science & Business Media, 2002).

[2] Georges Gonthier, "Formal Proof—The Four- Color Theorem" vol. 55, no. 11, 2008, Xavier Leroy, "Formal Verification of a Realistic Compiler", *Commun. ACM* vol. 52, no. 7, July 2009, https://doi.org/10.1145/1538788.1538814, pp. 107–15; Klein, Gerwin, et al. "seL4: Formal Verification of an OS Kernel". *Proceedings of the ACM SIGOPS 22nd Symposium on Operating Systems Principles*, SOSP '09, Oct. 2009, pp. 207–20. https://doi.org/10.1145/1629575.1629596; Gilles Barthe, et al., "EasyCrypt: A Tutorial", *Foundations of Security Analysis and Design VII: FOSAD 2012/2013 Tutorial Lectures*, edited by Alessandro Aldini et al. (Cham: Springer International Publishing, 2014) pp. 146–66.

[3] Minh, Frédéric Tran, et al. *Research Report - Proof Assistants for Teaching: A Survey.* Apr. 2024. Accessed 4 Sept. 2025.

[4] Cade Metz, "Is Math the Path to Chatbots That Don't Make Stuff Up?", *The New York Times*, Sept. 2024,

[5] Wiedijk, Freek. The De Bruijn Factor. 2000. www.cs.ru.nl/F.Wiedijk/factor/factor.pdf.

The main approach to tame this complexity has been to *automate* the various processes involved in formalization, which fall roughly within two categories: *elaboration*, concerned with the translation of requirements expressed in natural language or mathematical notations into precise logical specifications; and *synthesis*, where the system attempts to automatically generate (parts of) proofs and programs meeting these specifications. Progress on both fronts is currently being made with the help of state-of-the-art machine learning techniques, including LLMs[6]. More theoretical research has also been pursued in **type theory**, the field studying the logical formalisms at the foundation of all modern ITPs. They are the ultimate backbone on which relies our *trust* in the output of these systems, and thus a key differentiator with respect to purely probabilistic approaches to (generative) AI.

However, little attention has been devoted by the ITP community to another discipline closely related to type theory: **proof theory**. In particular, *structural* proof theory is its branch concerned with the study of combinatorial structures for representing and manipulating proofs. One can identify mainly three motivations for this study:

Challenge 1 (C1) is the fundamental problem of **proof identity**, also known as Hilbert's 24th problem[7]. It aims to answer the philosophical question "what is a proof?", and the mathematical question "when are two proofs equal?". It is thus intimately related to *homotopy type theory* which also investigates the structure of *equality*, a known weakness of many type theories.

Challenge 2 (C2) is to find proof systems for **non-standard logics** — such as *modal*, *intermediate*, *substructural* and *fixpoint* logics — satisfying good enough properties as to render an algorithmic treatment of these logics tractable. The most important property in this respect is that of **cut elimination**, which is essential both to reduce the complexity of *proof search* (proof *synthesis* in ITP terminology), and to ensure productivity of *program execution* through the **Curry-Howard correspondence** (CHC) between proofs and programs. The CHC is also at the heart of the *calculus of inductive constructions* (CoIC), which is the type theory used by the two leading proof assistants Rocq and Lean. Researchers are increasingly interested in type-theoretic formulations of these logics as they provide expressive languages for specifying behaviors of programs that go beyond pure functions, including effects (modal logics), resource-sensitivity (modal and substructural logics) and recursion (modal and fixpoint logics)[8].

Challenge 3 (C3) is to improve the **efficiency** of computational procedures on proofs, but also on programs through the CHC. A well-established principle in computer science and software engineering is that choosing appropriate data structures for a problem can lead to orders-of-magnitude improvements in efficiency. Finding the right data structures for such general classes of objects as proofs and programs is thus a very enticing goal with wide implications, including faster automation in ITPs.

In the past decades, two families of proof formalisms have emerged to tackle these challenges:

Graphical proof systems represent proof objects as *graphs* instead of *trees* of inference rules. *Proof nets*[9] are one of the first graphical proof formalisms, meant to reduce the bureauceacy of proofs in linear

[6]Lasse Blaauwbroek, et al., "Learning Guided Automated Reasoning: A Brief Survey", vol. 14560 (2024) pp. 54–83.

[7]Lutz Straßburger, "The problem of proof identity, and why computer scientists should care about Hilbert's 24th problem", *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* vol. 377, no. 2140, 11 Mar. 2019, https://doi.org/10.1098/rsta.2018.0038, p. 20180038.

