Compilation and Program Analysis (#4) : Types, and Typing MiniWhile

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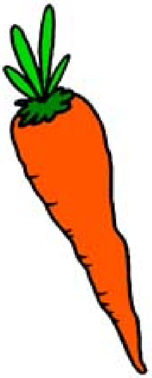
2021-2022

Typing



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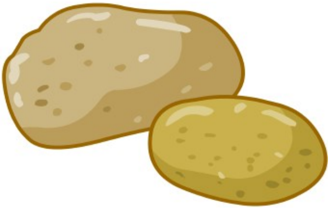
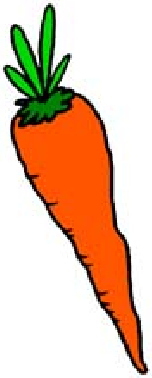


Typing



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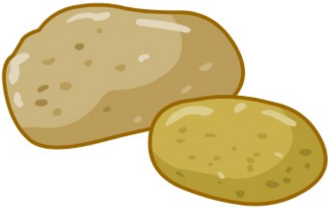

Typing

If you write: `"5" + 37`

what do you want to obtain

- a compilation error? (OCaml)
- an exec error? (Python)
- the int 42? (Visual Basic, PHP)
- the string `"537"`? (Java)
- anything else?

and what about `37 / "5"` ?

Typing

When is

```
e1 + e2
```

legal, and what are the semantic actions to perform ?

- Typing: an analysis that gives a type to each subexpression, and reject incoherent programs.

When

- Dynamic typing (during exec): Lisp, PHP, Python
- Static typing (at compile time): C, Java, OCaml

▶ Here: the second one.

Slogan

well typed programs do not go wrong

Typing objectives

- Should be **decidable**.
- It should reject programs like `(1 2)` in OCaml, or `1+"toto"` in C before an actual error in the evaluation of the expression: this is **safety**.

  The type system is related to the kind of error to be detected: **operations on basic types** / method invocation (message not understood) / correct synchronisation (e.g. session types) in concurrent programs / ...
- The type system should be expressive enough and not reject too many programs. (**expressivity**)

Principle

All sub-expressions of the program must be given a type

$$\texttt{fun}\ (x : \texttt{int}) \to \texttt{let}\ (y : \texttt{int}) = (+ :)(((x : \texttt{int}), (1 : \texttt{int})) : \texttt{int} \times \texttt{int})\ \texttt{in}$$

What does the programmer write?

- The type of all sub-expressions (like above) easy to verify, but tedious for the programmer
- Annotate only variable declarations (Pascal, C, Java, …)

$$\texttt{fun}\ (x : \texttt{int}) \to \texttt{let}\ (y : \texttt{int}) = +(x, 1)\ \texttt{in}\ y$$

- Only annotate function parameters

$$\texttt{fun}\ (x : \texttt{int}) \to \texttt{let}\ y = +(x, 1)\ \texttt{in}\ y$$

- Annotate nothing: complete inference : Ocaml, Haskell, …

Properties

- correction: “yes” implies the program is well typed.
- completeness: the converse.

(optional)

- principality : The most general type is computed.

Typing judgement

We will define how to compute **typing judgements** denoted by:

$$\Gamma \vdash e : \tau$$

and means "in environment Γ, expression e has type τ"

- Γ associates a type $\Gamma(x)$ to all free variables x in e.

Safety = well typed programs do not go wrong

In general a type-safety property looks like this:

Theorem (Safety)

If $\emptyset \vdash e : \tau$, then the reduction of e is infinite or terminates with a value.

Typing Safety

In general, a type-safety proof is based on two lemmas:

Lemme (progression)

If $\emptyset \vdash e : \tau$*, then* e *is a value or there exists* e' *such that* $e \to e'$*.*

Lemme (preservation)

If $\emptyset \vdash e : \tau$ *and* $e \to e'$ *then* $\emptyset \vdash e' : \tau$*.*

This works almost the same for small-step and big-step.

What is a good output for a type-checker?

- We do not want:

```
failwith "typing error"
```

  the origin of the problem should be clearly stated
- We keep the types for next phases.

In practice

- Input: Trees are decorated by source code lines.
- Output: Trees are decorated by types.

