Robust PI Controller Design using Graphical Approach for Blood Pressure Regulation

Gaurav Kumar Gupta (2101EE31)
Supervisor: Dr. Ahmad Ali

Department of Electrical Engineering
Indian Institute of Technology Patna, Bihar, India

October 4, 2024

Introduction

- During surgeries, to maintain the mean arterial blood pressure, a vasodilating drug is given to lower the blood pressure to control excessive bleeding.
- Its required amount and effects greatly vary on the type of patient, so we need to monitor the effects, too, continuously.
- Such precise drug-delivery systems require a controller to work on the patient response model.
- We will use a Graphical Approach based on Maximum Sensitivity with a PI controller.
- Compare the results of the PI controller to the FOPID controller of the reference paper.

Basic Control System

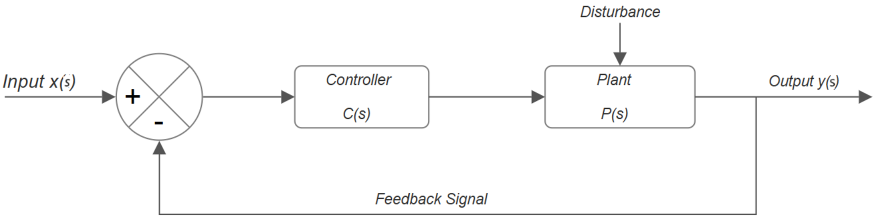

Figure: 1 : Control system Diagram.

Where:
P(s): Transfer function of the plant considered
C(s): Controller of the Plant

Problem Formulation

- For suboptimal control, maximum gain of S(sensitivity function) for frequency $\omega \in [0, \infty) \leq \gamma$ (max sensitivity).
- When $\gamma \geq 1$, ensures phase margin $\phi_m = 2\sin^{-1}\left(\frac{1}{2\gamma}\right)$ degrees, and gain margin $G_m = 20\log\left(\frac{\gamma}{\gamma-1}\right)$ dB.
- So we design a controller minimizing S, i.e.

$$\left\| \frac{1}{1 + P(s)C(s)} \right\|_\infty < \gamma \tag{1}$$

- The following equation of a human body plant for a drug response is considered for testing the method:

$$P(s) = \frac{Ke^{T_{\text{id}} s}(1 + \alpha e^{T_{\text{cd}} s})}{\tau s + 1} \tag{2}$$

Designing a Controller using using Graphical Approach based on Maximum Sensitivity

- Substituting $s = j\omega$, in $P(s)$ so that, $e^{Tj\omega} = \cos(T\omega) - j\sin(T\omega)$ gives:

$$P_r(\omega) = \frac{K}{t^2\omega^2 + 1}\left(\cos(T_i\omega) + a\cos(T_i\omega + T_c\omega) - t\omega\sin(T_i\omega) - at\omega\sin(T_i\omega + T_c\omega)\right) \tag{3}$$

$$\omega P_i(\omega) = \frac{-K}{t^2\omega^2 + 1}\left(\sin(T_i\omega) + a\sin(T_i\omega + T_c\omega) + t\omega\cos(T_i\omega) + at\omega\cos(T_i\omega + T_c\omega)\right) \tag{4}$$

- **A PI controller $C(s) = K_p - \frac{K_i}{s}$ has been used.**

- Considering $P(j\omega) = P_r(\omega) + j\omega P_i(\omega)$ and $C(j\omega) = K_p - \frac{K_i}{j\omega}$, eqn(1) is written as:

$$\left|(1 + P_r(\omega)K_p + P_i(\omega)K_i) + j\left(\omega P_i(\omega)K_p - P_r(\omega)\frac{K_i}{\omega}\right)\right| \geq \gamma, \qquad (5)$$

- On simplifying and rearranging equation (5) in the parametric form of an ellipse gives:

$$K_i = C_i(\omega) + h(\omega)\cos(\theta)$$
$$K_p = C_p(\omega) + k(\omega)\sin(\theta)$$
$$\theta \in [0, 2\pi)$$

