Problem Set 7

> **Due dates:** Electronic submission of the pdf file of this homework is due on **3/21/2025 before 11:59pm** on canvas. The homework must be typeset with LaTeX to receive any credit. All answers must be formulated in your own words.

Watch out for additional material that will appear on Thursday! Deadline is on Friday, as usual.
Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: _____

Read the chapters on "Elementary Graph Algorithms" and "Single-Source Shortest Paths" in our textbook before attempting to answer these questions.

Problem 1 (20 points)**.** Give an algorithm that determines whether or not a given undirected graph $G = (V, E)$ contains a cycle. Your algorithm should run in $O(V)$ time, independent of the number $|E|$ of edges.

Solution.

Problem 2 (20 points)**.** Given a weighted, directed graph $G = (V, E)$ with no negative-weight cycles, let m be the maximum over all vertices $v \in V$ of the minimum number of edges in a shortest path from the source s to v. (Here, the shortest path is by weight, not the number of edges.) Suggest a simple change to the Bellman-Ford algorithm that allows it to terminate in $m + 1$ passes, even if m is not known in advance.

Solution.

Problem 3 (20 points)**.** Suppose that we change line 6 of Dijkstra's algorithm in our textbook to the following.

6 **while** $|Q| > 1$

This change causes the while loop to execute $|V| - 1$ times instead of $|V|$ times. Is this proposed algorithm correct? Explain. [Use the version of Dijkstra's algorithm from the textbook]

Solution.

Problem 4 (40 points)**.** Help Professor Charlie Eppes find the most likely escape routes of thieves that robbed a bookstore on Texas Avenue in College Station. The map will be published on Thursday evening. In preparation, you might want to implement Dijkstra's single-source shortest path algorithm, so that you can join the manhunt on Thursday evening. Include your implementation of Dijkstra's algorithm and explain all details of your choice of the min-priority queue.

[Edge weight 1 means very desirable street, weight 2 means less desirable street]

Solution.

Make sure that you derive the solutions to this homework by yourself without any outside help. Searching for solutions on the internet or asking any form of AI is not allowed. Write the solutions in your own words. Use version control for your program development and be prepared to show and explain any version of your code.

Checklist:

- ☐ Did you add your name?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted)
- ☐ Did you sign that you followed the Aggie honor code?
- ☐ Did you solve all problems?
- ☐ Did you typeset your answers entirely in LaTeX?
- ☐ Did you submit the pdf file of your homework?