Investment Portfolio Optimisation

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1. Simulation of the mean-variance model

1. The 5 stocks I looked at was JP.Morgan (JPM), Uber, SNAP, Ford (F), and Citi (C). All of these stocks shares are in the New York Stock Exchange. Since there wasn't a transaction day on 31/12/2022 I used the previous day which was on 30/12/2022.

Stock	Stock exchange	30/12/2021	30/12/2022
JPM	NYSE	152.50	133.12
UBER	NYSE	42.66	24.73
SNAP	NYSE	48.03	8.95
F	NYSE	18.65	10.94
C	NYSE	57.53	44.79

2. I began by taking the adjusted close prices for each stock and continuously compounding them. The formula I used is shown below, with p being the previous adjusted close and n being the newer adjusted close. For example, to get the value 0.544 for JPM I did $(118.7998 - 118.1568)/118.1568$,

$$\frac{n-p}{p}$$

Below is a snippet of the calculated continuously compounded values,

JPM	UBER	SNAP	F	C	JPM	UBER	SNAP	F	C
118.1568	51.14	49.59	7.722801	55.55475	0.544%	5.612%	1.452%	1.526%	2.594%
118.7998	54.01	50.31	7.840638	56.99582	4.696%	-2.833%	-0.974%	2.197%	5.754%
124.3781	52.48	49.82	8.012859	60.27516	3.284%	6.955%	5.259%	2.489%	1.180%
128.4626	56.13	52.44	8.212275	60.98644	0.110%	-5.078%	0.553%	-0.662%	-0.985%
128.6044	53.28	52.73	8.157888	60.38601	1.492%	2.459%	3.129%	3.333%	1.637%
130.5237	54.59	54.38	8.429818	61.37442	1.572%	7.236%	3.807%	5.161%	0.346%

Figure 1: Piece of my stocks adjusted close prices

I then calculated the mean of the return for each stock with $\bar{r}_n = (\bar{r}_1, \bar{r}_2, \bar{r}_3, \bar{r}_4, \bar{r}_5)$ where $n = 1 =$ JPM, $n = 2 =$ UBER, $n = 3 =$ SNAP, $n = 4 =$ F, and $n = 5 =$ C. To find the mean you use the formula,

$$\bar{r}_n = \frac{\sum x_i}{n}$$

where x_i is the value of the continuously compounded adjusted price, where i is the place in the list, and n is the total number of adjusted prices.
Since I was working in excel I used the built in AVERAGE function to find the collective

means for each stock. Below is the values I calculated the mean of the returns to be for each stock.

	Daily returns
JPM	0.111%
UBER	-0.029%
SNAP	0.070%
FORD	0.387%
CITI	0.027%

Figure 2: Table of mean values

Therefore, the mean vector is $\bar{r}_n = (0.111, -0.029, 0.070, 0.387, 0.027)$.

The formula for the variance is,

$$var(x) = E(x^2) - \bar{x}^2$$

And to find the covariance the formula is,

$$cov(x_1, x_2) = E(x_1x_2) - \bar{x}_1\bar{x}_2$$

When calculating the covariance matrix for the stocks I used the excel formula shown below,

```
=COVARIANCE.S(INDIRECT(M$2),INDIRECT($L3))
```

Figure 3: Covariance excel code

This produces the results for the covariance matrix, which is shown below,

	Variance-Covariance matrix				
	JPM	UBER	SNAP	FORD	CITI
JPM	0.000182	0.00010656	5.93E-05	0.000142	0.000175
UBER	0.000107	0.00086964	0.000507	0.000147	0.000123
SNAP	5.93E-05	0.00050738	0.001636	0.000193	7.02E-05
FORD	0.000142	0.00014702	0.000193	0.00069	0.000161
CITI	0.000175	0.00012255	7.02E-05	0.000161	0.000262

Figure 4: Table of covariance matrix

3. I initially set all my weights to 0.2 for each stock to make them even due to the condition $\sum w_i = 1$ and that there is 5 stocks in total. I then calculated the return using the formula shown below, where $V3 : V7$ is the matrix of the weights. Transposing the weights matrix in order for the matrix multiplication to work and multiplying it by the mean of each stock $(S3 : S7)$.

```
{=MMULT(TRANSPOSE(V3:V7),S3:S7)}
```

Figure 5: Return excel code

To calculate the variance I transposed the weights and multiplied it by the matrix multiplication of the weights and the covariance matrix $(M3 : Q7)$. The formula is shown below. The standard deviation is just the square root value of the variance.

```
=MMULT(TRANSPOSE(V3:V7),MMULT(M3:Q7,V3:V7))
```

Figure 6: Variance excel code

Below is the return, variance and standard deviation (SD) for when the weights are all equal ($w_i = 0.2$ for all $i = 1, 2, .., 5$). Annualised the data to help with the computation by multiplying the return and variance by 365 and the standard deviation by $\sqrt{365}$. These values will change when excel solver is used.

