FOPI Controller Design for Blood Pressure Regulation using Hippopotamus Optimization Algorithm

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Abstract. Blood pressure readings indicate the force the heart exerts on arterial walls while pumping blood. Maintaining mean arterial blood pressure (MABP) during surgeries is critical to controlling excessive bleeding. Vasodilating drugs like sodium nitroprusside (SNP) are used to lower blood pressure, but the required dosage and effects vary significantly depending on the condition of the patient. This necessitates continuous monitoring and precise drug delivery. To achieve this, a fractional-order proportional integral (FOPI) controller is designed by defining the search space using a graphical approach based on the maximum sensitivity criterion, then utilizing the hippopotamus optimization algorithm (HOA) on the region to select the suboptimal parameters. The maximum sensitivity criteria ensure adequate phase margin (ϕ_m) and gain margin (G_m) for stability. And HOA optimizes the FOPI parameters by minimizing integral absolute error (IAE) and integral time absolute error (ITAE) taken as cost functions. MATLAB simulations demonstrate the controller's superior performance in integral error, settling time, steady-state error reduction, and real-time applicability compared to existing control strategies.

Keywords: Mean Artial Blood Pressure · Hippopotamus Algorithm · Fractional-Order Proportional Integral · H_∞ criteria · Integral Error

1 Introduction

Blood pressure (BP) readings measure the force the heart exerts on arterial walls during blood circulation. It is influenced by cardiac output and peripheral resistance. Vasodilators like Sodium Nitroprusside (SNP) are administered to hypertensive patients to lower blood pressure by reducing peripheral resistance [1]. Precise infusion control is crucial to avoid toxicity from SNP breakdown products and prevent adverse effects like shock from excessive BP reduction, especially in post-operative patients.

To address these challenges, Sheppard et al. developed a computer-based control system by analyzing the physiological responses of SNP, but the system did not account for patient sensitivity [2]. These early controllers lacked adaptability to real-world disturbances such as surgical stimuli, necessitating the development

of adaptive controllers. So, Slate and Sheppard developed a model-based adaptive controller incorporating patient sensitivity to the drug [3]. Internal Model Control (IMC) improved peak overshoot, settling time, and steady-state performance. Further tuning of IMC enhanced robustness and disturbance rejection through IMC filtering [4]. Hamamaci [6] introduced an algorithm for regulating MABP using a FOPID controller, which exhibited superior disturbance rejection and greater adaptability to uncertain conditions than conventional PID controllers. To enhance controller parameter tuning, optimization strategies have gained substantial attention. Several metaheuristic algorithms, such as the seeker optimization algorithm, whale optimization algorithm, particle swarm optimization, and human-inspired methods, are applied. Additionally, genetic algorithms are frequently used to optimize PID parameters [7].
This paper focuses on designing a 2-step FOPI controller combining H_∞ criteria [9] to define search space and then using hippopotamus optimization algorithm (HOA) [8] to tune the controller parameters for regulating MABP.

2 Patient Response Model

Slate et al. [3] investigated the impact of SNP infusion on patients by analyzing five parameters that exhibit significant variability in MABP in response to the drug and modeled the transfer function given below:

$$P(s) = \frac{\Delta P_d(s)}{I_n(s)} = \frac{Ke^{-T_{id}s}(1 + \alpha e^{-T_{cd}s})}{\tau s + 1} \tag{1}$$

where $\Delta P_d(s)$ represents the change in MABP (mmHg) and $I_n(s)$ is the drug infusion rate (ml/hr). The patients are classified into three categories based on their drug sensitivity, as shown in Table 1. Enbiya [5] developed a simulink model to represent inherent disturbances in biological systems shown in Fig 1.

