Part B-1

1 Excellence #@REL-EVA-RE@#

1.1 Quality and pertinence of the project's research and innovation objectives

Proof assistants — also called **interactive theorem provers (ITPs)** — are software tools used to rigorously verify formal modelling and reasoning. Contemporary systems such as *Rocq*, *Lean*, and *Isabelle*[1] offer powerful frameworks for constructing formal specifications and proofs: they have been used successfully in various applications, ranging from the verification of advanced theorems in mathematics to the certification of complex software artifacts such as programming language compilers, operating system kernels and cryptographic protocols[2]. Yet they remain notoriously difficult to learn and use, limiting broader adoption in various settings where they could bring transformative societal impact such as *mathematics education*, *formal verification* and *AI safety*.

The experienced researcher's (ER) work tackles the problem of usability from an original angle, centering around **human interaction**. Building on *Proof-by-Pointing* and *Proof-by-Linking*[3], the *Proof-by-Action* (PbA) paradigm introduced in the ER's PhD thesis enables the construction of proofs by directly manipulating logical statements in a *graphical user interface* (GUI) via natural gestures like *pointing* and *drag-and-drop*. This is still rooted in rigorous **proof theory** by drawing on the recent methodology of *deep inference*[4], and complements fully automated approaches by making **interactive refinement** of proofs more intuitive, thus lowering the barrier to entry.

In this direction, the **scroll nets** introduced recently by the ER offer a new **diagrammatic** framework rooted in Peirce's *existential graphs* (EGs). One performs proofs in EGs by rewriting diagrams in contexts of arbitrary depth (Fig. 1a), and scroll nets record these rewrites *statically* as arrows in a directed graph (Fig. 1b). This yields a representation of proofs that subsumes the *simply-typed λ-calculus* (STLC), the programming language underlying modern ITPs.

Motivated by this discovery, the SCROLLNETS project will pursue the following ambitious objective:

> *Establish **scroll nets** as the foundation for a new generation of **ITPs** that support **interactive refinement** of formal specifications, proofs and programs through **diagrammatic manipulations** in a GUI.*

Given the early stage of the theory of scroll nets and the expertise of the Theory of Computation group at Birmingham hosting the ER, the SCROLLNETS project will progress towards this vision by focusing on the following Key Objectives:

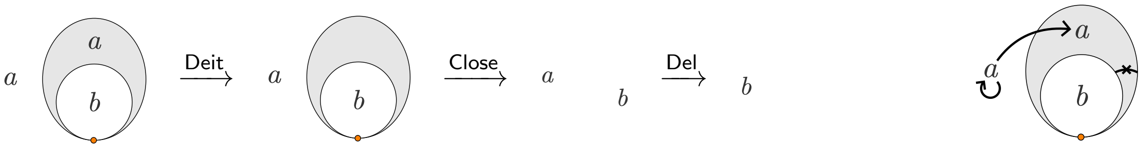
- **KO1.** extend the theory of scroll nets to account for **richer logics**, beyond minimal implicative logic;
- **KO2.** find natural and efficient **translations** of state-of-the-art (SOTA) proof systems and programming languages into scroll nets.

[1] Team, The Rocq Development. The Rocq Prover. Apr. 2025. https://doi.org/10.5281/zenodo.15149629; Moura, Leonardo de, and Sebastian Ullrich. *Automated Deduction – CADE 28*. Edited by André Platzer and Geoff Sutcliffe, 2021, pp. 625–35; Tobias Nipkow, et al., *Isabelle/HOL: a proof assistant for higher-order logic*, vol. 2283 (Springer Science & Business Media, 2002).

[2] Georges Gonthier, "Formal Proof—The Four- Color Theorem" vol. 55, no. 11, 2008, Xavier Leroy, "Formal Verification of a Realistic Compiler", *Commun. ACM* vol. 52, no. 7, July 2009, https://doi.org/10.1145/1538788.1538814, pp. 107–15; Klein, Gerwin, et al. "seL4: Formal Verification of an OS Kernel". *Proceedings of the ACM SIGOPS 22nd Symposium on Operating Systems Principles*, SOSP '09, Oct. 2009, pp. 207–20. https://doi.org/10.1145/1629575.1629596; Gilles Barthe, et al., "EasyCrypt: A Tutorial", *Foundations of Security Analysis and Design VII: FOSAD 2012/2013 Tutorial Lectures*, edited by Alessandro Aldini et al. (Cham: Springer International Publishing, 2014) pp. 146–66.

[3] Yves Bertot, et al., "Proof by pointing", *Theoretical Aspects of Computer Software*, edited by Masami Hagiya and John C. Mitchell, redacted by Gerhard Goos and Juris Hartmanis, vol. 789 (Berlin, Heidelberg: Springer Berlin Heidelberg, 1994) pp. 141–60; Kaustuv Chaudhuri, "Subformula Linking as an Interaction Method", *Interactive Theorem Proving*, edited by Sandrine Blazy et al., redacted by David Hutchison et al., vol. 7998 (Berlin, Heidelberg: Springer Berlin Heidelberg, 2013) pp. 386–401.

[4] Guglielmi, Alessio. Deep Inference. 2014. people.bath.ac.uk/ag248/p/DI.pdf. Accessed 2 Jan. 2024.