[8]Wenhao Tang, et al., "Modal Effect Types", *Proc. ACM Program. Lang.* vol. 9, OOPSLA1, Apr. 2025, https://doi.org/10.1145/3720476, Marshall, Danielle, et al. "Linearity and Uniqueness: An Entente Cordiale". *Programming Languages and Systems*, edited by Ilya Sergey, 2022, pp. 346–75. https://doi.org/10.1007/978-3-030-99336-8_13; Ranald Clouston, et al., "The Guarded Lambda-Calculus: Programming and Reasoning with Guarded Recursion for Coinductive Types", *Logical Methods in Computer Science* vol. Volume 12, Issue 3. *arXiv*, arxiv.org/abs/1606.09455, Apr. 2017, https://doi.org/10.2168/LMCS-12(3:7)2016, p. 2019.

[9]Jean-Yves Girard, "Linear logic", *Theoretical Computer Science* vol. 50, no. 1, 1987, https://doi.org/https://doi.org/10.1016/0304-3975(87)90045-4, pp. 1–101.

logic (C2) in order to identify their essence (C1). Further developments stemming from proof nets like the *geometry of interaction* and *interaction nets* have found applications in program optimization for hardware synthesis and parallelized execution[10] (C3). The *combinatorial proofs* of Hughes are direct decendants of proof nets mainly concerned with C1[11], and *string diagrams* from category theory have been applied to logic and programming languages, with well-known connections to proof nets[12].

Deep inference generalizes Gentzen-style proof systems by allowing inference rules to be applied at any depth inside of a formula, rather than being restricted to its top-level logical connective. The terminology of "deep inference" was proposed by Alessio Guglielmi, who invented the *calculus of structures* to overcome the inability of sequent calculus to capture the substructural logic MV[13]. Since then, calculi of structures and so-called *nested sequent calculi* have been introduced to give proof systems enjoying cut-elimination to most substructural, modal and intermediate logics[14] (C2). Deep inference has also been used in the study of proof complexity, providing in some cases exponential speedup over sequent calculus with respect to proof size, as well as quasipolynomial-time cut-elimination[15] (C3). Lastly, many deep inference formalisms enjoy CHC-style interpretations with variants of λ-calculus, which also improve space efficiency[16].

The experienced researcher (ER) has accumulated significant knowledge of graphical and deep inference proof systems, as well as their applications to ITPs. This expertise was developed during his PhD thesis[17] titled "Deep Inference for Graphical Theorem Proving", where he designed various proof formalisms that enable a novel approach to interactive theorem proving based on **direct manipulation** of logical statements in a graphical user interface (GUI). This extends earlier works on *Proof-by-Pointing*[18] and *Proof-by-Linking*[19] where proofs are constructed through *click* and *drag-and-drop* gestures on formulas, to a more encompassing paradigm termed *Proof-by-Action* (PbA). The goal is to improve accessibility and usability of ITPs by focusing on better principles for *human interaction*, complementing more mainstream research around machine automation. Applying a mixture of graphical and deep inference proof theory to that effect is a highly original endeavor, with no similar efforts in the contemporary research landscape.

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[10] Ghica, Dan R. "Geometry of Synthesis: A Structured Approach to VLSI Design". *Proceedings of the 34th Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages*, POPL '07, Jan. 2007, pp. 363–75. https://doi.org/10.1145/1190216.1190269; Mackie, Ian. "An Interaction Net Implementation of Closed Reduction". *Implementation and Application of Functional Languages*, edited by Sven-Bodo Scholz and Olaf Chitil, 2011, pp. 43–59. https://doi.org/10.1007/978-3-642-24452-0_3.

[11] Dominic Hughes, "Proofs without syntax", *Annals of Mathematics* vol. 164, no. 3, Nov. 2006, https://doi.org/10.4007/annals.2006.164.1065, pp. 1065–76.

[12] Robin Piedeleu and Fabio Zanasi, "An Introduction to String Diagrams for Computer Scientists", *Elements in Applied Category Theory*, May 2025, https://doi.org/10.1017/9781009625715,

[13] Guglielmi, Alessio. *A Calculus of Order and Interaction*. 1999. www.researchgate.net/publication/2807151_A_Calculus_of_Order_and_Interaction.

[14] Roman Kuznets and Lutz Straßburger, "Maehara-style modal nested calculi", *Archive for Mathematical Logic* vol. 58, nos. 3–4, May 2019, https://doi.org/10.1007/s00153-018-0636-1, pp. 359–85; Postniece, Linda. "Proof theory and proof search of bi-intuitionistic and tense logic". 2010, Artwork Size: vii, 228 leaves., vii, 228 leaves. https://doi.org/10.25911/5D5FCC3A4DB33.