Table 1: Parameter Values of Patients

Symbol	Parameter	Sensitive	Normal	Insensitive
K (mmHg/(ml/hr))	Drug sensitivity	-9	-0.7143	-0.1786
α	Recirculation constant	0	0.4	0.4
T_{id} (sec)	Initial time delay	20	30	60
T_{cd} (sec)	Recirculation time delay	30	45	75
τ (sec)	Time constant	30	40	60

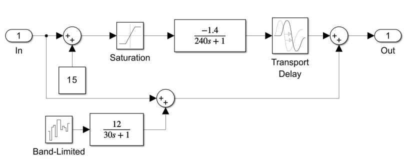

Fig. 1: Model of Disturbances.

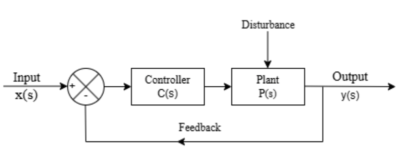

Fig. 2: Proposed Control Structure.

3 Controller Design Methodology

The FOPI controller should meet the following design requirements for a setpoint of -30 mmHg: a settling time of less than 500 seconds, an overshoot not exceeding 7 mmHg, and a steady-state error of less than 1 mmHg.

3.1 H_∞ Criterion based graphical approach

For the system shown in Fig. 2, for the suboptimal control, the maximum value of gain of sensitivity function must be less than γ (defined as maximum sensitivity). Hence, for the designed controller C(s), the condition 2 should be satisfied:

$$\left|\frac{1}{1+P(s)C(s)}\right|_\infty < \gamma \tag{2}$$

The plant transfer function in frequency domain is written as:

$$P(s) = P(j\omega) = P_r(\omega) + jP_i(\omega). \tag{3}$$

Also, a FOPI controller defined as

$$C(j\omega) = K_p - \frac{K_i}{j\omega^\lambda} \tag{4}$$

Putting (3) and (4) in (2) gives us

$$\left|1 + (P_r(\omega) + jP_i(\omega))\left(K_p - \frac{jK_i}{\omega^\lambda}\right)\right| > \frac{1}{\gamma} \tag{5}$$

Solving and rearranging (5) gives us the equation of an ellipse shown below:

$$\frac{(K_i - C_1(\omega))^2}{a^2} + \frac{(K_p - C_2(\omega))^2}{b^2} > 1 \quad \text{where:} \tag{6}$$

$$C_1(\omega) = -\frac{\omega\omega^\lambda P_i(\omega)}{|P(j\omega)|^2}, \qquad C_2(\omega) = -\frac{P_r(\omega)}{|P(j\omega)|^2}$$

$$a(\omega) = \frac{\omega^\lambda}{\gamma|P(j\omega)|}, \qquad b(\omega) = \frac{1}{\gamma|P(j\omega)|}$$

The ellipses from (6) are plotted in MATLAB. The plotted ellipses for a sensitive patient with the selected region given to HOA for optimization is shown in fig. 3.

3.2 Hippopotamus Optimization Algorithm

The(HOA) is a metaheuristic algorithm inspired by the social behavior of hippopotamuses. It simulates their interactions within a herd, including position updates, defense against predators, and escape mechanisms. HOA balances exploration (searching diverse areas) and exploitation (refining solutions in promising regions) to find optimal solutions. The algorithm works similar to behavioral patterns of hippos in different situations or phases. The pseudocode of HOA with equations is provided below:

