1. (a)

If we choose $\{(S,A),(S,C),(S,E)\}$, the total cost is $10+15+18=43$

If we choose $\{(S,A),(S,C),(E,F)\}$, the total cost is $10+15+15=40$

If we choose $\{(S,A),(S,C),(F,G)\}$, the total cost is $10+15+10=35$

If we choose $\{(S,A),(C,B),(S,E)\}$, the total cost is $10+15+18=43$

If we choose $\{(S,A),(C,B),(E,F)\}$, the total cost is $10+15+15=40$

If we choose $\{(S,A),(C,B),(F,G)\}$, the total cost is $10+15+10=35$

If we choose $\{(A,B),(S,C),(S,E)\}$, the total cost is $8+15+18=41$

If we choose $\{(A,B),(S,C),(E,F)\}$, the total cost is $8+15+15=38$

If we choose $\{(A,B),(S,C),(F,G)\}$, the total cost is $8+15+10=33$

If we choose $\{(A,B),(C,B),(S,E)\}$, the total cost is $8+15+18=41$

If we choose $\{(A,B),(C,B),(E,F)\}$, the total cost is $8+15+15=38$

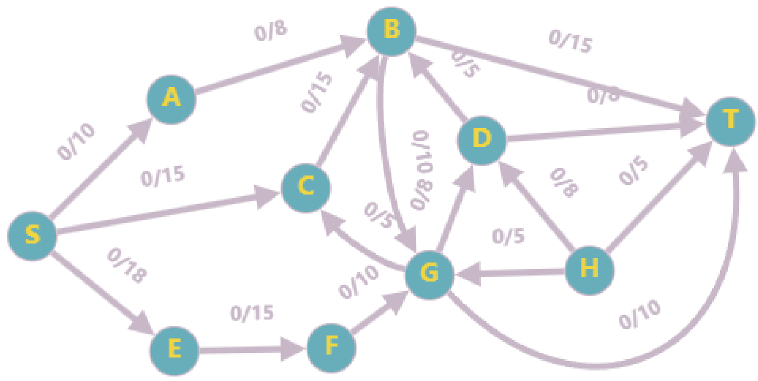
If we choose $\{(A,B),(C,B),(F,G)\}$, the total cost is $8+15+10=33$

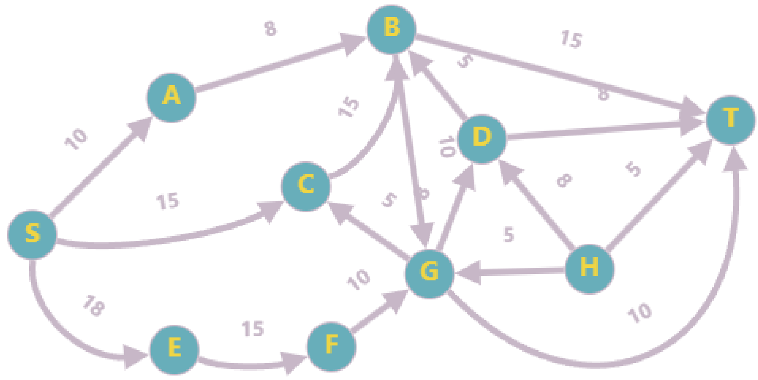
Among all the cases, the min-cut is $\{(A,B),(S,C),(F,G)\}$, which value is 33. According to the max-flow min-cur theorem, the max-flow is 33.

$(b)(i)$

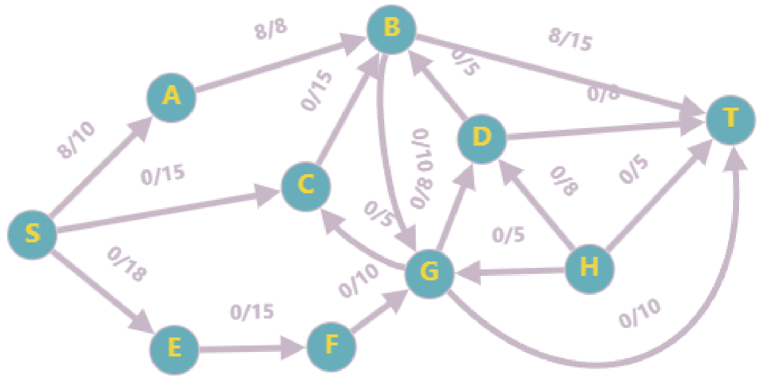
According to Fold-Fulkerson algorithm.

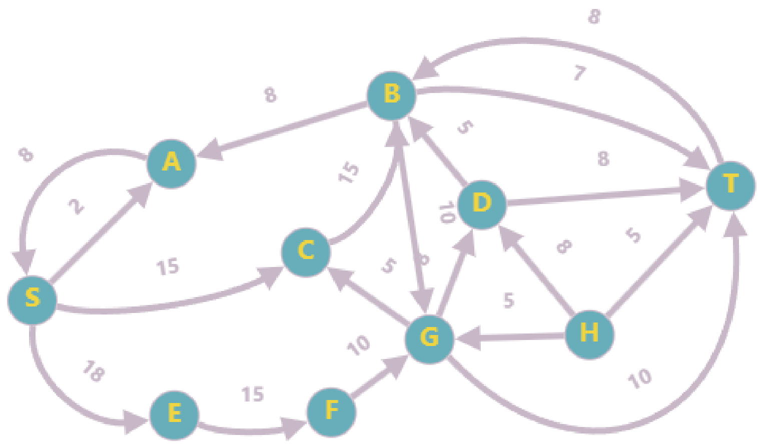
Initialization: $Flow=0$



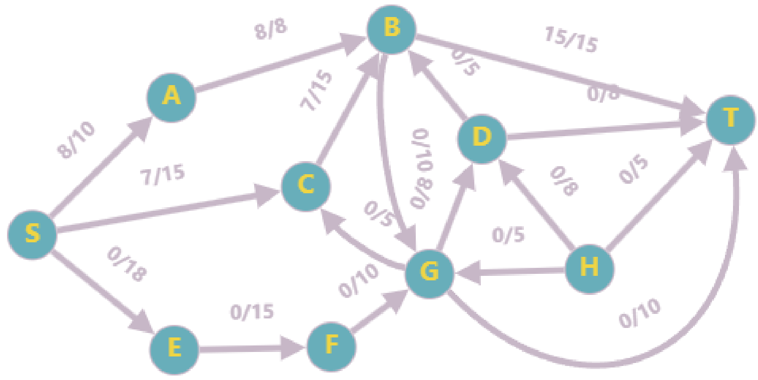


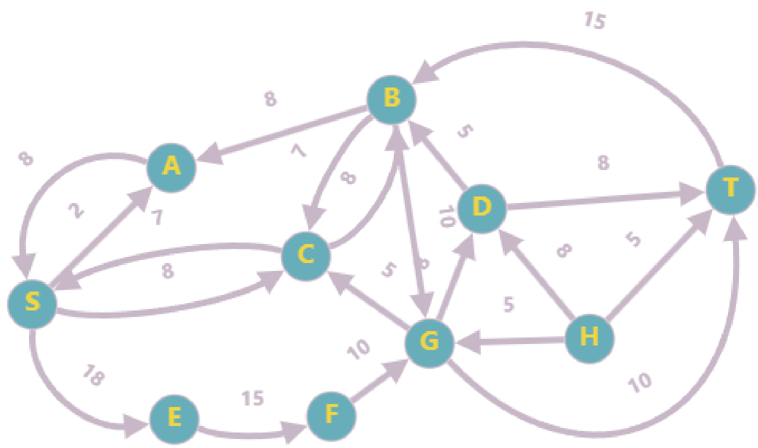
Iteration 1: Augmented Path 1: $S-A-B-T \rightarrow Flow=Flow+8=8$



