Graphical Approach based Robust PI Controller Design for Blood Pressure Regulation

A Project Report Submitted

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ABSTRACT

Blood pressure reading represents how much force the heart exerts when pumping blood to artery walls. During surgeries, to maintain the mean arterial blood pressure, a vasodilating drug like Sodium Nitroprusside (SNP) to lower the blood pressure to control excessive bleeding. Its required amount and effects greatly vary on the type of patient, so we need to continuously monitor the effects too. Such precise drug-delivery systems require a controller to work on the patient response model. The proposed PI controller uses a simple, maximum sensitivity criterion-based graphical approach of controller design with fixed maximum sensitivity (γ) to ensure minimum phase margin (ϕm) and gain margin(Gm). The design aims to explore how robustness and performance im-prove as compared to other existing control strategies.

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1. INTRODUCTION TO H-infinity (H∞) criterion

The **H-infinity (H∞) criterion loop shaping approach** is a robust control design technique aimed at achieving good performance and stability for systems in the presence of model uncertainties. When applied to the design of a **PI (Proportional-Integral) controller**, the approach helps balance between robustness (handling disturbances, noise, and uncertainties) and performance (tracking and regulation).

1.1 Why do we need H∞ Criteria ?

- **Robustness to Uncertainty and Disturbances:** The human body is a complex and dynamic system with various uncertainties such as patient-specific drug sensitivities, varying physiological responses, and external disturbances (e.g., reflex responses, respiratory effects). The H∞ criterion is particularly effective in handling uncertainties and disturbances by minimizing the worst-case scenario (infinity-norm) of system performance. This means it can adapt to a wider range of patient conditions, providing better control over blood pressure under variable conditions.
- **Better Handling of Delays:** The patient response model includes time delays, such as initial and recirculation delays. H∞ controllers are designed to maintain stability and performance in systems with inherent delays, as they can manage the system's frequency response more effectively. This could improve the controller's performance in preventing oscillations and erratic behavior during SNP infusion.
- **Optimized Trade-offs Between Performance and Robustness:** The H∞ PI controller optimizes the trade-off between performance (such as settling time, overshoot, and peak response) and robustness to disturbances. This is crucial in a clinical setting where blood pressure must be tightly controlled without causing excessive fluctuations, especially in post-operative patients where stable blood pressure is critical for recovery.
- **Increased Stability Margin:** The ellipse-forming method used in H∞ control focuses on maintaining stability over a wide range of operating conditions by shaping the frequency response. This would help prevent excessive blood pressure drops or increases during SNP infusion, ensuring the system remains stable even in the face of sudden change.

1.2 Steps for designing controller using Maximum Sensitivity

1. Define plant model and Controller equation.
2. If the equation contains delay in exponential form, put it in trigonometric form
3. Simplify the plant equation to separate real and imaginary parts.
4. Multiply it with the controller equation.
5. Put it in the formula of maximum sensitivity.
6. Simplify the formula to write it in the parametric form of an ellipse.
7. Plot the ellipses in Kp Ki plane. The outer region of the ellipses is the stable region.

8. Select a point from this region as the control parameters.
9. Place the values in the Simulink model.
10. Analyse the response.

1.3 PI controller design for a human body drug response model

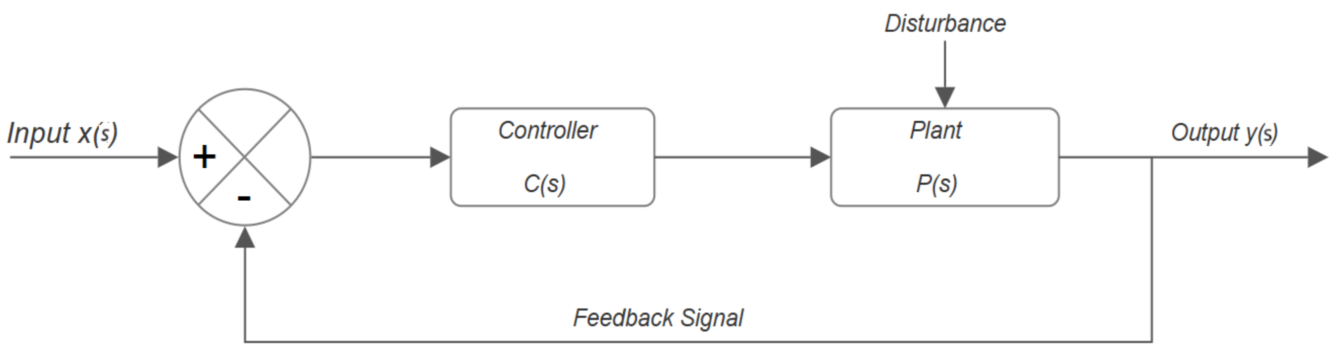


Figure 1.1: control system structure.