Where:

$$C_i(\omega) = -\frac{\omega^2 P_i(\omega)}{|P(j\omega)|^2}, \quad C_p(\omega) = -\frac{P_r(\omega)}{|P(j\omega)|^2},$$

$$h(\omega) = \gamma|P(j\omega)| \quad k(\omega) = \frac{\gamma|P(j\omega)|}{\omega},$$

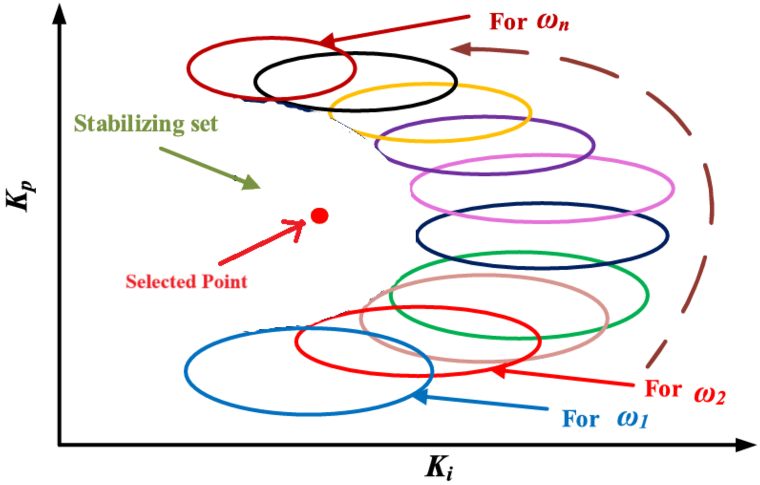

Figure: 2 : Stabilizing PI controller region for plant.

- From the figure 2, we can see that the region outside the ellipses is stable, and values of K_p and K_i can be selected from it.

Testing on a Sample Equation 1

Values for a sensitive patient are: $K = -9$ mmHg/(ml/hr), $\alpha = 0$, T_{id} (sec) $=$ 20, T_{cd} (sec) $= 30$, τ (sec) $= 30$, $\omega = [10^{-6}, 0.68]$

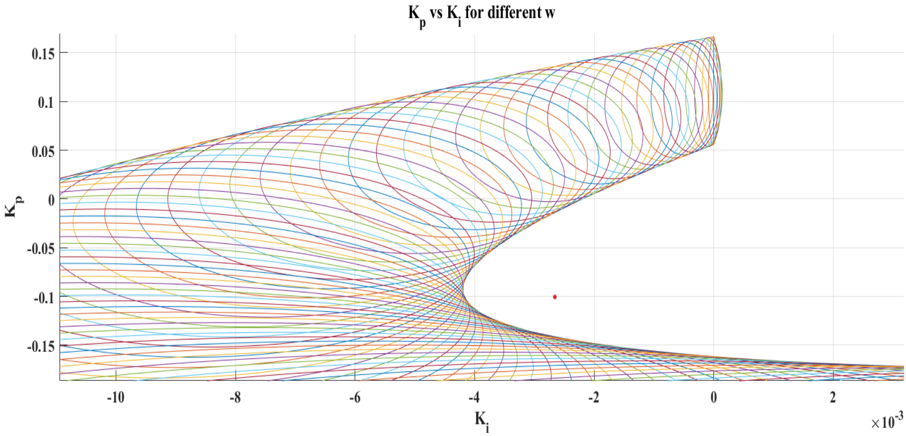

Figure: 3: Sensitive patient plot for $\gamma = 2$, Selected $K_i = -0.0025$, $K_p = -0.1$.

Testing on a Sample Equation 2

Values for a nominal patient are: $K = -0.7143$ mmHg/(ml/hr), $\alpha = 0.4$, T_{id} (sec) $= 30$, T_{cd} (sec) $= 45$, τ (sec) $= 40$, $\omega = [10^{-6}, 0.68]$