1. Generalities about typing

2. Typing ML - ENSL Only

3. Imperative languages (C, Mini-While)
   - Simple Type Checking for mini-while
   - A bit of implementation (for expr)
   - Safety ENS Only

Mini-While Syntax

Expressions:

e	$::=$	c	*constant*
	$\mid$	x	*variable*
	$\mid$	$e + e$	*addition*
	$\mid$	$e \times e$	*multiplication*
	$\mid$	$\ldots$	

Mini-while:

$S(Smt)$	$::=$	$x := expr$	assign
	$\mid$	$skip$	do nothing
	$\mid$	$S_1; S_2$	sequence
	$\mid$	$\texttt{if}\ b\ \texttt{then}\ S_1\ \texttt{else}\ S_2$	test
	$\mid$	$\texttt{while}\ b\ \texttt{do}\ S\ \texttt{done}$	loop

Typing rules for expr

Here types are basic types: Int|Bool

$$\frac{}{\Gamma \vdash x : \Gamma(x)} \qquad \frac{n \in \mathbb{Z}}{\Gamma \vdash n : \texttt{int}}$$ (or $\mathrm{tt}\colon\ \mathrm{bool}, \ldots$)

$$\frac{\Gamma \vdash e_1 : \texttt{int} \quad \Gamma \vdash e_2 : \texttt{int}}{\Gamma \vdash e_1 + e_2 : \texttt{int}}$$

Typing rules for statements: $\Gamma \vdash S$

A statement S is well-typed (there is no type for statements)

on board!

Typing While : recap

$$\frac{c \in \mathbb{Z}}{\Gamma \vdash c : \texttt{int}} \qquad \frac{\Gamma(x) = t \quad t \in \{\texttt{int}, \texttt{bool}\}}{\Gamma \vdash x : t}$$

$$\frac{\Gamma \vdash e_1 : \texttt{int} \quad \Gamma \vdash e_2 : \texttt{int}}{\Gamma \vdash e_1 + e_2 : \texttt{int}} \qquad \frac{\Gamma \vdash e_1 : \texttt{int} \quad \Gamma \vdash e_2 : \texttt{int}}{\Gamma \vdash e_1 < e_2 : \texttt{bool}}$$

$$\frac{\Gamma \vdash S_1 : \texttt{void} \quad \Gamma \vdash S_2 : \texttt{void}}{\Gamma \vdash S_1 ; S_2 : \texttt{void}} \qquad \frac{\Gamma \vdash e : t \quad \Gamma \vdash x : t \quad t \in \{\texttt{int}, \texttt{bool}\}}{\Gamma \vdash x = e : \texttt{void}}$$

$$\frac{\Gamma \vdash b : \texttt{bool} \quad \Gamma \vdash S : \texttt{void}}{\Gamma \vdash \texttt{while}\ b\ \texttt{do}\ S\ \texttt{done} : \texttt{void}}$$

$$\frac{\Gamma \vdash b : \texttt{bool} \quad \Gamma \vdash S_1 : \texttt{void} \quad \Gamma \vdash S_2 : \texttt{void}}{\Gamma \vdash \texttt{if}\ b\ \texttt{then}\ S_1\ \texttt{else}\ S_2 : \texttt{void}}$$

Typing: an example

Considering $\Gamma = \{x_1 \mapsto int\}$, prove that the given sequence of instructions is well typed:

```
x1 = 3 ;
x1 = x1+9 ;
```

on board!

Hybrid expressions

What if we have `1.2 + 42` ?

- reject?
- compute a float!

► This is **type coercion**. We will see how to implement it during a lab.

More complex expressions

What if we have types `pointer of bool` or `array of int`? We might want to check equivalence (for addition . . .).

- This is called **structural equivalence** (see Dragon Book, “type equivalence”). This is solved by a basic graph traversal checking that each element are equivalent/compatible.

Sub-typing ENSL Only

- A type can be more precise than another one, e.g.

$$int <: num$$

- Need additional rule to use sub-typing:

$$\frac{e : \tau \qquad \tau <: \tau'}{e : \tau'}$$

- Sometimes, rule to compose sub-types, e.g. functions or parametric types

$$\frac{e : Array[\tau] \qquad \tau <: \tau'}{e : Array[\tau']}$$

How to define subtyping for functions?

Note: subtyping is heavily used in OOP

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Principle

- Gamma is constructed with lexing information or parsing (variable declaration with types).
- Rules are semantic actions. The semantic actions are responsible for the evaluation order, as well as typing errors.

Type Checking V1 : Visitor

MuTypingVisitor.py

```
# now visit expr

def visitAtomExpr(self, ctx):
    return self.visit(ctx.atom())

def visitOrExpr(self, ctx):
    lvaltype = self.visit(ctx.expr(0))
    rvaltype = self.visit(ctx.expr(1))
    if (BaseType.Boolean == lvaltype) and (BaseType.Boolean == rvaltype):
        return BaseType.Boolean
    else:
        self._raise(ctx, 'boolean operands', lvaltype, rvaltype)
```

In practice for mini-C (lab sessions)

No annotation is added to the AST (everything is int or bool, no ambiguity)

We can create associating type to variables, directly from parsing

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Summary

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