(a) Dynamic rewriting from premiss $a \wedge (a \Rightarrow b)$ to conclusion b

(b) Static representation of rules as arrows in a bigraph

Figure 1: Proof of *modus ponens* $a \wedge (a \Rightarrow b) \vdash b$ in scroll nets. Intuitionistic implication $a \Rightarrow b$ is represented topologically by a *scroll*, i.e. two nested ellipses that intersect exactly at one point, where the outer gray *outloop* (resp. inner white *inloop*) contains the antecedant a (resp. consequent b).

1.1.1 State-of-the-art and contributions

Despite their potential, the adoption of ITPs is limited by both social and foundational factors. While community building, public communication and learning resources are crucial (as shown by the success of Lean and its Mathlib library), many researchers point to deeper issues. A key problem is the overwhelming **bureaucracy** of formalization: formal proofs demand a level of detail that makes them far more time-consuming than informal proofs, with proof size around four times bigger on average[5].

The primary strategy to manage this complexity is *automation*, covering both the *elaboration* of specifications from informal mathematical texts and the *synthesis* of proofs/programs satisfying these specifications. While research on both fronts progresses with the help of SOTA machine learning techniques[6], theoretical work in **type theory** remains fundamental. As the logical backbone of modern ITPs, type theory provides the trust and reliability that distinguishes these systems from purely probabilistic AI.

The modern version of type theory was born in the 1970s from Martin-Löf's work, following the **Curry-Howard correspondence** (CHC) between formal proofs (in the system of *natural deduction*) and computer programs (expressed in the STLC). Despite this remarkable connection, proof theory and type theory have been developed mostly as separate disciplines with different conceptual apparati, preventing cross-pollination of results. This has led the ITP community to devote little attention to *structural* proof theory, the branch of proof theory which studies combinatorial structures for representing and manipulating proofs. The latter could however bring much improvements to the foundations of ITPs, as illustrated by the three broad challenges that motivate its study:

C1. Challenge 1 is the fundamental problem of **proof identity**, also known as Hilbert's 24$^{\text{th}}$ problem[7]. It aims to answer the philosophical question "what is a proof?", and the mathematical question "when are two proofs equal?". It is thus intimately related to *homotopy type theory* which also investigates the structure of *equality*, a known weakness of many type theories.

C2. Challenge 2 is to find proof systems for **non-standard logics** — such as *modal*, *intermediate*, *substructural* and *fixpoint* logics — satisfying good enough properties as to render an algorithmic treatment of these logics tractable. The most important property in this respect is that of **cut elimination**, which is essential both to reduce the complexity of *proof search* (proof synthesis in ITP terminology), and to ensure productivity of *program execution* through the CHC. Researchers are increasingly interested in type-theoretic formulations of non-standard logics as they provide expressive languages for specifying behaviors of programs that go beyond pure functions, including effects (modal logics), resource-sensitivity (modal and substructural logics) and recursion (modal and fixpoint logics)[8].

[5] Wiedijk, Freek. The De Bruijn Factor. 2000. www.cs.ru.nl/F.Wiedijk/factor/factor.pdf.

[6] Lasse Blaauwbroek, et al., "Learning Guided Automated Reasoning: A Brief Survey", vol. 14560 (2024) pp. 54–83.

[7] Lutz Straßburger, "The problem of proof identity, and why computer scientists should care about Hilbert's 24th problem", *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* vol. 377, no. 2140, 11 Mar. 2019, https://doi.org/10.1098/rsta.2018.0038, p. 20180038.

[8] Wenhao Tang, et al., "Modal Effect Types", *Proc. ACM Program. Lang.* vol. 9, OOPSLA1, Apr. 2025, https://doi.org/10.1145/3720476, Marshall, Danielle, et al. "Linearity and Uniqueness: An Entente Cordiale". *Programming Languages and Systems*, edited by Ilya Sergey, 2022, pp. 346–75. https://doi.org/10.1007/978-3-030-99336-8_13; Ranald Clouston, et al., "The Guarded Lambda-Calculus: Programming and Reasoning with Guarded Recursion for Coinductive Types", *Logical Methods in Computer Science* vol. Volume 12, Issue 3. *arXiv*, arxiv.org/abs/1606.09455, Apr. 2017, https://doi.org/10.2168/LMCS-12(3:7)2016, p. 2019.

C3. Challenge 3 is to improve the **efficiency** of computational procedures on proofs, but also on programs through the CHC. A well-established principle in computer science and software engineering is that choosing appropriate data structures for a problem can lead to orders-of-magnitude improvements in efficiency. Finding the right data structures for such general classes of objects as proofs and programs is thus a very enticing goal with wide implications, including faster automation in ITPs.

In the past decades, two families of proof formalisms have emerged to tackle these challenges:

Graphical proof systems represent proof objects as *graphs* instead of *trees* of inference rules. *Proof nets*[9] are one of the first graphical proof formalisms, meant to reduce the bureauceacy of proofs in linear logic (C2) in order to identify their essence (C1). Further developments stemming from proof nets like the *geometry of interaction* and *interaction nets* have found applications in program optimization for hardware synthesis and parallelized execution[10] (C3). The *combinatorial proofs* of Hughes are direct decendants of proof nets mainly concerned with C1[11], and *string diagrams* from category theory have been applied to logic and programming languages, with well-known connections to proof nets[12].

