1.		A and B are two events in a sample space. Given that $P(A) = 0.4$ and $P(A \cup B) = 0.7$. (i) Find the probability that neither A nor B occurs. (ii) Find the value of $P(B)$ for which A and B are mutually exclusive. (iii) Find the value of $P(B)$ for which A and B are independent.	**[3+3+4]**

Sol: **(a) The probability that neither A nor B occurs is the COMPLEMENT to the probability P (A or B).**

For P (A or B), it is given: P (A or B) = 0.7.

THEREFORE, P (neither A nor B) = 1 - P (A or B) = 1 - 0.7 = 0.3. It is the <u>ANSWER</u> to question (a).

(b) If A and B are mutually exclusive, it means that they are disjoint. i.e., P (A and B) = 0.

It implies that P (A or B) = P(A) + P(B),

and, substituting the given data, we have 0.7 = 0.4 + P(B),

which implies P(B) = 0.7 - 0.4 = 0.3. It is the <u>ANSWER</u> to question (b).

(c) A and B are independent (by the definition) if and only if P (A and B) = P(A)*P(B).

Next, we use the general formula
P (A or B) = P(A) + P(B) - P (A and B)
and substitute there P (A and B) = P(A)*P(B). It gives us

P (A or B) = P(A) + P(B) - P(A)*P(B).

Substitute here the given values. You will get

0.7 = 0.4 + P(B) - 0.4*P(B).

Simplify and get P(B)

0.7 - 0.4 = P(B) - 0.4*P(B)

0.3 = 0.6*P(B)

P(B) = $\frac{0.3}{0.6}$ **=** $\frac{1}{2}$ **= 0.5. It is the <u>ANSWER</u> to question (c).**

2.		It is observed that 50% of mails are spam. There is a software that filters spam mail before reaching the inbox with the accuracy of 99% and chances of tagging a non-spam mail as spam mail is 5%. If a certain mail is tagged as spam then find the probability that it is not a spam mail.	**[10]**

Sol:

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= 4/9.

Question 6:

It is observed that 50% of mails are spam. There is a software that filters spam mail before reaching the inbox. It accuracy for detecting a spam mail is 99% and chances of tagging a non-spam mail as spam mail is 5%. If a certain mail is tagged as spam find the probability that it is not a spam mail.

Solution:

Let E_1 = event of spam mail

E_2 = event of non-spam mail

A = event of detecting a spam mail

Now, $P(E_1) = 0.5$ and $P(E_2) = 0.5$

$P(A|E_1) = 0.99$ and $P(A|E_2) = 0.05$

Then,

$$P(E_2|A) = \frac{P(A|E_2)P(E_2)}{P(A|E_1)P(E_1)+P(A|E_2)P(E_2)}$$

$$= \frac{0.05\times0.5}{0.99\times0.5+0.05\times0.5} = \frac{0.025}{0.520}$$

$= 5/104 \approx 4.8\%$

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3.	a)	For two events A and B, we have $P(\overline{A \cup B}) = 1/6$, $P(A \cap B) = 1/4$ and $P(A) = 3/4$. Prove or disprove that events are independent but not equally likely.	**[5]**

Sol:

$P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$,

and $P(\overline{A \cup B}) = \frac{1}{6}$

$\Rightarrow 1 - P(A \cup B) = 1/6$ {Such that $p(A) + p(\bar{A}) = 1$}

$\Rightarrow 1 - P(A) - P(B) + P(A \cap B) = 1/6$

Such that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(\bar{A}) - P(B) + 1/4 = 1/6$

$\Rightarrow P(B) = 1/4 + 1/4 - 1/6$

$\Rightarrow P(B) = 1/3$ and $P(A) = 3/4$

Now $P(A \cap B) = 1/4 = 3/4 \times 1/3 = P(A)\,P(B)$

Hence the events A and B are independent events.

But $P(A) \neq P(B)$, so these two events are not equally likely.

3.

	b)	Three terminals on an on-line computer system are attached to a communication line to the central computer system. The probability that any terminal is ready to transmit is 95%. Let X be the number of ready terminals at any point of time. Draw the graph of the cumulative distribution function (CDF) of the random variable X.	**[5]**

Sol: Here $n = 3, p = 0.95, q = 0.05$.

So:
$$P(X=0) = \binom{3}{0}(0.95)^0(0.05)^3 = 0.000125,$$
$$P(X=1) = \binom{3}{1}(0.95)^1(0.05)^2 = 0.007125,$$
$$P(X=2) = \binom{3}{2}(0.95)^2(0.05)^1 = 0.135375,$$
$$P(X=3) = \binom{3}{3}(0.95)^3(0.05)^0 = 0.857375.$$

Hence $F(x) = \begin{cases} 0, & x < 0 \\ 0.000125, & 0 \le x < 1 \\ 0.00725, & 1 \le x < 2 \\ 0.142625, & 2 \le x < 3 \\ 1, & x \ge 3 \end{cases}$

4		An interactive system consists of 10 terminals that are connected to the central computer. At any time, each terminal is ready to transmit with probability 0.7, independently of other terminals. (i) Find the probability that exactly 8 terminals are ready to transmit at 12 noon. (ii) Find the probability that at least 8 terminals are ready to transmit at 12 noon.	**[5+5]**

Sol: Here $n = 10,\ p = 0.7,\ q = 0.3$. So:

(i) $P(X=8) = \binom{10}{8}(0.7)^8(0.3)^2 = 0.2335$

and (ii) $P(X \ge 8) = \binom{10}{8}(0.7)^8(0.3)^2 + \binom{10}{9}(0.7)^9(0.3) + \binom{10}{10}(0.7)^{10} = 0.3828$.

5.		Mass-produced computer RAMs are packed in boxes of 1000. It is believed that 1 of the RAMs in 2000 on average is substandard. (i) What is the probability that a box contains exactly 2 defective RAMs. (ii) What is the probability that a box contains at most 2 defective RAMs.	**[5+5]**

Sol: Here $n = 1000, p = \frac{1}{2000}$. Since p is very small and n is very high, so we'll apply Poisson's distribution to answer this with $\lambda = np = 1/2$.

Hence $P(X=2) = \frac{e^{-\lambda}\lambda^2}{2!} = 0.075816$,

And $P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= 0.606531 + 0.303265 + 0.075816 = 0.985612$.