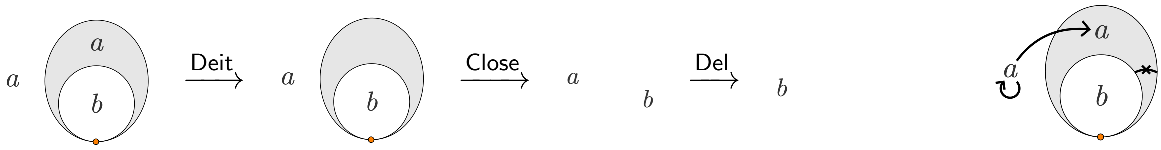
[15] Anupam Das, "On the Relative Proof Complexity of Deep Inference via Atomic Flows", *Logical Methods in Computer Science* vol. Volume 11, Issue 1, Mar. 2015, https://doi.org/10.2168/LMCS-11(1:4)2015, p. 735; Paola Bruscoli, et al., "Quasipolynomial Normalisation in Deep Inference via Atomic Flows and Threshold Formulae", *Logical Methods in Computer Science* vol. Volume 12, Issue 2, May 2016, https://doi.org/10.2168/LMCS-12(2:5)2016, p. 1637.

[16] Nicolas Guenot, "Nested Deduction in Logical Foundations for Computation", PhD dissertation, Ecole Polytechnique X, 2013; Gundersen, Tom, et al. "Atomic Lambda Calculus: A Typed Lambda-Calculus with Explicit Sharing". *28th Annual ACM/IEEE Symposium on Logic in Computer Science*, June 2013, pp. 311–20. https://doi.org/10.1109/LICS.2013.37.

[17] Pablo Donato, "Deep Inference for Graphical Theorem Proving", PhD dissertation, Institut Polytechnique de Paris, 2024.

[18] Yves Bertot, et al., "Proof by pointing", *Theoretical Aspects of Computer Software*, edited by Masami Hagiya and John C. Mitchell, redacted by Gerhard Goos and Juris Hartmanis, vol. 789 (Berlin, Heidelberg: Springer Berlin Heidelberg, 1994) pp. 141–60.

[19] Kaustuv Chaudhuri, "Subformula Linking as an Interaction Method", *Interactive Theorem Proving*, edited by Sandrine Blazy et al., redacted by David Hutchison et al., vol. 7998 (Berlin, Heidelberg: Springer Berlin Heidelberg, 2013) pp. 386–401.



(a) Dynamic rewriting from premiss $a \wedge (a \Rightarrow b)$ to conclusion b

(b) Static representation of rules as arrows in a bigraph

Figure 1: Proof of *modus ponens* $a \wedge (a \Rightarrow b) \vdash b$ in scroll nets. Intuitionistic implication $a \Rightarrow b$ is represented topologically by a *scroll*, i.e. two nested ellipses that intersect exactly at one point, where the outer gray *outloop* (resp. inner white *inloop*) contains the antecedant a (resp. consequent b).

In continuation of this programme, the ER has introduced in his last preprint[20] a new graphical framework called **scroll nets**. It is based on a long forgotten diagrammatic proof system called *existential graphs* (EGs), invented by the famous philosopher and logician Charles Sanders Peirce at the dusk of the 19th century — thus predating the very existence of proof theory and computer science. Proofs in EGs are defined by a small set of inference rules that *dynamically* rewrite diagrams in contexts of arbitrary depth (Fig. 1a), thus combining features of both deep inference and string diagrams[21]. Scroll nets provide a *static* way to represent proofs in EGs by recording explicitly applications of inference rules in a *directed graph* (Fig. 1b), combining the type-theoretic methodology of internalizing inference rules inside judgments with a graphical structure similar to proof nets and combinatorial proofs. Crucially, this graph shares the same nodes as the (tree-shaped) statements involved in the proof, making scroll nets a more compact representation than other graphical proof structures, but also surprisingly a variant of the notion of *bigraph*. Bigraphs were introduced by Milner as a foundational combinatorial structure encompassing most models of concurrent/parallel computation, including Petri nets and his own CCS and π-calculus[22]. In his preprint, the ER shows how scroll nets naturally subsume the simply-typed λ-calculus — the common kernel of all type theories used in ITPs — by identifying a generalization of the notion of *detour* from natural deduction that abstracts away the shape of formulas.

