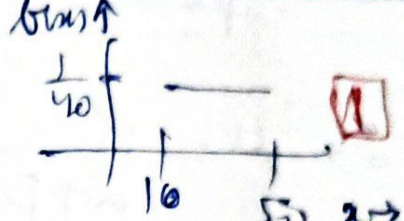
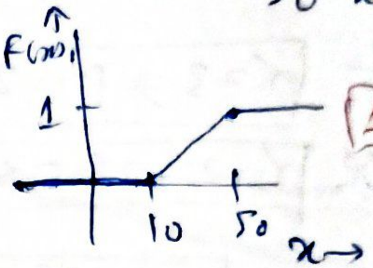
Sol. (1)(i): The PDF $f(x) = \begin{cases} \dfrac{1}{50-10}, & x \in (10, 50) \\ 0, & x \notin (10, 50) \end{cases}$



& the CDF $F(x) = \begin{cases} 0, & x \leq 10 \\ \dfrac{x-10}{50-40}, & x \in (10, 50) \\ 1, & x \geq 50 \end{cases}$



(ii) $P(X < 23 \mid X > 15) = \dfrac{P(X < 23 \;\&\; X > 15)}{P(X > 15)} = \dfrac{P(15 < X < 23)}{P(X > 15)}$

$$= \frac{\frac{1}{40}\int_{15}^{23} dx}{1 - P(X < 15)} = \frac{\frac{1}{40}(23-15)}{1 - \frac{1}{40}\int_{10}^{15} dx}$$

$$= \frac{\frac{1}{40}(23-15)}{1 - \frac{1}{40}(15-10)} = \frac{\frac{1}{40}(8)}{\frac{40-5}{40}} = \frac{8/40}{35/40}$$

$$= 8/35 \quad \text{Ans}$$

Sol. (2)(i) $Z(t) = \alpha\beta t^{\beta-1} = t^{-2/3} \Rightarrow \alpha\beta = 1 \;\&\; \beta - 1 = -\frac{2}{3} \Rightarrow \boxed{\beta = \frac{1}{3}}$

$\Rightarrow \boxed{\alpha = 3}$

(ii) $F(t) = 1 - e^{-\alpha t^{\beta}} \Rightarrow F(10) = 1 - e^{-3t^{1/3}}\Big|_{t=10} = 1 - e^{(-3)(10^{1/3})} = 1 - 0.00156$

Hence $P(T > 10) = 1 - F(10) = 1 - (1 - 0.00156) = 0.00156$

Sol^n (3): Here $\alpha+\beta=5$ & $\sigma=\frac{1}{5} \Rightarrow \sigma^2=\frac{1}{25}$

~~$\therefore \mu=\frac{\alpha}{\alpha+\beta}$ & $\sigma^2=\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}=\frac{1}{25}$~~

~~$\Rightarrow \boxed{\alpha=(5)(\alpha+\beta)=(5)(5)=25}$ $\Rightarrow \boxed{\beta=\frac{1}{25}}$~~

$\therefore \mu=\frac{\alpha}{\alpha+\beta}$ & $\sigma^2=\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}=\frac{1}{25}$

$\Rightarrow \frac{\alpha\beta}{(5^2)(6)}=\frac{1}{25}$

$\Rightarrow \boxed{\alpha\beta=6.}$

$\Rightarrow \alpha(5-\alpha)=6$

$\Rightarrow 5\alpha-\alpha^2=6$

$\Rightarrow \alpha^2-5\alpha+6=0.$

$\Rightarrow \alpha^2-3\alpha-2\alpha+6=0$

$\Rightarrow \alpha(\alpha-3)-2(\alpha-3)=0.$

$\Rightarrow \alpha=3$ or $\alpha=2$ 2

If $\boxed{\alpha=3 \Rightarrow \beta=2}$

OR $\boxed{\alpha=2 \Rightarrow \beta=3.}$ 2

Hence $\boxed{\mu=\frac{3}{5} \text{ OR } \frac{2}{5}}$ 2

(ii) $P(X>\frac{1}{3})=\frac{1}{B(3,2)}\int_{1/3}^{1} x^{3-1}(1-x)^{2-1}dx$

$=\frac{1}{\frac{\Gamma(3)\Gamma(2)}{\Gamma(3+2)}}\int_{\frac{1}{3}}^{1} x^2(1-x)\,dx$