1.3.1 Working on the human Body Plant.

We have taken the human body plant $P(s) = \frac{Ke^{T_{\mathrm{id}}s}(1+\alpha e^{T_{\mathrm{cd}}s})}{\tau s+1}$ -(1)
Substituting $s = j\omega$, in P(s), so that $e^{Tj\omega} = \cos(T\omega) - j\sin(T\omega)$ in the above equation (1) and separating its real and imaginary parts give-

$$P_r(\omega) = \frac{K}{t^2\omega^2+1}(\cos\ (T_i\omega) + a\cos(T_i\omega + T_c\omega) - t\omega\sin(T_i\omega) - at\omega\sin(T_i\omega + T_c\omega)$$
$$\omega P_i(\omega) = \frac{-K}{t^2\omega^2+1}(\sin(T_i\omega) + a\sin(T_i\omega + T_c\omega) + t\omega\cos(T_i\omega) + at\omega\cos(T_i\omega + T_c\omega)$$

And we are taking a PI controller defined as

$$C(j\omega) = K_p - \frac{K_i}{j\omega}$$

Considering the plant

$$P(j\omega) = P_r(\omega) + j\omega P_i(\omega)$$

And the maximum sensitivity equation

$$|\frac{1}{1 + \mathrm{P(s)C(s)}}|_\infty < \gamma$$

Putting the values of P(s) and C(s) in the equation gives us

$$|1+\left(P_r(\omega) + jP_i(\omega)\right)\left(K_p - j\frac{K_i}{\omega}\right)| > 1/\gamma\text{^}2$$

Solving and rearranging the equation and substituting the values of $P_r(\omega)$ *and* $P_i(\omega)$ gives us the parametric equation of an ellipse.

$$K_i = C_i(\omega) + h(\omega)\cos(\theta)$$

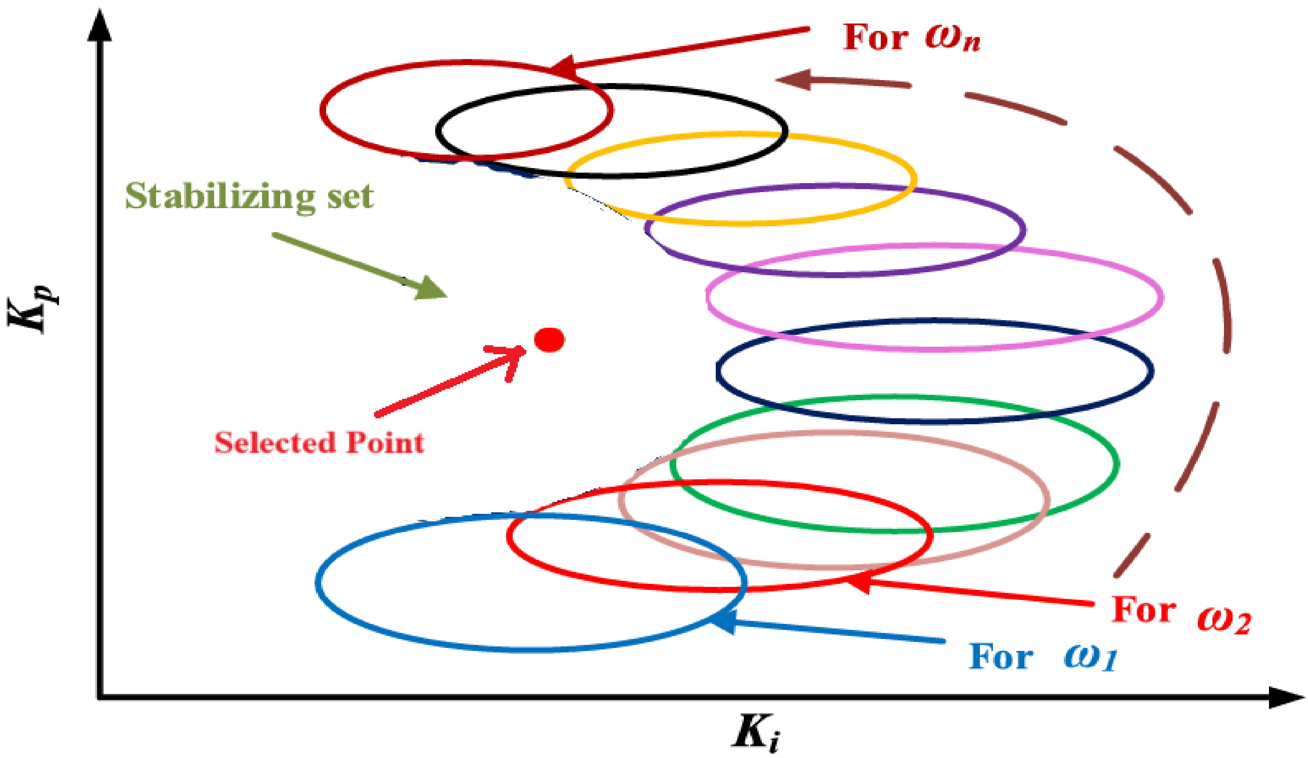
$$K_p = C_p(\omega) + k(\omega)sin(\theta) \qquad \theta \in [0,2\pi)$$

Where,

$$C_i(\omega) = -\frac{\omega^2 P_i(\omega)}{|P(j\omega)|^2}, \quad C_p(\omega) = -\frac{P_r(\omega)}{|P(j\omega)|^2},$$

$$h(\omega) = \gamma|P(j\omega)| \quad k(\omega) = \frac{\gamma|P(j\omega)|}{\omega},$$

1.3.2 Plotting the ellipses.



From the region outside the ellipses, we will select a point (Ki,Kp) as the PI controller parameters for our controller.

1.3.3 Choosing the best region for the plant.

We plot the Nyquist plot of the closed loop gain P(s)C(s) to check the gain margin and phase margin to better understand the regions we need to select the point from.

