COMP3011: Design and Analysis of Algorithms Due Date: 13 Oct, 2024

Assignment 1

The Hong Kong Polytechnic University

Problem 1 (15 points)

In each of the following situations, indicate whether $f = O(g)$, or $f = \Omega(g)$, or both (in which case $f = \Theta(g)$).

	$f(n)$	$g(n)$
(a)	$n^{1.01}$	$n \log^2 n$
(b)	$n2^n$	3^n
(c)	2^n	2^{n+1}

Problem 2 (15 points)

Let G be an undirected graph on n nodes. Prove any two of the following statements imply the third:

1. G is connected.
2. G does not contain a cycle.
3. G has $n - 1$ edges.

Problem 3 (10 points) (Appeared in Tutorial W4 without presenting detailed proof)

The diameter of a tree $T = (V, E)$ is defined as the largest of all shortest path distances in the tree. Give an efficient algorithm to compute the diameter of a tree, and prove its correctness.

Problem 4 (15 points)

Use Dijkstra's algorithm to find the shortest path from s to v for the following graph in Figure 1. Backtracking is required.

Problem 5 (15 points)

Suppose we are given an undirected graph $G = (V, E)$, and we identify two nodes v and w in G. Give an algorithm that computes the number of shortest $v - w$ paths in G. (The algorithm should not list all the paths; just the number suffices.) Analyze the running time of your algorithm.

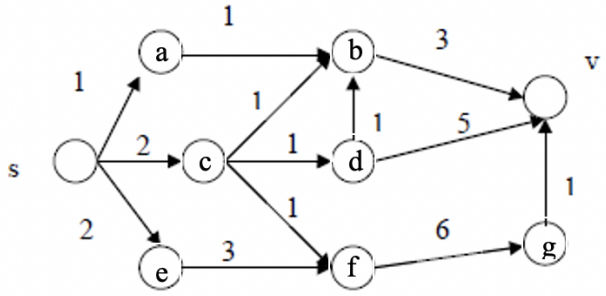

Figure 1: Problem 4

Problem 6 (15 points)

A thief robbing a store finds n items. The ith item is worth v_i dollars and weighs w_i pounds, where v_i and w_i are integers. The thief wants to take as valuable a load as possible, but he can carry at most W pounds in his knapsack, for some integer W. The thief can take fractions of the items. Which items should he take?

Give an efficient algorithm that achieves this goal and prove its correctness.

Problem 7 (15 points)

Write a Java, Python, C, or C++ program that uses Prim's algorithm to compute a minimum spanning tree for the graph in Figure 2. Your program needs to output the intermediate steps. Both code and result should be submitted.

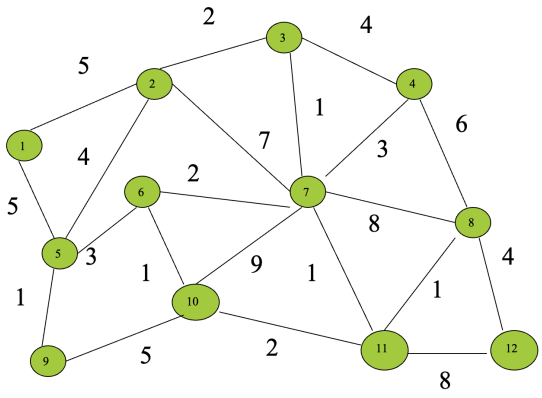

Figure 2: Problem 7