All these discoveries hint toward the potential of scroll nets as a very expressive framework for both proof theory and type theory, unifying features found in most of the formalisms that have emerged from the two disciplines in a natural and efficient way. Moreover, the ER's vision is to exploit the diagrammatic nature of scroll nets to redesign not only the *backend* but also the *frontend* of ITPs, making them the *interaction substrate*[23] of a new kind of GUI in the PbA paradigm. Indeed, what crucially distinguishes scroll nets from other deep inference or graphical proof formalisms is their suitability for *interactive manipulation*: every inference rule possesses a natural spatial interpretation as a *gesture* on a logical statement, either inserting/erasing a statement at a designated location through *pointing* (looping arrows in Fig. 1b), or (un)copying to/from some location through *drag-and-drop* (non-looping arrows in Fig. 1b). This natural semantics of gestures as a means of manipulating located objects is very familiar to users of modern GUIs in virtually every domain, and could lead to a **groundbreaking "no-code" approach to interactive theorem proving** that is much more intuitive than current textual interfaces.

Given the early stage of the theory of scroll nets and the expertise of the Theory of Computation group at Birmingham hosting the ER, the SCROLLNETS project will progress towards this vision by focusing on the

[20] Pablo Donato, "Scroll Nets", July 2025, https://doi.org/10.48550/arXiv.2507.19689,

[21] Bonchi, Filippo, et al. "Diagrammatic Algebra of First Order Logic". *Proceedings of the 39th Annual ACM/IEEE Symposium on Logic in Computer Science*, LICS '24, 2024. https://doi.org/10.1145/3661814.3662078.

[22] Milner, Robin. "Bigraphical Reactive Systems". *Proceedings of the 12th International Conference on Concurrency Theory*, CONCUR '01, Aug. 2001, pp. 16–35. Accessed 1 July 2025.

[23] Mackay, Wendy E., and Michel Beaudouin-Lafon. "Interaction Substrates: Combining Power and Simplicity in Interactive Systems". *Proceedings of the 2025 CHI Conference on Human Factors in Computing Systems*, Apr. 2025, pp. 1–16. https://doi.org/10.1145/3706598.3714006.

following Key Objectives:

(KO1) extend the theory of scroll nets to account for **richer logics**, beyond minimal implicative logic;

(KO2) find natural and efficient **translations** of state-of-the-art proof systems and typed models of computation into scroll nets.

This should support the following very ambitious long-term goal of the project:

> *Establish **scroll nets** as the foundation for a new generation of ITPs that support **interactive refinement** of formal specifications, proofs and programs through **diagrammatic manipulations**.*

1.2 Soundness of the proposed methodology

The research objectives of ScrollNets are organised into five technical work packages, detailed below. The core theory will be developed in WP1 and then extended to richer logics in WP2, WP3 and WP4, meeting KO1. This will culminate in WP5 with the study of translations from state-of-the-art models of computation, achieving KO2.

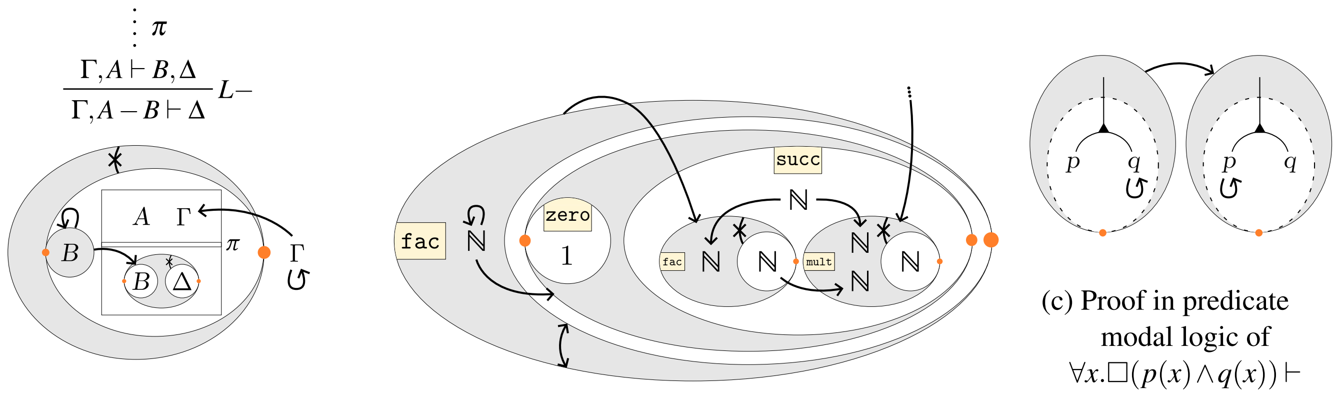
1. WP1 will work in the restricted and well-understood setting of minimal implicative logic, with the aim to prove formally two key meta-theoretic properties of scroll nets:

