

Seeing Inside: The Radon Transform, MATLAB Experiments, and Biomedical Imaging Applications

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Abstract

The Radon transform links pure mathematics with life-saving medical technology. This report introduces the transform, demonstrates analytic and numerical examples in MATLAB, and explains how filtered back-projection underpins modern computed tomography (CT). All experiments and figures were produced with a MATLAB Live Script; figure file names are indicated for reproducibility.

Contents

1 Introduction

Computed tomography (CT) revolutionised diagnostic radiology by allowing physicians to reconstruct cross-sectional images of internal anatomy from line-integral measurements of X-ray attenuation. Mathematically, these line integrals form the *Radon transform*, introduced by Johann Radon in 1917 [?]. This report summarises the theory, reproduces textbook examples, and documents hands-on MATLAB explorations relevant to biomedical engineering practice.

2 Theory of the Radon Transform

For a function $f(x, y)$ on \mathbb{R}^2 , its Radon transform $\mathcal{R}f$ is the set of line integrals

$$(\mathcal{R}f)(p, \varphi) = \int_{-\infty}^{\infty} f(p \cos \varphi - s \sin \varphi, p \sin \varphi + s \cos \varphi) \, ds, \quad (1)$$

where p is the signed distance of the line from the origin and φ is the angle the line's normal makes with the x -axis. Recovering f from $\mathcal{R}f$ is the *inverse Radon transform*. The Fourier Slice Theorem shows that a 1-D inverse Fourier transform of each projection, followed by a 2-D inverse transform, re-creates f . Numerically this leads to *filtered back-projection* (FBP), implemented in MATLAB as `iradon`.

3 Analytic Experiments

3.1 Radially symmetric Gaussian

Using the Symbolic Math Toolbox we verified that a centred Gaussian

$$f(x, y) = \exp(-(x^2 + y^2))$$

has a Radon transform independent of φ : $(\mathcal{R}f)(p, \varphi) = \sqrt{\pi} \exp(-p^2)$. Figure ?? juxtaposes the surface plot of f and the graph of its transform.

