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1. $f'(x) = e^x + 2\sin(2x) + 2\cos x$

since $e^x > 0$ on $[0, \frac{\pi}{4}]$, $\sin(2x) \geq 0$ on $[0, \frac{\pi}{4}]$, $\cos x > 0$ on $[0, \frac{\pi}{4}]$

$\therefore f'(x) > 0$ on $[0, \frac{\pi}{4}]$ $\Rightarrow$ $f(x)$ is one to one in the inteval $[0, \frac{\pi}{4}]$

when $y = f(x) = -\frac{1}{2}$, $x = 0$

$$\Rightarrow (f^{-1})'(-\tfrac{1}{2}) = \frac{1}{f'(0)} = \frac{1}{3}$$

2. $\cos(x^2+2y) + 5xe^y = \tan^{-1}(y) + 1 + 6y$

$$\Rightarrow \frac{d[\cos(x^2+2y) + 5xe^y]}{dx} = \frac{d[\tan^{-1}(y) + 1 + 6y]}{dx}$$

$$\Rightarrow -\sin(x^2+2y)\cdot\frac{d(x^2+2y)}{dx} + 5e^y + 5x\cdot\frac{de^y}{dx} = \frac{d(\tan^{-1}y)}{dx} + \frac{d(6y)}{dx}$$

$$\Rightarrow -\sin(x^2+2y)(2x + 2\frac{dy}{dx}) + 5e^y + 5x\cdot e^y\cdot\frac{dy}{dx} = \frac{1}{y^2+1}\frac{dy}{dx} + 6\frac{dy}{dx}$$

when $x=0$, $y=0$ $\Rightarrow$

$$5 = 7\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{5}{7} \Rightarrow y' \text{ at } (x,y) = (0,0) \text{ is } \frac{5}{7}$$

3. $f'(x) = -6x^2 - 6x + 12 = -6(x+2)(x-1)$

when $x_1 = -2$, $x_2 = 1$, $f'(x_1) = f'(x_2) = 0$

when $x \in [-4, -2)$, $(1, 2]$, $f'(x) < 0$

$x \in (-2, 1)$, $f'(x) > 0$

$\Rightarrow$ $x = -2$ is a local minima, $f(x) = -27$

$x = 1$ is a local maxima, $f(x) = 0$

when $x = 2$, $f(x) = -11$, $x = -4$, $f(x) = 25$

$\Rightarrow$ the global maximum is 25, ~~f(x)=~~ when $x = -4$

the global minimum is -27, when $x = -2$

4. (a) $\int \left(\frac{2x^3-4x+7}{x^2}\right)dx = \int \left(2x - \frac{4}{x} + \frac{7}{x^2}\right)dx$

$= x^2 - 4\ln x - \frac{7}{x} + C$

(b) $\int \frac{x}{\sqrt[3]{x+8}}dx$, let $t = \sqrt[3]{x+8} \Rightarrow x = t^3 - 8$

$= \int \frac{t^3-8}{t} d(t^3-8) = \int \left(t^2 - \frac{8}{t}\right)(3t^2)dt$

$= \int (3t^4 - 8t)dt = \frac{3}{5}t^5 - 4t^2 + C = \frac{3}{5}(x+8)^{\frac{5}{3}} - 4(x+8)^{\frac{2}{3}} + C$

(C) let $t = e^{2x} \geq 0$, $x = \frac{\ln t}{2}$

$\int \frac{e^{2x}}{\sqrt{1-e^{4x}}}dx = \int \frac{t}{\sqrt{1-t^2}} \frac{1}{2t}dt = \int \frac{1}{2\sqrt{1-t^2}}dt$

$\cancel{\text{let } u = \sqrt{1-t^2} \Rightarrow t = \sqrt{1-u^2} \quad t=}$

$\cancel{\int \frac{1}{2\sqrt{1-t^2}}dt = \int \frac{1}{2u} d(1-u^2)^{\frac{1}{2}} = \int \frac{-2u}{2u}(1-u^2)^{-\frac{1}{2}}du}$

$\int \frac{1}{2\sqrt{1-t^2}}dt = \frac{1}{2}\sin^{-1}(t) + C = \frac{1}{2}\sin^{-1}(e^{2x}) + C$

(d) $\int \frac{\sin(2x)}{5+\cos^2(x)}dx = -\frac{1}{2}\int \frac{1}{5+\frac{1+\cos(2x)}{2}} d\cos(2x)$

let $t = \cos(2x)$, $\Rightarrow = -\frac{1}{2}\int \frac{1}{11+t}d(t+11) = -\frac{1}{2}\ln(t+11) + C$

$= -\frac{1}{2}\ln|\cos 2x + 11| + C$

(e) let $t = \sqrt{x}$

$\Rightarrow \int \frac{1}{\sqrt{x}(4-x)}dx = \int \frac{2t}{t(4-t^2)}dt = \int \frac{2}{4-t^2}dt$

$= \frac{1}{2}\ln\left|\frac{2+t}{2-t}\right| + C = \frac{1}{2}\ln\left|\frac{2+\sqrt{x}}{2-\sqrt{x}}\right| + C$