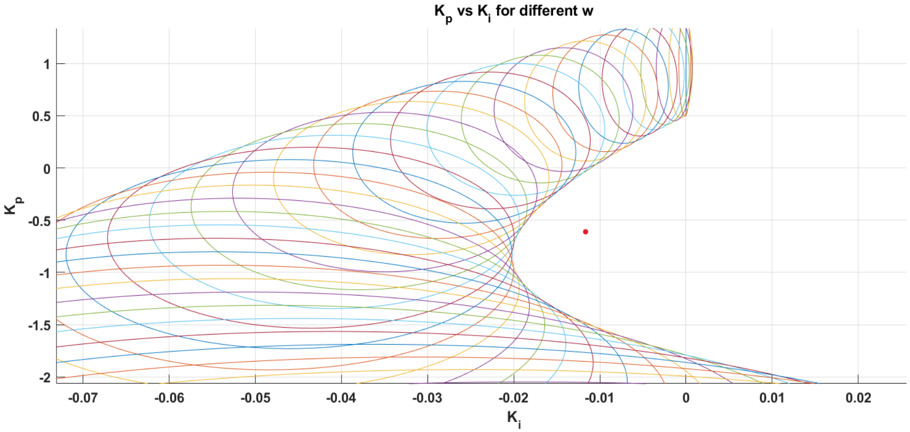

Figure: 4: Nominal patient plot for $\gamma = 2$, Selected $K_i = -0.012$, $K_p = -0.62$.

Choosing the best region for the plant

- We will use different techniques to optimize the selection of the region to choose the point.
- The Nyquist plot is analysed for encirclements of the critical point $-1 + j0$ to determine the closed-loop stability of the PI controller.

Sensitive ITAE FOPID Parameters

Kp = -0.048 , Ki = -0.0047, $\lambda = 0.92$, $Kd = -0.9754$, $\mu = 0.6695$

Nominal IAE FOPID parameters

Kp = -0.1534 , Ki = -0.0054, $\lambda = 0.8584$, $Kd = -9.266$, $\mu = 0.4605$

Simulation and Comparison

Figure 6 and 7 compares the FOPID and proposed PI controllers' performances.

Nyquist plot (Sensitive Patient)

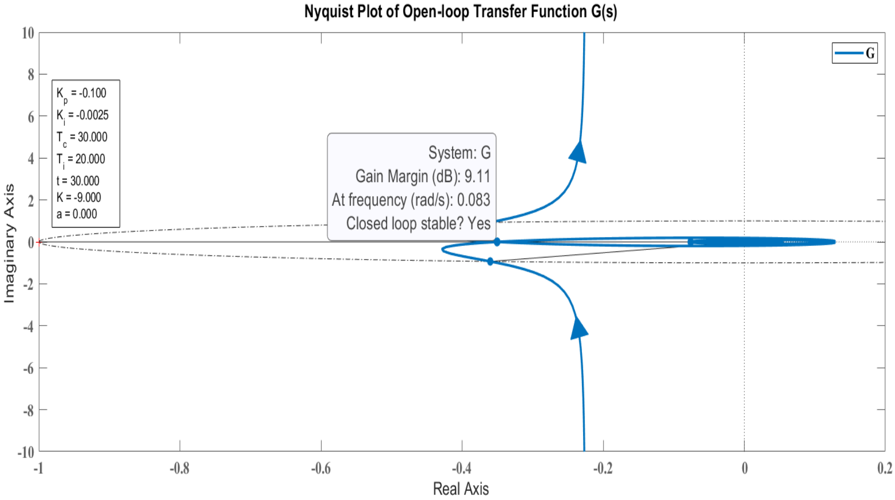

Figure: 4: Nyquist Plot for checking the Stability at the selected Kp and Ki.

Nyquist plot (Nominal Patient)

