VIT Vellore Institute of Technology

Reg. No. : 23MCA1030

Final Assessment Test (FAT) - November/December 2023

Programme	**M.C.A.**	Semester	**FALL SEMESTER 2023 - 24**
Course Title	**PROBABILITY AND STATISTICS**	Course Code	**PMAT501L**
Faculty Name	**Prof. Saroj Kumar Dash**	Slot	**E1+TE1**
		Class Nbr	**CH2023240101711**
Time	**3 Hours**	Max. Marks	**100**

PART-A (10 X 10 Marks)
Answer any 10 questions

01. A random sample of 200 adults are classified below with their educational qualifications. **[10]**

Education	Male	Female
Elementary	38	45
Secondary	28	50
College	22	17

If a person is picked at random from this group, find the probability that
(a) the person is a male, given that the person has a secondary; (5 Marks)
(b) the person does not have a college degree, given that the person is a female. (5 Marks)

02. (a) A manufacturer of a flu vaccine is concerned about the quality of its flu serum. Batches of serum are processed by three different departments having rejection rates of 0.1, 0.08 and 0.12 respectively. The inspections by the three departments are sequential and independent. **[10]**
(i) What is the probability that a batch of serum survives the first departmental inspection but is rejected by the second department? [2-Marks]
(ii) What is the probability that a batch of serum is rejected by the third department? [3-Marks]
(b) Suppose a certain type of small data processing firm is so specialized that some have difficulty making a profit in their first year of operation. The probability density function that characterizes the proportion Y that makes a profit is given by $f(y) = 280y^4(1-y)^3,\ 0 < y < 1$ and $f(y) = 0$ for other values of y. Find $P(Y \leq 0.5)$. [5-Marks]

03. Determine the value(s) of c so that the following function represent joint probability distribution of the random variables X and Y. **[10]**

$$f(x, y) = cxy, \text{ for } x = 1, 2, 3;\ y = 1, 2, 3.$$

[2-Marks]
Hence evaluate the following probabilities:
(i) $P(X \leq 2, Y = 1)$; [2-Marks]
(ii) $P(X > 2, Y \leq 1)$; [2-Marks]
(iii) $P(X > Y)$; [2-Marks]
(iv) $P(X + Y = 4)$. [2-Marks]

04. A coin is biased such that a head is three times as likely to occur as a tail. Find the expected number of tails when this coin is tossed twice. **[10]**

05. The Mathematics marks (in percentage) of few students on a midterm examination (x) and on the final examination (y) are given below: [10]

x	77	50	71	72	81	94	96	99	67
y	82	66	78	34	47	85	99	99	68

(i) Estimate the linear regression line. [8-Marks]

(ii) Estimate the final examination mark of a student if he scored 85% marks on the midterm examination. [2-Marks]

06. (a) On an average 3 traffic accidents per month occur at a certain city junction. What is the probability that in any given month at this junction [10]

(i) exactly 5 accidents will occur? [1-Mark]

(ii) fewer than 3 accidents will occur? [2-Marks]

(iii) at least 2 accidents will occur? [2-Marks]

(b) The following data represent the number of hours of flight training received by the following student pilots from a certain instructor prior to their first solo flight:

9, 12, 18, 14, 12, 14, 12, 10, 16, 11, 9, 11, 13, 11, 13, 15, 13, 14.

Using binomial probabilities, perform a sign test at the 0.02 level of significance to test the instructor's claim that the median time required before his students' solo flight is 12 hours of flight training. [5-Marks]

07. The length of time between breakdowns of an essential piece of equipment is important in the decision of the use of auxiliary equipment. An engineer thinks that the best model for the time between breakdowns of a generator is the exponential distribution with a mean of 15 days. [10]

(a) If the generator has just broken down, what is the probability that it will break down in the next 21 days? [8-Marks]

(b) What is the probability that the generator will operate for 30 days without a breakdown? [2-Marks]

08. An electrical firm manufactures light bulbs that have a lifetime which is approximately normally distributed with a mean of 800 hours and a standard deviation of 40 hours. Test the hypothesis that $\mu = 800$ hours against the alternative, $\mu \neq 800$ hours, if a random sample of 30 bulbs has an average life of 788 hours. [10]

09. A study was conducted to see if increasing the substrate concentration has an appreciable effect on the velocity of a chemical reaction. With a substrate concentration of 1.5 moles per Liter, the reaction was run 15 times, with an average velocity of 7.5 micro-moles per 30 minutes and a standard deviation of 1.5. With a substrate concentration of 2.0 moles per Liter, 12 runs were made, yielding an average velocity of 8.8 micromoles per 30 minutes and a sample standard deviation of 1.2. Is there any reason to believe that this increase in substrate concentration causes an increase in the mean velocity of the reaction of more than 0.5 micromole per 30 minutes? Use a 0.01 level of significance and assume the populations to be approximately normally distributed with equal variances [10]

10. A soft-drink dispensing machine is said to be out of control if the variance of the contents exceeds 1.15 deciliters. If a random sample of 25 drinks from this machine has a variance of 2.03 deciliters, does this indicate at the 0.05 level of significance that the machine is out of control? [10]

11. In a study analyzed by the Statistics Consulting Center at Virginia Tech, a group of subjects was asked to complete a certain task on the computer. The response measured was the time to completion. The purpose of the experiment was to test a set of facilitation tools developed by the Department of Computer Science at the university. There were 10 subjects involved. With a random assignment, five were given a standard procedure using Fortran language for completion of the task. The other five were asked to do the task with the use of the facilitation tools. The data on the completion times for the task are given here. **[10]**

Group 1 (Standard Procedure)	161	169	174	158	163
Group 2 (Facilitation Tool)	132	162	134	138	133

Assuming that the population distributions are normal and variances are the same for the two groups, support or refute the conjecture that the facilitation tools increase the speed with which the task can be accomplished.

12. Two types of instruments for measuring the amount of Sulfur Monoxide in the atmosphere are being compared in an air-pollution experiment. The following readings were recorded daily for a period of 2 weeks: **[10]**

Sulfur Monoxide

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Instrument A	0.96	0.82	0.75	0.61	0.89	0.64	0.81	0.68	0.65	0.84	0.59	0.94	0.91	0.77
Instrument B	0.87	0.74	0.63	0.55	0.76	0.7	0.69	0.57	0.53	0.88	0.51	0.79	0.84	0.63

Using the normal approximation to the binomial distribution, perform a sign test to determine whether the different instruments lead to different results with the 5% level of significance.

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