1.3.4 Selecting a point and analyzing the controller response.

The values of Kp and Ki are put in the Simulink simulation and the output is analysed and compared to the output of the FOPID controller of the reference paper. The Model is shown below:-

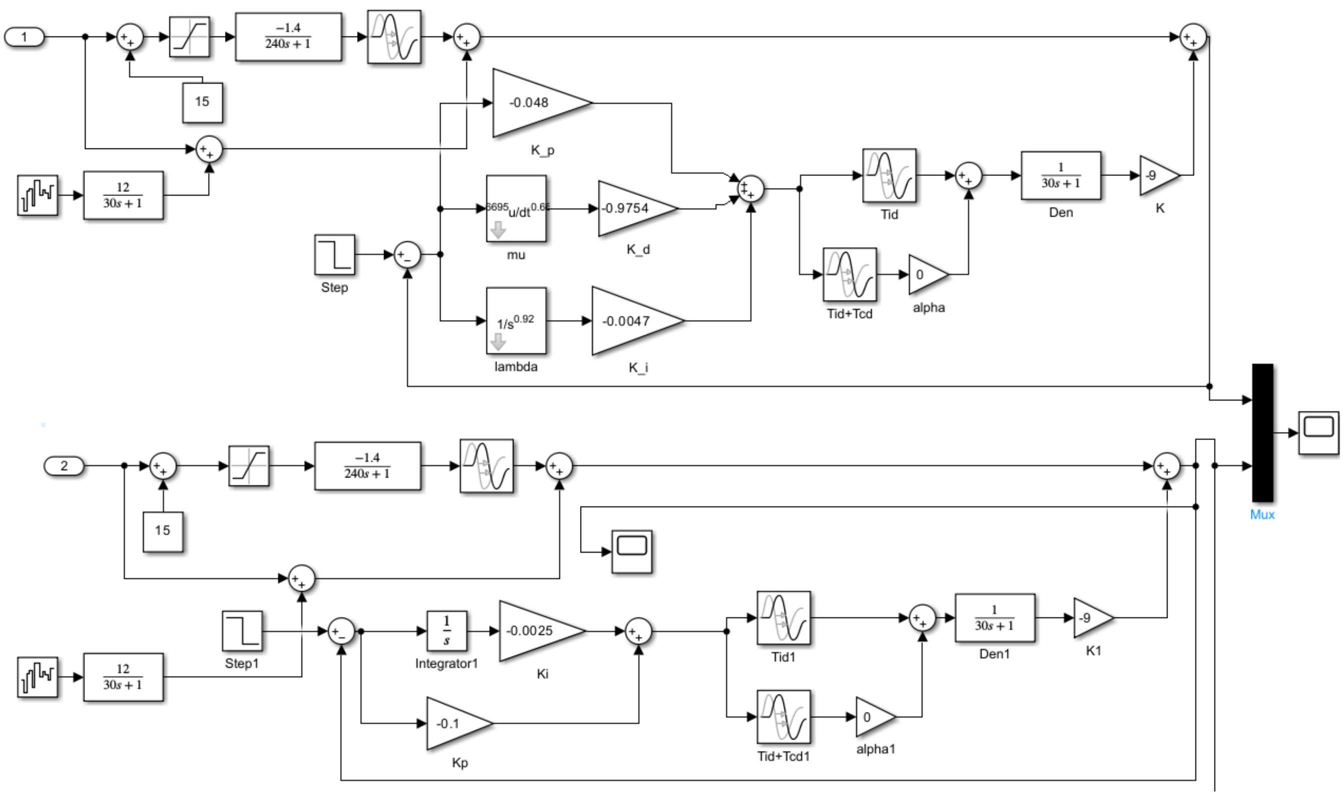


Figure: Simulink model for comparing the output of FOPID to the proposed PI controller

1.3.5 Analysis of the robustness of the PI controller

The Plant is given a ±10% disturbance and the controller's performance is analyzed for the case of the new plant.

