FOPI Controller for Blood Pressure Regulation using Hippopotamus Algorithm

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Abstract. Blood pressure readings indicate the force the heart exerts on arterial walls while pumping blood. Maintaining mean arterial blood pressure during surgeries is critical to controlling excessive bleeding. Vasodilating drugs like Sodium Nitroprusside (SNP) are used to lower blood pressure, but their required dosage and effects vary significantly depending on the condition of the patient. This necessitates continuous monitoring and precise drug delivery. To achieve this, a fractional-order proportional integral (FOPI) controller is designed using a graphical approach based on the maximum sensitivity criterion, ensuring adequate phase margin (ϕ_m) and gain margin (G_m) for stability. The FOPI parameters are then optimized using the hippopotamus optimization algorithm (HOA)), a metaheuristic technique that mimics hippopotamus behavior in the wild. This algorithm minimizes integral absolute error (IAE) and integral time absolute error (ITAE), which are taken as cost functions. This study evaluates the robustness and performance of the proposed controller compared to existing control strategies. A MATLAB environment is used to simulate the system and implement the algorithm, demonstrating that the proposed controller ensures lesser settling time and steady-state error while being more practical for real-time implementation.

Keywords: Mean Artial Blood Pressure · Hippopotamus Algorithm · Fractional-Order Proportional Integral · H_∞ criteria · Integral Error

1 Introduction

Blood pressure readings measure the force exerted by the heart on arterial walls during blood circulation. Blood pressure is influenced by cardiac output and peripheral resistance. Vasodilators like Sodium Nitroprusside (SNP) are administered to hypertensive patients to lower blood pressure by reducing peripheral resistance. However, precise control of the infusion rate and duration is crucial to prevent toxicity from byproducts such as nitric oxide, thiocyanates, and cyanides, which form when SNP breaks down in the body. Excessive blood pressure reduction can lead to adverse effects, particularly in post-operative patients, increasing the risk of shock. Therefore, accurate infusion rate control is essential. Clinical SNP administration remains challenging due to patient variability and the need for controlled drug release. To address this, automated drug delivery systems are required to dynamically adjust infusion rates and maintain optimal blood pressure levels. [1].

Sheppard et al. developed a computer-based control system by analyzing the physiological responses of SNP in post-operative patients. However, the system did not account for patient sensitivity [2]. These early controllers lacked adaptability to real-world disturbances, such as surgical stimuli, necessitating the development of adaptive controllers. To address these limitations, Slate and Sheppard proposed a model-based adaptive controller that incorporated patient sensitivity to the drug [3]. Internal Model Control (IMC) improved peak overshoot, settling time, and steady-state performance. Further tuning of IMC enhanced robustness and disturbance rejection through IMC filtering [4]. Although optimal IMC provided certain advantages, PID controllers demonstrated shorter settling times [5]. IMC-based PID controllers further refined response characteristics compared to standard IMC controllers. Hamamaci [6] introduced an algorithm for regulating mean arterial pressure using a fractional-order time-delay system. This approach utilized fractional-order PID controllers, which exhibited superior disturbance rejection and greater adaptability to uncertain conditions compared to conventional PID controllers. To enhance controller parameter tuning, optimization strategies have gained substantial attention. Both heuristic and metaheuristic techniques are widely employed for this purpose. Several metaheuristic algorithms, such as the seeker optimization algorithm, whale optimization algorithm, multi-strategy modified INFO algorithm, logarithmic spiral search-based arithmetic algorithm, particle swarm optimization, and human-inspired methods, have been applied. Additionally, genetic algorithms are frequently used to optimize PID parameters [7].

This paper focuses on designing a FOPI controller for regulating MABP. The control parameter search space for K_p and K_i is determined using a graphical approach based on the H_∞ criterion [10]. Within this search space, the hippopotamus optimization algorithm (HOA) is employed to identify suboptimal parameters by minimizing integral errors namely, integral absolute error (IAE) and integral time absolute error(ITAE), taken as cost functions. The algorithm operates through a three-phase model that mathematically simulates group dynamics, including positional updates in aquatic environments, predator defense mechanisms, and evasion strategies [8].