	Daily	Annualised
Return	0.113%	41.331%
Variance	0.00028	0.10231473
SD	1.674%	31.99%

Figure 7: Values found for return, variance and SD

To calculate how much to invest into each stock given the expected return I multiplied the calculated portfolio weights (by using excel solver) by one million. This can be seen in the table below. The 0.2 for each weight is used as a basis before excel solver changes its value in the computation.

		How much money invested into each stock		
Portfolio weights		**r=0.04**	**r=0.75**	**r=0.15**
JPM	0.20	560946.67	1053788	637302.1
UBER	0.20	123288.54	-14165.4	101993.1
SNAP	0.20	55700.10	46694.05	54304.73
FORD	0.20	-162957.06	241355.2	-100316
CITI	0.20	423021.75	-327672	306716.4
	1.00	1,000,000	1000000	1000000

Figure 8: Table of how much to invest into each stock

The excel solver is shown below with the inputs used for each computation of $\bar{r}$. The $N11$ represents the value of the variance (annualised), this is the objective function as that's what is stated in the mean-variance (MV) model. $V3 : V7$ is the portfolio weights, when they are set as 0.2, this is the changing variable as we are finding how much should be invested into each stock in order to achieve a given expected return. The constraints $N10$ and $V8$ is the expected returns (annualised) and the total of the portfolio weights, respectively. The expected returns change to $0.04, 0.75, 0.15$ as these were the values I decided to work with given the required intervals. The total portfolio weights is set to one as this is a set constraint within the MV model. I also allowed the option for negative results as this can be used to show whether we can short sell.

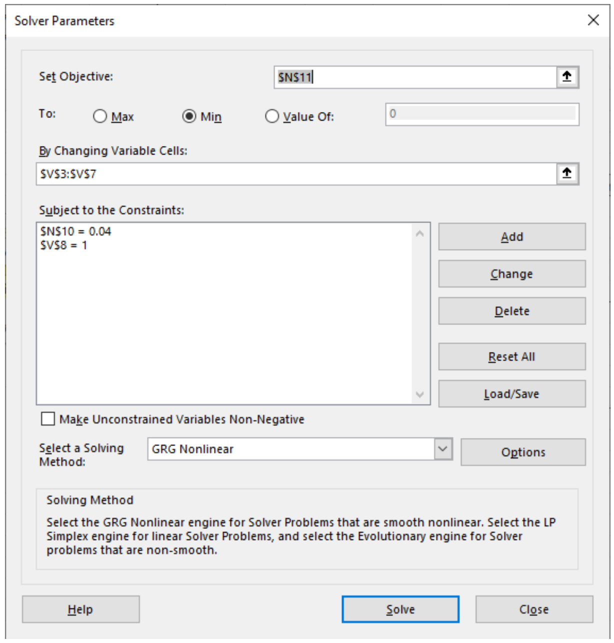

Figure 9: Demonstration of inputs used for excel solver

Below are the results I found for each value of the expected return after using excel solver. To calculate the number of shares to invest in each stock, I divided how much money to invest in each stock by the closing price on 30/12/2021. The values with negatives are a short sell.

Expected return	Variance	Weights		no. shares to invest
0.04	0.075829	0.56	**JPM**	3678.23
		0.12	**UBER**	2890.03
		0.06	**SNAP**	1159.69
		-0.16	**FORD**	-8738.54
		0.42	**CITI**	7353.41
0.75	0.074576	1.05	**JPM**	6909.89
		-0.01	**UBER**	-332.05
		0.05	**SNAP**	972.19
		0.24	**FORD**	12942.62
		-0.33	**CITI**	-5695.94
0.15	0.068718	0.64	**JPM**	4178.91
		0.10	**UBER**	2390.84
		0.05	**SNAP**	1130.64
		-0.10	**FORD**	-5379.44
		0.31	**CITI**	5331.67

Figure 10: Table of final results

4. Below I have calculated what each invested stock will be worth after one year. My method was to multiply the number of shares invested into each stock by the adjusted closing value on 31/12/2022. The values underneath the found profit is the total including the negatives, whilst the one on the right doesn't include the negative values. Since the negatives are just sold/selling stocks we can ignore the values since it will just deduct the final value of the portfolio. Therefore, the final value of the portfolio for the expected returns $0.04, 0.75, 0.15$ is just £900830.45, £1070118.64, £823039.71 respectively. The portfolio that produces the highest final value after one year is the portfolio that has an expected return of 0.75.

Expected return		Profit from investments	
0.04	JPM	489634.2	
	UBER	71470.36	
	SNAP	10379.26	
	FORD	-95602.3	
	CITI	329346.7	
		805228.2	900830.4482
0.75	JPM	919821.3	
	UBER	-8211.7	
	SNAP	8701.057	
	FORD	141596.3	
	CITI	-255111	
		806795.5	1070118.644
0.15	JPM	556282.7	
	UBER	27960.77	
	SNAP	-48146	
	FORD	-58852.8	
	CITI	238796.3	
		716040.9	823039.7075

Figure 11: Table of portfolio's worth after 1 year