Deep inference generalizes Gentzen-style proof systems by allowing inference rules to be applied at any depth inside of a formula, rather than being restricted to its top-level logical connective. Calculi of structures and so-called *nested sequent calculi* have been introduced to give proof systems enjoying cut-elimination to most substructural, modal and intermediate logics[13] (C2). Deep inference has also been used in the study of proof complexity, providing in some cases exponential speedup over sequent calculus with respect to proof size, as well as quasipolynomial-time cut-elimination[14] (C3). Lastly, many deep inference formalisms enjoy CHC-style interpretations with variants of λ-calculus, which also improve space efficiency[15].

It is in continuation of this line of research and of his PhD thesis[16] that the ER introduced in his last preprint the graphical framework of **scroll nets**[17]. It is based on a long forgotten diagrammatic proof system called *existential graphs* (EGs), invented by the famous philosopher and logician Charles Sanders Peirce at the dusk of the 19$^{\text{th}}$ century — thus predating the very existence of proof theory and computer science. Scroll nets combine the type-theoretic methodology of internalizing inference rules inside judgments with a graphical structure comparable to proof nets and combinatorial proofs. This graph shares the same nodes as the propositions involved in the proof, making scroll nets a more compact representation than these graphical proof structures, but also surprisingly a variant of the notion of *bigraph*. Bigraphs were introduced by Milner as a foundational combinatorial structure encompassing most models of concurrent/parallel computation, including Petri nets and his own CCS and π-calculus[18]. Contrary to proof nets which stem from sequent calculus, scroll nets also enjoy a notion of *detour* arising from the interplay of introduction and elimination rules, making them closer to natural deduction. Although they were developed completely independently,

[9] Jean-Yves Girard, "Linear logic", *Theoretical Computer Science* vol. 50, no. 1, 1987, https://doi.org/https://doi.org/10.1016/0304-3975(87)90045-4, pp. 1–101.

[10] Ghica, Dan R. "Geometry of Synthesis: A Structured Approach to VLSI Design". *Proceedings of the 34th Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages*, POPL '07, Jan. 2007, pp. 363–75. https://doi.org/10.1145/1190216.1190269; Mackie, Ian. "An Interaction Net Implementation of Closed Reduction". *Implementation and Application of Functional Languages*, edited by Sven-Bodo Scholz and Olaf Chitil, 2011, pp. 43–59. https://doi.org/10.1007/978-3-642-24452-0_3.

[11] Dominic Hughes, "Proofs without syntax", *Annals of Mathematics* vol. 164, no. 3, Nov. 2006, https://doi.org/10.4007/annals.2006.164.1065, pp. 1065–76.

[12] Robin Piedeleu and Fabio Zanasi, "An Introduction to String Diagrams for Computer Scientists", *Elements in Applied Category Theory*, May 2025, https://doi.org/10.1017/9781009625715,

[13] Roman Kuznets and Lutz Straßburger, "Maehara-style modal nested calculi", *Archive for Mathematical Logic* vol. 58, nos. 3–4, May 2019, https://doi.org/10.1007/s00153-018-0636-1, pp. 359–85; Postniece, Linda. "Proof theory and proof search of bi-intuitionistic and tense logic". 2010, Artwork Size: vii, 228 leaves., vii, 228 leaves. https://doi.org/10.25911/5D5FCC3A4DB33.

[14] Anupam Das, "On the Relative Proof Complexity of Deep Inference via Atomic Flows", *Logical Methods in Computer Science* vol. Volume 11, Issue 1, Mar. 2015, https://doi.org/10.2168/LMCS-11(1:4)2015, p. 735; Paola Bruscoli, et al., "Quasipolynomial Normalisation in Deep Inference via Atomic Flows and Threshold Formulae", *Logical Methods in Computer Science* vol. Volume 12, Issue 2, May 2016, https://doi.org/10.2168/LMCS-12(2:5)2016, p. 1637.

[15] Nicolas Guenot, "Nested Deduction in Logical Foundations for Computation", PhD dissertation, Ecole Polytechnique X, 2013; Gundersen, Tom, et al. "Atomic Lambda Calculus: A Typed Lambda-Calculus with Explicit Sharing". *28th Annual ACM/IEEE Symposium on Logic in Computer Science*, June 2013, pp. 311–20. https://doi.org/10.1109/LICS.2013.37.

[16] Donato, Pablo. Deep Inference for Graphical Theorem Proving. 29 May 2024. theses.hal.science/tel-04698985.

[17] Pablo Donato, "Scroll Nets", July 2025, https://doi.org/10.48550/arXiv.2507.19689,

[18] Milner, Robin. "Bigraphical Reactive Systems". *Proceedings of the 12th International Conference on Concurrency Theory*, CONCUR '01, Aug. 2001, pp. 16–35. Accessed 1 July 2025.

EGs and natural deduction were both born from the desire to construct a formalism that comes as close as possible to actual reasoning, and is thus more amenable to human manipulation.

