PEP 527 – Mathematical Methods for Physicists and Engineers 1

Lecture 1	*Intro & prerequisites review*. Geometric Series. Simple ODEs. Fourier and Laplace transforms.
Lecture 2	*Tensor analysis 1.* Rotation matrices. Moment of inertia tensor.
Lecture 3	*Tensor analysis 2.* Formation of higher order tensors. Invariant tensors: Levi-Civita and Kronecker. Symmetry of tensor components.
Lecture 4	*Tensor analysis 3.* Tensor fields. Tensor derivatives. Derivative operator theorems.
Lecture 5	*Curvilinear coordinates*. Metric tensor and scale factors. Derivative operators in orthogonal curvilinear coords.
Lecture 6	*Rotating coordinate systems.* Euler's equations. Precession in free rotation.
Lecture 7	*Calculus of variations.* Euler-Lagrange equations. First integrals. Lagrange multipliers. Geodesics. Brachistochrone. Catenary.
Lecture 8	*Complex vector space 1*. Matrix inverse. Unitary and Hermitian matrices. Eigenproblem of Hermitian matrix. Resolution of identity. Eigenproblem of rotation matrix.
Lecture 9	*Complex vector space 2*. Projectors. Gram-Schmidt process. Baker-Campbell-Hausdorff lemma. Dirac notation.
Lecture 10	*Perturbation theory*. Regular and singular perturbations. Principle of dominant balance. Petrurbation in eigenproblems. WKB approximation. Lindstet-Poincare expansion.
Lecture 11	*Complex analysis 1*. Cauchy-Riemann conditions. Cauchy's integral formula. Taylor expansion.
Lecture 12	*Complex analysis 2.* Laurent expansions. Calculus of residues. Calculation of integrals.
Lecture 13	*Complex analysis 3*. Application of residues: Series summation. Implicit equations. Conformal mapping and 2D Laplace equation.
Lecture 14	*Complex analysis 4*. Inverse Laplace transform. Branch cuts. Principal values. Kramers-Kronig relations.

Course outcomes for PEP 527:

1. Simplify the combination of derivative operators acting on a combination of scalars/vectors/tensors, e.g. $\nabla \times (\vec{A} \times \vec{B})$.
2. Find the derivative operators in arbitrary curvilinear coordinates, e.g. $\nabla \times \vec{A}$ in oblate spheroidal coordinates.
3. Use calculus of variations to minimize given quantity (with or without constraints), e.g. find the shape water surface in the rotating bucket.
4. Cast a system of equations in the matrix form and find the eigenmodes/energies, e.g. find the vibrational spectrum of a CO_2 molecule.
5. Use perturbation theory to find the change in the response of the system with a small perturbation, e.g. find the changes in the vibrational spectrum of CO_2 if ^{12}C is substituted by ^{13}C.
6. Calculate the integrals using complex residues, e.g. solve the differential equation using Fourier and inverse Fourier transform.
7. Use calculus of residues to calculate infinite sums, e.g. find the Riemann zeta-function $\zeta(4)$.

Assessment:

Homework assignments (7-8) + take-home final exam.

Course objectives for PEP 527 and PEP 528:

The series of two courses introduces the mathematical tools needed for advanced physics and engineering problems. The students will learn the techniques for solving physical equations in the exact form (linear operator eigenproblems, complex analysis) and using the approximate methods (perturbations, variations, asymptotics). The partial differential equations of the Laplace and Helmholtz type are studied in the spherical and cylindrical coordinates; the special functions appearing in their solutions are studied in detail. The problems with the source are studied using Green's functions and integral equations. The students will also learn to solve the problems in general curvilinear coordinate systems.

Textbooks:

Mathematical Methods for Physicists by Arfken, Weber

Mathematics of Classical and Quantum Physics by Byron, Fuller

Mathematical Methods for Physics and Engineering by Riley, Hobson, Bence

Class lecture notes.