   - **Normalization** is a standard property of structural proof systems that asserts the existence (and sometimes unicity) of a *normal form* associated to every proof, often exhibited constructively by a terminating (sometimes confluent) rewriting system. The ER has already sketched in his preprint such a rewriting system, called *detour elimination*. It will need a rigorous mathematical analysis to reach its definitive form and obtain more insights on its computational properties. We expect that detour elimination will be both terminating and confluent as well as highly parallelizable, in analogy with normalization procedures for other graphical formalisms like proof nets and interaction nets.

   - **Sequentialization** originates from proof nets: it is a fundamental theorem establishing the *canonicity* of proof nets as a representation of proofs in (some fragments of) linear logic, quotienting proofs in sequent calculus modulo rule permutations. The ER has observed a similar phenomenon in scroll nets, where multiple *proof traces* (as in Fig. 1a) which only vary in the order of inferences can build the same scroll net (as in Fig. 1b). Moreover, there is a bijective correspondence between the inferences in a proof trace and their static representation as edges in the associated scroll net, which preserves enough information to enable efficient reconstruction of possible traces from the latter. WP1 will workout the details of the sequentialization theorem and define formally such a reconstruction algorithm, which is not known to have any equivalent in the proof-theoretic landscape.

2. Oostra already identified how to capture intuitionistic disjunction in EGs, by considering a *horizontal* generalization of the scroll where one can have an arbitrary number h of inloops attached to the same outloop[24]. The ER has performed preliminary experiments on a further *vertical* generalization of the scroll where inloops can be recursively attached to other inloops, leading to a notion of (h,v)-scroll with h (resp. v) counting the maximum number of inloops in horizontal (resp. vertical) dimension. Then intuitionistic disjunction and falsehood correspond to the cases where $h > 1$ and $h < 1$, while intuitionistic *subtraction* (also called *exclusion* or *co-implication*) and negation correspond to the cases where $v > 1$ and $v < 1$. This is illustrated in Fig. 2a, which shows the natural encoding of the left introduction rule for subtraction in sequent calculus into scroll nets. WP2 will prove that generalized

[24] Arnold Oostra, *Los gráficos Alfa de Peirce aplicados a la lógica intuicionista*, Cuadernos de Sistemática Peirceana 2 (Centro de Sistemática Peirceana, 2010).



(a) Encoding intuitionistic subtraction

(b) Encoding natural numbers and the factorial function

(c) Proof in predicate modal logic of $\forall x.\Box(p(x) \wedge q(x)) \vdash (\forall x.\Box p(x)) \wedge (\forall x.\Box q(x))$

Figure 2: Extensions of scroll nets to richer logics (WP2 – WP4)

scrolls lead to a sound and complete characterization of intuitionistic, bi-intuitionistic and classical logic by considering various restrictions on their shapes, as well as the topological properties of the graphs of inference rules that ensue. It will also ensure that the sequentialization and normalization results of WP1 extend well to this setting, which would constitute important contributions to both C1 and C2 by providing a non-trivial equational theory for classical and bi-intuitionistic proofs.

3. Once disjunction is properly handled by the results of WP2, adding *smallest* and *greatest fixpoints* of propositions is a natural step to take, that would bring the computational power of state-of-the-art functional programming languages. Indeed, the combination of these two constructs corresponds to *(co-)recursive algebraic data types* through the CHC, which are essential to express (co-)recursive computation on (infinite) tree-structured data. Fig. 2b gives a proof-of-concept of how this could be implemented in scroll nets through the classic example of the factorial function. The idea is to enlarge the space of correct proofs by allowing *self-reference* in inference rules, here represented by the arrow that calls the `fac` scroll into itself. WP3 will explore this idea further, addressing both C2 and KO2.