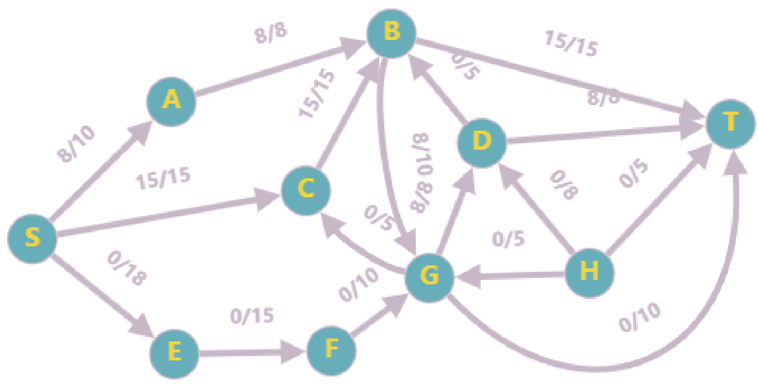


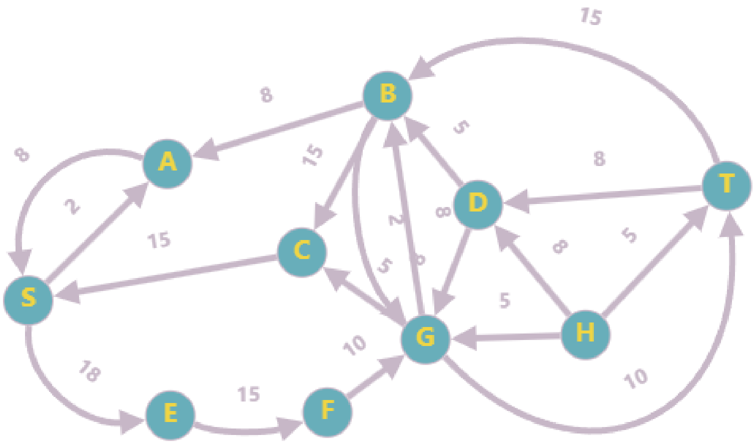
Iteration 2: Augmented Path 2: $S - C - B - T \rightarrow Flow = Flow + 7 = 15$



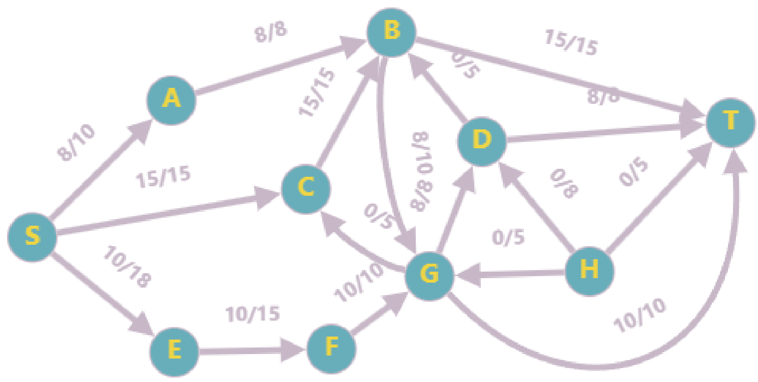


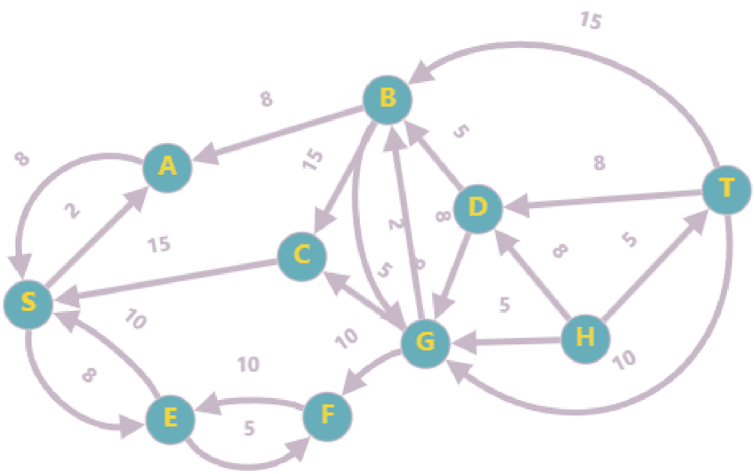
Iteration 3: Augmented Path 3: $S - C - B - G - D - T \rightarrow Flow = Flow + 8 = 23$





Iteration 4: Augmented Path 4: $S - E - F - G - T \rightarrow Flow = Flow + 10 = 33$





No more possible augmented paths left

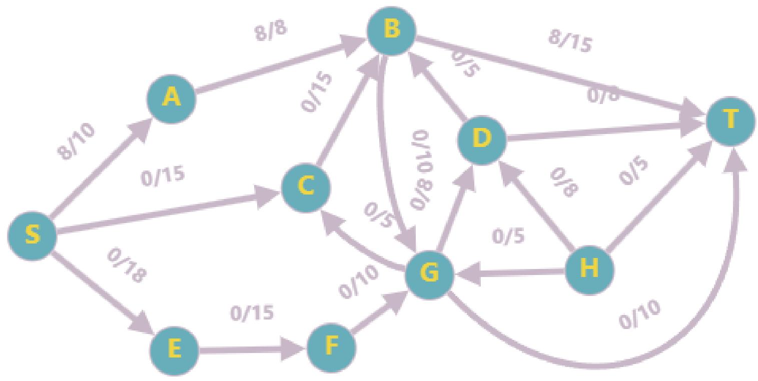
Hence, maximum $Flow = 33$

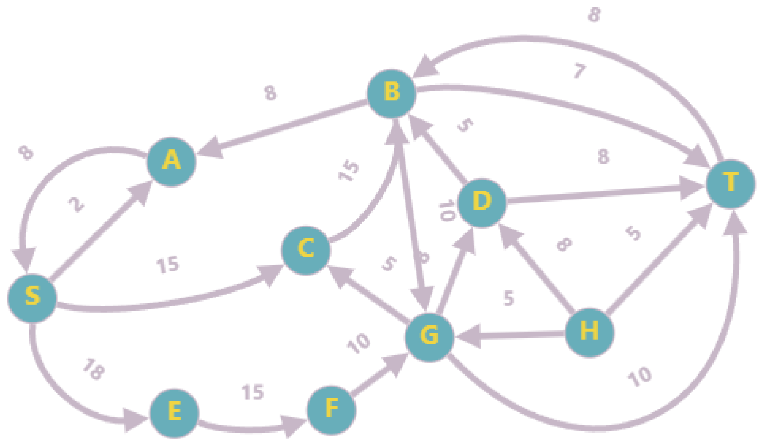
(ii)

