Problem Set 5

Due date: Electronic submission of the pdf file of this homework is due on **2/23/2025 before 11:59pm** on canvas.

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Resources. Class Textbook
Wikipedia
GeeksForGeeks (Dynamic Programming Article)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: Chayce Leonard ____________________________

Make sure that you describe all solutions in your own words. Typesetting in LaTeX is required. Read chapters 14 and 15 in our textbook.

Problem 1 (20 points)**.** Determine an LCS of $X = \langle R, H, U, B, A, R, B\rangle$ and $Y = \langle S, T, R, A, W, B, E, R, R, Y\rangle$ using the dynamic programming algorithm that was discussed in class. Make sure that you explain your answer step-by-step, in detail, rather than just giving an LCS. [Make sure that you list X vertically, and Y horizontally when constructing the table. If $x_i \neq y_j$, and $c[i-1, j] = c[i, j-1]$, choose $c[i-1, j]$. Explain row-by-row how the table is constructed.]

Why do we care? For every algorithm, you should make sure that you can work it out step-by-step.

Solution. test

Problem 2 (20 points)**.** Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is $\langle 5, 10, 3, 12, 5, 50, 6\rangle$. Explain how you found the solution. [Hint: You might find it worthwhile to implement the dynamic programming algorithm, and print the relevant tables to aid in your explanations. However, you should make sure that you are able to solve it by hand as well.]

Solution.

Problem 3 (20 points)**.** Use a proof by induction to show that the solution to the recurrence

$$P(n) = \begin{cases} 1 & \text{if } n = 1, \\ \sum_{k=1}^{n-1} P(k)P(n-k) & \text{if } n \geq 2. \end{cases}$$

is $\Omega(2^n)$.

Why do we care? This is a simple lower bound on the number of parenthesizations of a chain of n matrices. It will remind you about the true meaning of $\Omega(2^n)$. The result serves as a reminder why a brute-force solution is not attractive.

Solution.

Problem 4 (20 points)**.** Let $R(i, j)$ be the number of times that table entry $m[i, j]$ is referenced while computing other table entries in a call of MATRIX-CHAIN-ORDER, see [CLRS, Section 14.2]. Show that the total number of references for the entire table is

$$\sum_{i=1}^{n} \sum_{j=i}^{n} R(i, j) = \frac{n^3 - n}{3}.$$

[Hint: Re-read the definition of $R(i, j)$ a couple of times.]

Why do we care? This is a key statistics for the run-time of the dynamic-programming solution. The manipulation of such sums is an essential skill that you need to train.

Solution.

Problem 5 (20 points)**.** Give an $O(n^2)$-time algorithm to find the longest monotonically increasing subsequence of a sequence of n numbers.

Why do we care? You should learn how to apply the algorithms from the lecture. This is a good opportunity to hone your problem solving skills. Make sure that you solve it yourself without any help!

Solution.

Discussions on canvas are always encouraged, especially to clarify concepts that were introduced in the lecture. However, discussions of homework problems on canvas should not contain spoilers. It is okay to ask for clarifications concerning homework questions if needed. Make sure that you write the solutions in your own words.

Checklist:
☐ Did you add your name?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit the pdf file of your homework?