All these discoveries hint toward the potential of scroll nets as a very expressive framework for both proof theory and type theory, unifying features found in most of the formalisms that have emerged from the two disciplines in a natural and efficient way. The ER's vision is to exploit the diagrammatic nature of scroll nets to redesign both the *frontend* and *backend* of ITPs, making them the *interaction substrate*[19] of a new kind of GUI in the PbA paradigm. Indeed, what crucially distinguishes scroll nets from other graphical and deep inference proof systems is their suitability to *interactive manipulation*: every inference rule possesses a natural interpretation as a *gesture* on logical statements, either *pointing* at a statement to be inserted or deleted (looping arrow in Fig. 1b), or (un)copying a designated statement through *drag-and-drop* (non-looping arrow in Fig. 1b). This natural semantics of gestures as a means of moving objects in space is very familiar to users of modern GUIs, and the ER believes it could lead to a **groundbreaking "no-code" approach to formal specification** that is much more intuitive than current textual interfaces.

1.2 Soundness of the proposed methodology

The research objectives of ScrollNets are organised into four technical work packages, detailed below. The core theory will be developed in WP1 and then extended to richer logics in WP2, WP3 and WP4, meeting KO1. WP3 and WP4 will deal with crucial features found in SOTA proving and programming systems like recursion and dependent types, contributing to KO2.

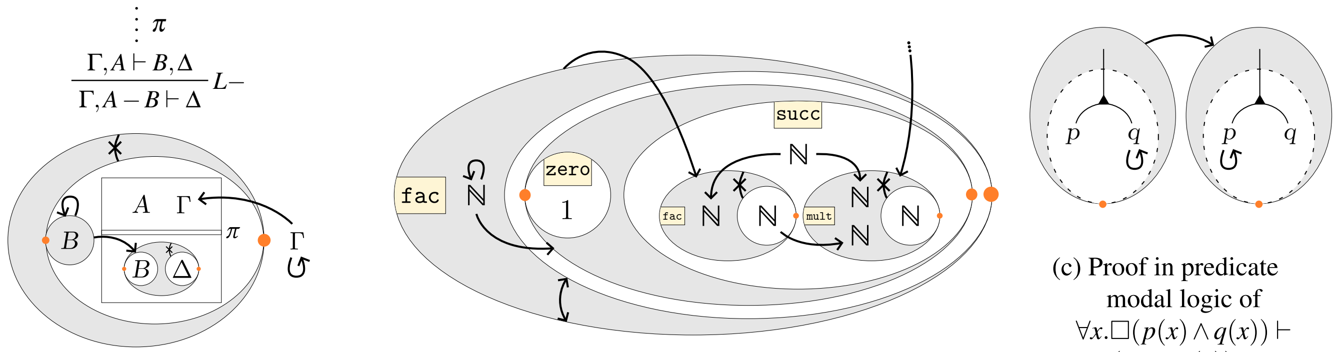
WP1. ScrollNets will start in the restricted and well-understood setting of minimal implicative logic, with the aim to prove formally two key meta-theoretic properties of scroll nets:

- **Normalization** is a standard property of structural proof systems that asserts the existence (and sometimes unicity) of a *normal form* associated to every proof, often exhibited constructively by a terminating (sometimes confluent) rewriting system. The ER has already sketched in his preprint such a rewriting system, called *detour elimination*. It will need a rigorous mathematical analysis to reach its definitive form and obtain more insights on its computational properties. We expect that detour elimination will be both terminating and confluent as well as highly parallelizable, in analogy with normalization procedures for other graphical formalisms like proof nets and interaction nets.
- **Sequentialization** originates from proof nets: it is a fundamental theorem establishing the *canonicity* of proof nets as a representation of proofs in (some fragments of) linear logic, quotienting proofs in sequent calculus modulo rule permutations. The ER has observed a similar phenomenon in scroll nets, where multiple *proof traces* (as in Fig. 1a) which only vary in the order of inferences can build the same scroll net (as in Fig. 1b). Moreover, there is a bijective correspondence between the inferences in a proof trace and their static representation as edges in the associated scroll net, which preserves enough information to enable efficient reconstruction of possible traces from the latter. WP1 will workout the details of the sequentialization theorem and define formally such a reconstruction algorithm, which is not known to have any equivalent in the proof-theoretic landscape.

WP2. Oostra already identified how to capture intuitionistic disjunction in EGs, by considering a *horizontal* generalization of the scroll where one can have an arbitrary number h of inloops attached to the same outloop[20]. The ER has performed preliminary experiments on a further *vertical* generalization of the scroll where inloops can be recursively attached to other inloops, leading to a notion of (h, v)-scroll with h (resp. v) counting the maximum number of inloops in horizontal (resp. vertical) dimension. Then intuitionistic disjunction and falsehood correspond to the cases where $h > 1$ and $h < 1$, while intuitionistic *subtraction* (also called *exclusion* or *co-implication*) and negation correspond to the cases where $v > 1$ and $v < 1$. This is illustrated in Fig. 2a, which shows the natural encoding of the left introduction rule for subtraction in sequent calculus into scroll nets. WP2 will prove that generalized scrolls lead to a sound and complete characterization of intuitionistic, bi-intuitionistic and classical logic by considering

[19] Mackay, Wendy E., and Michel Beaudouin-Lafon. "Interaction Substrates: Combining Power and Simplicity in Interactive Systems". *Proceedings of the 2025 CHI Conference on Human Factors in Computing Systems*, Apr. 2025, pp. 1–16. https://doi.org/10.1145/3706598.3714006.

