Problem Set 8

Due dates: Electronic submission of this homework is due on **Friday 4/4/2025 before 11:59pm** on canvas.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ____________________________

This homework needs to be typeset in LaTeX to receive any credit. All answers need to be formulated in your own words.

You need to gain some familiarity with conditional probabilities, as they figure prominently in many arguments about randomized algorithms. The next two exercises give some opportunity to explore conditional probabilities.

Problem 1 (20 points)**.** Let $B_1, B_2, \ldots, B_t$ denote a partition of the sample space Ω.
(a) Prove that $\Pr[A] = \sum_{k=1}^{t} \Pr[A \mid B_k] \Pr[B_k]$.
(b) Deduce that $\Pr[A] \leq \max_{1\leq k\leq t} \Pr[A \mid B_k]$.

Solution.

Problem 2 (20 points)**.** Consider an experiment, where you toss two fair coins. Give examples of events where (a) $\Pr[A_1 \mid B_1] < \Pr[A_1]$, (b) $\Pr[A_2 \mid B_2] = \Pr[A_2]$, and (c) $\Pr[A_3 \mid B_3] > \Pr[A_3]$. Make sure that your proofs are complete and self-contained.

Solution.

Problem 3 (20 points)**.** Research the Schwartz-Zippel Randomized Polynomial Identity Test. (a) Explain the method. (b) Explain why randomized algorithms are better suited for this problem than deterministic algorithms.

Hint: To get started, you can read:
`https://web.stanford.edu/class/archive/cs/cs265/cs265.1212/Lectures/Lecture1/l1.pdf`

Solution.

Problem 4 (20 points)**.** There may be several different min-cut sets in a graph. Using the analysis of the randomized min-cut algorithm, argue that there can be at most $n(n-1)/2$ distinct min-cut sets.

Solution.

Problem 5 (20 points)**.** A popular choice for pivot selection in Quicksort is the median of three randomly selected elements. Approximate the probability of obtaining at worst an a-to-$(1-a)$ split in the partition (assuming that a is a real number in the range $0 < a < 1/2$). [Hint: Suppose that the median-of-three is the m-th smallest element of the array. Then it gives at worst an a-to-$(1-a)$ split if and only if $an \leq m \leq (1-a)n$. Now count how many sets of three elements can lead to the the pivot (= median-of-three) being the m-th smallest element.]

Solution.

Checklist:
☐ Did you add your name?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit the pdf file of your homework?