BINARY SEARCH TREES

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Agenda

1 Trees

2 Dictionaries implemented as BSTs

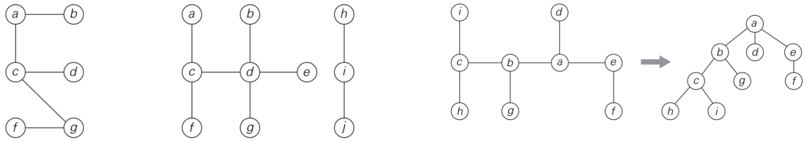
3 Binary tree traversals

4 Bibliography

Trees[1]

A free tree: a connected acyclic graph

- Forest: acyclic graph not necessarily connected
- Rooted tree (root typically on the top)



Applications: implement dictionaries, fault analysis, etc.

[1] Source: A. Levitin. Introduction to the Design and Analysis of Algorithms. 2011.

Trees: terminology for rooted trees

- Parent
- Siblings
- Subtree
- Internal vertices
- Ancestors / descendants (proper)

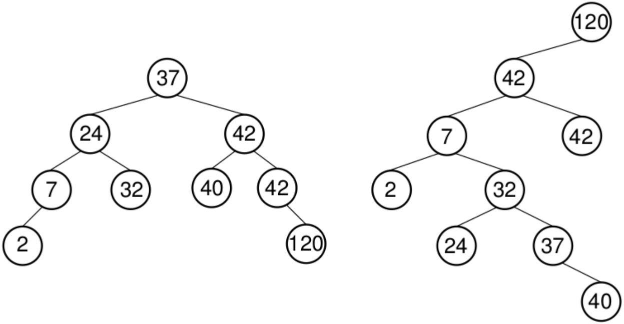
Trees: terminology for rooted trees

- Leaf: no children
- Depth/level of v: length of the unique path from the root to v
- Height: length of the longest unique path from the node to a leaf
- m-ary tree: every internal vertex has no more than m children
  - Complete m-ary tree: levels filled from top to bottom, left to right
  - Full m-ary tree: exactly m children
  - m-ary tree, where $m = 2$: binary tree
- Ordered tree
  - Ordered binary tree: binary search tree (BST)
- Balanced tree

Binary search trees[2]

Left tree: insertion order = 37, 24, 42, 7, 2, 40, 42, 32, 120

Right tree: insertion order = 120, 42, 42, 7, 2, 32, 37, 24, 40



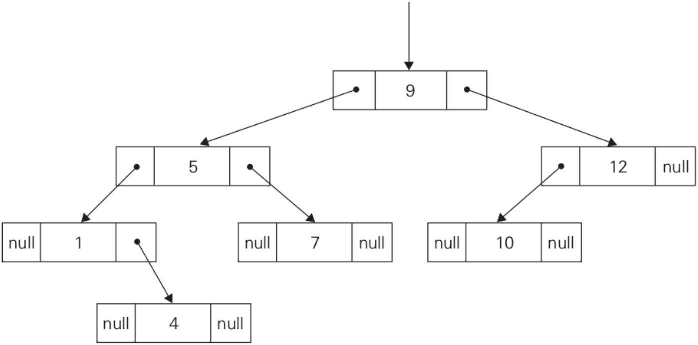
[2] Source: C. Shaffer. Data Structures and Algorithm Analysis. 2013.

Agenda

1. Trees
2. Dictionaries implemented as BSTs
3. Binary tree traversals
4. Bibliography

Dictionaries implemented as BSTs[3]

Typical implementation: based on references (pointers)



[3] Source: A. Levitin. Introduction to the Design and Analysis of Algorithms. 2011.

Dictionaries implemented as BSTs

Composite type (BSTNode):

```
Key key;
E element;
BSTNode left;                                   // left child
BSTNode right ;                                 // right child
```

Algorithm: BSTNode create_bstnode(Key k, E e)

```
$n.key \leftarrow k$;
$n.element \leftarrow e$;
$n.left \leftarrow n.right \leftarrow NULL$;
return $n$;
```

Dictionaries implemented as BSTs

Composite type (BST):

```
BSTNode $root$;
int $nodecount$;                    // number of elements
```

Algorithm: BST create_bst()

```
$bst.root \leftarrow NULL$;
$bst.nodecount \leftarrow 0$;
return $bst$;
```

Dictionaries implemented as BSTs

Algorithm: E find(BST bst, Key k)

```
return $findhelp(bst.root, k)$;
```

Algorithm: E findhelp(BSTNode rt, Key k)

