Seeing Inside: The Radon Transform, MATLAB Experiments, and Biomedical Imaging Applications

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Abstract

The Radon transform links pure mathematics with life–saving medical technology. This report introduces the transform, demonstrates analytic and numerical examples in MATLAB, and explains how filtered back-projection underpins modern computed tomography (CT). All experiments and figures were produced with a MATLAB Live Script; figure file names are indicated for reproducibility.

Contents

1 Introduction

Computed tomography (CT) revolutionised diagnostic radiology by allowing physicians to reconstruct cross-sectional images of internal anatomy from line–integral measurements of X-ray attenuation. Mathematically, these line integrals form the *Radon transform*, introduced by Johann Radon in 1917 [?]. This report summarises the theory, reproduces textbook examples, and documents hands-on MATLAB explorations relevant to biomedical engineering practice.

2 Theory of the Radon Transform

For a function $f(x, y)$ on $\mathbb{R}^2$, its Radon transform $\mathcal{R}f$ is the set of line integrals

$$\big(\mathcal{R}f\big)(p, \varphi) \;=\; \int_{-\infty}^{\infty} f\big(p\cos\varphi - s\sin\varphi,\; p\sin\varphi + s\cos\varphi\big)\,\mathrm{d}s, \tag{1}$$

where p is the signed distance of the line from the origin and φ is the angle the line's normal makes with the x-axis. Recovering f from $\mathcal{R}f$ is the *inverse Radon transform.* The Fourier Slice Theorem shows that a 1-D inverse Fourier transform of each projection, followed by a 2-D inverse transform, re-creates f. Numerically this leads to *filtered back-projection* (FBP), implemented in MATLAB as `iradon`.

3 Analytic Experiments

3.1 Radially symmetric Gaussian

Using the Symbolic Math Toolbox we verified that a centred Gaussian

$$f(x, y) = \exp\big(-(x^2 + y^2)\big)$$

has a Radon transform independent of φ: $(\mathcal{R}f)(p, \varphi) = \sqrt{\pi}\,\exp(-p^2)$. Figure **??** juxtaposes the surface plot of f and the graph of its transform.

gaussian_surface.png

(a) 3-D surface of f.

gaussian_radon.png

(b) Radon transform vs. p.

Figure 1: Radially symmetric example.

3.2 Asymmetric polynomial–Gaussian

Replacing the integrand by $g(x, y) = (x^2 + 3y^2)e^{-(x^2+y^2)}$ breaks the circular symmetry; the resulting transform depends on both p and φ (analytical expression omitted for brevity). Figure **??** shows density plots of g and $\mathcal{R}g$.

4 Discrete Radon Transform Experiments

All remaining experiments use MATLAB's discrete `radon` and `iradon` functions.

4.1 Binary disk phantom

Figure **??** demonstrates how 180 projections of a simple disk (representing, e.g. a cross-section of an organ) produce a *sinogram*, which FBP then reconstructs nearly perfectly.

4.2 Shepp–Logan head phantom

The Shepp–Logan phantom is a synthetic yet realistic CT head slice. With 180 projections we obtain the results in Figure **??**. Qualitatively the reconstruction captures fine structures such as the ventricles, illustrating clinical usefulness.

5 Biomedical Imaging Context

5.1 From mathematics to CT scanners

In practice, modern scanners collect fan- or cone-beam projections while rotating around the patient. Proprietary reconstruction pipelines apply noise correction, calibration, and iterative or FBP algorithms. Nevertheless, Equation (**??**) and its inversion remain the core. Figure **??** uses MATLAB's built-in `mri` volume to visualise slices akin to clinical data.

asymmetric_function_and_radon.png

Figure 2: Density maps of the asymmetric test function (left) and its Radon transform (right).

5.2 Impact and future directions

Beyond CT, Radon-type transforms appear in:

- **Positron Emission Tomography (PET)**—employing coincident γ-ray detection.
- **Optical projection tomography** for mesoscopic specimens.
- **Electron microscopy** tilt-series reconstruction.
- **Barcode decoding** (1-D Radon in scanners).

Emerging research combines deep learning with classical FBP to reduce radiation dose while preserving diagnostic quality.

disk_original.png

(a) Original image.

disk_sinogram.png

(b) Sinogram.

disk_reconstruction.png

(c) FBP reconstruction.

Figure 3: Parallel-beam CT simulation of a disk phantom.

phantom_original.png

(a) Ground truth.

phantom_sinogram.png

(b) Sinogram.

phantom_reconstruction.png

(c) Reconstruction.

Figure 4: Shepp–Logan head phantom experiment.

6 Conclusion

Through analytic proofs and MATLAB simulations I gained intuition for how line integrals over an object encode its structure, and how inverse methods—especially filtered back-projection—turn those measurements into images that guide clinical decisions. The Radon transform thus beautifully exemplifies the synergy between mathematics, computation, and biomedical engineering.

References

[1] J. Radon, *Über die Bestimmung von Funktionen durch ihre Integralwerte längs gewisser Mannigfaltigkeiten*, Ber. Sächs. Akad. Wiss. Leipzig, Math.-Phys. Kl. **69** (1917), 262–277.

[2] A. Cormack and G. N. Hounsfield, *Nobel Lectures in Physiology or Medicine 1979*, Nobel Foundation, Stockholm, 1980.

mri_volume.png

Figure 5: Orthogonal slices of MATLAB's demo MRI data. Each axial slice can be reconstructed individually via the inverse Radon transform when acquired in projection space.