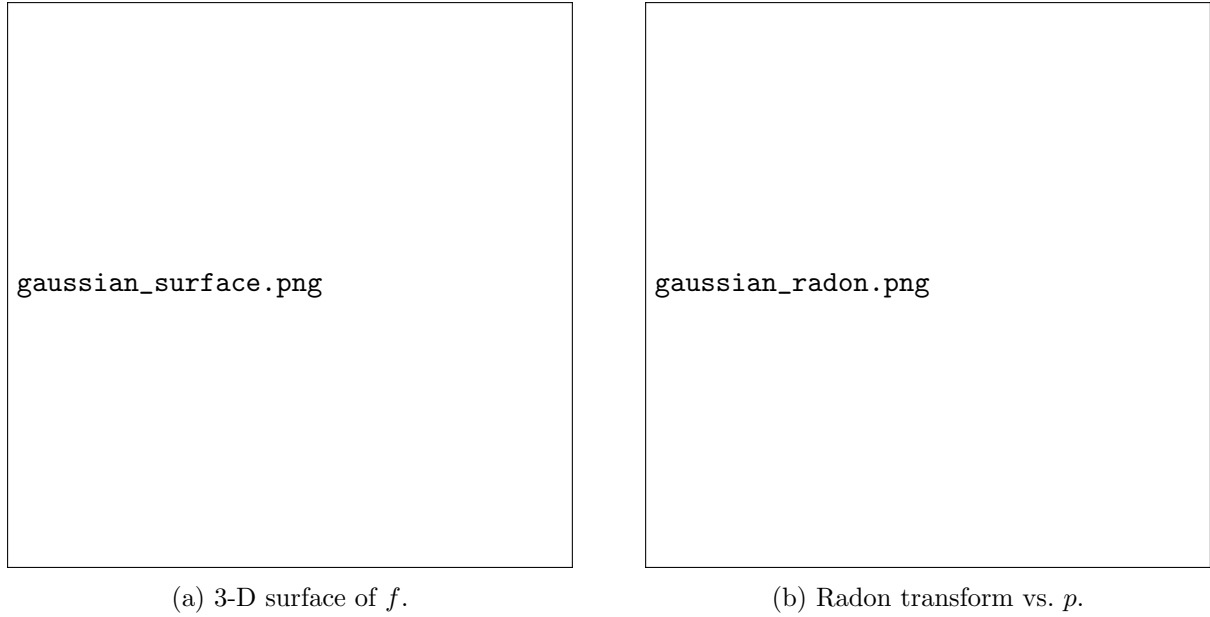


Figure 1: Radially symmetric example.

3.2 Asymmetric polynomial–Gaussian

Replacing the integrand by $g(x, y) = (x^2 + 3y^2)e^{-(x^2+y^2)}$ breaks the circular symmetry; the resulting transform depends on both p and φ (analytical expression omitted for brevity). Figure ?? shows density plots of g and $\mathcal{R}g$.

4 Discrete Radon Transform Experiments

All remaining experiments use MATLAB’s discrete `radon` and `iradon` functions.

4.1 Binary disk phantom

Figure ?? demonstrates how 180 projections of a simple disk (representing, e.g. a cross-section of an organ) produce a *sinogram*, which FBP then reconstructs nearly perfectly.

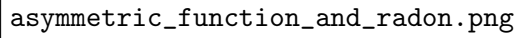
4.2 Shepp–Logan head phantom

The Shepp–Logan phantom is a synthetic yet realistic CT head slice. With 180 projections we obtain the results in Figure ?. Qualitatively the reconstruction captures fine structures such as the ventricles, illustrating clinical usefulness.

5 Biomedical Imaging Context

5.1 From mathematics to CT scanners

In practice, modern scanners collect fan- or cone-beam projections while rotating around the patient. Proprietary reconstruction pipelines apply noise correction, calibration, and iterative or FBP algorithms. Nevertheless, Equation (??) and its inversion remain the core. Figure ?? uses MATLAB’s built-in `mri` volume to visualise slices akin to clinical data.

The image shows two side-by-side density maps. The left map represents an asymmetric test function, which is a 2D plot with a complex, non-symmetric shape. The right map represents its Radon transform, which is a 2D plot showing the function's projection onto various angles, resulting in a more symmetric, elongated shape. The text 'asymmetric_function_and_radon.png' is overlaid on the left map.

asymmetric_function_and_radon.png

Figure 2: Density maps of the asymmetric test function (left) and its Radon transform (right).

5.2 Impact and future directions

Beyond CT, Radon-type transforms appear in:

- **Positron Emission Tomography (PET)**—employing coincident γ -ray detection.
- **Optical projection tomography** for mesoscopic specimens.
- **Electron microscopy** tilt-series reconstruction.
- **Barcode decoding** (1-D Radon in scanners).

Emerging research combines deep learning with classical FBP to reduce radiation dose while preserving diagnostic quality.