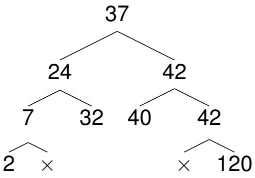
```
if $rt = NULL$ then return $NULL$ ;
if $rt.key > k$ then
    return $findhelp(rt.left, k)$;
else if $rt.key = k$ then
    return $rt.element$;
else
    return $findhelp(rt.right, k)$;
```

Dictionaries implemented as BSTs

```
Algorithm: E find(BST bst, Key k)
return findhelp(bst.root, k);
```

```
Algorithm: E findhelp(BSTNode rt, Key k)
if $rt = NULL$ then return NULL ;
if $rt.key > k$ then
    return findhelp(rt.left, k);
else if $rt.key = k$ then
    return rt.element;
else
    return findhelp(rt.right, k);
```

find(_,32)



Dictionaries implemented as BSTs

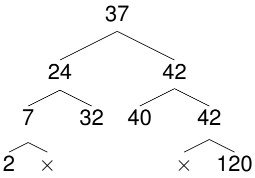
Algorithm: E find(BST bst, Key k)

```
return findhelp(bst.root, k);
```

Algorithm: E findhelp(BSTNode rt, Key k)

```
if $rt = NULL$ then  return $NULL$ ;
if $rt.key > k$ then
    return $findhelp(rt.left, k)$;
else if $rt.key = k$ then
    return $rt.element$;
else
    return $findhelp(rt.right, k)$;
```

find(_,32)



Dictionaries implemented as BSTs

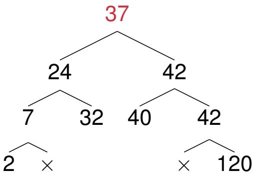
Algorithm: E find(BST bst, Key k)

```
return findhelp(bst.root, k);
```

Algorithm: E findhelp(BSTNode rt, Key k)

```
if $rt = NULL$ then return NULL ;
if $rt.key > k$ then
    return findhelp(rt.left, k);
else if $rt.key = k$ then
    return rt.element;
else
    return findhelp(rt.right, k);
```

find(_,32)



Dictionaries implemented as BSTs

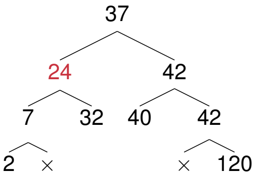
Algorithm: E find(BST bst, Key k)

```
return findhelp($bst.root$, $k$);
```

Algorithm: E findhelp(BSTNode rt, Key k)

```
if $rt = NULL$ then return $NULL$ ;
if $rt.key > k$ then
    return findhelp($rt.left$, $k$);
else if $rt.key = k$ then
    return $rt.element$;
else
    return findhelp($rt.right$, $k$);
```

find(_,32)

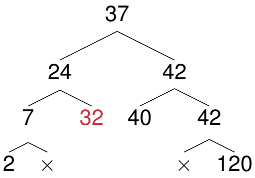


Dictionaries implemented as BSTs

```
Algorithm: E find(BST bst, Key k)
    return findhelp(bst.root, k);

Algorithm: E findhelp(BSTNode rt, Key k)
    if $rt = NULL$ then return $NULL$ ;
    if $rt.key > k$ then
        return findhelp(rt.left, k);
    else if $rt.key = k$ then
        return rt.element;
    else
        return findhelp(rt.right, k);
```

find(_,32)



Returning element associated with $k = 32$

Dictionaries implemented as BSTs

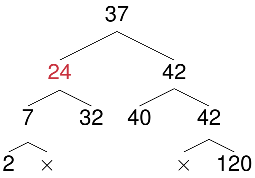
```
Algorithm: E find(BST bst, Key k)

return $findhelp(bst.root, k)$;
```

```
Algorithm: E findhelp(BSTNode rt, Key k)

if $rt = NULL$ then return $NULL$ ;
if $rt.key > k$ then
    return $findhelp(rt.left, k)$;
else if $rt.key = k$ then
    return $rt.element$;
else
    return $findhelp(rt.right, k)$;
```

find(_,32)



Returning element associated with $k = 32$

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Dictionaries implemented as BSTs

Algorithm: E find(BST bst, Key k)

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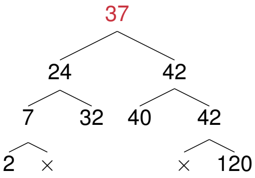
return $findhelp(bst.root, k)$;
```

Algorithm: E findhelp(BSTNode rt, Key k)