The key contributions of this work are as follows:

- Development of a two-stage FOPI controller.
- Finding search space using H_∞ criterion based graphical approach.
- Tuning the FOPI parameters using HOA by minimizing integral error.
- Analysis of simulation results showing reduced steady-state error and settling time.
- The FOPI controller, having 2 less parameters than a FOPID controller is easier to implement in real-time and also has lesser computational burden.

The rest of the paper is structured as follows: Section II presents the patient response model, which serves as the plant. Section III details the controller design and the H_∞ criterion. Section IV introduces the HOA method, followed by Section V, which discusses simulation results and analysis. Finally, Section VI concludes the study with key findings and future directions.

2 Patient Response Model

Slate et al. [1] investigated the impact of SNP infusion on patients by analyzing five parameters that exhibit significant variability in MABP in response to the drug.

The transfer function model used by Slate is:

$$P(s) = \frac{\Delta P_d(s)}{I_n(s)} = \frac{Ke^{-T_{id}s}(1+\alpha e^{-T_{cd}s})}{\tau s+1} \tag{1}$$

where $\Delta P_d(s)$ represents the change in MABP (mmHg), $I_n(s)$ is the drug infusion rate (ml/hr), The patients are classified into three categories based on their sensitivity to the drug, as shown in Table 1.

Table 1. Parameter Values of Patients

Symbol	Parameter Name	Sensitive	Normal	Insensitive
K (mmHg/(ml/hr))	Drug sensitivity	-9	-0.7143	-0.1786
α	Recirculation constant	0	0.4	0.4
T_{id} (sec)	Initial time delay	20	30	60
T_{cd} (sec)	Recirculation time delay	60	30	45
τ (sec)	Time constant	75	30	40

Biological systems are inherently influenced by disturbances [9]. To achieve greater accuracy, these disturbances should be incorporated into the system model. Enbiya [5] developed a Simulink model to represent these disturbances as shown in Fig 1.

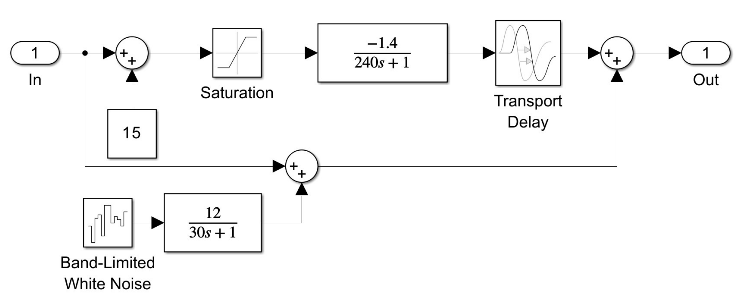

Fig. 1. Model of Disturbances.

3 Controller Design

We are going to design a FOPI controller using a graphical approach based on H_∞ criteria to find the search space for implementing HOA. The control structure using this FOPI controller is shown in Fig. 2.

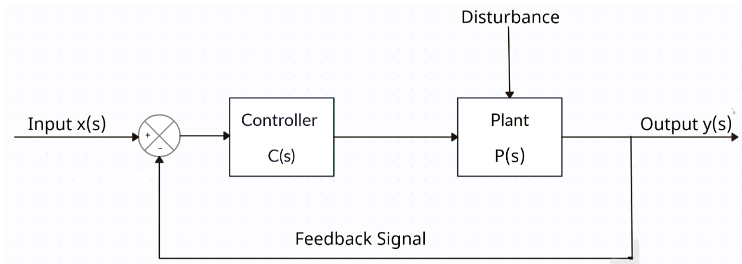

Fig. 2. Proposed Control Structure.

The objective of this research is to find the search space and then tune the parameters of the FOPI controller to achieve precise tracking of the patient model output to the desired setpoint. We employ HOA to minimize the error, specifically using Integral Absolute Error (IAE) and Integral Time Absolute Error (ITAE) as cost functions.