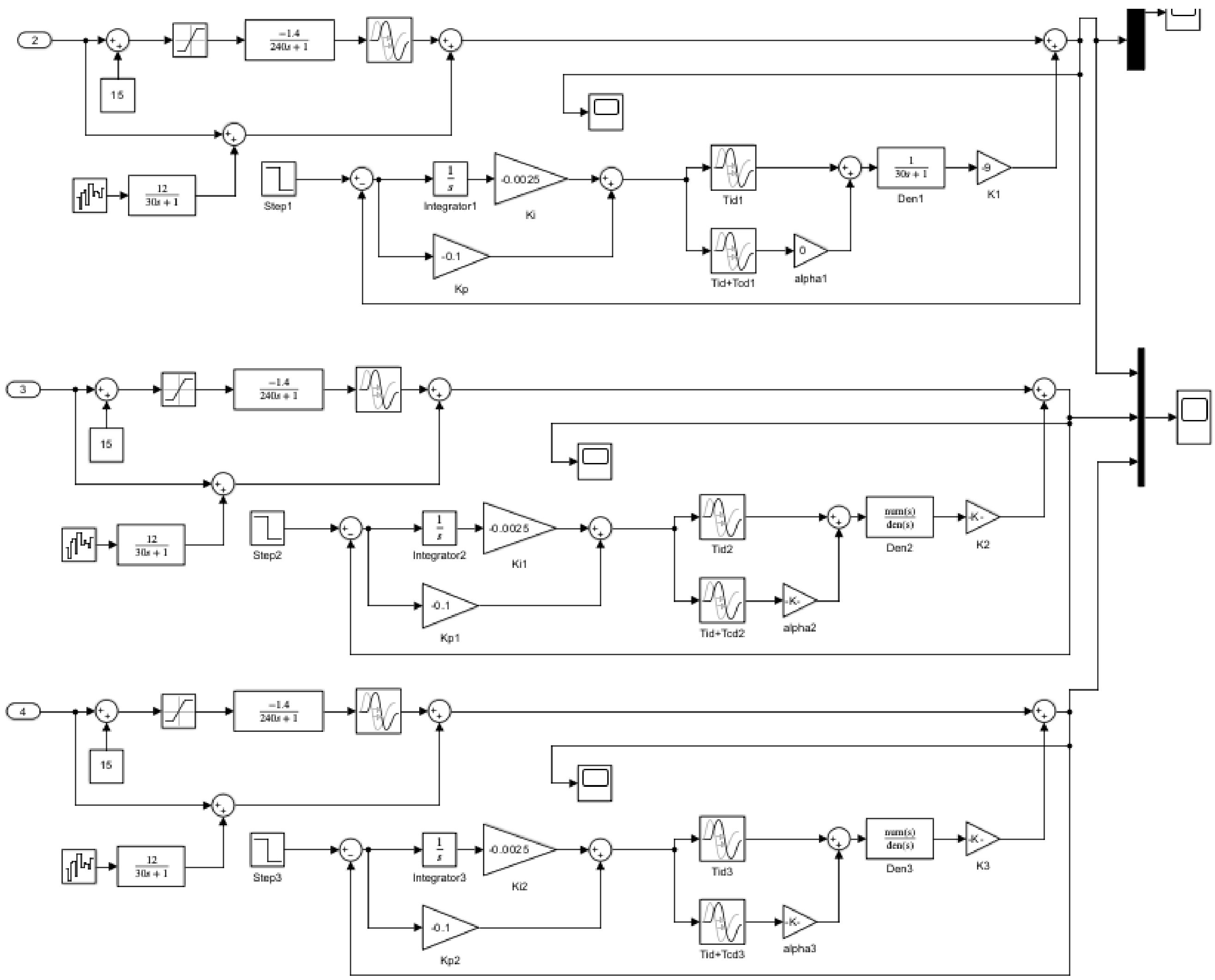


Figure: Simulink model for comparing the output of PI controller with ±10% perturbation

2. Simulation results

2.1 Plotting the Ellipses for a sensitive and a nominal patient model

2.1.1 For a sensitive patient, the parameters of P(s) are K = −9 mmHg/(ml/hr), α = 0, Tid(sec)=20, Tcd (sec) = 30, τ (sec) = 30, ω = [10−6,0.68]

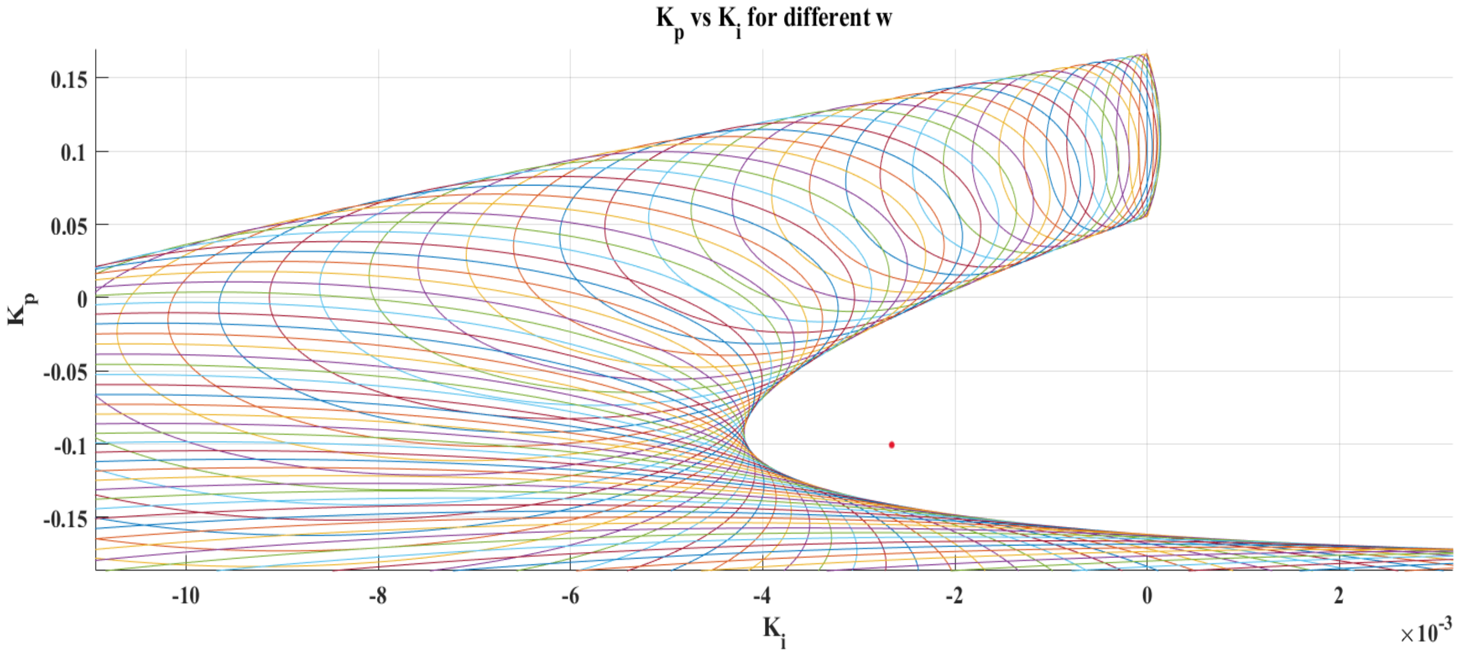


Figure: Sensitive patient plot for γ = 2, Selected Ki = −0.0025, Kp = −0.1.