```
if $rt = NULL$ then return $NULL$ ;
if $rt.key > k$ then
    return $findhelp(rt.left, k)$;
else if $rt.key = k$ then
    return $rt.element$;
else
    return $findhelp(rt.right, k)$;
```

`find(_,32)`



Returning element associated with $k = 32$

Dictionaries implemented as BSTs

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return $findhelp(bst.root, k)$;
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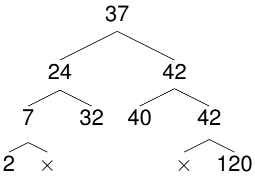
Algorithm: E findhelp(BSTNode rt, Key k)

```
if $rt = NULL$ then return $NULL$ ;
if $rt.key > k$ then
    return $findhelp(rt.left, k)$;
else if $rt.key = k$ then
    return $rt.element$;
else
    return $findhelp(rt.right, k)$;
```

find(_,32)



Returning element associated with $k = 32$

Dictionaries implemented as BSTs

Algorithm: void insert(BST bst, Key k, E e)

```
$bst.root \leftarrow inserthelp(bst.root, k, e)$;
$bst.nodecount{+}{+}$;
```

Algorithm: BSTNode inserthelp(BSTNode rt, Key k, E e)

```
if $rt = NULL$ then return $create\_bstnode(k, e)$ ;
if $rt.key > k$ then
    $rt.left \leftarrow inserthelp(rt.left, k, e)$;
else
    $rt.right \leftarrow inserthelp(rt.right, k, e)$;
return $rt$;
```

Important: repeated keys
go to the right subtree

Dictionaries implemented as BSTs

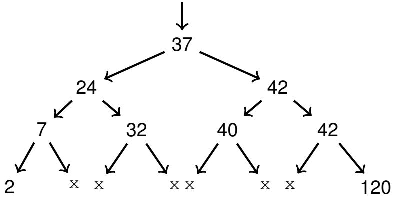
Algorithm: void insert(BST bst, Key k, E e)

```
bst.root ← inserthelp(bst.root, k, e);
bst.nodecount++;
```

Algorithm: BSTNode inserthelp(BSTNode rt, Key k, E e)

```
if rt = NULL then return create_bstnode(k, e) ;
if rt.key > k then
    rt.left ← inserthelp(rt.left, k, e);
else
    rt.right ← inserthelp(rt.right, k, e);
return rt;
```

`insert(_,25,_)`



Dictionaries implemented as BSTs

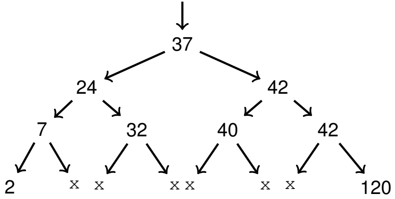
Algorithm: void insert(BST bst, Key k, E e)

```
bst.root ← inserthelp(bst.root, k, e);
bst.nodecount++;
```

Algorithm: BSTNode inserthelp(BSTNode rt, Key k, E e)

```
if $rt = NULL$ then return create_bstnode(k, e) ;
if $rt.key > k$ then
    rt.left ← inserthelp(rt.left, k, e);
else
    rt.right ← inserthelp(rt.right, k, e);
return rt;
```

insert(_,25,_)



Dictionaries implemented as BSTs

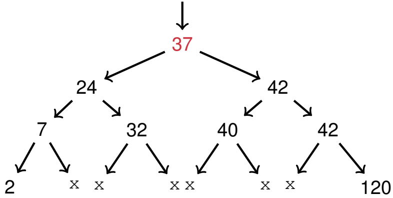
```
Algorithm: void insert(BST bst, Key k, E e)

$bst.root \leftarrow inserthelp(bst.root, k, e)$;
$bst.nodecount$++;
```

```
Algorithm: BSTNode inserthelp(BSTNode rt, Key k, E e)

if $rt = NULL$ then return $create\_bstnode(k, e)$ ;
if $rt.key > k$ then
    $rt.left \leftarrow inserthelp(rt.left, k, e)$;
else
    $rt.right \leftarrow inserthelp(rt.right, k, e)$;
return $rt$;
```

insert(_,25,_)



Dictionaries implemented as BSTs

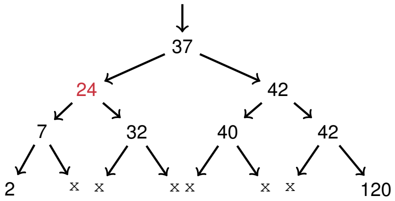
Algorithm: void insert(BST bst, Key k, E e)

```
bst.root ← inserthelp(bst.root, k, e);
bst.nodecount++;
```

Algorithm: BSTNode inserthelp(BSTNode rt, Key k, E e)