[20] Arnold Oostra, *Los gráficos Alfa de Peirce aplicados a la lógica intuicionista*, Cuadernos de Sistemática Peirceana 2 (Centro de Sistemática Peirceana, 2010).



(a) Encoding intuitionistic subtraction (b) Encoding natural numbers and the factorial function (c) Proof in predicate modal logic of $\forall x.\Box(p(x) \wedge q(x)) \vdash (\forall x.\Box p(x)) \wedge (\forall x.\Box q(x))$

Figure 2: Extensions of scroll nets to richer logics (WP2 – WP4)

various restrictions on their shapes, as well as the topological properties of the graphs of inference rules that ensue. It will also ensure that the sequentialization and normalization results of WP1 extend well to this setting, which would constitute important contributions to both C1 and C2 by providing a non-trivial equational theory for classical and bi-intuitionistic proofs.

WP3. Once disjunction is properly handled by the results of WP2, adding *least* and *greatest fixpoints* of propositions is a natural step to take, that would bring the computational power of SOTA functional programming languages. Indeed, the combination of these two constructs corresponds to *(co-)recursive algebraic data types* through the CHC, which are essential to express (co-)recursive computation on (infinite) tree-structured data. Fig. 2b gives a proof-of-concept of how this could be implemented in scroll nets through the classic example of the factorial function. The idea is to enlarge the space of correct proofs by allowing *self-reference* in inference rules, here represented by the arrow that calls the `fac` scroll into itself. WP3 will explore this idea further as well as its connections to the emerging technology of *cyclic proofs*, addressing both C2 and KO2.

WP4. Work packages WP1 to WP3 deal with various flavours of propositional logic, corresponding to Peirce's system Alpha of EGs. However, Peirce also explored extensions of EGs with new diagrammatic constructs that go beyond propositional logic: system Beta used so-called *lines of identity* (LoI) to capture both quantifiers and equality in *predicate logic*; while system Gamma was a more experimental attempt at capturing both *modal* and *higher-order* logics with a special kind of epistemic negation called "broken cut", represented by a dashed ellipse whose content should be interpreted as "possibly not true"[21]. This is illustrated in Fig. 2c, where the left scroll can be read classically as $\neg\exists x.\diamond\neg(p(x) \wedge q(x))$ which is classically equivalent to the premiss $\forall x.\Box(p(x) \wedge q(x))$: the LoI connecting p, q and the outloop represents the universally quantified variable x, while the dashed inloop represents necessity. WP4 will explore the precise inference rules governing these constructs, trying to accomodate the partial descriptions in Peirce's writings to the modern CHC conception of SCROLLNETS. We expect that LoI can express *dependent* types and *identity* types, while broken cuts could capture either type-theoretic *modalities* or type-theoretic *universes* (or both), thus contributing greatly to KO1. WP4 will also be essential to reach the necessary expressive power to simulate the *calculus of inductive constructions* — the type theory underlying the leading proof assistants Rocq and Lean — and thus situate scroll nets as a realistic foundation for ITPs.

Interdisciplinarity By incorporating the CHC at the heart of its methodology, SCROLLNETS is by its very nature interdisciplinary, placing itself at the crossroad of **mathematical logic** and **programming language theory**. Although not the focus of this project, the long-term vision of exploiting scroll nets as a medium for powerful and intuitive GUIs in ITPs warrants future intersections with the field of **human-computer interaction** (HCI). Through its dealing with the very foundations of the notion of formal proof, SCROLLNETS also exhibits a strong **philosophical** flavor, which will be technically realized through its systematic hermeneutics of the original writings of C. S. Peirce on EGs. Peirce was probably one of the last polymathic genius in the history of western science, and his broad holistic vision of logic as the formal investigation of the principles of scientific inquiry certainly influences the ER's

[21] Minghui Ma and Ahti-Veikko Pietarinen, "Gamma Graph Calculi for Modal Logics", *Synthese* vol. 195, no. 8, Aug. 2018, https://doi.org/10.1007/s11229-017-1390-3, pp. 3621–50.

own vision, putting SCROLLNETS in the realm of bordering areas of philosophy like *epistemology*, *semiotics* and *metaphysics*.

Gender Not applicable, because the research of SCROLLNETS is of an abstract and theoretical nature.

Open science and research data management All the results of SCROLLNETS will be made available as arXiv pre-prints, with the aim of encouraging informal evaluation from the scientific community. Moreover, the ER will (co-)organise a workshop to communicate results and encourage feedback from other researchers (Sec. 2.2). The project deliverables will be published in the proceedings of high-rated and peer-reviewed conferences and in scientific journals (Sec. 2.2). Proceeding publications will be made available on arXiv, and journal contributions will be published in Open Access format, for which funding is foreseen. SCROLLNETS does not envisage the collection of any kind of data, and eventual software prototypes experimenting with an implementation of scroll nets will all be open-sourced.

1.3 Quality of the supervision, training and of the two-way transfer of knowledge between the researcher and the host

Quality of the supervision The project will be supervised by Anupam Das, associate professor at the School of Computer Science of the University of Birmingham (UoB) in the Theory of Computation (ToC) group. Prof. Das is a leading expert in proof theory, who is nowadays particularly active in the area of cyclic proofs for fixpoint logics (WP3). He has a strong background in deep inference, linear logic and their applications to complexity theory, and has also made multiple contributions to the study of intuitionistic modal logics (WP4). His supervision will thus be essential to the success of SCROLLNETS.