4. Work packages WP1 to WP3 deal with various flavours of propositional logic, corresponding to Peirce's system Alpha of EGs. However, Peirce also explored extensions of EGs with new diagrammatic constructs that go beyond the language of propositional logic: system Beta used so-called *lines of identity* (LoI) to capture both quantifiers and equality in *predicate logic*; while system Gamma was a more experimental attempt at capturing both *modal* and *higher-order* logics with a special kind of epistemic negation called "broken cut", represented by a dashed ellipse whose content should be interpreted as "possibly not true"[25]. This is illustrated in Fig. 2c, where the left scroll can be read classically as $\neg\exists x.\diamond\neg(p(x) \wedge q(x))$ which is classically equivalent to the premiss $\forall x.\Box(p(x) \wedge q(x))$: the LoI connecting p, q and the outloop represents the universally quantified variable x, while the dashed inloop represents necessity. WP4 will explore the precise inference rules governing these constructs, trying to accomodate the partial descriptions left in Peirce's writings to the modern CHC conception inherent to scroll nets. We expect that LoI can express *dependent product/sum* types and *identity* types, while broken cuts could capture either type-theoretic *modalities* or type-theoretic *universes* (or both), thus contributing greatly to KO1. WP4 will also be essential to reach the necessary expressive power to simulate CoIC, and thus situate scroll nets as a realistic new foundation for ITPs.

5. The Functional Machine Calculus (FMC) and Relational Machine Calculus (RMC) are two foundational models of computation introduced very recently by W. Heijltjes, also co-inventor of intuition-

[25] Minghui Ma and Ahti-Veikko Pietarinen, "Gamma Graph Calculi for Modal Logics", *Synthese* vol. 195, no. 8, Aug. 2018, https://doi.org/10.1007/s11229-017-1390-3, pp. 3621–50.

istic combinatorial proofs[26]. They achieve respectively a unification of the two main paradigms of declarative programming, *functional* and *logic* programming, with the more mainstream paradigm of *imperative* programming. An open problem is to find a suitable combination of the FMC and RMC that could subsume all three paradigms. WP5 will explore the potential of scroll nets as a solution in the well-typed fragment, by devising translations of the FMC and RMC that preserve both its denotational and operational semantics. This will build on WP1 for the operational semantics given by detour elimination; WP2 to account for non-determinism and failure with $(n,1)$-scrolls for sum types ($n > 1$) and empty types ($n < 1$); WP3 to account for recursion of the Kleene star operator with smallest and greatest fixpoints; and WP4 to account for unification with LoI for dependent types.

Interdisciplinarity By incorporating the CHC at the heart of its methodology, ScrollNets is by its very nature interdisciplinary, placing itself at the crossroad of **mathematical logic** and **programming language theory**. Although not the focus of this project, the long-term vision of exploiting scroll nets as a medium for powerful and intuitive GUIs in ITPs warrants future intersections with the field of **human-computer interaction** (HCI). Through its dealing with the very foundations of the notion of formal proof, ScrollNets also exhibits a strong **philosophical** flavor, which will be technically realized through its systematic hermeneutics of the original writings of C. S. Peirce on EGs. Peirce was probably one of the last polymathic genius in the history of western science, and his broad holistic vision of logic as the formal investigation of the principles of scientific inquiry certainly influences the ER's own vision, putting ScrollNets in the realm of bordering areas of philosophy like *epistemology*, *semiotics* and *metaphysics*.

Gender Not applicable, because the research of ScrollNets is of an abstract and theoretical nature.

Open science and research data management All the results of ScrollNets will be made available as arXiv pre-prints, with the aim of encouraging informal evaluation from the scientific community. Moreover, the ER will (co-)organise a workshop to communicate results and encourage feedback from other researchers (Sec. 2.2). The project deliverables will be published in the proceedings of high-rated and peer-reviewed conferences and in scientific journals (Sec. 2.2). Proceeding publications will be made available on arXiv, and journal contributions will be published in Open Access format, for which funding is foreseen. ScrollNets does not envisage the collection of any kind of data, and eventual software prototypes experimenting with an implementation of scroll nets will all be open-sourced.

1.3 Quality of the supervision, training and of the two-way transfer of knowledge between the researcher and the host

Quality of the supervision The project will be supervised by Anupam Das, associate professor at the School of Computer Science of the University of Birmingham in the Theory of Computation (ToC) group. Prof. Das is a leading expert in proof theory, who is nowadays particularly active in the area of cyclic proofs for fixpoint logics (WP3). He has a strong background in deep inference, linear logic and their applications to complexity theory, and has also made multiple contributions to the study of intuitionistic modal logics (WP4). His supervision will thus be essential to the success of ScrollNets.

[26] Willem Heijltjes, "The Functional Machine Calculus", *Electronic Notes in Theoretical Informatics and Computer Science* vol. Volume 1 - Proceedings of... *arXiv*, arxiv.org/abs/2212.08177, 22 Feb. 2023, https://doi.org/10.46298/entics.10513, p. 10513; Barrett, Chris, et al. "The Relational Machine Calculus". *Proceedings of the 39th Annual ACM/IEEE Symposium on Logic in Computer Science*, 8 July 2024, pp. 1–15. https://doi.org/10.1145/3661814.3662091; Heijltjes, Willem B., et al. "Intuitionistic proofs without syntax". *2019 34th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS)*, June 2019, pp. 1–13. https://doi.org/10.1109/LICS.2019.8785827.