2.1.2 For a nominal patient, the parameters of P(s) are K = −0.7143 mmHg/(ml/hr), α = 0.4, Tid(sec) = 30, Tcd (sec) = 45, τ (sec) = 40, ω = [10−6, 0.68].

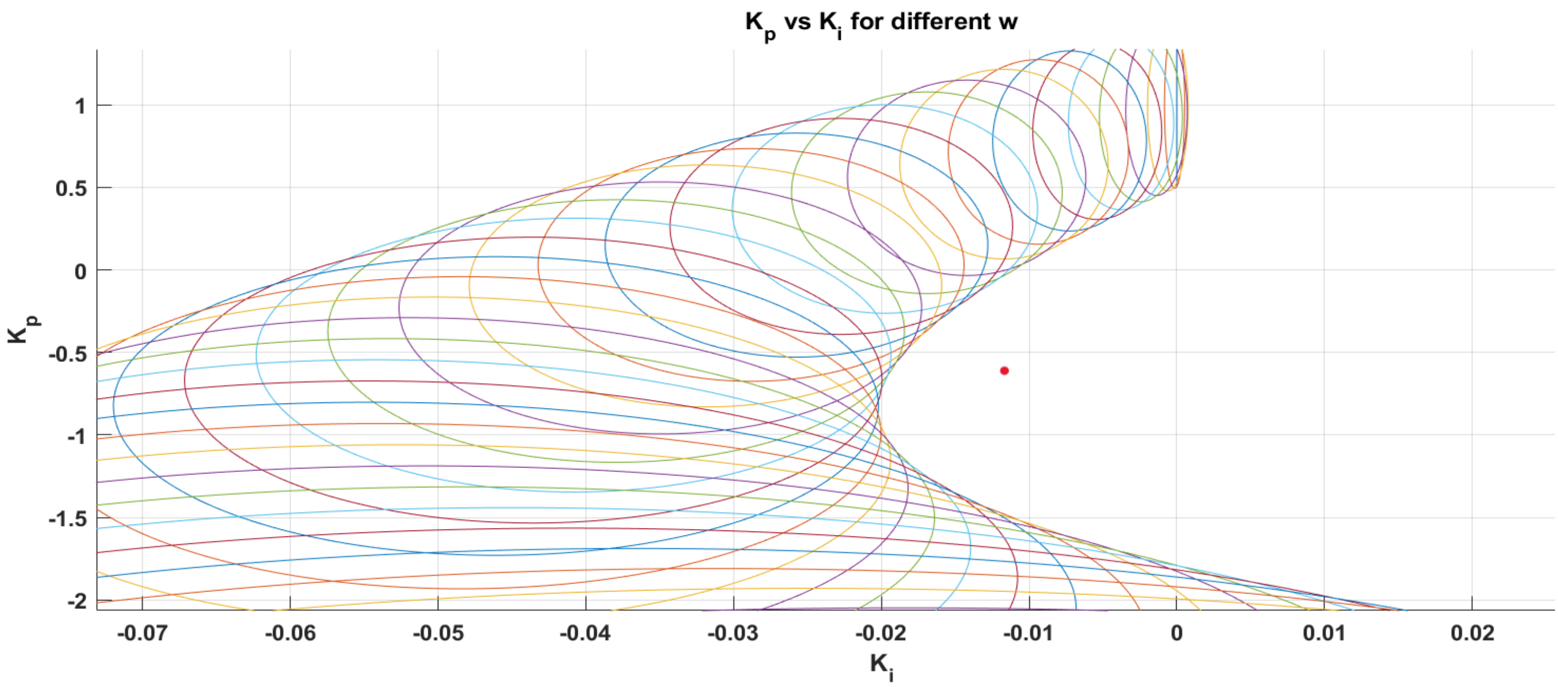


Figure: 4: Nominal patient plot for γ = 2, Selected Ki = −0.012, Kp = −0.62.

