Quantum Mechanics and Waves & Oscillations

Assignment - 4

Assignment due date is 28/11/2024. Each question carries 5 marks.

Q.1: Two equal masses are connected, as shown in Figure 1, with two identical massless springs of spring constant k. Considering only motion in the vertical direction, show that the angular frequencies of the two normal modes are given by $\omega^2 = \left(3 \pm \sqrt{5}\right) k/2m$ and hence that the ratio of the normal mode frequencies is $\left(\sqrt{5}+1\right) / \left(\sqrt{5}-1\right)$. Find the ratio of amplitudes of the two masses in each separate mode. (Note: You need not consider the gravitational forces acting on the masses, because they are independent of the displacements and hence do not contribute to the restoring forces that cause the oscillations. The gravitational forces merely cause a shift in the equilibrium positions of the masses, and you do not have to find what those shifts are.)

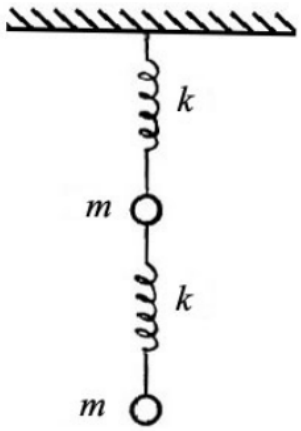

Figure 1:

Q.2: (a) Find the Fourier series of the function shown in the Figure 2
(b) If the release takes place at $t = 0$. What will the string look like ($f(x,t)$) at time t?

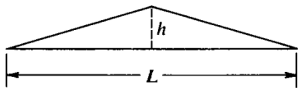

Figure 2:

Q.3: Find the Fourier series for the following functions ($0 \leq x \leq L$)
(a) $y(x) = Ax(L - x)$
(b) $y(x) = A\sin(\pi x/L)$

Q.4: A wave group consists of two wavelengths λ and $\lambda + \Delta\lambda$ where $\Delta\lambda/\lambda$ is very small. Show that the number of wavelengths λ contained between two successive zeros of the modulating envelope is $\approx \lambda/\Delta\lambda$.
b?