```
if $rt = NULL$ then return create_bstnode($k, e$) ;
if $rt.key > k$ then
    $rt.left \leftarrow inserthelp(rt.left, k, e)$;
else
    $rt.right \leftarrow inserthelp(rt.right, k, e)$;
return $rt$;
```

`insert(_,25,_)`



Dictionaries implemented as BSTs

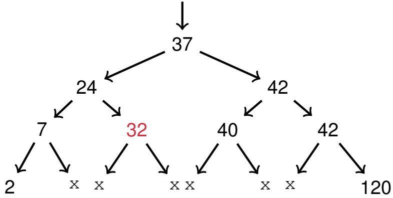
Algorithm: void insert(BST bst, Key k, E e)

```
bst.root ← inserthelp(bst.root, k, e);
bst.nodecount++;
```

Algorithm: BSTNode inserthelp(BSTNode rt, Key k, E e)

```
if $rt = NULL$ then return create_bstnode(k, e) ;
if $rt.key > k$ then
    rt.left ← inserthelp(rt.left, k, e);
else
    rt.right ← inserthelp(rt.right, k, e);
return rt;
```

insert(_,25,_)

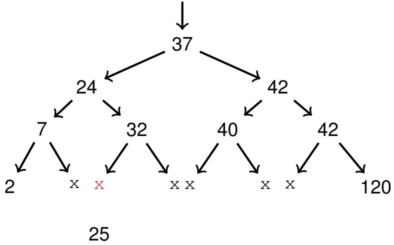


Dictionaries implemented as BSTs

```
Algorithm: void
insert(BST bst, Key k, E e)
  $bst.root \leftarrow inserthelp(bst.root, k, e)$;
  $bst.nodecount$++;
```

```
Algorithm: BSTNode
inserthelp(BSTNode rt, Key k, E e)
  if $rt = NULL$ then return $create\_bstnode(k, e)$ ;
  if $rt.key > k$ then
      $rt.left \leftarrow inserthelp(rt.left, k, e)$;
  else
      $rt.right \leftarrow inserthelp(rt.right, k, e)$;
  return $rt$;
```

insert(_,25,_)



Returning the reference to the new node

Dictionaries implemented as BSTs

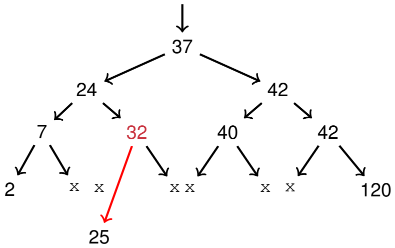
Algorithm: void insert(BST bst, Key k, E e)

```
$bst.root \leftarrow inserthelp(bst.root, k, e)$;
$bst.nodecount$++;
```

Algorithm: BSTNode inserthelp(BSTNode rt, Key k, E e)

```
if $rt = NULL$ then return $create\_bstnode(k, e)$ ;
if $rt.key > k$ then
    $rt.left \leftarrow inserthelp(rt.left, k, e)$;
else
    $rt.right \leftarrow inserthelp(rt.right, k, e)$;
return $rt$;
```

insert(_,25,_)



Returning the reference to the current node

Dictionaries implemented as BSTs

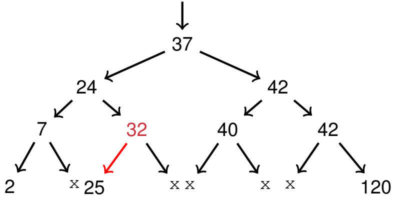
Algorithm: void insert(BST bst, Key k, E e)

```
bst.root ← inserthelp(bst.root, k, e);
bst.nodecount++;
```

Algorithm: BSTNode inserthelp(BSTNode rt, Key k, E e)

```
if $rt = NULL$ then return create_bstnode(k, e) ;
if $rt.key > k$ then
    rt.left ← inserthelp(rt.left, k, e);
else
    rt.right ← inserthelp(rt.right, k, e);
return rt;
```

`insert(_,25,_)`



Better presented this way

Returning the reference to the current node

Dictionaries implemented as BSTs

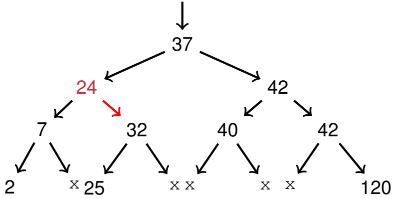
Algorithm: void insert(BST bst, Key k, E e)

```
bst.root ← inserthelp(bst.root, k, e);
bst.nodecount++;
```

Algorithm: BSTNode inserthelp(BSTNode rt, Key k, E e)

