

The Hong Kong Polytechnic University

COMP2012 Discrete Mathematics

Assignment 2

(Due: 23:59, 24<sup>th</sup> November, 2024)

**Guideline:**

- This is an individual assignment;
- No need to copy questions; simply write your answer;
- Submit the soft copy of your answer to Blackboard (as a doc/docx/pdf file);
- Beware of the late penalty policy (for details, please refer to Blackboard);
- Total marks: 100 marks.

**Questions:**

**Question 1**

**[30 marks]**

Determine the maximum flow of the network  $G$  in Figure 1-1,

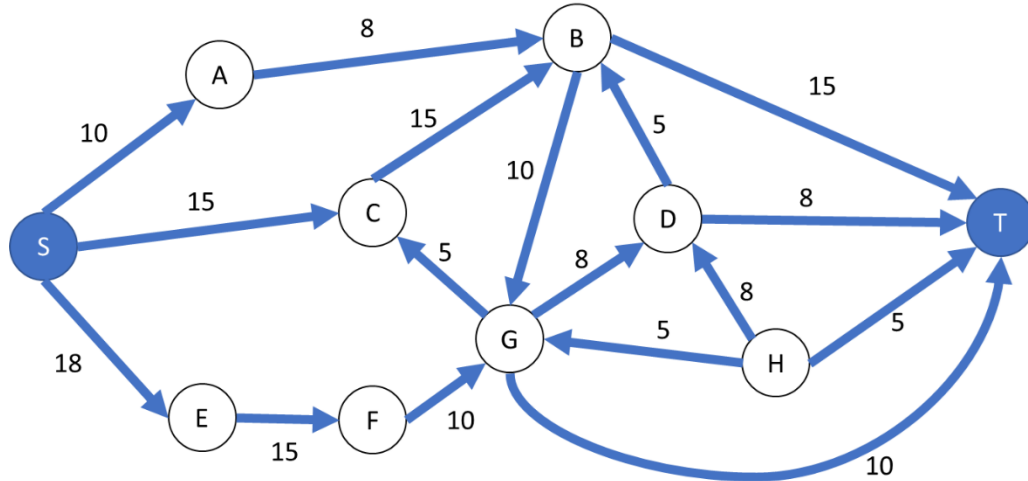


Figure 1-1

- 1(a) Using the *max-flow min-cut* theorem, consider only the edges formed between vertices S, A, B, C, E, F and G (5 marks)
- 1(b) Using (i) *Folk-Fulkerson* algorithm, and then (ii) *Edmonds-Karp* algorithm. (20 marks)
- 1(c) Suppose a network  $G'$  is an undirected graph with the same vertices and edges (without directions) as in  $G$ . Find the minimum spanning tree (MST) from the network  $G'$  with Kruskal's Algorithm. (5 marks)

**Question 2****[15 marks]**

- 2(a)** Computing students must complete a capstone project during their final year of study. Please create a bipartite graph based on the information provided below, with one set of vertices representing the students and the other representing the available capstone projects. Connect the students to the projects using edges that reflect their interests. (5 marks)

<u><b>Student Interests:</b></u>	<u><b>Available Capstone Projects:</b></u>
Adam: Interested in metaverse projects	TJ1: Metaverse, AR
Bob: Interested in AR projects	TJ2: Metaverse, VR
Cathy: Interested in VR projects	TJ3: HCI, VR
David: Interested in HCI and Fintech projects	TJ4: Fintech, Blockchain
Elise: Interested in Data Science projects	TJ5: Fintech, Data Science

- 2(b)** Determine the student-project match using an appropriate maximum flow algorithm (such as the Ford-Fulkerson or Edmonds-Karp algorithm). Ensure that each student is paired with only one project, and that each project is assigned to only one student. (10 marks)

*Hint: consider the edge weights as 1.*

**Question 3.****[25 marks]**

- 3(a)** Given the array of integers below, draw a Binary Search Tree (BST). (5 marks)

46, 9, 42, 45, 99, 56, 6, 21, 76, 82

- 3(b)** Is this BST a balanced tree? (1 mark) Give your justification (2 marks)
- 3(c)** List nodes in a *pre-order traversal*. (5 marks)
- 3(d)** List nodes in a *post-order traversal*. (5 marks)
- 3(e)** List nodes in an *in-order traversal*. (5 marks)
- 3(f)** On the BST, show the steps to delete node 46? (2 marks)

**Question 4****[15 marks]**

- 4(a) Simplify  $F(A, B, C) = \bar{A}BC + A\bar{B}C + ABC\bar{C} + ABC$  by Karnaugh map. (5 marks)
- 4(b) As an example of a combinational circuit, we learnt the logic of a half adder in Lecture 12. By connecting two half-adders, a full adder circuit is constructed. A complete adder includes three binary digit inputs (two input bits and one carry input bit), as well as two binary digit outputs: the sum bit (S) and the carry output bit (Cout):