Prof. Das has an excellent record of almost 60 publications in top-tier journals (LMCS, JAR) and peer-reviewed conference proceedings (LICS, CSL, FSCD, IJCAR, TABLEAUX) in logic, theoretical computer science and automated theorem proving. He also has a good track record in mentoring young researchers: he co-supervised 2 Ph.D. students with Prof. Escardo and Prof. Paul Levy — with whom the ER expects collaborations on WP4 and WP5 — and supervised 5 postdocs thanks to the funding that he obtained as a recipient of the UKRI Future Leaders Fellowship *Structure vs. Invariants in Proofs* (2020–2024, renewed for 2024–2027). He was himself an MSCA postdoctoral fellow from 2017 to 2019.

Training The ER's overarching career goal is to secure a computer science professorship in Europe. In the medium term, he aspires to obtain an unsupervised temporary position and funding by an ERC starting grant. The acquisition of a prestigious MSCA grant would significantly enhance his prospects. The proposed training program is structured into three Training Objectives:

TO1. **Gaining scientific knowledge** During his tenure at the host institution, the ER will expand his scientific horizon by immersing himself in the English school of proof theory, the historical centre for the development of the deep inference methodology. Here, he will gain further knowledge of advanced SOTA topics such as cyclic proofs, intuitionistic modal logics and homotopy type theory. At least weekly meetings with his supervisor will serve as forums to assess project progress, explore scientific inquiries, and exchange innovative ideas. The ER will also actively participate in the ToC's weekly seminar series, featuring speakers from renowned global research institutions.

TO2. **Enhance management, leadership and grant writing skills** The ER will participate in several management and leadership skills courses offered by UoB's People and Organization Development (POD) division, including Research Team Leader, Effective Financial Management, and other programs on Health & Safety in the Workplace, Ethics & Governance in Research, Scientific Communication, Public Engagement & Dissemination, and Personal Effectiveness. The ER will also attend the POD Grant Writing workshop. These skills will enrich the ER's ability to adapt to changes and undertake a variety of activities to support his high-level interdisciplinary research capability.

TO3. **Enhance supervision and teaching skills** The ER will actively engage in mentoring the supervisor's students, providing individual guidance on their research projects, and offering courses in specialised subjects such as structural proof theory and interactive theorem proving. He will also take on responsibilities such as crafting research internship proposals and overseeing interns during 2-3 month periods of study.

Applicant-to-host transfer of knowledge The project will greatly benefit from the combined expertise of several leading researchers at ToC. For WP2, the ER will collaborate with Dr. Shillito, a postdoctoral fellow with internationally recognized expertise in the syntax and semantics of bi-intuitionistic logic. For WP4, the ER will also collaborate with Dr. Marin and Prof. Escardo, eminent specialists in modal logics and dependent types/homotopy type theory/ITPs, respectively. For the foundational work in WP1, the ER may collaborate with Prof. Ghica, who has been applying graphical formalisms to model computations for several decades in both academia and industry.

Applicant-to-host transfer of knowledge The ER will bring to ToC his worldwide unique expertise on EGs and their proof theory, as well as the application of deep inference and graphical proof theory to interactive theorem proving. He will share his knowledge through one-to-one interactions with ToC members and students, by speaking at seminars, by supervising Master projects and by (co-)organising a workshop on SCROLLNETS topics (Sec. 2.2).

1.4 Quality and appropriateness of the researcher's professional experience, competences and skills

The ER is an internationally recognised expert in proof theory and interactive theorem proving. His PhD thesis, defended in May 2024, revolves around the design and implementation of novel proof systems that enable the construction of formal proofs through direct manipulation in a GUI. In particular, Chapters 9 and 10 provide a deep 100 pages analysis of the proof theory of EGs both in propositional/first-order and intuitionistic/classical logic, and describe a prototype of GUI for ITPs based on this theory called Flower Prover, which is publicly accessible and usable online[22]. Thus the SCROLLNETS project constitutes a direct continuation and substantial extension of the ER's doctoral research and benefits from his personal interdisciplinary background in computer science, mathematics and philosophy. Indeed, although the ER has been mainly trained in computer science during his studies, he also attended the 1st year of the Université Paris 1's master in Logic and Philosophy of Science (LOPHISC). After finishing his PhD, the ER has secured a 1-year postdoctoral position at the Grothendieck Institute, working with Olivia Caramello and Fields medalist Laurent Lafforgue on the formalization of topos theory in the Lean proof assistant, thus broadening significantly his mathematical knowledge. Topos theory also has well-known applications to the semantics of higher-order and dependent type theories which have been investigated by researchers of ToC like Steve Vickers, making this acquired knowledge relevant to WP4 of SCROLLNETS.

The ER has proven his potential to carry out interesting and impactful research witnessed by two publications at top-tier conferences (CPP, FSCD), as well as his strong independence by being the sole author of his second article. He is an experienced speaker, having delivered ? conference talks, ? workshop talks, and ? invited seminar talks to date. Moreover, he has been invited by Ahti-Veikko Pietarinen — one of the most prolific Peirce scholars in the world with an h-index of 35 — to contribute a chapter to the forthcoming book *Peirce's Philosophy of Notation* in the Peirceana series[23], demonstrating recognition of his interdisciplinary and international expertise on EGs. The ER has been involved in teaching during every year of his PhD: he taught various courses on programming languages at undergraduate and graduate levels, both at Université Paris 7 and École Polytechnique. This involved teaching classes to groups of 50 students and performing duties such as exam grading, project review and practical sessions mentoring.