Start HO

1. Define the optimization problem.
2. Set the maximum number of iterations (T) and hippopotamuses (N).
3. Generate the initial positions of all hippopotamuses and evaluate the objective function for them using the equation: $\chi_i : x_{i,j} = lb_j + r \cdot (ub_j - lb_j)$
4. For $t = 1$ to T
5. Update the dominant hippo position based on the objective function value.
6. **Phase 1**: Hippopotamus position update (Exploration Phase)
7. For $i = 1$ to $N/2$
8. Calculate the new position for the ith hippo using the equations:
$$\chi_i^{M_{hippo}} : x_{i,j}^{M_{hippo}} = x_{i,j} + y_1 \cdot (D_{hippo} - I_1 x_{i,j}) \quad \text{and}$$
$$X_i^{FB_{hippo}} : x_{i,j}^{FB_{hippo}} = \begin{cases} x_{i,j} + h_1 \cdot (D_{hippo} - I_2 \cdot MG_i), & T > 0.6 \\ x_{i,j} + k_2 \cdot (M_{Gi} - D_{hippo}), & T < 0.6 \;\text{ and }\; r_6 > 0.5 \\ lb_j + r_7 \cdot (ub_j - lb_j), & \text{otherwise} \end{cases}$$
9. Update the position of the ith hippopotamus using the equations:
$$\chi_i = \begin{cases} \chi_i^{M_{hippo}}, & F_i^{M_{hippo}} < F_i \\ \chi_i, & \text{otherwise} \end{cases} \quad \text{and} \quad \chi_i = \begin{cases} \chi_i^{FB_{hippo}}, & F_i^{FB_{hippo}} < F_i \\ \chi_i, & \text{otherwise} \end{cases}$$
10. End For
11. **Phase 2**: Hippopotamus defense against predators (Exploration Phase)
12. For $i = 1 + N/2$ to N
13. Generate a random position for the predator using the equation:
$$Predator_j = lb_j + r_8 \cdot (ub_j - lb_j)$$
14. Calculate the new position for the ith hippopotamus using the equation:
$$\chi_i^{\text{HippoR}} = \begin{cases} \mathbf{RL} \oplus \text{Predator}_j + \frac{f}{c - d \cdot \cos(2\pi g)} \cdot \frac{1}{\mathbf{D}}, & F_{\text{Predator}_j} < F_i, \\ \mathbf{RL} \oplus \text{Predator}_j + \frac{f}{c - d \cdot \cos(2\pi g)} \cdot \frac{1}{2.\mathbf{D} + r_9}, & F_{\text{Predator}_j} \geq F_i, \end{cases}$$
15. Update the position of the ith hippopotamus using the equation:
$$\chi_i = \begin{cases} \chi_i^{HippoR} & F_i^{HippoR} < F_i \\ \chi_i & F_i^{HippoR} \geq F_i \end{cases}$$
16. End For
17. **Phase 3**: Hippopotamus escaping from the predator (Exploitation Phase)
18. For $i = 1$ to N
19. Calculate new bounds for the decision variables using the equations:
$$\text{lb}_j^{\text{local}} = \frac{\text{lb}_j}{t}, \quad \text{ub}_j^{\text{local}} = \frac{\text{ub}_j}{t}$$
20. Calculate the new position for the ith hippopotamus using the equation:
$$x_{i,j}^{\text{HippoE}} = x_{i,j} + r_{10} \cdot \text{lb}_j^{\text{local}} + s_1 \cdot \left(\text{ub}_j^{\text{local}} - \text{lb}_j^{\text{local}}\right)$$
21. Update the position of the ith hippopotamus using the equation:
$$\chi_i = \begin{cases} \chi_i^{HippoE} & F_i^{HippoE} < F_i \\ \chi_i & F_i^{HippoE} \geq F_i \end{cases}$$
22. End For
23. Save the best candidate solution found so far.
24. End For
25. Output the best solution of the objective function found by HO.

End HO

3.3 Designing the Controller

The controller design process begins by defining the plant model with appropriate patient parameters and setting $\gamma = 2$ followed by plotting the ellipses for the closed-loop function. Next, the range of (K_p, K_i) is selected from the region bounded by these ellipses, and a range for λ is chosen, preferably between 0 and 2. The HO algorithm is then executed in MATLAB over the specified range using 25 search agents and 200 iterations. If the simulation encounters an error, the values of K_p, K_i and λ are checked in Simulink, as one or more of these parameters may have caused instability; the range is adjusted accordingly, and the algorithm is rerun. If the final result lies on the boundary of the specified region, the range is expanded, and the algorithm is executed again. This controller design approach is applied to all three types of patients, and the corresponding controller parameters tuned by HOA are presented in Table 2.