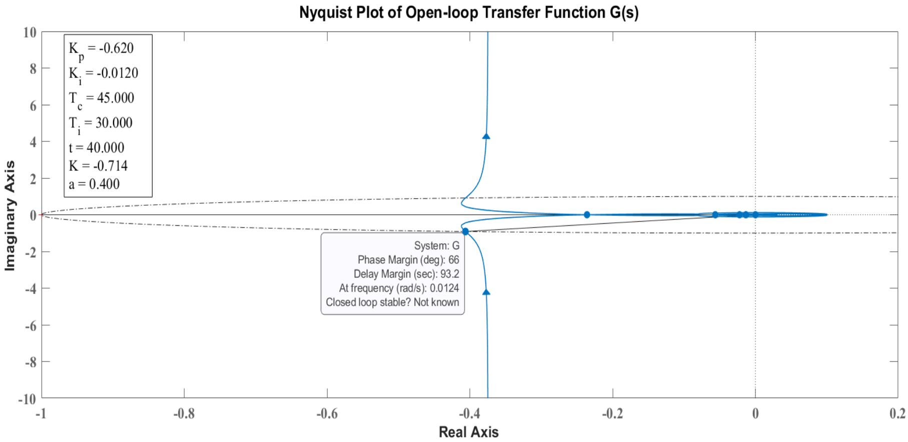

Figure: 5: Nyquist Plot for checking the Stability at the selected Kp and Ki.

Proposed Controller Performance comparison (Sensitive)

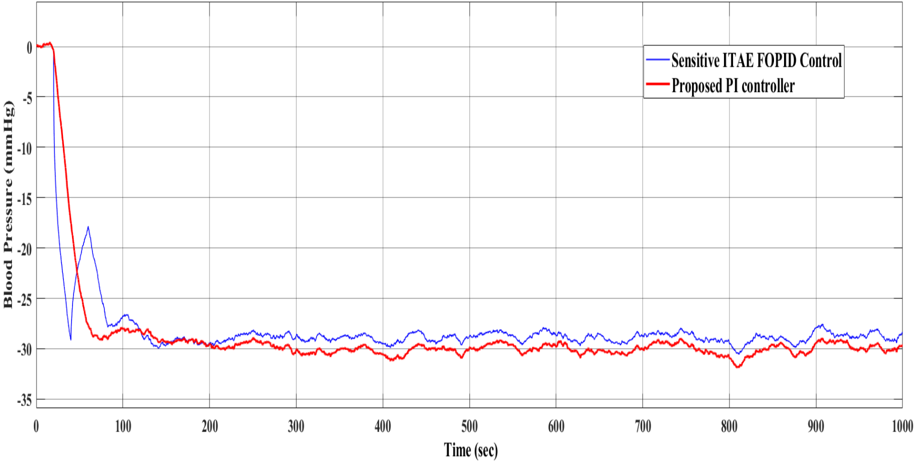

Figure: 6: Comparison between FOPID and proposed PI controller performance (Sensitive).

Proposed Controller Performance comparison (Nominal)

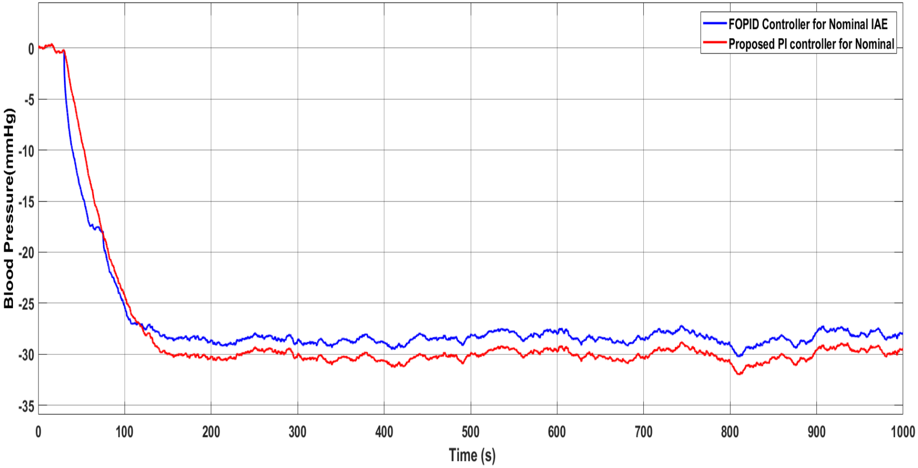

Figure: 7: Comparison between FOPID and proposed PI controller performance (Nominal).