According to the Edmonds-Karp algorithm

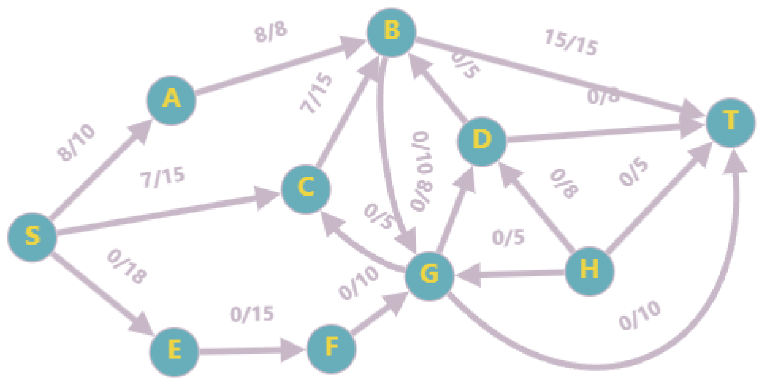
Initialization $Flow = 0$

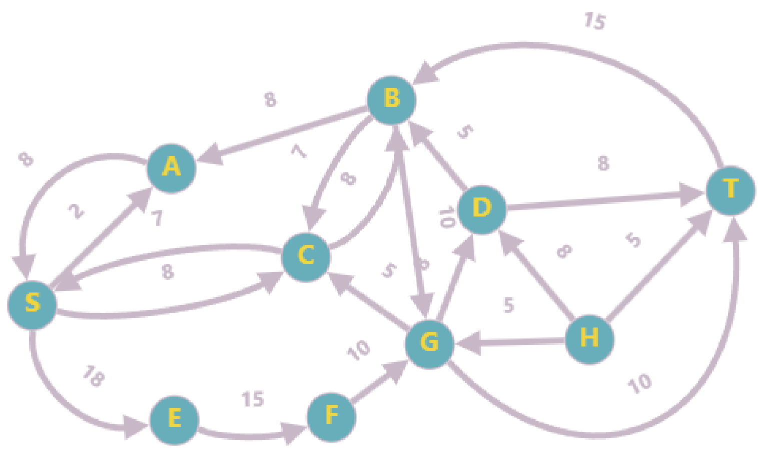
Iteration 1: Augmented Path 1: $S - A - B - T \rightarrow Flow = Flow + 8 = 8$



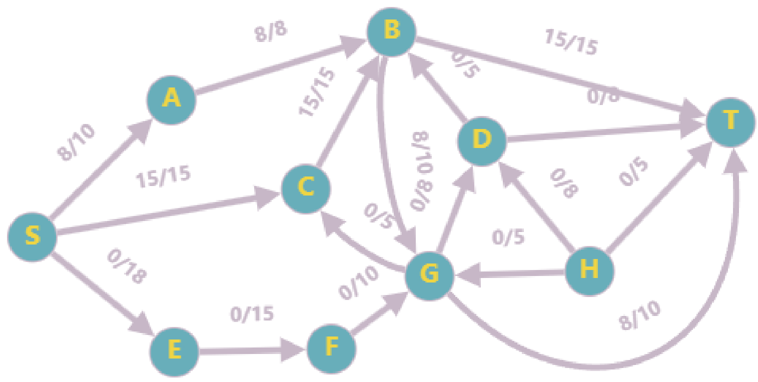


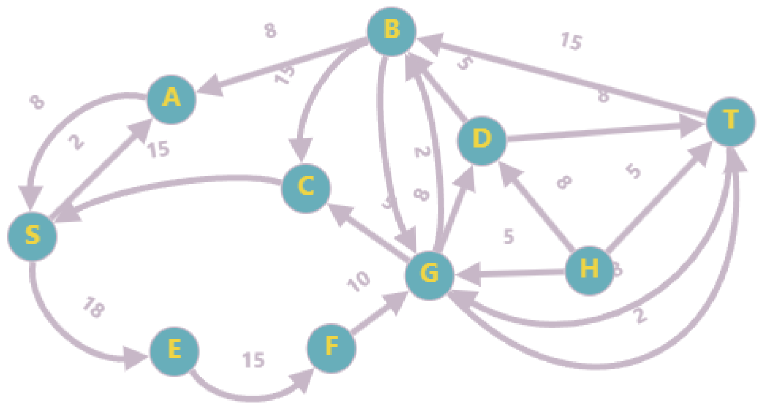
Iteration 2: Augmented Path 2: $S - C - B - T \rightarrow Flow = Flow + 7 = 15$



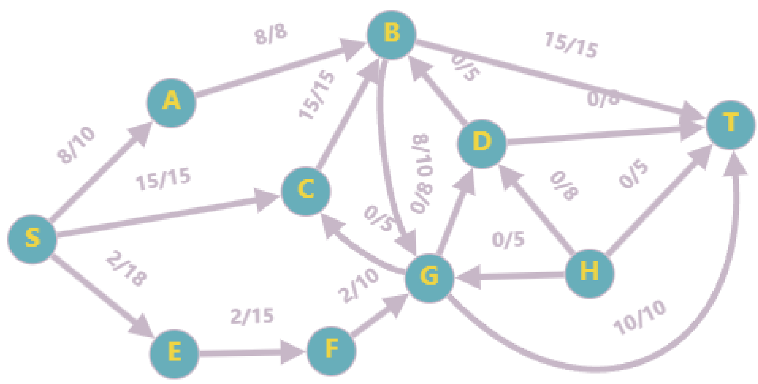


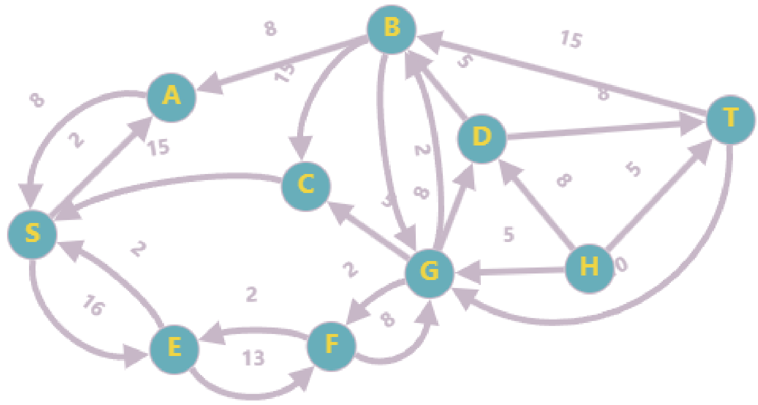
Iteration 3: Augmented Path 3: $S - C - B - G - T \rightarrow Flow = Flow + 8 = 23$



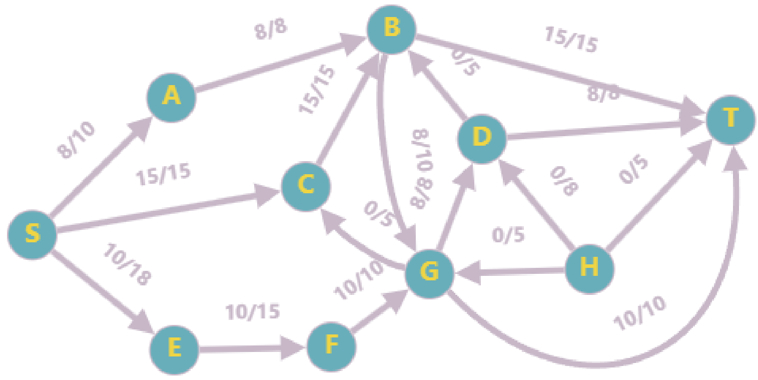


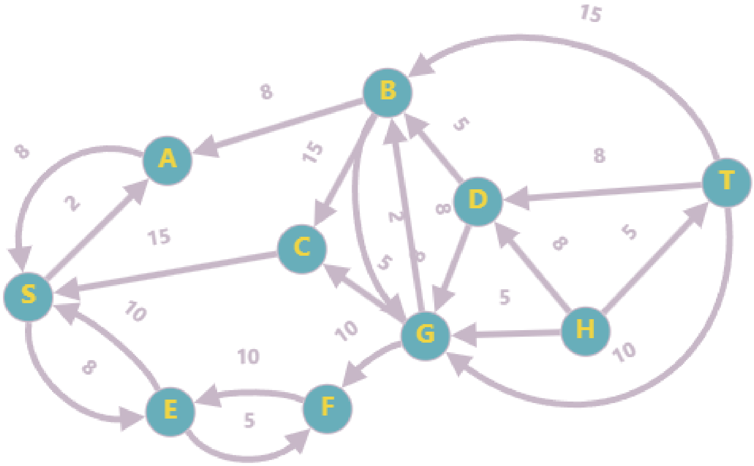
Iteration 4: Augmented Path 4: $S - E - F - G - T \rightarrow Flow = Flow + 2 = 25$