Table 2: HOA Tuned FOPI Parameters.

Patient	Error	K_p	K_i	λ
Sensitive	IAE	-0.12644	-0.0022915	1.0505
	ITAE	-0.103	-0.0023486	1.0295
Nominal	IAE	-1.6097	-0.009579	1.0919
	ITAE	-1.0323	-0.012667	1.0169
Insensitive	IAE	-3.9225	-0.023564	1.0627
	ITAE	-3.6109	-0.023459	1.0562

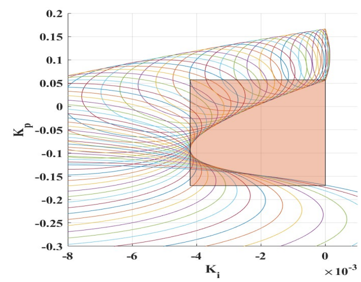

Fig. 3: Sensitive patient ellipses with the selected region.

4 Simulation Results and Discussion

The proposed FOPI (HOA) controller demonstrates superior performance compared to the FOPID (GA) controller [10] across all three patient models. The set point of -30 mmHg was used to evaluate the effectiveness of the controllers, with step responses compared in Fig. 4, Fig. 6 and Fig. 8, and control efforts illustrated in Fig. 5, Fig. 7, and Fig. 9.

The results, as summarized in Table 3, highlight significant improvements in steady-state error, settling time, and integral errors. Specifically, the settling time for the sensitive patient model was reduced by nearly 35%, while for the nominal patient model with the ITAE performance index, the reduction was as high as 80%. The FOPI (HOA) controller achieved minimal steady-state errors, ensuring compliance with the design requirements, whereas the FOPID (GA) controller exhibited steady-state errors exceeding $\pm 5\%$ of the setpoint for insensitive and nominal patient models with the IAE performance index so the settling time in these cases were not taken. Furthermore, the ITAE cost function was optimized by 50% to 70% across all patient types, more than the IAE cost function optimization range of 25% to 45%. Additionally, the control effort

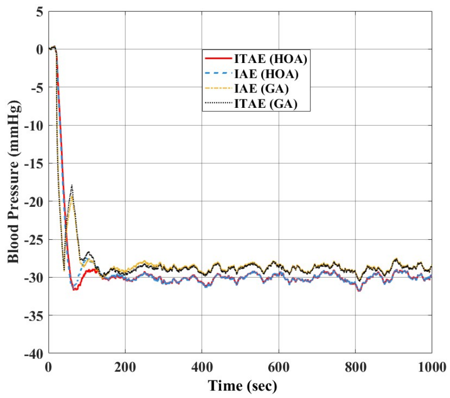

Fig. 4: Sensitive Patient Results.

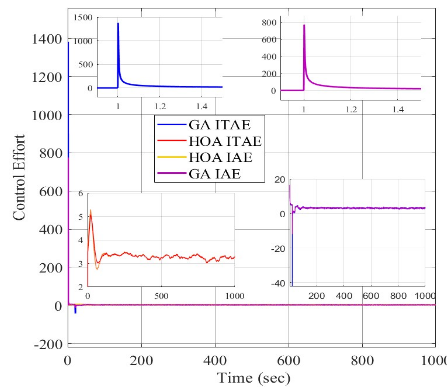

Fig. 5: Control Effort for Sensitive.

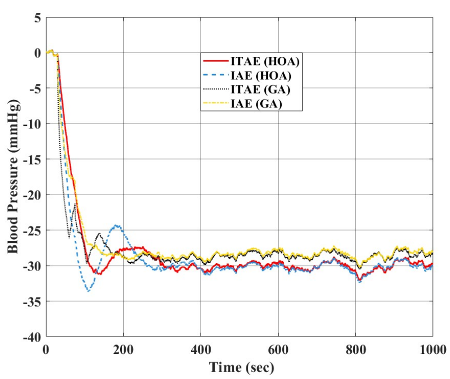

Fig. 6: Nominal Patient Results.