(f) $\int \sin(8x)\sin(4x)dx = \int 2\sin^2(4x)\cos(4x)dx$

$= 2\int \sin^2(4x)\, d\sin(4x)$

$= \frac{2}{3}\sin^3(4x) + C$

(g) $\int 32\sin^2 x\cos^2 x\,dx$

$\cancel{= 32\int \frac{1}{3}\sin x\, d[\cos^3 x]}$

$\cancel{= \frac{32}{3}(\sin x\cos^3 x - \int \cos^3 x\, d\sin x)}$

$= \int 8\sin^2(2x)\,dx$

$= \int (4-\cos 4x)\,dx$

$= 4x - \frac{1}{\cancel{4}}\sin 4x + C$

(h) $\int e^{\sin^2 x}\sin(2x)\,dx$

$= 2\int e^{\sin^2 x}\sin x\, d\sin x$

let $u = \sin x$

$= 2\int e^{u^2}u\,du$

$= \int e^{u^2}du^2$

$= e^{u^2} + C = e^{\sin^2 x} + C$

(i) $\int \sqrt{x}\ln x\,dx$

$= \frac{2}{3}\int \ln x\, d\,x^{\frac{3}{2}}$

$= \frac{2}{3}(x^{\frac{3}{2}}\ln x - \int x^{\frac{3}{2}}\, d\ln x)$

$= \frac{2}{3}(x^{\frac{3}{2}}\ln x - \int x^{\frac{1}{2}}\,dx)$

$= \frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{4}{9}x^{\frac{3}{2}} + C$

(j) $\int x\cos^5 x\,dx$

$\int \cos^5 x\,dx = \int \cos^4 x\, d\sin x$ $\cancel{= \sin x\cos^4 x - \int \cos^3 x\sin x\,dx}$

$= \int (1-\sin^2 x)^2 d\sin x$ $\cancel{= \sin x\cos^4 x - \int 2\sin 2x\, d\sin x}$ $\cancel{\int 2\cos^2 x\, d\sin^2 x}$

$= \frac{1}{5}\sin^5 x - \frac{2}{3}\sin^3 x$ $\cancel{= \sin x\cos^4 x - \int 2(1-\sin^2 x)\, d\sin^2 x}$

$+ \sin x + C$ $\cancel{= \sin x\cos^4 x - 2\sin^2 x + \sin^4 x + C}$

$\cancel{\int x\cos^5 x\,dx}$

$\int x\cos^5 x\,dx = \int x\, d(\frac{1}{5}\sin^5 x - \frac{2}{3}\sin^3 x + \sin x)$

$= (\frac{1}{5}\sin^5 x - \frac{2}{3}\sin^3 x + \sin x)x - \int (\frac{1}{5}\sin^5 x - \frac{2}{3}\sin^3 x + \sin x)dx$

$= (\frac{1}{5}\sin^5 x - \frac{2}{3}\sin^3 x + \sin x)x + \int [\frac{1}{5}(1-\cos^2 x)^2 - \frac{2}{3}(1-\cos^2 x) + 1]\, d\cos x$

$= (\frac{1}{5}\sin^5 x - \frac{2}{3}\sin^3 x + \sin x)x + \frac{1}{25}\cos^5 x + \frac{4}{45}\cos^3 x + \frac{8}{15}\cos x + C$

(k) $\frac{x^2}{(x-1)(x-2)^2} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$

$\Rightarrow \times (x-2)^2$, let $x=2 \Rightarrow \frac{4}{1} = C \Rightarrow C=4$

$\times (x-1)$, let $x=1 \Rightarrow \frac{1}{(1-2)^2} = A \Rightarrow A=1$

let $x=0 \Rightarrow -A - \frac{B}{2} + \frac{C}{4} = 0 \Rightarrow B=0$

$\Rightarrow \int \frac{x^2}{(x-1)(x-2)^2} dx = \int \frac{1}{x-1} dx + \int \frac{4}{(x-2)^2} dx$

$= \int \frac{1}{x-1} d(x-1) + \int \frac{4}{(x-2)^2} d(x-2)$

$= \ln|x-1| - \frac{4}{(x-2)} + C$

$= \ln|x-1| - \frac{4}{x-2} + C$

(l) $\frac{2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$\Rightarrow \times (x)$, let $x=0 \Rightarrow \frac{2}{1} = A \Rightarrow A=2$

$\times (x^2+1)$, let $x=1 \Rightarrow \frac{2}{1} = 2A+B+C \Rightarrow B+C=-2$

$\times (x^2+1)$, let $x=-1 \Rightarrow -2 = -2A-B+C \Rightarrow C-B = 2$

$\Rightarrow B=-2,\ C=0$

$\Rightarrow \int \frac{2}{x(x^2+1)} dx = \int \frac{2}{x} dx + \int \frac{-2x}{x^2+1} dx$

$= 2\ln|x| + \int \frac{-1}{x^2+1} d\,x^2+1$

$= 2\ln|x| - \ln(x^2+1) + C$