2.2 Verifying the Nyquist stability of the selected point

2.2.1 Nyquist Plot for sensitive patient

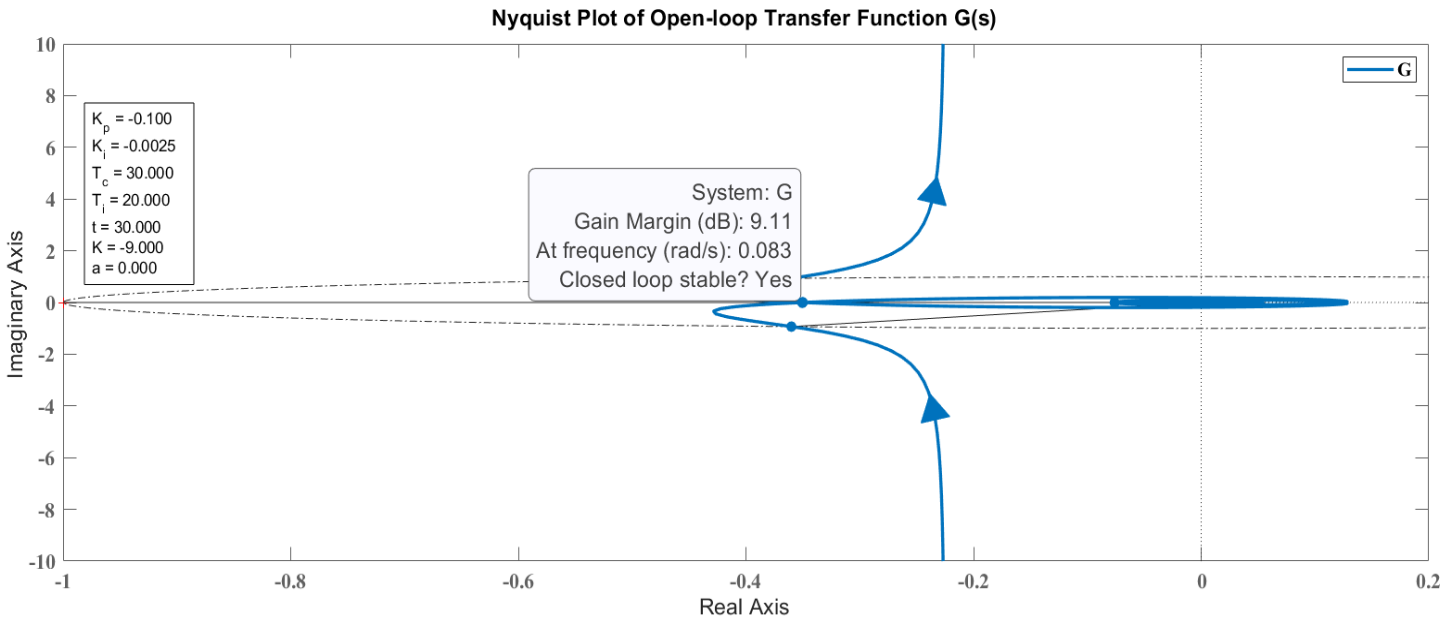


Figure: Nyquist plot shows the closed loop stability of the PI controller

2.2.2 Nyquist plot for Nominal patient

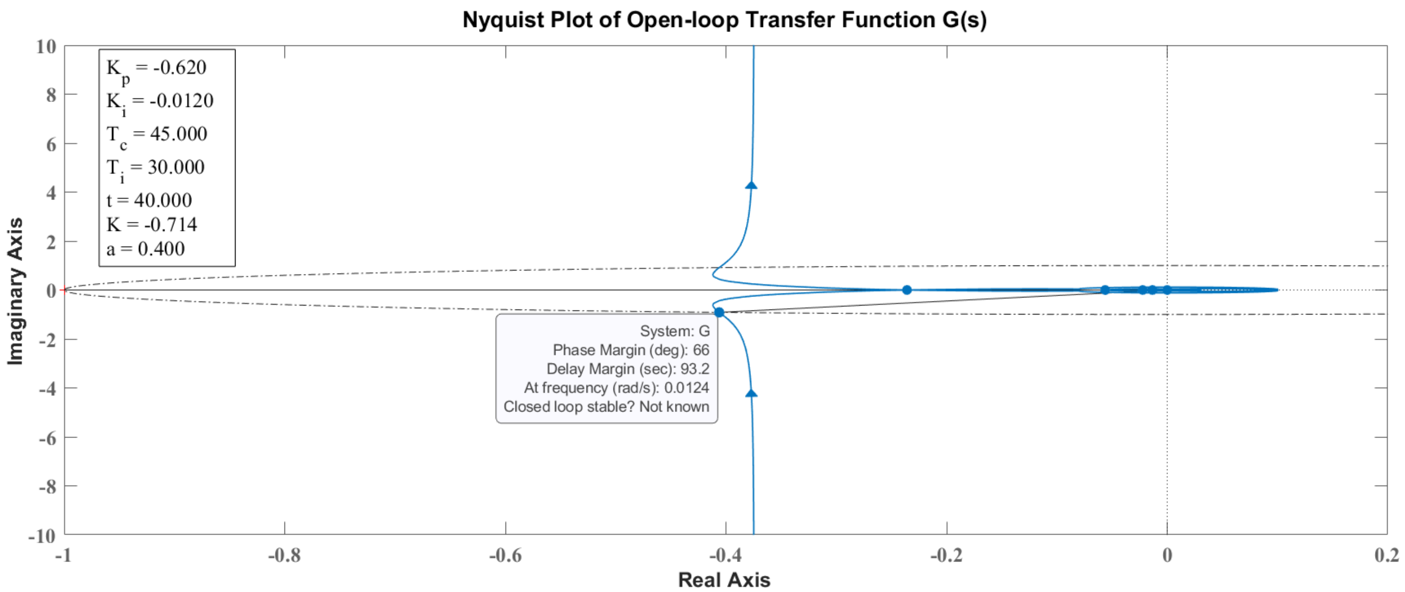


Figure: The phase and gain margin are always positive, confirming stability

2.3 Comparing the outputs to the FOPID controller's

2.3.1 FOPID vs PI controller performance for sensitive patient

Sensitive ITAE(Inverse Time Absolute Error) FOPID Parameters:
Kp = -0.048 , Ki = -0.0047, λ = 0.92, Kd = −0.9754, μ = 0.6695