```
if $rt = NULL$ then return create_bstnode(k, e) ;
if $rt.key > k$ then
    rt.left ← inserthelp(rt.left, k, e);
else
    rt.right ← inserthelp(rt.right, k, e);
return rt;
```

insert(_,25,_)



Returning the reference to the current node

Dictionaries implemented as BSTs

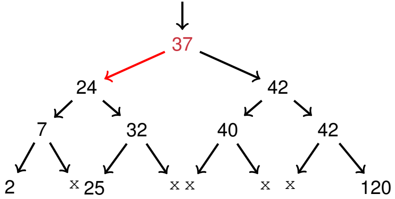
Algorithm: void insert(BST bst, Key k, E e)

```
bst.root ← inserthelp(bst.root, k, e);
bst.nodecount++;
```

Algorithm: BSTNode inserthelp(BSTNode rt, Key k, E e)

```
if rt = NULL then return create_bstnode(k, e) ;
if rt.key > k then
    rt.left ← inserthelp(rt.left, k, e);
else
    rt.right ← inserthelp(rt.right, k, e);
return rt;
```

insert(_,25,_)



Returning the reference

to the current node (root)

Dictionaries implemented as BSTs

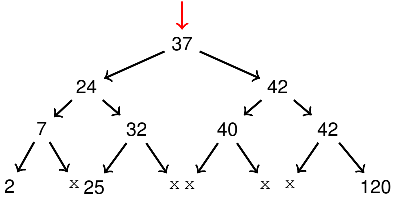
Algorithm: void insert(BST bst, Key k, E e)

```
bst.root ← inserthelp(bst.root, k, e);
bst.nodecount++;
```

Algorithm: BSTNode inserthelp(BSTNode rt, Key k, E e)

```
if $rt = NULL$ then return create_bstnode(k, e) ;
if $rt.key > k$ then
    rt.left ← inserthelp(rt.left, k, e);
else
    rt.right ← inserthelp(rt.right, k, e);
return rt;
```

`insert(_,25,_)`



Updating the reference to the root node

Dictionaries implemented as BSTs

Algorithm: E remove(BST bst, Key k)

```
E temp ← findhelp(bst.root, k);
if $temp \neq NULL$ then
    bst.root ← removehelp(bst.root, k);
    bst.nodecount--;
return temp;
```

Dictionaries implemented as BSTs

Algorithm: BSTNode removehelp(BSTNode rt, Key k)

```
if $rt = NULL$ then return $NULL$;
if $rt.key > k$ then
    $rt.left \leftarrow removehelp(rt.left, k)$;
else if $rt.key < k$ then
    $rt.right \leftarrow removehelp(rt.right, k)$;
else
    if $rt.left = NULL$ then return $rt.right$ ;
    else if $rt.right = NULL$ then return $rt.left$ ;
    else
        BSTNode $temp \leftarrow getmin(rt.right)$;
        $rt.element \leftarrow temp.element$;
        $rt.key \leftarrow temp.key$;
        $rt.right \leftarrow deletemin(rt.right)$;
return $rt$;
```

Dictionaries implemented as BSTs

Algorithm: BSTNode removehelp(BSTNode rt, Key k)

```
if $rt = NULL$ then return $NULL$;
if $rt.key > k$ then
    $rt.left \leftarrow removehelp(rt.left, k)$;
else if $rt.key < k$ then
    $rt.right \leftarrow removehelp(rt.right, k)$;
else
    if $rt.left = NULL$ then return $rt.right$;
    else if $rt.right = NULL$ then return $rt.left$ ;
    else
        BSTNode $temp \leftarrow getmin(rt.right)$;
        $rt.element \leftarrow temp.element$;
        $rt.key \leftarrow temp.key$;
        $rt.right \leftarrow deletemin(rt.right)$;
return $rt$;
```

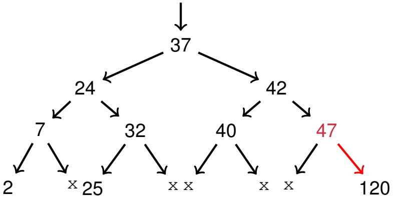
Let *rt* be the node to be removed, three cases:

(i) $rt.left = NULL$,

(ii) $rt.right = NULL$, and

(iii) otherwise.

`remove(_,47)`



Dictionaries implemented as BSTs

Algorithm: BSTNode removehelp(BSTNode rt, Key k)