10% Uncertanity in Plant Parameter (Sensitive)

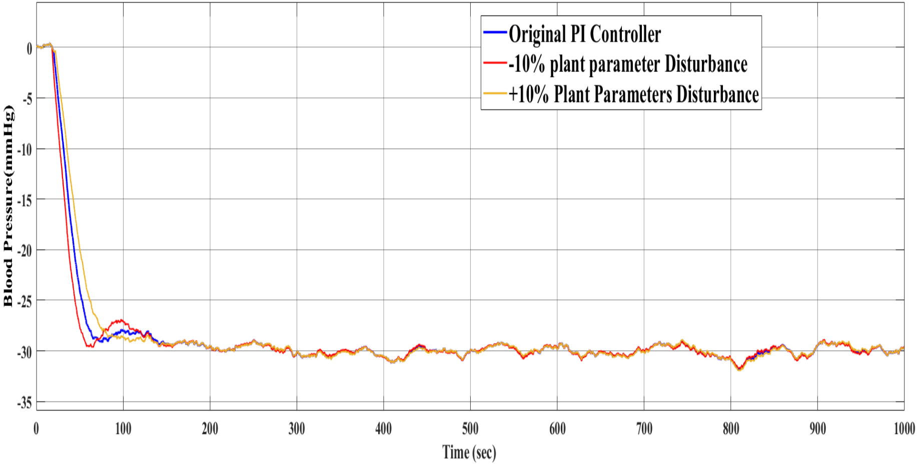

Figure: 8: Step Response with ±10% uncertainty in Sensitive Patient.

10% Uncertanity in Plant Parameter (Nominal)

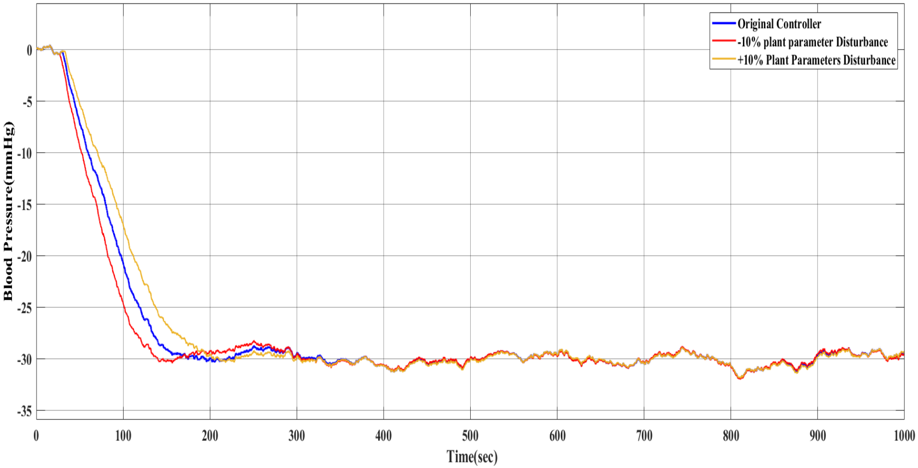

Figure: 9: Step Response with ±10% uncertainty in Normal Patient.

Conclusion

- PI controller designed using **H∞ criterion** for a system with delay.
- Controller parameters selected through **trial-and-error** method from stability region.
- Stability, closed-loop gain, and phase margins analyzed using **Nyquist plot**.
- Achieved **less steady-state error** and **shorter settling time**.
- **Linear controller** ensures easy real-time implementation.
- Fewer parameters reduce the **computational burden**.
- **Future work** involves using optimization techniques for better control settings.

References

[1] P. Krishna and G. K. R. P. V, "Fractional-order PID controller for blood pressure regulation using genetic algorithm," Biomedical Signal Processing and Control, vol. 88, p. 105564, Feb. 2024. [Online]. Available: https://doi.org/10.1016/j.bspc.2023.105564

[2] S. Saxena and Y. Hote, "A simulation study on optimal IMC-based PI/PID controller for mean arterial blood pressure," Biomedical Engineering Letters, vol. 2, Dec. 2013. [Online]. Available: https://doi.org/10.1007/s13534-012-0077-4

[3] S. R. Kumar, P. Verma, M. Ram, S. Somanshu, and M. N. Anwar, "H_∞ criterion based PI controller for DC-DC boost converter," Dec. 2022. [Online]. Available: https://doi.org/10.1109/ONCON56984.2022.10126845

Thank You!

Special thanks to
Mr. Sumit Ranjan Kumar
Miss. Akanksha Dwivedi
for their guidance