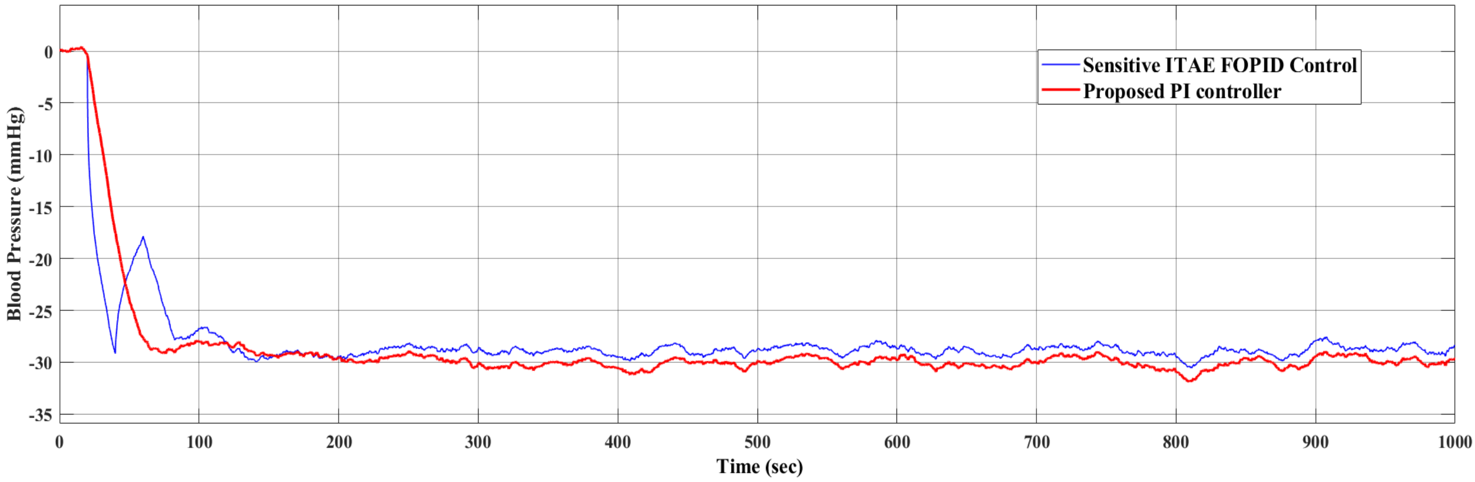


Figure: FOPID vs PI controller response for sensitive patient.

2.3.2 FOPID vs PI controller performance for nominal patient

Nominal IAE(Inverse Absolute Error) FOPID parameters:
Kp = -0.1534 , Ki = -0.0054, λ = 0.8584, Kd = −9.266, μ = 0.4605

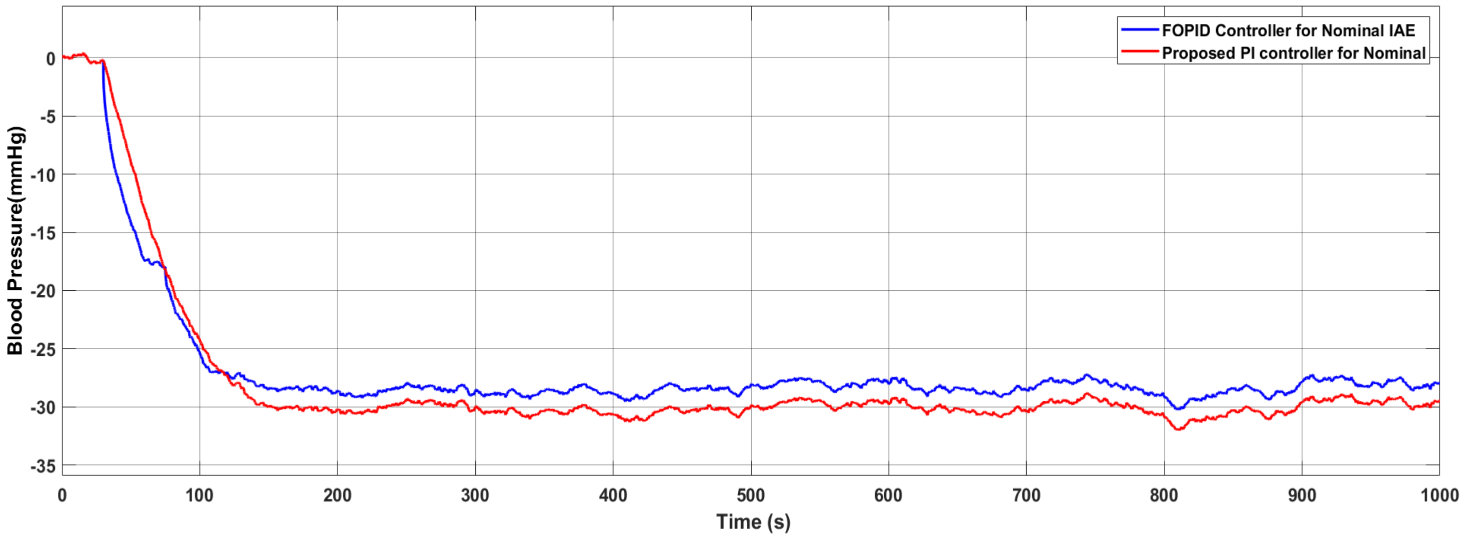
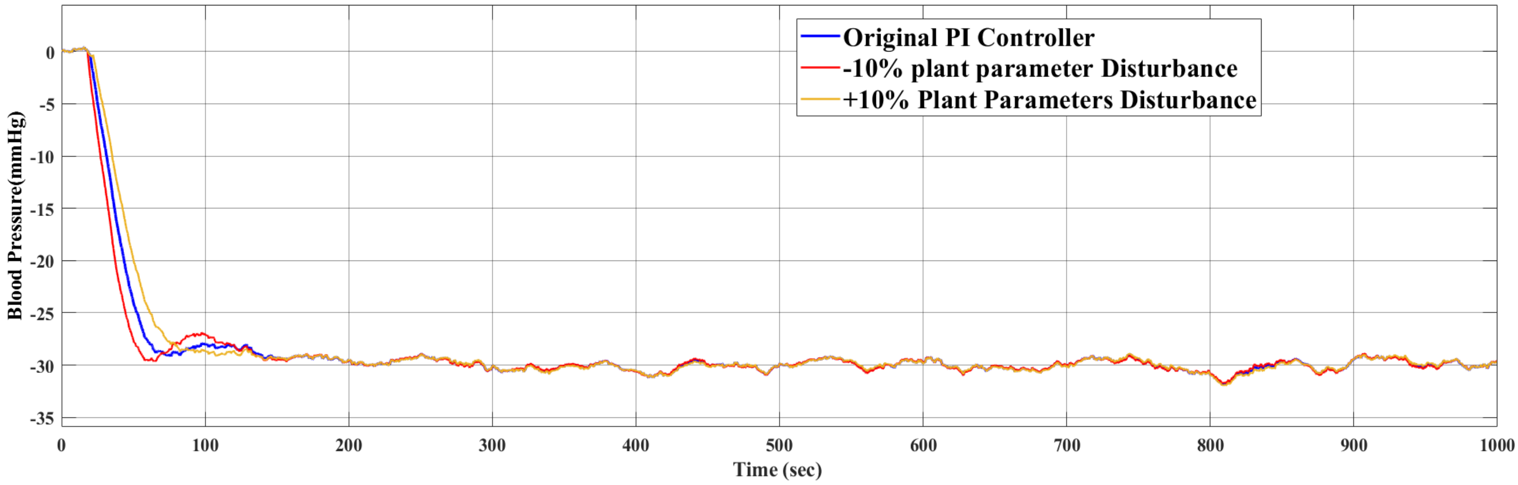