```
if $rt = NULL$ then return $NULL$;
if $rt.key > k$ then
    $rt.left \leftarrow removehelp(rt.left, k)$;
else if $rt.key < k$ then
    $rt.right \leftarrow removehelp(rt.right, k)$;
else
    if $rt.left = NULL$ then return $rt.right$ ;
    else if $rt.right = NULL$ then return $rt.left$ ;
    else
        BSTNode $temp \leftarrow getmin(rt.right)$;
        $rt.element \leftarrow temp.element$;
        $rt.key \leftarrow temp.key$;
        $rt.right \leftarrow deletemin(rt.right)$;
return $rt$;
```

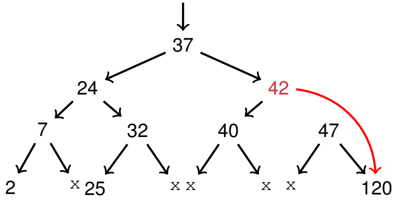
Let *rt* be the node to be removed, three cases:

(i) $rt.left = NULL$,

(ii) $rt.right = NULL$, and

(iii) otherwise.

`remove(_,47)`



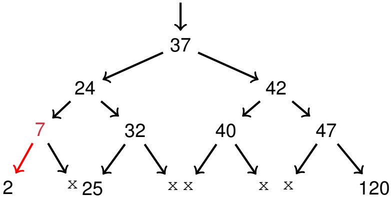
Dictionaries implemented as BSTs

Algorithm: BSTNode removehelp(BSTNode rt, Key k)

```
if $rt = NULL$ then return $NULL$;
if $rt.key > k$ then
    $rt.left \leftarrow removehelp(rt.left, k)$;
else if $rt.key < k$ then
    $rt.right \leftarrow$
      $removehelp(rt.right, k)$;
else
    if $rt.left = NULL$ then return
      $rt.right$ ;
    else if $rt.right = NULL$ then
      return $rt.left$;
    else
        BSTNode
          $temp \leftarrow getmin(rt.right)$;
        $rt.element \leftarrow temp.element$;
        $rt.key \leftarrow temp.key$;
        $rt.right \leftarrow$
          $deletemin(rt.right)$;
return $rt$;
```

Let *rt* be the node to be removed, three cases:
(i) $rt.left = NULL$,
(ii) $rt.right = NULL$, and
(iii) otherwise.

`remove(_,7)`



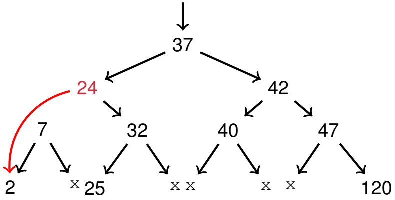
Dictionaries implemented as BSTs

```
Algorithm: BSTNode removehelp(BSTNode rt, Key k)

if $rt = NULL$ then return $NULL$;
if $rt.key > k$ then
    $rt.left \leftarrow removehelp(rt.left, k)$;
else if $rt.key < k$ then
    $rt.right \leftarrow removehelp(rt.right, k)$;
else
    if $rt.left = NULL$ then return $rt.right$ ;
    else if $rt.right = NULL$ then return $rt.left$ ;
    else
        BSTNode $temp \leftarrow getmin(rt.right)$;
        $rt.element \leftarrow temp.element$;
        $rt.key \leftarrow temp.key$;
        $rt.right \leftarrow deletemin(rt.right)$;
return $rt$;
```

Let *rt* be the node to be removed, three cases:
(i) $rt.left = NULL$,
(ii) $rt.right = NULL$, and
(iii) otherwise.

`remove(_,7)`



Dictionaries implemented as BSTs

```
Algorithm: BSTNode removehelp(BSTNode rt, Key k)
if $rt = NULL$ then return $NULL$;
if $rt.key > k$ then
    $rt.left \leftarrow removehelp(rt.left, k)$;
else if $rt.key < k$ then
    $rt.right \leftarrow removehelp(rt.right, k)$;
else
    if $rt.left = NULL$ then return $rt.right$;
    else if $rt.right = NULL$ then return $rt.left$;
    else
        BSTNode $temp \leftarrow getmin(rt.right)$;
        $rt.element \leftarrow temp.element$;
        $rt.key \leftarrow temp.key$;
        $rt.right \leftarrow deletemin(rt.right)$;
return $rt$;
```

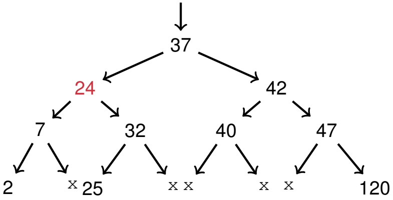
Let *rt* be the node to be removed, three cases:

(i) $rt.left = NULL$,

(ii) $rt.right = NULL$, and

(iii) otherwise.

`remove(_,24)`



temp = reference to node 25

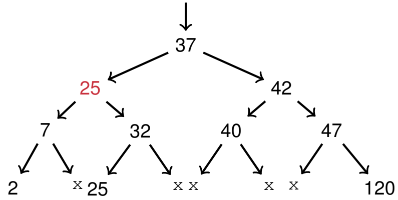
Dictionaries implemented as BSTs

Algorithm: BSTNode removehelp(BSTNode rt, Key k)

