PEP 528 – Mathematical Methods for Physicists and Engineers 2

Lecture 1	Gaussian integrals. Gamma and Beta functions. Stirling's approximation. N-dimensional sphere. Ideal gas in statistical mechanics.
Lecture 2	*Sturm-Liouville Theory 1.* Scalar product and Hilbert space. Convergence, polynomials and Weierstrass theorem. Orthogonal polynomials. Linear operators. Delta function. Self-adjoint ODEs and boundary conditions.
Lecture 3	*Sturm-Liouville Theory 2*. S-L eigenproblem and eigenbasis. Recurrent relations. Rodrigues' formula. Wronskian and second solution.
Lecture 4	*Sturm-Liouville Theory 3*: Regular and singular points. Series solutions and Frobenius method. Finite and infinite solutions. Generating functionals.
Lecture 5	*PDEs and Special Functions 1.* Laplace eqn in spherical coords 1: Legendre polynomials. Multipole expansion. Example: sphere in an external field
Lecture 6	*PDEs and Special Functions 2.* Laplace eqn in spherical coords 2: Associated Legendre functions and spherical harmonics. Spherically symmetric PDE.
Lecture 7	*PDEs and Special Functions 3*. Laplace and Helmholtz eqns. in cylindrical coords. Bessel ODE and Bessel (and related) functions. Helmholtz eqn. in spherical coords, and spherical Bessel functions. Example: Fraunhofer diffraction.
Lecture 8	*PDEs and Special Functions 4*.Hermite functions. Eigenfunctions of Fourier transform. LHO in quantum mechanics. Laguerre functions. Hydrogen atom.
Lecture 9	*Green's Functions 1.* Inhomogeneous ODEs and definition of GF. Continuity conditions for GF. Calculating the GF by matched homogeneous solutions. GF as a sum over the eigenvalues.
Lecture 10	*Green's Functions 2*. GF for damped harmonic oscillator and contour integrals in Fourier-space. GF as a sum of general solutions.

Lecture 11	*Green's Functions 3.* GF for Helmhotz and Wave equations. Diffusion equation GF. Three dimensional GF as a sum of eigenfunctions.
Lecture 12	*Integral Equations 1*. Classification of integral equation. Fredholm theory. Sum of separable kernels. Fredholm alternative. Born series.
Lecture 13	*Integral Equations 2*. Differential and integral equations. Sturm-Liouville operators in integral form. Hilbert-Schmidt theory.
Lecture 14	*Asymptotic analysis.* Matched asymptotic expansion. Singular perturbations and boundary layers.

Course outcomes:

1. Use Gram-Schmidt process to form a set of orthogonal polynomials on the given Hilbert space, e.g. for (-1,1) interval with weight w=1, show that the resulting functions are proportional to Legendre polynomials.
2. Solve the differential equation using Frobenius method, and find the condition for non-divergent solutions, e.g. show that the constant in the Legendre differential equation must be of the form *n(n+1)* with *n* integer.
3. Solve the Laplace equation in spherical coordinates using separation of variables and Legendre polynomials/spherical harmonics, e.g. find the velocity of the potential fluid flow around a sphere.
4. Solve the Laplace/Helmholtz equation in cylindrical coordinates using separation of variables and Bessel functions, e.g. stdudy the oscillations of a drum with given initial shape.
5. Solve the inhomogeneous differential equation using Green's functions, e.g. find the time evolution of a damped linear harmonic oscillator with the given driving force.
6. Solve the Fredholm integral equations, e.g. show that eigenfunctions of the Fourier transform are the Hermite functions.
7. Find the asymptotic expansion of the integrals using the Laplace method or method of stationary phase, e.g. find the asymptotic behavior or spherical Bessel functions.

Assessment:

Homework assignments (7-8) + take-home final exam.

Course objectives for PEP 527 and PEP 528:

The series of two courses introduces the mathematical tools needed for advanced physics and engineering problems. The students will learn the techniques for solving physical equations in the exact form (linear operator eigenproblems, complex analysis) and using the approximate methods (perturbations, variations, asymptotics). The partial differential equations of the Laplace and Helmholtz type are studied in the spherical and cylindrical coordinates; the special functions appearing in their solutions are studied in detail. The problems with the source are studied using Green's functions and integral equations. The students will also learn to solve the problems in general curvilinear coordinate systems.

Textbooks:

Mathematical Methods for Physicists by Arfken, Weber

Mathematics of Classical and Quantum Physics by Byron, Fuller

Mathematical Methods for Physics and Engineering by Riley, Hobson, Bence

Class lecture notes.