Iteration 5: Augmented Path 5: $S - E - F - G - D - T \rightarrow Flow = Flow + 8 = 33$





No more possible augmented paths left

Hence, maximum $Flow = 33$

(c)

Initial

Vertex Sets: $\{S\},\{A\},\{B\},\{C\},\{D\},\{E\},\{F\},\{G\},\{H\},\{T\}$

Edge chose: $\{\}$

Iteration 1: For edge $(D, B) = 5$, B and D are not in a set, choose.

Vertex Sets: $\{S\},\{A\},\{B,D\},\{C\},\{E\},\{F\},\{G\},\{H\},\{T\}$

Edge chose: $\{(D,B)\}$

Iteration 2: For edge $(G, C) = 5$, G and C are not in a set, choose.

Vertex Sets: $\{S\},\{A\},\{B,D\},\{C,G\},\{E\},\{F\},\{H\},\{T\}$

Edge chose: $\{(D,B),(G,C)\}$

Iteration 3: For edge $(H, G) = 5$, H and G are not in a set, choose.

Vertex Sets: $\{S\},\{A\},\{B,D\},\{C,G,H\},\{E\},\{F\},\{T\}$

Edge chose: $\{(D,B),(G,C),(H,G)\}$

Iteration 4: For edge $(H, T) = 5$, H and T are not in a set, choose.

Vertex Sets: $\{S\},\{A\},\{B,D\},\{C,G,H,T\},\{E\},\{F\}$

Edge chose: $\{(D,B),(G,C),(H,G),(H,T)\}$

Iteration 5: For edge $(A, B) = 8$, A and B are not in a set, choose.

Vertex Sets: $\{S\},\{A,B,D\},\{C,G,H,T\},\{E\},\{F\}$

Edge chose: $\{(D,B),(G,C),(H,G),(H,T),(A,B)\}$

Iteration 6: For edge $(G, D) = 8$, G and D are not in a set, choose.

Vertex Sets: $\{S\},\{A,B,C,D,G,H,T\},\{E\},\{F\}$

Edge chose: $\{(D,B),(G,C),(H,G),(H,T),(A,B),(G,D)\}$

Iteration 7: For edge $(D, T) = 8$, D and T are in a set, not choose.

Iteration 8: For edge $(H, D) = 8$, H and D are in a set, not choose.

Iteration 9: For edge $(S, A) = 10$, S and A are not in a set, choose.

Vertex Sets: $\{S, A, B, C, D, G, H, T\}, \{E\}, \{F\}$

Edge chose: $\{(D, B), (G, C), (H, G), (H, T), (A, B), (G, D), (S, A)\}$

Iteration 10: For edge $(B, G) = 10$, B and G are in a set, not choose.

Iteration 11: For edge $(F, G) = 10$, F and G are not in a set, choose.

Vertex Sets: $\{S, A, B, C, D, F, G, H, T\}, \{E\}$

Edge chose: $\{(D, B), (G, C), (H, G), (H, T), (A, B), (G, D), (S, A), (F, G)\}$

Iteration 12: For edge $(G, T) = 10$, G and T are in a set, not choose.

Iteration 13: For edge $(S, C) = 15$, S and C are in a set, not choose.

Iteration 14: For edge $(C, B) = 15$, C and B are in a set, not choose.

Iteration 15: For edge $(E, F) = 15$, E and F are not in a set, choose.

Vertex Sets: $\{S, A, B, C, D, E, F, G, H, T\}$

Edge chose: $\{(D, B), (G, C), (H, G), (H, T), (A, B), (G, D), (S, A), (F, G), (E, F)\}$

Now, all the vertexes are in a set.

We obtain the final spanning tree is $(D, B), (G, C), (H, G), (H, T), (A, B), (G, D), (S, A), (F, G), (E, F)$

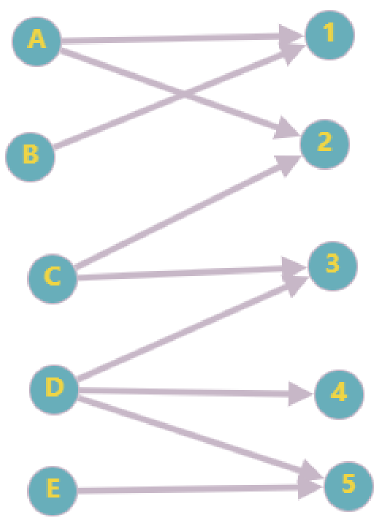
whose value is $5 + 5 + 5 + 5 + 8 + 8 + 10 + 10 + 15 = 71$

2. (a)

We use vertexes A, B, C, D, E to represent **Adam**, **Bob**, **Cathy**, **David**, **Elise** respectively.

And we use vertexes $1, 2, 3, 4, 5$ to represent **TJ1**, **TJ2**, **TJ3**, **TJ4**, **TJ5** respectively.

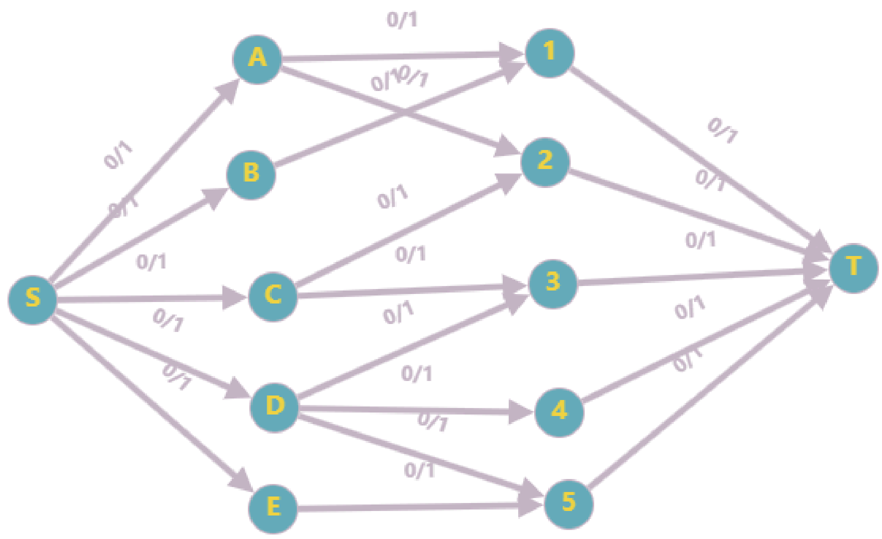
We can get graph as below.