Prof. Das has an excellent record of almost 60 publications in top-tier journals (LMCS, JAR) and peer-reviewed conference proceedings (LICS, CSL, FSCD, IJCAR, TABLEAUX) in logic, theoretical computer science and automated theorem proving. He also has a good track record in mentoring young researchers: he co-supervised 2 Ph.D. students with Prof. Escardo and Prof. Paul Levy — with whom the ER expects collaborations on WP4 and WP5 — and supervised 5 postdocs thanks to the funding that he obtained as a recipient of the UKRI Future Leaders Fellowship *Structure vs. Invariants in Proofs* (2020–2024, renewed for 2024–2027). He was himself an MSCA postdoctoral fellow from 2017 to 2019.

Host-to-applicant transfer of knowledge The project will greatly benefit from the combined expertise of several leading researchers at ToC. For WP2, the ER will collaborate with Dr. Shillito, a postdoctoral fellow with internationally recognized expertise in the syntax and semantics of bi-intuitionistic logic. For WP4, the ER will also collaborate with Dr. Marin and Prof. Escardo, eminent specialists in modal logics and dependent types/homotopy type theory/ITPs, respectively. For the foundational work in WP1 and its application in WP5, the ER will collaborate with Prof. Ghica, who has been applying graphical formalisms to model computations for several decades in both academia and industry. For WP5, the ER will additionally collaborate with Prof. Paul Levy, a renowned expert in effectful programming languages. His groundbreaking work in this field was recognized with the 2025 Alonzo Church Award for Outstanding Contributions to Logic and Computation.

Applicant-to-host transfer of knowledge The ER will bring to ToC his worldwide unique expertise on EGs and their proof theory, as well as the application of deep inference and graphical proof theory to interactive theorem proving. He will share his knowledge through one-to-one interactions with ToC members and students, by speaking at seminars, by supervising Master projects and by (co-)organising a workshop on SCROLLNETS topics (Sec. 2.2).

Training The ER's overarching career goal is to secure a computer science professorship in Europe. In the medium term, he aspires to obtain an unsupervised temporary position and funding by an ERC starting grant. The acquisition of a prestigious MSCA grant would significantly enhance his prospects. The proposed training program developed with the supervisor is structured into five Training Objectives:

TO1. **Gaining scientific knowledge** The SCROLLNETS project will advance the ER's expertise in theoretical domains, complementing his practical software engineering skills obtained during his PhD through substantial implementation work of ITP technology. During his tenure at the host institution, the ER will expand his scientific horizon by immersing himself in the English school of proof theory, the historical centre for the development of the deep inference methodology. Here, he will gain further knowledge of advanced state-of-the-art topics such as cyclic proofs, intuitionistic modal logics and homotopy type theory. At least weekly meetings with his supervisor will serve as forums to assess project progress, explore scientific inquiries, and exchange innovative ideas.

TO2. **Developing communication and public outreach skills**

TO3. **Enhance management and leadership skills**

TO4. **Enhance supervision and teaching skills**

The ER will actively engage in mentoring the supervisor's students, providing individual guidance on their research projects, and offering courses in specialised subjects such as structural proof theory and interactive theorem proving. He will also take on responsibilities such as crafting research internship proposals and overseeing interns during 2-3 month periods of study. The ER will actively participate in the ToC's weekly seminar series, featuring speakers from renowned global research institutions. Moreover, the ER will be an active participant in the national monthly "Chocola" meetings held in Lyon. As a fellowship recipient, the

ER will also acquire grant management skills and enhance his grant proposal writing abilities, supported by the Inria cell of the European Research Network.

1.4 Quality and appropriateness of the researcher's professional experience, competences and skills

Discuss the quality and appropriateness of the researcher's existing professional experience in relation to the proposed research project.

2 Impact #@IMP-ACT-IA@#

2.1 Credibility of the measures to enhance the career perspectives and employability of the researcher and contribution to his/her skills development

At a minimum, address the following aspects:

- Specific measures to enhance career perspectives and employability of the researcher inside and/or outside academia.
- *Expected* contribution of proposed skills development to the future career of the researcher.