```
if $rt = NULL$ then return $NULL$;
if $rt.key > k$ then
    $rt.left \leftarrow removehelp(rt.left, k)$;
else if $rt.key < k$ then
    $rt.right \leftarrow removehelp(rt.right, k)$;
else
    if $rt.left = NULL$ then return $rt.right$ ;
    else if $rt.right = NULL$ then return $rt.left$ ;
    else
        BSTNode $temp \leftarrow getmin(rt.right)$;
        $rt.element \leftarrow temp.element$;
        $rt.key \leftarrow temp.key$;
        $rt.right \leftarrow deletemin(rt.right)$;
return $rt$;
```

Let *rt* be the node to be removed, three cases:
(i) $rt.left = NULL$,
(ii) $rt.right = NULL$, and
(iii) otherwise.

`remove(_,24)`



temp = reference to node 25

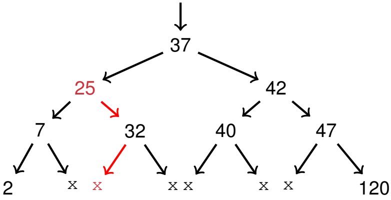
Dictionaries implemented as BSTs

Algorithm: BSTNode removehelp(BSTNode rt, Key k)

```
if $rt = NULL$ then return $NULL$;
if $rt.key > k$ then
    $rt.left \leftarrow removehelp(rt.left, k)$;
else if $rt.key < k$ then
    $rt.right \leftarrow$
     $removehelp(rt.right, k)$;
else
    if $rt.left = NULL$ then return
     $rt.right$ ;
    else if $rt.right = NULL$ then
     return $rt.left$ ;
    else
        BSTNode
         $temp \leftarrow getmin(rt.right)$;
        $rt.element \leftarrow temp.element$;
        $rt.key \leftarrow temp.key$;
        $rt.right \leftarrow$
         $deletemin(rt.right)$;
return $rt$;
```

Let *rt* be the node to be removed, three cases:
(i) $rt.left = NULL$,
(ii) $rt.right = NULL$, and
(iii) otherwise.

`remove(_,24)`



temp = reference to node 25

Dictionaries implemented as BSTs

Algorithm: BSTNode getmin(BSTNode rt)

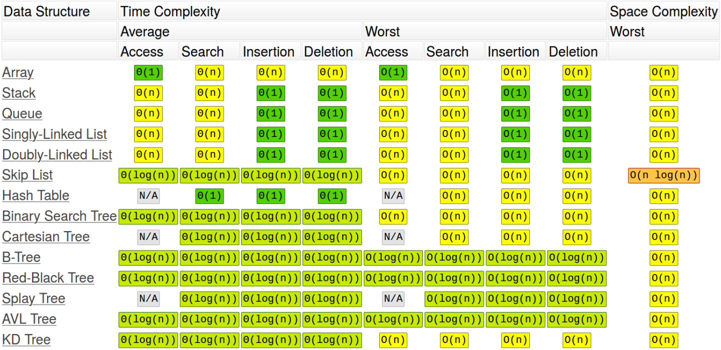
```
if $rt.left = NULL$ then return $rt$;
return $getmin(rt.left)$;
```

Algorithm: BSTNode deletemin(BSTNode rt)