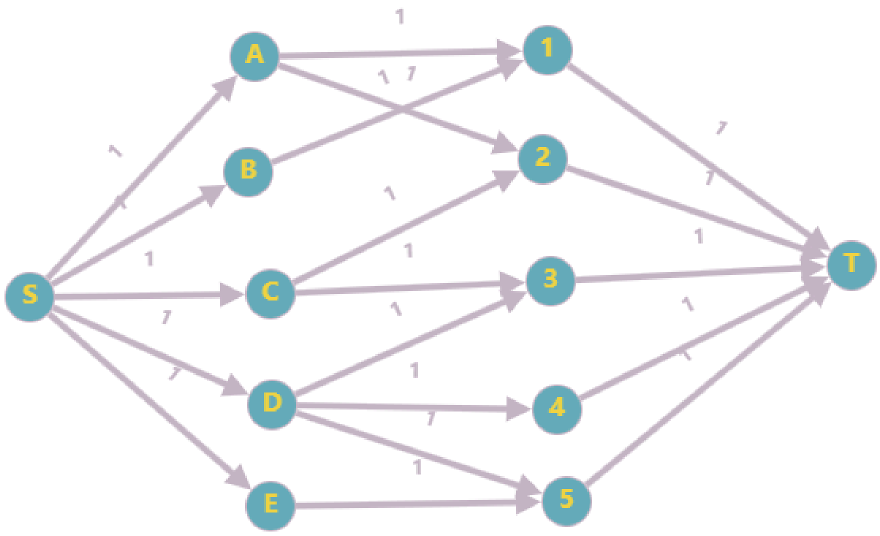


(b)

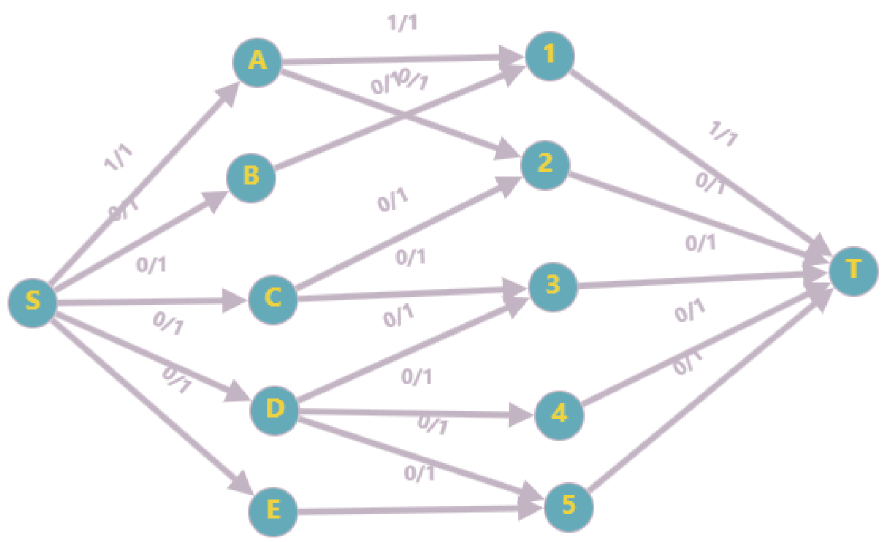
We use the Ford-Fulkerson algorithm because the maximum flow is not exceed 5. We can create the initial graph below

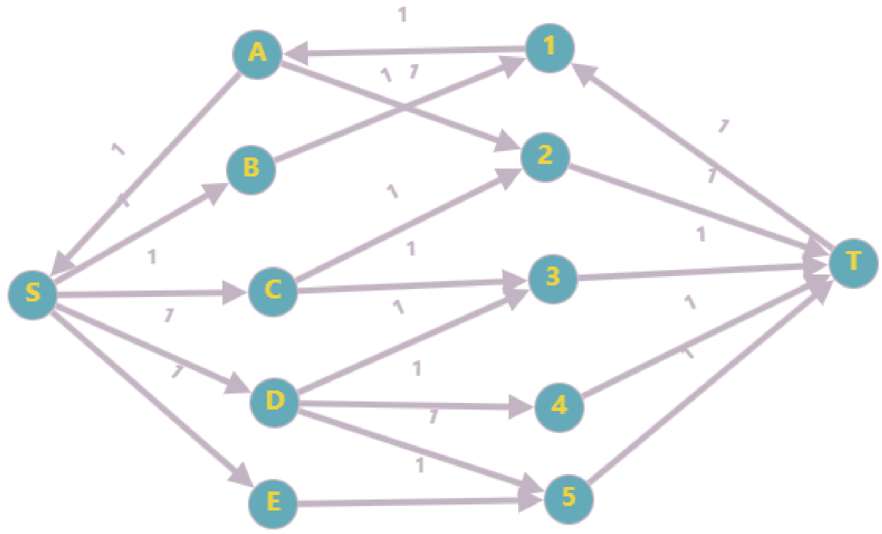
Initial $Flow = 0$



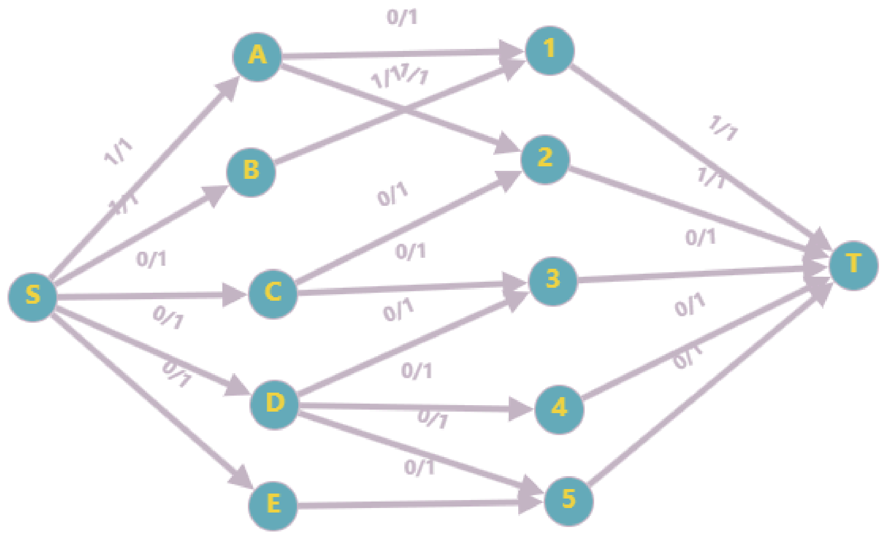


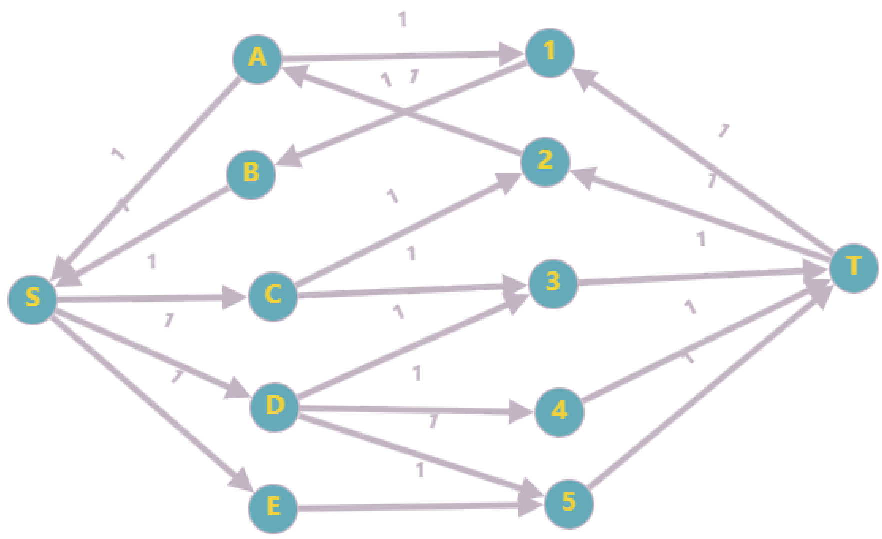
Iteration 1: Augmented path 1: $S - A - 1 - T \rightarrow Flow = Flow + 1 = 1$