2.2 Suitability and quality of the measures to maximise expected outcomes and impacts, as set out in the dissemination and exploitation plan, including communication activities #@COM-DIS-VIS-CDV@#

At a minimum, address the following aspects:

- *Plan for the dissemination and exploitation activities, including communication activities*[27]: Describe the planned measures to maximize the impact of your project by providing a first version of your 'plan for the dissemination and exploitation including communication activities'. Describe the dissemination, exploitation measures that are planned, and the target group(s) addressed (e.g. scientific community, end users, financial actors, public at large). Regarding communication measures and public engagement strategy, the aim is to inform and reach out to society and show the activities performed, and the use and the benefits the project will have for citizens. Activities must be strategically planned, with clear objectives, start at the outset and continue through the lifetime of the project. The description of the communication activities needs to state the main messages as well as the tools and channels that will be used to reach out to each of the chosen target groups.
- *Strategy for the management of intellectual property, foreseen protection measures*: if relevant, discuss the strategy for the management of intellectual property, foreseen protection measures, such as patents, design rights, copyright, trade secrets, etc., and how these would be used to support exploitation.
- All measures should be proportionate to the scale of the project, and should contain concrete actions to be implemented both during and after the end of the project.

[27] In case your proposal is selected for funding, a more detailed Dissemination and Exploitation plan will need to be provided as a mandatory project deliverable during project implementation.

Risk	
The FMC or RMC's expressivity makes it difficult to find an efficient simulation in scroll nets.	Close collaborati

2.3 *The magnitude and importance of the project's contribution to the expected scientific, societal and economic impacts*

- Provide a narrative explaining how the project's results are expected to make a difference in terms of impact, beyond the immediate scope and duration of the project. The narrative should include the components below, tailored to your project.
- Be specific, referring to the effects of your project, and not R&I in general in this field. State the target groups that would benefit.
- The impacts of your project may be:
  - *Scientific*: e.g. contributing to specific scientific advances, across and within disciplines, creating new knowledge, reinforcing scientific equipment and instruments, computing systems (i.e. research infrastructures);
  - *Economic/technological*: e.g. bringing new products, services, business processes to the market, increasing efficiency, decreasing costs, increasing profits, contributing to standards' setting, etc.
  - *Societal*: e.g. decreasing CO2 emissions, decreasing avoidable mortality, improving policies and decision-making, raising consumer awareness.
- Only include such outcomes and impacts where your project would make a significant and direct contribution. Avoid describing very tenuous links to wider impacts.
- Give an indication of the magnitude and importance of the project's contribution to the expected outcomes and impacts, should the project be successful. Provide credible quantified estimates where possible and meaningful.

"Magnitude" refers to how widespread the outcomes and impacts are likely to be. For example, in terms of the size of the target group, or the proportion of that group, that should benefit over time.

"Importance" refers to the value of those benefits. For example, number of additional healthy life years; efficiency savings in energy supply.

#§COM-DIS-VIS-CDV§# #§IMP-ACT-IA§#

3 *Quality and Efficiency of the Implementation* #@QUA-LIT-QL@# #@WRK-PLA-WP@# #@CON-SOR-CS@# #@PRJ-MGT-PM@#

3.1 *Quality and effectiveness of the work plan, assessment of risks and appropriateness of the effort assigned to work packages*

At a minimum, address the following aspects:

- Brief presentation of the overall structure of the work plan, including deliverables and milestones.
- Timing of the different work packages and their components;
- Mechanisms in place to assess and mitigate risks (of research and/or administrative nature).

A Gantt chart must be included and should indicate the proposed Work Packages (WP), major deliverables, milestones, secondments, placements, if applicable. This Gantt chart counts towards the 10-page limit.

The schedule in the Gantt chart should indicate the number of months elapsed from the start of the action (Month 1, Month 2, etc.), but no actual dates.

3.2 Quality and capacity of the host institutions and participating organisations, including hosting arrangements

At a minimum, address the following aspects:

- Hosting arrangements, including integration in the team/institution(s) and support services available to the researcher.
- Quality and capacity of the participating organisations, including infrastructure, logistics and facilities. Additional information should be outlined in Part B-2 Section 5 ("Capacity of the Participating Organisations").

Note that for GF, both the quality and capacity of the outgoing Third Country host and the return host should be outlined.

Associated partners linked to a beneficiary [28]

If applicable, outline here the involvement of any 'associated partners linked to a beneficiary' (in particular, the name of the entity, the type of link with the beneficiary and the tasks to be carried out).

#§CON-SOR-CS§# #§PRJ-MGT-PM§# #§QUA-LIT-QL§# #§WRK-PLA-WP§#

[28] See the definitions section of the MSCA Work Programme for further information.