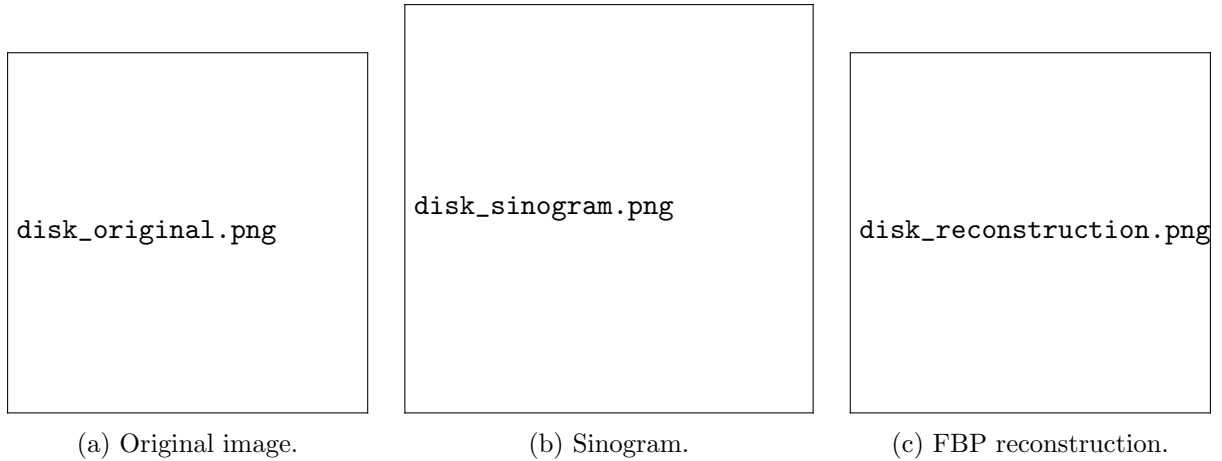


Figure 3: Parallel-beam CT simulation of a disk phantom.

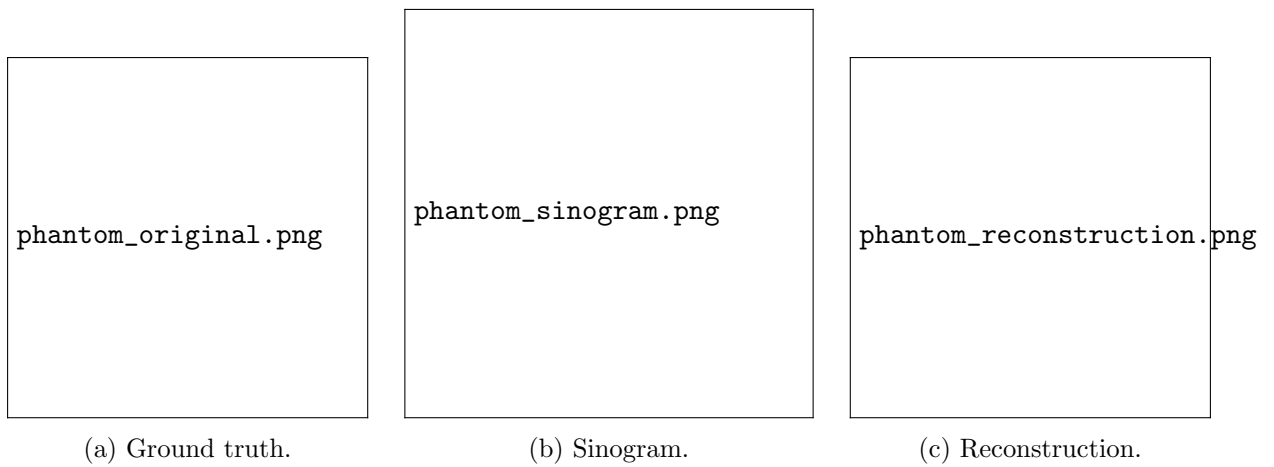


Figure 4: Shepp–Logan head phantom experiment.

6 Conclusion

Through analytic proofs and MATLAB simulations I gained intuition for how line integrals over an object encode its structure, and how inverse methods—especially filtered back-projection—turn those measurements into images that guide clinical decisions. The Radon transform thus beautifully exemplifies the synergy between mathematics, computation, and biomedical engineering.

References

- [1] J. Radon, *Über die Bestimmung von Funktionen durch ihre Integralwerte längs gewisser Mannigfaltigkeiten*, Ber. Sächs. Akad. Wiss. Leipzig, Math.-Phys. Kl. **69** (1917), 262–277.
- [2] A. Cormack and G. N. Hounsfield, *Nobel Lectures in Physiology or Medicine 1979*, Nobel Foundation, Stockholm, 1980.



Figure 5: Orthogonal slices of MATLAB's demo MRI data. Each axial slice can be reconstructed individually via the inverse Radon transform when acquired in projection space.