$=\frac{4!}{2!\,1!}\left[\frac{1}{3}x^3-\frac{1}{4}x^4\right]_{1/3}^{1}$ 2

$=12\left[\left(\frac{1}{3}-\frac{1}{4}\right)-\left(\frac{1}{3}\left(\frac{1}{3}\right)^3-\frac{1}{4}\left(\frac{1}{3}\right)^4\right)\right]$

$=12\left[\frac{4-3}{12}-\frac{1}{3^4}+\frac{1}{4(3)^4}\right]$

$=12\left[\frac{1}{12}-\frac{1}{3^4}\left(1-\frac{1}{4}\right)\right]=12\left[\frac{1}{12}-\left(\frac{1}{3^4}\right)\left(\frac{3}{4}\right)\right]=(12)\left(\frac{1}{12}\right)\left[1-\frac{1}{9}\right]=\frac{8}{9}$ Ans 2

Sol^n (4): (i) $X \sim N(2, 4)$, where X = Distance from the target.

The mission is successful if $|X| < 10$, i.e. $-10 < X < 10$. 1

~~⇔ The ... is failed if |X| > 10, i.e~~

$$\Rightarrow \frac{-10-2}{4} < \frac{X-2}{4} < \frac{10-2}{4} \quad 1$$

$$\Rightarrow -3 < Z < 2 \quad 1$$

$\therefore P(-3 < Z < 2)$ = Prob of mission is successful.

$\Rightarrow$ Prob. of mission is failed $= 1 - P(-3 < Z < 2)$

$$= 1 - [\Phi(2) - \Phi(-3)]$$

$$= 1 - [0.9772 - 0.0013]$$

2 $$= 1 - [0.9759] = 0.0241.$$

$= P(\text{failed mission})$, Ans

(ii) $n = 5$, $p = 0.0241$ & $q = 0.9759$.
1 1 1

Hence P(at most one mission fails out of 5)

1 $= P$(ZERO failed mission) $+ P$(Only one failed mission)

$$= \binom{5}{0} p^0 q^5 + \binom{5}{1} p^1 q^4$$

$$= (0.9759)^5 + (5)(0.0241)(0.9759)^4$$

2 $= 0.8852 + 0.1093 = 0.9945$ Ans

(5.) Normal $(15, 3)$

$\mu = 15$

$\sigma = 3$

i) Probability of success to reach B at 9 am –

$$P(X < x) = 0.99$$

$$\text{or, } P\left(Z < \frac{x-15}{3}\right) = 0.99$$

$$\text{or, } \Phi^{-1}(0.99) = \frac{x-15}{3}$$

$$\text{or, } x = 15 + 3 \times (2.33)$$

$$= 21.99$$

$$\simeq 22 \text{ min.}$$

The traveller must start at 8:38 am to reach at 9 am.

ii) Probability of failure to reach B at 9 am –

$$P(X > x) = 0.99$$

$$\text{or, } 1 - P(X < x) = 0.99$$

$$\text{or, } P(X < x) = 0.01$$

or, $\cancel{\Phi^{-1}(0.01)}$ $P\left(Z < \frac{x-15}{3}\right) = 0.01$

or, $x = 15 + 3 \times \Phi^{-1}(0.01) = 15 + 3 \times (-2.33)$

$= 8.01 \simeq 8$ min

The traveller can start at 8:52 am to get 99% failure to reach the place B at 9am.

— x —