2 *Impact* #@IMP-ACT-IA@#

2.1 Credibility of the measures to enhance the career perspectives and employability of the researcher and contribution to his/her skills development

The MSCA fellowship is a crucial step towards the ER's long-term goal of securing a professorship in Europe. It will directly support his medium-term ambition of obtaining an ERC starting grant by providing the ideal environment to mature his research programme on scroll nets from a promising initial discovery into a cornerstone of next-generation ITPs. The project will significantly enhance his academic track record through high-impact publications and presentations at top-tier venues, demonstrating his capacity for independent and groundbreaking research.

[22] Donato, Pablo. flower-prover. Oct. 2023. github.com/Champitoad/flower-prover. Accessed 8 Dec. 2023.
[23] https://www.amazon.fr/Peirces-Philosophy-of-Notation/dp/3110649500

The fellowship at UoB offers unparalleled networking opportunities. The ER will be integrated into the vibrant ToC group, fostering daily interactions and collaborations with his supervisor and its members, whose expertise is vital for the success of all five work packages (see Sec. 1.3). More generally, the UK is known to be one of the epicenters for research in proof theory and programming language theory, with leading researchers in these two fields at top universities; this includes the *Mathematical foundations of computation* group at University of Bath, the *Programming Principles, Logic, and Verification* group at University College London, and the *Programming, Logic, and Semantics* group at University of Cambridge. This strategic positioning will expand the ER collaborative network, solidifying his standing in the international community and paving the way for future joint projects. In particular, a collaboration on WP5 is expected with Dr. Heijltjes from University of Bath (see Sec. 1.2). The outcomes of SCROLLNETS, disseminated through open-access publications, pre-prints, and a dedicated workshop (Sec. 2.2), will cement the ER's reputation as a leader in the field, boosting his employability and career prospects.

2.2 Suitability and quality of the measures to maximise expected outcomes and impacts, as set out in the dissemination and exploitation plan, including communication activities #@COM-DIS-VIS-CDV@#

Scientific events The results of the SCROLLNETS project are planned to be published in high-impact journals such as LMCS and TCS, and in the proceedings of conferences like LICS, CSL, FSCD, FroCoS, TABLEAUX and DIAGRAMS. All of these are international top-tier venues focusing on logical topics in computer science, to the exception of DIAGRAMS which is interdisciplinary and open to all fields that study diagrammatic notations, including mathematics, cognitive science, psychology, philosophy and linguistics. DIAGRAMS 2026 will be hosted by Pawel Sobocinski's group in Tallinn, internationally renowned for their expertise in string diagrams, with Francesco Bellucci — co-editor with Pietarinen of the Peirceana book series (see Sec. 1.4) — as program chair. FroCoS is also specifically interested in frameworks that can combine multiple logics and thus particularly relevant to SCROLLNETS. Additionally, the ER will report about the status of the SCROLLNETS project frequently at international workshops like LFMTP or TYPES. He will propose to co-organize with colleagues Dr. Haydon (expert in EGs) and Dr. Di Giorgio (expert in string diagrams) a new "Graphical Logic Workshop" (GLW) affiliated with the Federated Logic Conference (FLoC), filling the absence of a federated community of researchers on SCROLLNETS topics. All these measures will enable a wide range of possible future research to be conducted by contributors, external collaborators, and independent research groups. Regarding the management of intellectual property, as explained in Sec. 1.2, all results of SCROLLNETS will be publicly available online to maximize outreach.

Public communication As the ultimate goal of SCROLLNETS is to propose a new notation for logic and programming that is more accessible to non-experts, the ER will undertake multiple initiatives to engage audiences outside of academia. A personal blog dedicated to scroll nets will be created to explain the basic principles of the formalism, assuming little to no background in logic and programming. This will serve as a direct test for the accessibility and intuitiveness of the notation, and may include interactive animations in the web browser inspired by the popular online learning platform Brilliant.org, whose slogan "Learn by doing" fits nicely with the ER's Proof-by-Action paradigm. To reach a more established audience, the ER will also contribute to Prof. Anupam Das's existing proof theory blog[24]. Furthermore, a YouTube channel may be launched to explore the concepts of visual logic and programming, leveraging the video format to provide dynamic and engaging explanations. Finally, the ER will actively participate in science popularization events such as *The Big Bang Fair*[25] in Birmingham and *Computer Science in Action* in London[26], presenting the project's ideas to students, teachers, and the general public to inspire the next generation of computer scientists.

2.3 The magnitude and importance of the project's contribution to the expected scientific, societal and economic impacts

The core vision of SCROLLNETS is to make formal reasoning more accessible. If successful, its impact will extend far beyond the project's immediate scope, transforming domains where rigorous verification is critical but currently hindered by the steep learning curve of ITPs.