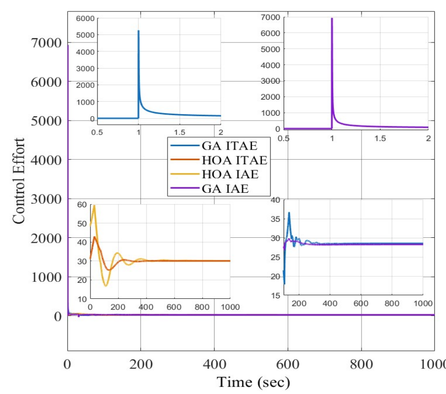

Fig. 7: Control Effort for Nominal.

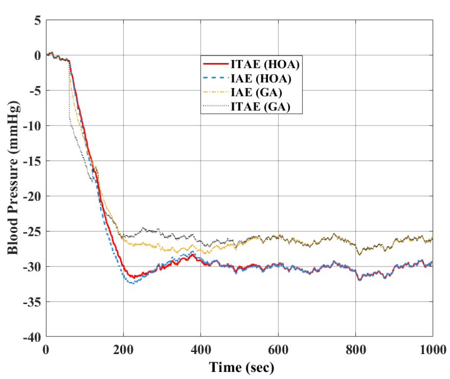

Fig. 8: Insensitive Patient Results.

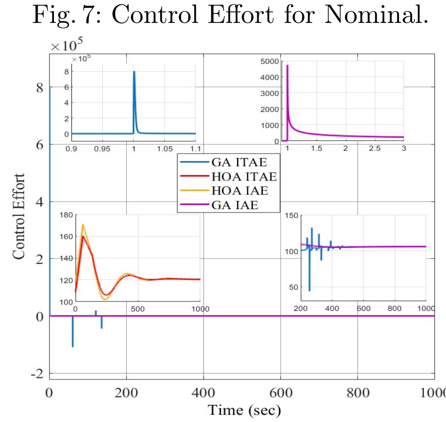

Fig. 9: Control Effort for insensitive.

required by the FOPI (HOA) controller was significantly lower compared to the FOPID (GA) controller, further emphasizing its efficiency and robustness. These findings underscore the potential of the proposed FOPI controller as a reliable and effective solution for managing patient-specific physiological responses.

Table 3: Performance Indices for Patient Model

		Integral Error		Steady State Error		Settling Time (s)	
Patient	Cost	FOPID GA	FOPI HOA	FOPID GA	FOPI HOA	FOPID GA	FOPI HOA
Sens.	IAE	2.36e3	1.3e3	-1.27	0.023	81	53
	ITAE	0.59e6	0.23e6	-1.17	0.016	114	78
Nom.	IAE	3.2e3	2.4e3	-1.734	0.103	–	232
	ITAE	0.73e6	0.35e6	-1.458	0.022	583	104
Insens.	IAE	6.2e3	4.0e3	-3.545	0.172	–	429
	ITAE	1.94e6	0.49e6	-3.533	0.170	–	422

5 Conclusion

This study successfully designed a fractional-order proportional-integral (FOPI) controller for regulating MABP across three distinct patient models. A novel approach, combining an H_∞ criteria-based graphical method with HOA, was employed. The graphical approach, utilizing a two-dimensional K_p vs K_i parameter space, facilitated the identification of suitable parameter ranges, while HOA subsequently optimized these parameters to achieve the desired performance. This combined methodology effectively balances performance and robustness in the face of model uncertainties, a critical consideration in complex physiological systems like the human cardiovascular system. The inherent robustness of H_∞ control is well-suited for blood pressure regulation, addressing patient-specific sensitivities, variable physiological responses, and potential external disturbances. Furthermore, the approach effectively accommodates the time delays associated with drug responses within the body, optimizes the trade-off between performance metrics such as settling time and robustness, and ensures stability across a range of operational conditions, thereby mitigating the risk of excessive blood pressure fluctuations.

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