4 H_∞ Criterion

The H_∞ criteria is a robust control design technique that converts a control problem statement to a mathematical optimization problem. For the suboptimal control, the maximum value of gain of sensitivity function must be less than γ (defined as maximum sensitivity). Therefore our problem is to find the controller minimizing this sensitivity value:

$$\left| \frac{1}{1 + P(s)C(s)} \right|_\infty < \gamma \tag{2}$$

. To apply the H_∞ criteria on the human body plant, its equation is taken with real and imaginary parts separated:

$$P(s) = P(j\omega) = P_r(\omega) + jP_i(\omega) \tag{3}$$

And a FOPI controller defined as

$$C(j\omega) = K_p - \frac{K_i}{j\omega^\lambda} \tag{4}$$

Putting the values of $P(s)$ and $C(s)$ in eqn. 2 gives us

$$\left| 1 + (P_r(\omega) + jP_i(\omega)) \left(K_p - \frac{jK_i}{\omega^\lambda} \right) \right| > \frac{1}{\gamma} \tag{5}$$

Solving and rearranging the above equation gives us the equation of an ellipse:

$$\frac{(K_i - C_1(\omega))^2}{a^2} + \frac{(K_p - C_2(\omega))^2}{b^2} > 1 \quad (13) \tag{6}$$

where

$$C_1(\omega) = -\frac{\omega\omega^\lambda P_i(\omega)}{|P(j\omega)|^2}, \qquad C_2(\omega) = -\frac{P_r(\omega)}{|P(j\omega)|^2}$$

$$a(\omega) = \frac{\omega^\lambda}{\gamma|P(j\omega)|}, \qquad b(\omega) = \frac{1}{\gamma|P(j\omega)|}$$

The ellipses from these equations are plotted in MATLAB. From the region bounded by, yet outside the ellipses,a range of (K_i,K_p) is selected and then HOA is applied on that range with lambda taken between 0 to 2 to select the parameters for FOPI controller which minimizes the integral error cost functions. The plotted ellipses for a sensitive patiend with the bounded region given to HOA is shown in fig. 3.

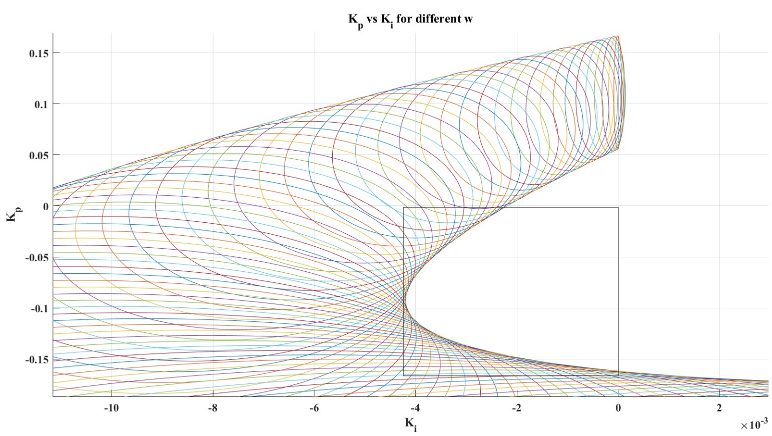

Fig. 3. Ellipses for a sensitive patient and the selected region.

5 Hippopotamus Optimization Algorithm

The Hippopotamus Optimization Algorithm (HOA) is a metaheuristic algorithm inspired by the social behavior of hippopotamuses. It simulates their interactions within a herd, including position updates, defense against predators, and escape mechanisms. HOA balances exploration (searching diverse areas) and exploitation (refining solutions in promising regions) to find optimal solutions. The algorithm works similar to behavioral patterns of hippos in different situations or phases.

5.1 Phases of Hippopotamus Optimization Algorithm

Position update of the hippopotamuses in the river or pond: This phase simulates the movement of hippopotamuses within the herd, driven by the influence of the dominant hippopotamus and the average position of randomly selected individuals. This phase fosters exploration and promotes diversity in the algorithm.
Hippopotamus defense mechanism against predators: In this phase, the defensive behavior of hippopotamuses is modeled when they encounter a predator, such as a lion or a hyena. The hippopotamuses turn towards the threat, open their powerful jaws, and produce loud vocalizations to deter the predator. This phase enhances the exploitation capabilities and accelerates convergence in the algorithm.
Hippopotamus evasion from predators: This phase represents the evasive actions of hippopotamuses when confronted by multiple predators or when their defense strategies fail. In this case, the hippopotamuses flee from the predator and move toward the nearest water body, such as a river or a pond. This phase helps prevent the algorithm from becoming trapped in local optima and ensures a balance between exploration and exploitation.