[24] https://prooftheory.blog/about-the-proof-theory-blog/
[25] https://www.thebigbang.org.uk/
[26] https://educationinaction.org.uk/study-day/computer-science-in-action-27-11-2024/

Scientific/Societal Impact in Mathematics Education ScrollNets will provide a “no-code” platform for students to interactively explore mathematical concepts. The diagrammatic nature of scroll nets offers a more intuitive way to grasp abstract proofs than traditional symbolic logic, fostering deeper understanding and engagement[27]. This could democratize access to advanced mathematics and computer science, benefiting students and educators from secondary to higher education by providing a collaborative and visual learning medium.

Economic/Technological Impact in Formal Verification By lowering the barrier to entry, ScrollNets will enable wider adoption of formal methods in industry. The PbA paradigm will allow domain experts (e.g., engineers, systems architects) to directly participate in the verification process without extensive training in ITPs. This will reduce costs and development time while increasing the reliability of safety-critical systems in sectors like aerospace, automotive, and healthcare, leading to significant economic and safety benefits.

Scientific/Technological Impact in AI Safety ScrollNets will contribute to the development of more reliable neurosymbolic AI systems. By providing a transparent and verifiable logical reasoning component, scroll nets can complement the probabilistic nature of LLMs, mitigating the risk of “hallucinations” and ensuring that AI-driven scientific discovery is grounded in rigorous, auditable proof. This enhances the trustworthiness of AI systems in high-stakes research and decision-making.

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3 *Quality and Efficiency of the Implementation* #@QUA-LIT-QL@# #@WRK-PLA-WP@# #@CON-SOR-CS@# #@PRJ-MGT-PM@#

3.1 *Quality and effectiveness of the work plan, assessment of risks and appropriateness of the effort assigned to work packages*

The project is structured into four technical work packages (WP1-WP4) to be completed over the 24-month fellowship. The work plan is designed to front-load the foundational theoretical work in the first year, allowing the second year to focus on extensions, applications, and dissemination.

Deliverables Each technical work package will culminate in a major scientific publication (D1-D4). An additional deliverable (D5) will be the creation of the project website/blog for public communication.

Milestones Key milestones include the completion of the foundational theory (M1), the successful extension to intermediate and fixpoint logics (M2), and the development of the theory for dependent types (M3).

Risks and Mitigation Potential risks to the project have been identified and mitigation strategies are in place to ensure its successful completion. Regular meetings with the supervisor will allow for continuous risk assessment.

Table 1: Risk Assessment and Mitigation Plan

Risk	Mitigation Strategy	Severity
R1: Foundational theory in WP1 (normalization/sequentialization) proves more complex than anticipated, delaying subsequent WPs.	Allocate an extra month for WP1. If major issues arise, consult with external experts in graphical proof theory, such as Dr. Heijltjes.	Medium
R2: The proposed extensions in WP2-WP3 do not yield sound and complete systems or fail to extend the core meta-theorems.	The expertise of the supervisor (cyclic proofs) and collaborators (modal/bi-intuitionistic logic) will be leveraged. If a specific extension fails, focus will shift to the more promising ones.	Medium
R3: Failure to extend the theory to dependent types in WP4.	Intensify collaboration with Prof. Escardo, an expert in dependent types. If significant roadblocks appear, focus will shift to the less speculative modal logic aspects of WP4.	Medium

[27] Minh, Frédéric Tran, et al. *Research Report - Proof Assistants for Teaching: A Survey.* Apr. 2024. Accessed 4 Sept. 2025.

3.2 Quality and capacity of the host institutions and participating organisations, including hosting arrangements

3.2.1 Quality and capacity of the host institution

The University of Birmingham (UoB) provides an ideal environment for SCROLLNETS, being a leading research institution ranked 76th globally in the 2025 QS World University Rankings. Its Theory of Computation (ToC) group is a world-renowned centre for theoretical computer science, featuring prominent researchers like Paul Blain Levy (2025 Alonzo Church awardee), Martin Escardo, and the supervisor, Anupam Das. The university's robust infrastructure and demonstrated success in hosting MSCA Fellows (see Part B-2, Section 5) ensure comprehensive support for the project. UoB's administrative services, including a dedicated Horizon Europe support team, will manage all financial and contractual aspects, from the Grant Agreement to final reporting, allowing the ER to focus on research. The project will adhere to the European Code of Conduct for Research Integrity, with clear protocols for scientific conduct and ethics overseen by the supervisor.

3.2.2 Hosting arrangements

The ER will be fully integrated as an employee at UoB, benefiting from its established support system for MSCA Fellows. This includes access to university services, health and safety resources, and professional development opportunities through the People and Organisational Development (POD) and the Centre for Learning and Academic Development (CLAD). The ER's professional growth will be supported by Personal Development Reviews (PDRs), ensuring alignment with career objectives. UoB is committed to upholding the principles of the European Charter for Researchers and the HR Excellence in Research Award.

Hosted within the ToC group, the ER will join a dynamic community of over 40 researchers, including faculty, postdoctoral fellows, and PhD students. This environment will foster intellectual exchange and collaboration. The group's weekly seminars and working groups will provide a platform for presenting progress on SCROLLNETS, refining milestones, and preparing for dissemination activities. Regular meetings with the supervisor will ensure close guidance and mentorship. The School of Computer Science will provide all necessary resources, including dedicated office space, high-performance computing, and full access to digital libraries and scientific publications from all major publishers.

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