Problem Set 4

Due dates: Electronic submission of the pdf file of this homework is due on **2/14/2025 before 11.59pm** on canvas.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework. The solutions given in this homework are my own work.

Signature: ____________________________

Make sure that you describe all solutions in your own words. Typeset your solutions in LaTeX. Read chapter 30 on "Polynomials and the FFT" and chapter 15 on "Greedy Algorithms" in our textbook.

Problem 1 (20 points)**.** The polynomial $A(x) = 1 + x + x^2$ can be represented by the vector $(1, 1, 1, 0)^t$. (a) Use a 4×4 DFT and transform this vector. (b) Explicitly describe the resulting vector in terms of a vector of the form $(A_(x_0), A(x_1), A(x_2), A(x_3))^t$ for some complex numbers x_0, x_1, x_2, and x_3. Make sure that you verify your result.

Solution.

Problem 2. (20 points) Let ω be a primitive nth root of unity. The fast Fourier transform implements the multiplication with the matrix

$$F = (\omega^{ij})_{i,j\in[0..n-1]}.$$

Show that the inverse of the matrix F is given by

$$F^{-1} = \frac{1}{n}(\omega^{-jk})_{j,k\in[0..n-1]}$$

[Hint: $x^n - 1 = (x - 1)(x^{n-1} + \cdots + x + 1)$, so any power $\omega^\ell \neq 1$ must be a root of $x^{n-1} + \cdots + x + 1$.] Thus, the inverse FFT, called IFFT, is nothing but the FFT using ω^{-1} instead of ω, and multiplying the result with $1/n$.

Solution.

Problem 3. (20 points) Describe in your own words how to do a polynomial multiplication using the FFT and IFFT for polynomials $A(x)$ and $B(x)$ of degree $\leq n - 1$. Make sure that you describe the length of the FFT and IFFT needed for this task. Be concise and precise. Illustrate how to multiply the polynomials $A(x) = x^2 + 2x + 1$ and $B(x) = x^3 + 2x^2 + 1$ using this approach.

Solution.

Problem 4. (20 points) How can you modify the polynomial multiplication algorithm based on FFT and IFFT to do multiplication of long integers in base 10? Make sure that you take care of carries in a proper way. Write your algorithm in pseudocode and give a brief explanation.

Solution.

Problem 5 (20 points)**.** Describe an efficient algorithm that, given a set

$$\{x_1, x_2, \ldots, x_n\}$$

of n points on the real line, determines the smallest set of unit-length closed intervals that contains all of the given points. Argue that your algorithm is correct.

Why do we care? This is nice opportunity to design a simple greedy algorithm. Arguing the correctness of a greedy algorithm is essential, and this is not too difficult to do in this case.

Solution.

Checklist:

- ☐ Did you add your name?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted)
- ☐ Did you sign that you followed the Aggie honor code?
- ☐ Did you solve all problems?
- ☐ Did you submit the pdf file of your homework?