```
if $rt.left = NULL$ then return $rt.right$;
$rt.left \leftarrow deletemin(rt.left)$;
return $rt$;
```

Asymptotic efficiency of BSTs[4]

Average case: considering a balanced BST



Data Structure	Time Complexity								Space Complexity
	Average				Worst				Worst
	Access	Search	Insertion	Deletion	Access	Search	Insertion	Deletion	
Array	Θ(1)	Θ(n)	Θ(n)	Θ(n)	O(1)	O(n)	O(n)	O(n)	O(n)
Stack	Θ(n)	Θ(n)	Θ(1)	Θ(1)	O(n)	O(n)	O(1)	O(1)	O(n)
Queue	Θ(n)	Θ(n)	Θ(1)	Θ(1)	O(n)	O(n)	O(1)	O(1)	O(n)
Singly-Linked List	Θ(n)	Θ(n)	Θ(1)	Θ(1)	O(n)	O(n)	O(1)	O(1)	O(n)
Doubly-Linked List	Θ(n)	Θ(n)	Θ(1)	Θ(1)	O(n)	O(n)	O(1)	O(1)	O(n)
Skip List	Θ(log(n))	Θ(log(n))	Θ(log(n))	Θ(log(n))	O(n)	O(n)	O(n)	O(n)	O(n log(n))
Hash Table	N/A	Θ(1)	Θ(1)	Θ(1)	N/A	O(n)	O(n)	O(n)	O(n)
Binary Search Tree	Θ(log(n))	Θ(log(n))	Θ(log(n))	Θ(log(n))	O(n)	O(n)	O(n)	O(n)	O(n)
Cartesian Tree	N/A	Θ(log(n))	Θ(log(n))	Θ(log(n))	N/A	O(n)	O(n)	O(n)	O(n)
B-Tree	Θ(log(n))	Θ(log(n))	Θ(log(n))	Θ(log(n))	O(log(n))	O(log(n))	O(log(n))	O(log(n))	O(n)
Red-Black Tree	Θ(log(n))	Θ(log(n))	Θ(log(n))	Θ(log(n))	O(log(n))	O(log(n))	O(log(n))	O(log(n))	O(n)
Splay Tree	N/A	Θ(log(n))	Θ(log(n))	Θ(log(n))	N/A	O(log(n))	O(log(n))	O(log(n))	O(n)
AVL Tree	Θ(log(n))	Θ(log(n))	Θ(log(n))	Θ(log(n))	O(log(n))	O(log(n))	O(log(n))	O(log(n))	O(n)
KD Tree	Θ(log(n))	Θ(log(n))	Θ(log(n))	Θ(log(n))	O(n)	O(n)	O(n)	O(n)	O(n)

[4] Source: http://bigocheatsheet.com/

Agenda

1 Trees

2 Dictionaries implemented as BSTs

3 Binary tree traversals

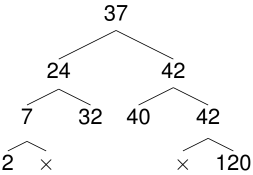
4 Bibliography

Binary tree traversals: preorder

The root is processed before the left and the right subtrees

Algorithm: void preorder(BSTNode rt)

```
if $rt \neq NULL$ then
    // do something with $rt$
    preorder($rt.left$);
    preorder($rt.right$);
```



Let do something be printing the root's key

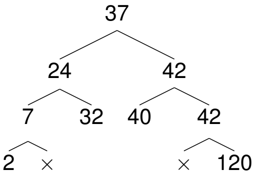
37, 24, 7, 2, 32, 42, 40, 42, 120

Binary tree traversals: inorder

The root is processed after the left subtree, but before the right subtree

Algorithm: void inorder(BSTNode rt)

```
if $rt \neq NULL$ then
    inorder($rt.left$);
    // do something with $rt$
    inorder($rt.right$);
```



Let do something be printing the root's key

$2, 7, 24, 32, 37, 40, 42, 42, 120$

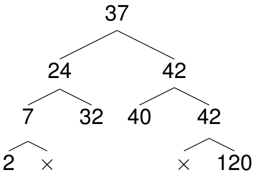
An inorder traversal visits the keys in a non-decreasing order

Binary tree traversals: posorder

The root is processed after the left and the right subtrees

Algorithm: void posorder(BSTNode rt)

```
if $rt \neq NULL$ then
    posorder($rt.left$);
    posorder($rt.right$);
    // do something with $rt$
```



Let do something be printing the root's key

2, 7, 32, 24, 40, 120, 42, 42, 37

Agenda

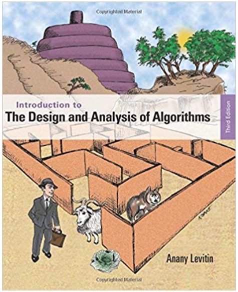
1 Trees

2 Dictionaries implemented as BSTs

3 Binary tree traversals

4 Bibliography

Bibliography



Chapter 1 (pp. 31–35)
Chapter 4 (pp. 163–164)
Chapter 5 (pp. 182–185)
Anany Levitin.
Introduction to the Design and Analysis of Algorithms.
3rd edition. Pearson. 2011.

Chapter 5 (pp. 145–170)
Clifford Shaffer.
Data Structures and Algorithm Analysis. Dover, 2013.

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