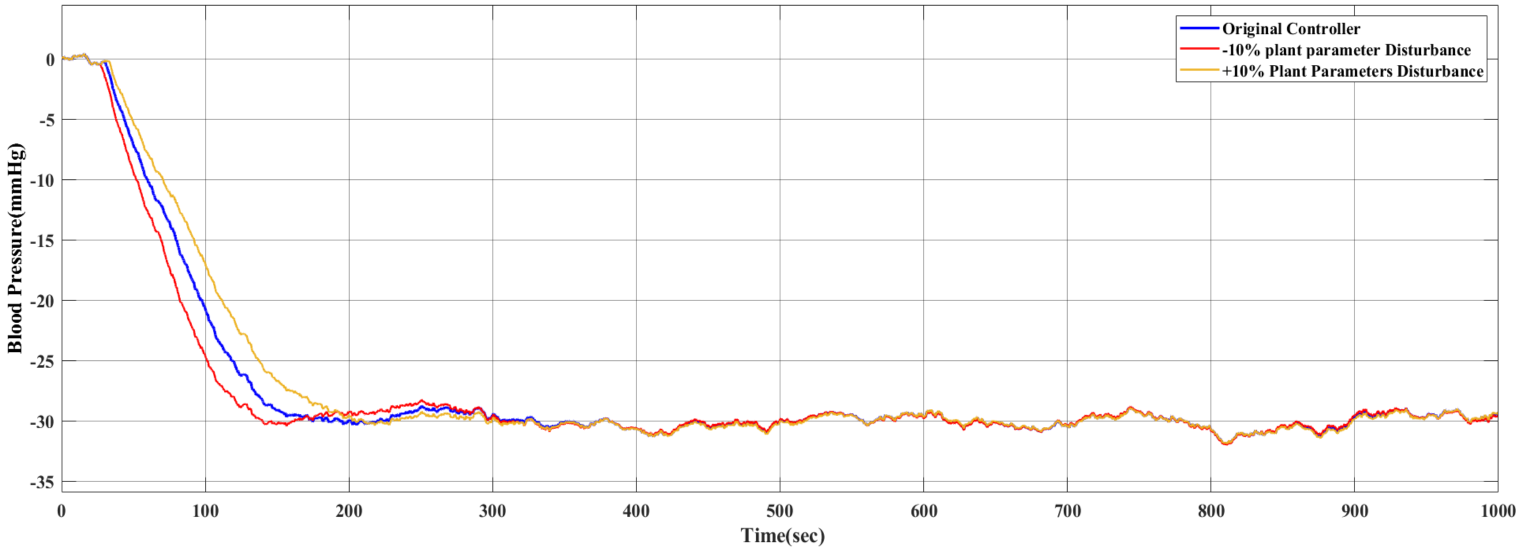
Figure: FOPID vs PI controller response for nominal patient.

2.4 Robustness Analysis

2.4.1 PI controller for sensitive patient model



2.4.2 PI controller for nominal patient model



3 Conclusion

A PI controller is designed graphically for the system with delay based on H inf Criteria. The final controller is chosen using a trial-and-error method from the stability region. And used the Nyquist plot of the open loop system to analyze the stability, closed-loop gain, and phase margins. The proposed controller satisfactorily tracks the desired reference with comparative less steady-state error and settling time. The proposed controller is linear, so it is easy to implement in real time. The parameter to be designed is less; hence, it has a smaller computational burden. In future, optimisation techniques will be used to find the best control settings.

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