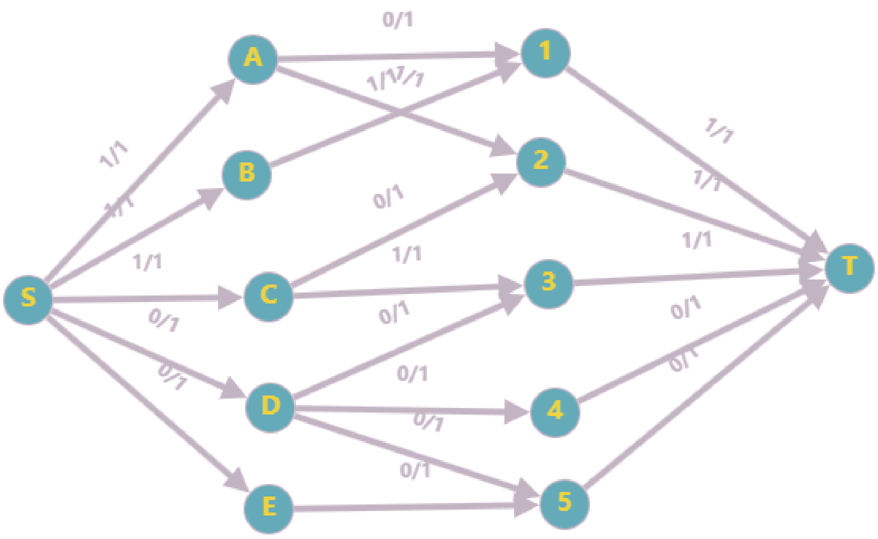


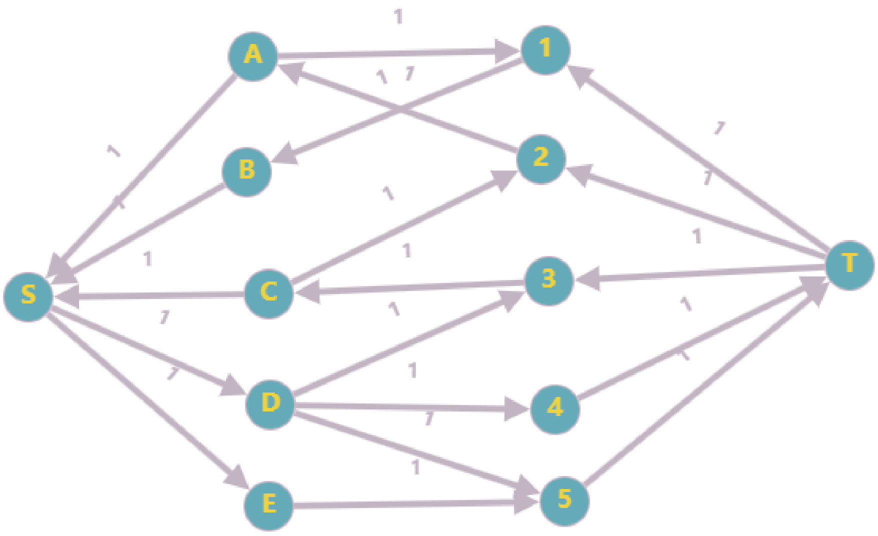
Iteration 2: Augmented path 1: $S - B - 1 - A - 2 - T \rightarrow Flow = Flow + 1 = 2$



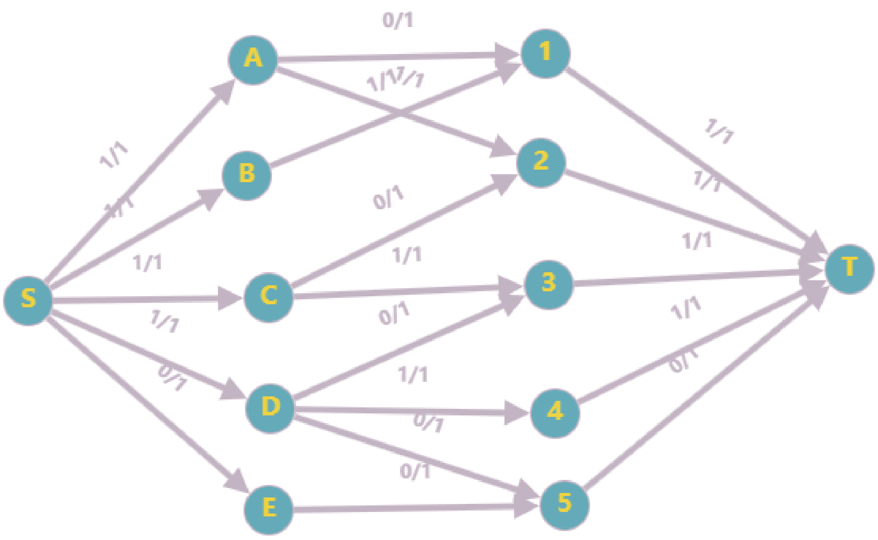


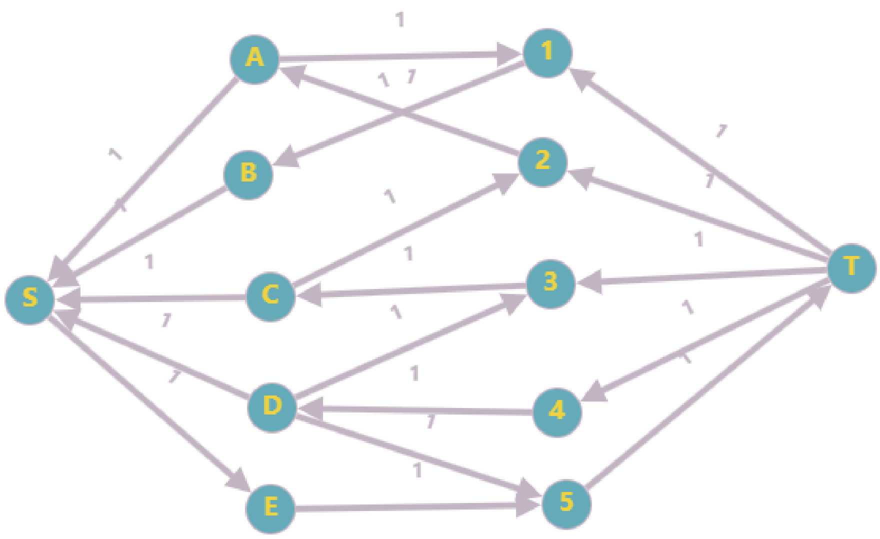
Iteration 3: Augmented path 1: $S - C - 3 - T \rightarrow Flow = Flow + 1 = 3$