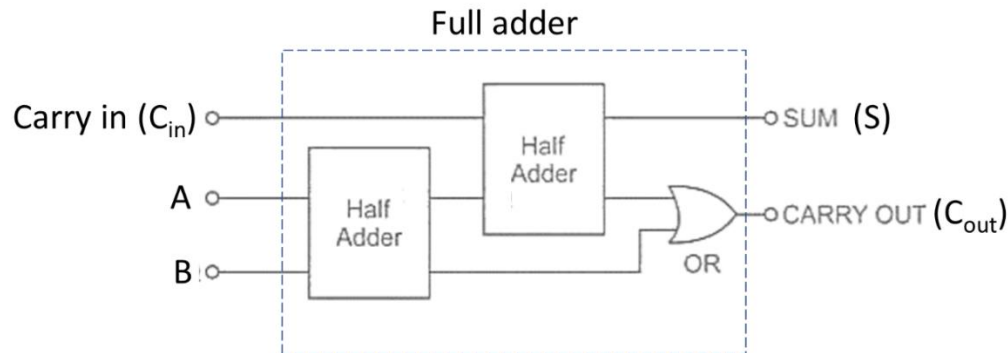


Figure 4-1

- (i) Determine the truth table of a full adder logic. (4 marks)
- (ii) Determine the logic of Sum output bit of a full adder. (2 marks)
- (iii) Determine the logic of Carry output bit of a full adder. (2 marks)
- (iv) Draw the combinational circuit of the full adder. (2 marks)

**Question 5.****[15 marks]**

Consider the campus map of the Hong Kong Polytechnic University shown in Figure 5-1:

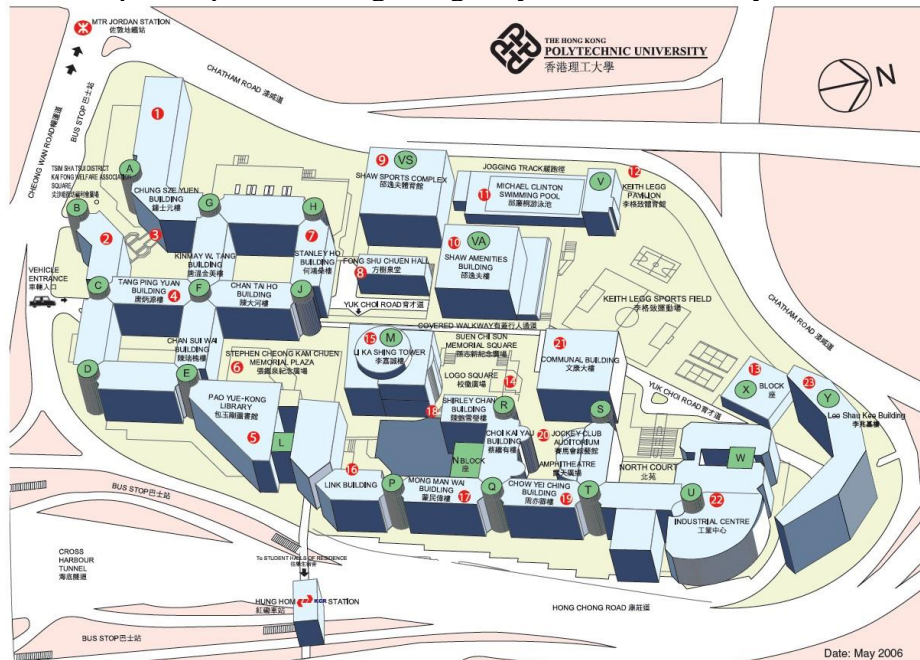


Figure 5-1

- 5(a) Draw a graph  $G$  with the following information given (5 marks):

**The distances between Cores/Blocks/Towers:**

M to R: 10 metres,    Q to R: 40 metres,    Q to M: 20 metres,  
Q to T: 50 metres,    S to T: 30 metres,    T to W: 80 metres,  
S to W: 50 metres,    M to VA: 70 metres,    R to VA: 70 metres,  
S to VA: 60 metres    M to S: 15 metres

- 5(b) Suppose today is the student orientation day and the whole area between Block M and VA are occupied by Academic Registry (AR). With the graph  $G$  obtained in 5(a), deduce the lowest cost distances from Core Q to the all destinations. (8 marks)
- 5(c) Consider that you are currently at W202 and then you are going to take the COMP2012 tutorial class at QR602. Assume at Block R, all the stairs and elevators are under maintenance, write down the shortest path with the result obtained in 5(b). (2 marks)

End of Assignment 2