5.2 Steps for Designing the Controller

The following steps outline the process for designing the controller:

1. Define the plant model with suitable patient parameters and the value of $\gamma = 2$ and plot the ellipses for the closed loop function.
2. Select the range of (K_p, K_i) from the region outside yet bounded by the ellipses. And choose a range for λ preferably between 0 and 2.
3. Run HO algorithm in MATLAB on the specified range.
4. If the code stops after giving error, check the value of K_p, K_i and λ in the simulation in Simulink. One or more of these parameters have made the simulation unstable so edit the range accordingly and rerun the algorithm.
5. If the final result is achieved at the boundary of the specified region then increase the range and rerun the algorithm.

5.3 Tuned Parameters by HOA

HOA was used to find the suboptimal parameters by simulating the controller in SIMULINK, which minimized the cost function (ITAE and IAE). Its range was chosen from the boundary made by the ellipses taking γ=2 with some additional regions around it too, in cases the algorithm reaches the boundary of the area provided. Table 2 lists the tuned parameters by HOA.

Table 2. Tuned Parameters of FOPI Controller using HOA.

Patient Type	Cost Function	K_p	K_i	λ
Sensitive	IAE	-0.12644	-0.0022915	1.0505
	ITAE	-0.103	-0.0023486	1.0295
Nominal	IAE	-1.6097	-0.009579	1.0919
	ITAE	-1.0323	-0.012667	1.0169
Insensitive	IAE	-3.9225	-0.023564	1.0627
	ITAE	-3.6109	-0.023459	1.0562

6 Simulation Results and Discussion

The set point is taken as -30mmHg to evaluate the effectiveness of the proposed controller. For the sensitive patient, the control effort is shown in Fig xyz and the step response is shown in Fig 4, for the nominal patient, the control effort is shown in Fig abc and the step response is shown in Fig 5 and for the insensitive patient, the control effort is shown in Fig pqr and the step response is shown in Fig 6

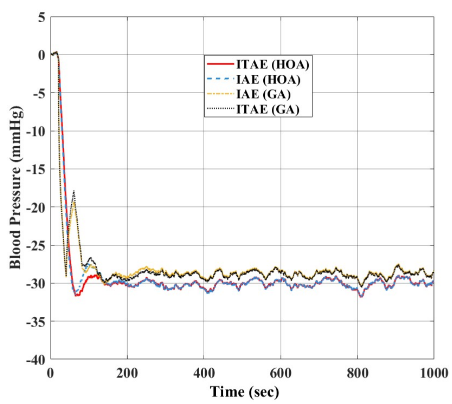

Fig. 4. Sensitive Patient Results.

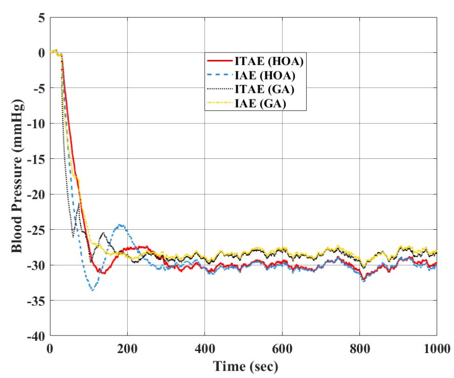

Fig. 5. Nominal Patient Results.

This approach balances performance and stability in the presence of model uncertainties, making it particularly useful for designing FOPI controllers in complex systems like the human body. H_∞ control is crucial for blood pressure regulation due to its inherent robustness, which allows it to handle patient-specific sensitivities, varying physiological responses, and external disturbances. Furthermore, it effectively manages time delays inherent in the response to drugs in the body, optimizes the trade-off between performance (e.g., settling time) and robustness, and maintains stability across a range of conditions, preventing excessive blood pressure fluctuations.

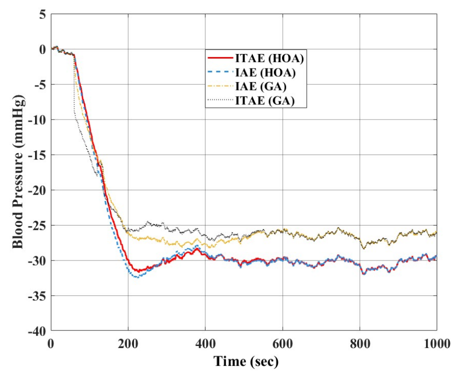

Fig. 6. Insensitive Patient Results.