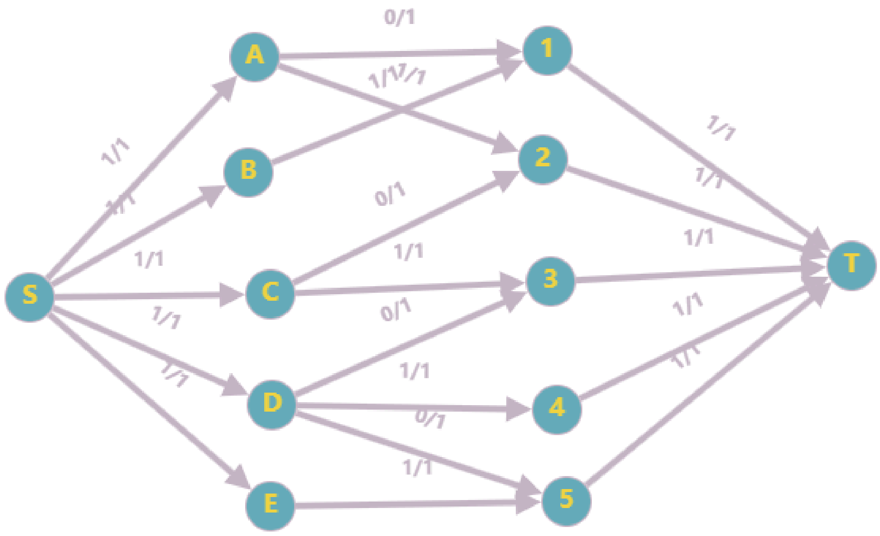


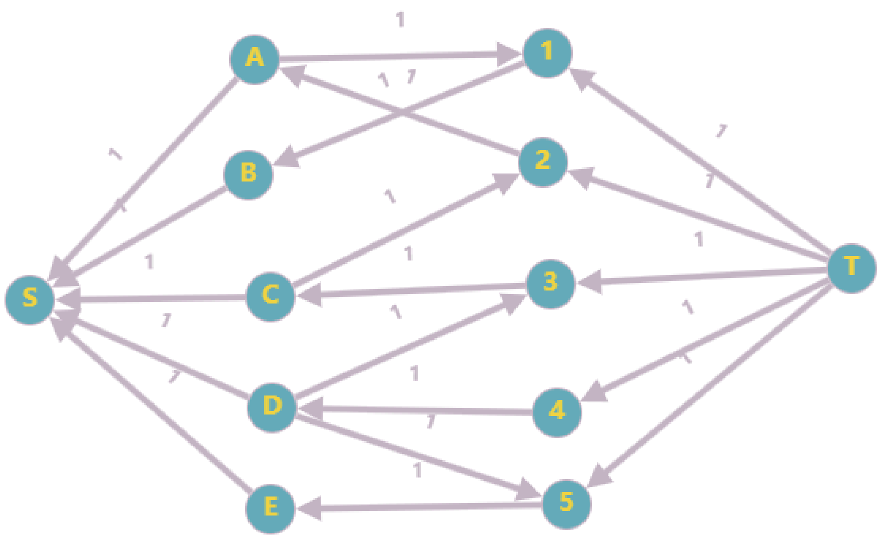
Iteration 4: Augmented path 1: $S - D - 4 - T \rightarrow Flow = Flow + 1 = 4$





Iteration 5: Augmented path 1: $S - E - 5 - T \rightarrow Flow = Flow + 1 = 5$



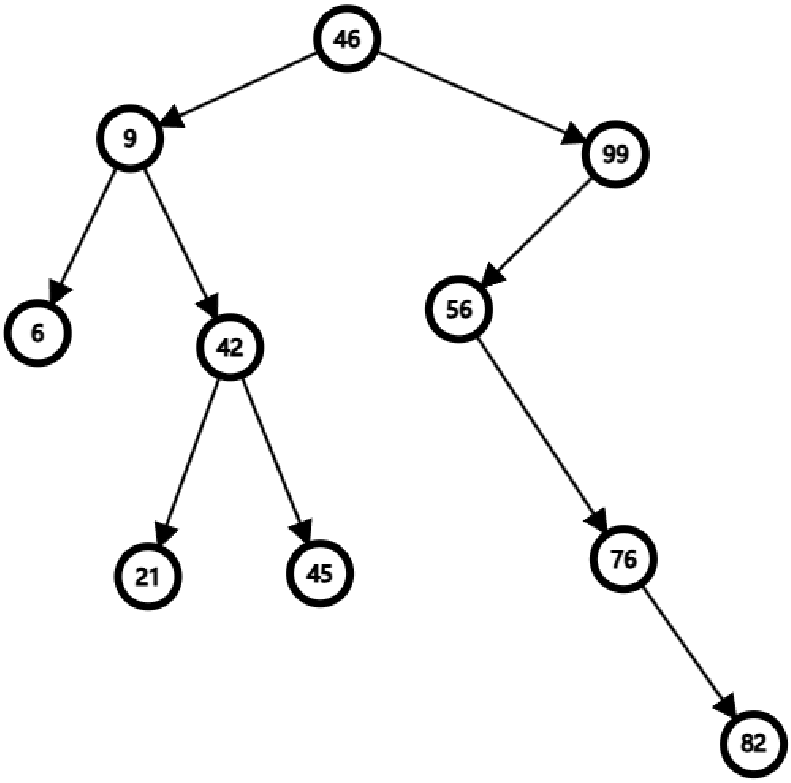


No more possible augmented paths left

Hence, maximum $Flow = 5$

Thus, the final assignment is $(Adam \rightarrow TJ2), (Bob \rightarrow TJ1), (Cathy \rightarrow TJ3), (David \rightarrow TJ4), (Elise \rightarrow TJ5)$

3. (a)



(b)

It's not a balanced tree. Because for leaves 6 are not at height $h = 4$ or $h - 1 = 3$, their height is 2

(c)

pre-order: $46, 9, 6, 42, 21, 45, 99, 56, 76, 82$

(d)

post-order: $6, 21, 45, 42, 9, 82, 76, 56, 99, 46$

(e)

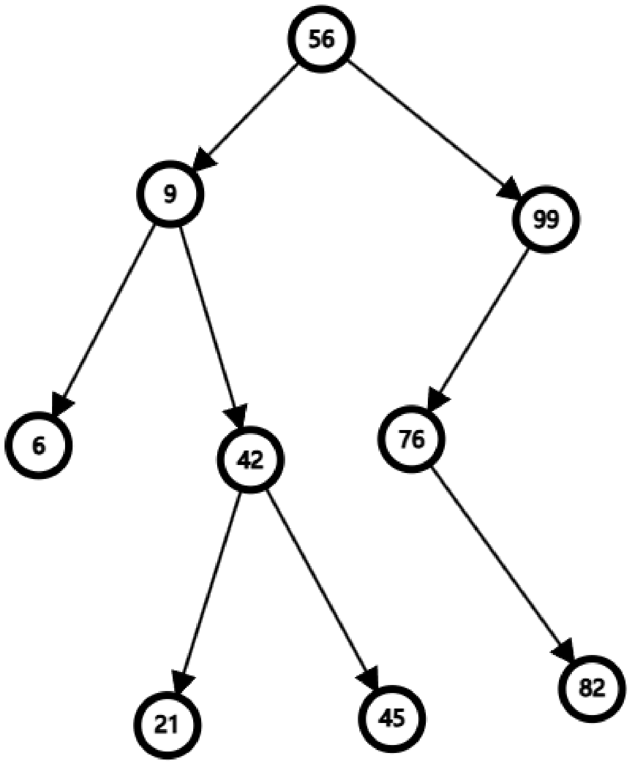
in-order: $6, 9, 21, 42, 45, 46, 56, 76, 82, 99$

