Linear control system design SSY285

Assignment M1: Dynamic model of DC motor with flywheel

Problem

Consider the system shown in Figure 1 which consists of an electric DC motor (having separate magnetization) that drives a flywheel and that is influenced by external torque.

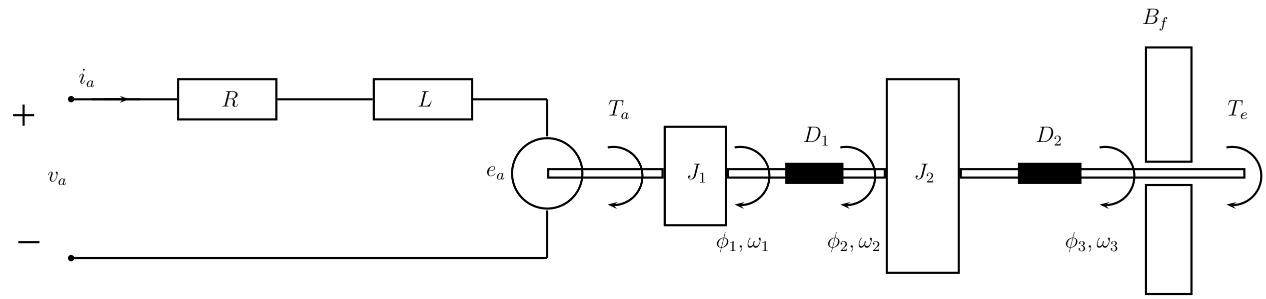

Figure 1: DC motor with flywheel

The system is characterized by the following variables: the external voltage applied to the rotor v_a, the rotor current i_a, the induced rotor voltage e_a, the torque produced by rotor T_a, and the external torque applied to flywheel axis T_e. Additionally, the axis is characterised by angles $\phi_1,\ \phi_2,\ \phi_3$ and angular velocities $\omega_1,\ \omega_2,\ \omega_3$.

The system is also characterized by electrical parameters: L is the rotor inductance and R is the rotor resistance. Furthermore, K_E is the coefficient which relates induced voltage e_a, to rotor velocity ω_1, and K_T is the rotor torque constant (driving torque to rotor current i_a). There are also mechanical parameters: J_1 is the rotor inertia and J_2 is the flywheel inertia. Both D_1 and D_2 represent the flexibility of the axis (modelled as torsion springs) on each side of flywheel. Finally, B_f denotes dynamic (linear) friction proportional to the angular velocity ω_3.

Questions

a) Given the relations $e_a = K_E \cdot \omega_1$ and $T_a = K_T \cdot i_a$, formulate a mathematical model for this system, using basic laws of electricity and mechanics. How many linear differential equations and of what order are needed to describe this system completely? Which are the system inputs in this case?

b) Assume that the inductance is very small, $L \approx 0$, then choose state variables to compose a state vector $x(t)$, and inputs for vector $u(t)$ of the underlying system. Pay attention to select the state and control inputs such that you formulate a continuous time state equation under the form of $\dot{x}(t) = Ax(t) + Bu(t)$. What are the matrices A and B?

c) The output $y(t)$ of the system in a state space model is related to state and input, as $y(t) = Cx(t) + Du(t)$. Give the matrices C and D for the following two cases: **(1)** $y_1(t) = \phi_2,\ y_2(t) = \omega_2$ and **(2)** $y_1(t) = i_a,\ y_2(t) = \omega_3$.

d) Assume the following parameter values:

$$R = 1\Omega,\ \ K_E = 10^{-1}Vs/rad,\ \ K_T = 10^{-1}Nm/A,\ \ J_1 = 10^{-5}kgm^2,\ \ J_2 = 4\cdot 10^{-5}kgm^2$$
$$B_f = 2\cdot 10^{-3}Nms,\ \ D_1 = 20Nm/rad,\ \ D_2 = 2Nm/rad$$

Calculate (with `Matlab`) the eigenvalues of the A-matrix. Is the system input-output stable for both cases **(1)** and **(2)** of *c*)? Investigate if pole-zero cancellation occurs.

e) Assume that the external torque is zero. Give the transfer function from the input, applied rotor voltage, to the output, defined in case **(2)** of subproblem *c*). Calculate the poles and transmission zeros in this case. Is the system of minimum phase?

f) Suppose that in the initial state, the applied rotor voltage and the external torque are all zero. Then the applied rotor voltage is stepwise changed from $0V$ to $10V$. Suppose also that once all angular velocities have reached the steady state, suddenly an external (velocity reducing) torque of $0.1Nm$ is applied stepwise. Investigate by simulation, how the state vector components evolve in time. Is it reasonable that the angles are increasing unboundedly? Plot also the outputs of for both cases in subproblem c) as a function of time.

Pre-approval of solution is mandatory before submission (by TA in tutorial session)