MIS

Imagine we have 3 different estimates, created with three different sampling techniques, for the integral of a function F:

$$\frac{F_1(x_1)}{p_1(x_1)} + \frac{F_2(x_2)}{p_2(x_2)} + \frac{F_3(x_3)}{p_3(x_3)}$$

we obviously can't just sum all togheter, since the final integral would result 3 times as large, so let's add some weights:

$$0.333\ \frac{F_1(x_1)}{p_1(x_1)} + 0.333\ \frac{F_2(x_2)}{p_2(x_2)} + 0.333\ \frac{F_3(x_3)}{p_3(x_3)}$$

This is unfortunately not going to help much to resolve variance related issues, because if one of the three estimators has terrible variance, the final estimator will also have terrible variance. page 12 of Veach's thesis

Given that we need to find the optimal weights for these three techniques:

$$w_1\ \frac{F_1}{p_1} + w_2\ \frac{F_2}{p_2} + w_3\ \frac{F_3}{p_3}$$

A good strategy is to use the balance heuristic:

$$w_1(x_1) = \frac{p_1(x_1)}{p_1(x_1) + p_2(x_1) + p_3(x_1)} = \frac{p_1(x_1)}{\sum_k\ p_k(x_1)}$$

and it can also be simplified:

$$w_1\ \frac{F_1}{p_1} = \frac{p_1(x_1)}{p_1(x_1) + p_2(x_1) + p_3(x_1)}\ \frac{F_1(x_1)}{p_1(x_1)} = \frac{F_1(x_1)}{\sum_k\ p_k(x_1)}$$

Difference between the standard model and the one-sample model, where we just randomly choose one of the existing sampling strategies:

```
} else if (type == 'normal mis') {
  // now let's do MIS, this is the normal model
  let sX = samplePdf1();
  let pX = pdf1(sX);
  let fX = sampleF(sX);
  let wX = pdf1(sX) / (pdf1(sX) + pdf2(sX));

  let sX2 = samplePdf2();
  let pX2 = pdf2(sX2);
  let fX2 = sampleF(sX2);
  let wX2 = pdf2(sX2) / (pdf1(sX2) + pdf2(sX2));

  sum += wX * (fX / pX) + wX2 * (fX2 / pX2);
} else if (type == 'one-sample model mis') {
  if (Math.random() < 0.5) {
    // now let's do MIS, but with the one-sample model
    let sX = samplePdf1();
    let fX = sampleF(sX);
    let sumPdfs = pdf1(sX) + pdf2(sX);

    // it works both ways, wether I simplify or not
    // sum += wX * (fX / pX) * 2;
    sum += (fX / sumPdfs) * 2;
  } else {
    let sX2 = samplePdf2();
    let fX2 = sampleF(sX2);
    let sumPdfs = pdf1(sX2) + pdf2(sX2);

    // it works both ways, wether I simplify or not
    // sum += wX2 * (fX2 / pX2) * 2;
    sum += (fX2 / sumPdfs) * 2;
  }
}
```

Given that $w_1(x_1)$ is computed as:

$$w_1(x_1) = \frac{p_1(x_1)}{p_1(x_1) + p_2(x_1) + p_3(x_1)} = \frac{p_1(x_1)}{\sum_k p_k(x_1)}$$

If I'm sampling the $F_1(x_1)$ value for the BRDF, and x_1 is the direction picked according to p_1 , then if I'm also using a light sample with pdf p_2 , to compute the weight for the BRDF sample I still need to use the direction x_1 that we picked from p_1 :

```
var brdfSamplePdf = 1 / (2 * PI);

// sampling the BRDF results in this new direction "x1"
(*ray).direction = normalize(Nt * nd.x + N * nd.y + Nb * nd.z);

if (config.MIS_TYPE == ONE_SAMPLE_MODEL || config.MIS_TYPE == NEXT_EVENT_ESTIMATION) {
  let ires = bvhIntersect(*ray);
  // ... intersection checking and pdf conversion ...
  // notice that the light sample pdf is created using the same ray
  // sampled by the BRDF (direction "x1")
  lightSamplePdf = r2 / (lNolD * ires.triangle.area);
```

Wait, light sampling might result in some direction never being sampled ...

... how can that possibly converge to a correct estimate of the integral?

`MIS-integration-test.js` tests exactly for this case by doing this estimate:

this is the integral:

```
   | y axis
   |
   |              80 ____
   |                 |  |
   |                 |  |
8  |________________|  |
   |___________________|______
   0              0.9  1            x axis



// the integral is 15.2
function sampleF(x) {
  if (x < 0.9) return 8;
```

```
    return 80;
  }
```

The first pdf is equally sampling the domain between `0 ... 1`
the second pdf is exclusively sampling the `0.9 ... 1` portion of the x axis

On its own, the second pdf converges to the wrong answer
With MIS and the balance heuristic, it converges to the correct answer

Intuitively, if the first technique sampled the value of F at `x = 0.5`, we have:

$$w_1(x_1) = \frac{p_1(x_1)}{p_1(x_1) + p_2(x_1)} = \frac{1}{1+0} = 1$$

Basically on every element in the `0 ... 0.9` range the value of $w_1(x_1)$ will be 1, and on every element in the `0.9 ... 1` range it will be:

$$w_1(x_1) = \frac{p_1(x_1)}{p_1(x_1) + p_2(x_1)} = \frac{1}{1+10} \approx 0.09$$

When we sample the second technique, since we're restricted to the `0.9 ... 1` range, the value of $w_2(x_2)$ will always be:

$$w_2(x_2) = \frac{p_2(x_2)}{p_1(x_2) + p_2(x_2)} = \frac{10}{1+10} \approx 0.9$$

Putting all of these togheter, we're looking at this type of computation:

$$w_1 \frac{F_1}{p_1} + w_2 \frac{F_2}{p_2} = 1\frac{8}{1} + 0.9\,\frac{80}{10} = 8 + (0.9 \cdot 8)$$

when the p_1 sample is in the `0 ... 0.9` range, and

$$w_1 \frac{F_1}{p_1} + w_2 \frac{F_2}{p_2} = 0.09\frac{80}{1} + 0.9\,\frac{80}{10} = 7.2 + (0.9 \cdot 8)$$

when the p_1 sample is in the `0.9 ... 1` range.

On a final note, with the one sample model, it almost looks like:

$$k \cdot \frac{F(x)}{\sum_k p_k(x)} = \frac{F}{\frac{1}{k} \cdot \sum_k p_k(x)} = \frac{F}{[p_1(x) + p_2(x)] \cdot 0.5} = \frac{F}{p_{mis}}$$

can be considered like a new pdf!

So in essence MIS is almost like creating a new pdf, that combines all of the techniques being used, even techniques that on their own would result in a pdf that doesn't converge to the right answer