(f)

First, when we delete 46, we need to replace it by the minimum node in its right subtree: 56

When we need to delete 56, it has one child, we replace 56 by 76

Thus the final tree is



4. (a)

The K-map can be represented as below

	BC	$\overline{B}C$	$\overline{BC}$	$B\overline{C}$
A	1	1		1
$\overline{A}$	1			

Thus, $F(A,B,C)$ can be simplify as $F(A,B,C) = BC + AB + AC$

$(b)(i)$

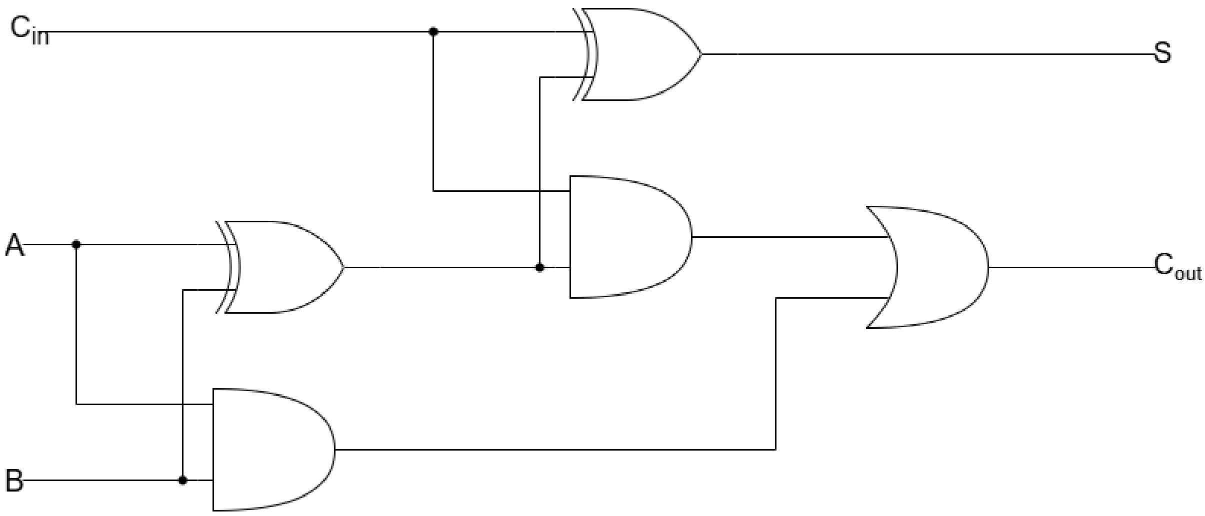
The table can be represented as below

A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

(ii) The logic of S is $S = A \oplus B \oplus C_{in}$

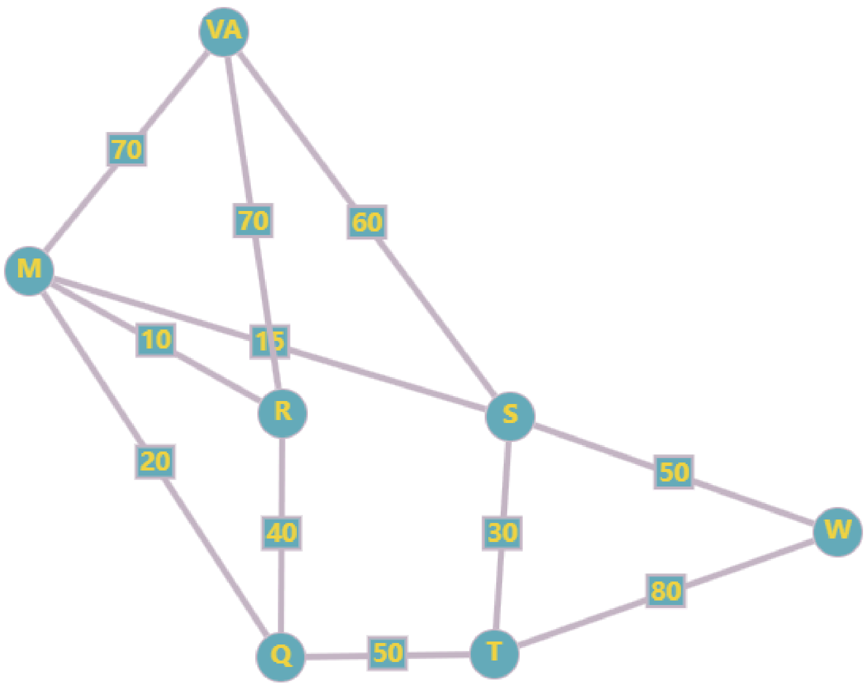
(iii) The logic of C_{out} is $C_{out} = AB + BC_{in} + AC_{in} = C_{in}(A \oplus B) + AB$, which is the same as $4(a)$

(iv) It can be represented by the graph below.



5. (a)

It can be represented by the graph below (*Note that all the unit of length is **Meter(s)***)



(b)

Suppose that in this situation M cannot directly go to VA

We use Dijkstra's algorithm start from point Q

Initialization:

$Dis = (Q : 0), (M : \infty), (VA : \infty), (R : \infty), (S : \infty), (T : \infty), (W : \infty)$

Iteration 1:

Exact vertex Q, update vertex M, R, T

$Q : d = 0\ meter$

$Dis = (M : 20), (VA : \infty), (R : 40), (S : \infty), (T : 50), (W : \infty)$

Iteration 2:

Exact vertex M, update vertex R, S

$Q : d = 0\ meter, M : d = 20\ meters$

$Dis = (VA : \infty), (R : 30), (S : 35), (T : 50), (W : \infty)$

Iteration 3:

Exact vertex R, update vertex VA

$Q : d = 0\ meter, M : d = 20\ meters, R : d = 30\ meters$

$Dis = (VA : 100), (S : 35), (T : 50), (W : \infty)$

Iteration 4:

Exact vertex S, update vertex VA, W, T

$Q : d = 0\ meter, M : d = 20\ meters, R : d = 30\ meters, S = 35\ meters$

$Dis = (VA : 95), (T : 50), (W : 85)$

Iteration 5:

Exact vertex T, update vertex W

$Q : d = 0\ meter, M : d = 20\ meters, R : d = 30\ meters, S = 35\ meters, T = 50\ meters$

$Dis = (VA : 95), (W : 85)$

Iteration 6:

Exact vertex W

$Q : d = 0\ meter, M : d = 20\ meters, R : d = 30\ meters, S = 35\ meters, T = 50\ meters, W = 85\ meters$

$Dis = (VA : 95)$

Iteration 7:

Exact vertex VA

$Q : d = 0\ meter, M : d = 20\ meters, R : d = 30\ meters, S = 35\ meters, T = 50\ meters, W = 85\ meters, VA = 95\ meters$

So the final result is:

$Q : d = 0\ meter, M : d = 20\ meters, R : d = 30\ meters, S = 35\ meters, T = 50\ meters, W = 85\ meters, VA = 95\ meters$

(c)

According to the description, we need to go from W to Q.

Because of all the path is undirected. So the minimum distance from $W \to Q$ is equal to the minimum distance from $Q \to W$. According to the steps in (b),
$d_W = d_S + d(S, W) = d_M + d(M, S) + d(S, W) = d_Q + d(Q, M) + d(M, S) + d(S, W)$

So the path from W to Q is $W \to S \to M \to Q$, whose length is $85\ meters$.