The results demonstrate effective blood pressure regulation for all patient types. The HOA successfully optimized the FOPI controller parameters, achieving minimal ITAE and IAE, as shown in Figures 4, 5, and 6. We achieved a smaller settling time and less steady-state error along with reduction in the Error values as shown in Table 3.

Table 3. Performance Indices for Patient Model

Patient Type	Performance Index	FOPID GA (Reference)	FOPI HOA (Proposed)
Sensitive	IAE ($\times 10^3$)	2.36	1.3
	ITAE ($\times 10^6$)	0.59	0.23
	Settling Time (s)	A	B
	Steady State Error	C	D
	Max Overshoot (%)	E	F
Nominal	IAE ($\times 10^3$)	3.2	2.4
	ITAE ($\times 10^6$)	0.73	0.35
	Settling Time (s)	G	H
	Steady State Error	I	J
	Max Overshoot (%)	K	L
Insensitive	IAE ($\times 10^3$)	6.2	4.0
	ITAE ($\times 10^6$)	1.94	0.49
	Settling Time (s)	M	N
	Steady State Error	O	P
	Max Overshoot (%)	Q	R

7 Conclusion

A FOPI controller is designed to control the mean artrial blood pressure of three different patient models. A H_∞ criteria-based graphical approach was used to find the range for the parameters of the controller and the parameters were tuned by hippopotamus optimization algorithm. The controller used a 2-D graphical approach of a K_p vs K_i graph to get the range of parameters. To get even better results, future research aims to use other techniques to estimate the range of the Kd parameters and fine tune it using HOA to design a FOPID controller.

References

1. Behbehani, K., Cross, R.R.: A controller for regulation of mean arterial blood pressure using optimum nitroprusside infusion rate. IEEE Trans. Biomed. Eng. **38**(6), 513–521 (1991).
2. Sheppard, L.C., Shotts, J.F., Roberson, N.F., Wallace, F.D., Kouchoukos, N.T.: Computer controlled infusion of vasoactive drugs in post cardiac surgical patients. In: IEEE/1979 Frontiers of Engineering in Health Care (IEEE CH1440-7), pp. 280–284 (1979).
3. Slate, J.B., Sheppard, L.C.: A model based adaptive blood pressure controller. In: 6th IFAC Conference on Identification System Parameter Estimation, Washington, DC, pp. 1437–1442 (1982).
4. Rao, P.V.G.K., Subramanyam, M.V., Satyaprasad, K.: Robust Design of PID Controller Using IMC Technique for Integrating Process Based on Maximum Sensitivity. J. Control Autom. Elect. Syst. **26**(5), 466–475 (2015).
5. Enbiya, E., Hossain, E., Mahieddine, F.: Performance of optimal IMC and PID Controllers for blood Pressure control. In: Proceedings of IFMBE, Miami, Florida, pp. 89–94 (2009).
6. Hamamci, S.E.: An algorithm for stabilizing fractional-order time delay systems using fractional-order PID controllers. IEEE Trans. Autom. Control **52**(10), 1964–1969 (2007).
7. Rao, P.V.G.K., Subramanyam, M.V., Satyaprasad, K.: Performance Comparison of PID Controller Tuned using Classical and Genetic Algorithm Methods. Int. J. Appl. Eng. Res. **6**(14), 1757–1766 (2011).
8. Amiri, M.H., Mehrabi Hashjin, N., Montazeri, S., Mirjalili, S., et al.: Hippopotamus optimization algorithm: a novel nature-inspired optimization algorithm. Sci Rep **14**, 5032 (2024).
9. Sondhi, S., Hote, Y.V.: Fractional-Order PI Controller with Specific Gain-Phase Margin for MABP Control. IETE J. Res. (2015).
10. Kumar S.R., Anwar M.N., Verma P., Somanshu S.R.: H_∞ Criterion Based PI Controller Design for DC-DC Step-Up Converters. IEEE Journal of Emerging and Selected Topics in Industrial Electronics, vol. 5, no. 2, (2024).