# HASH TABLES

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# Agenda



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# Dictionary ADT

A set of elements with the options of searching, insertion and deletion

- Elements are indexed by comparable keys

Operations:

- void clear(Dictionary d);
- void insert(Dictionary d, Key k, E e); // reflect on: multiple entries
- E remove(Dictionary d, Key k); // reflect on: multiple entries
- E removeAny(Dictionary d); // alternative: getKeys
- E find(Dictionary d, Key k); // reflect on: multiple entries
- int size(Dictionary d);

Implementations: array, linked list, hash tables, and balanced trees

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# Agenda

# Hashing

Space and time trade-off

- Worse space efficiency to get a better time efficiency

Hashing: distributing keys among a one-dimensional array dicionários

- Very efficient way[1] to implement dictionaries
- Hash table: $H[0..m-1]$
- Hash function: $h : Key \rightarrow 0..m-1$

$h$ needs to satisfy somewhat conflicting requirements

- Adequate relation between $m$ and $|Key|$
- Distribute keys as evenly as possible
- Has to be easy to compute

[1] In terms of temporal efficiency

# Hashing

Limitations:

- Not ideal in the presence of multiple entries per key
- Not ideal for accessing elements based on some order
- Not ideal for searching based on a range of keys

# Hash functions for integers

A trivial attempt[2]: $h(K) = K \bmod m$

- Let $m = 100$ (hash addresses within 0..99)
- Let $K = 4567$, then $h(K) = 4567 \bmod 100 = 67$

A better approach: the mid-square method

- Compute $K^2$, and select the $r$ middle digits, such that $10^r - 1 < m$
- Let $m = 100$ (hash addresses within 0..99), then $r = 2$
- Let $K = 4567$, $K^2 = 20857489$, then $h(K) = 57$

[2] **Attention** to the implementation of mod : $\mathbb{Z} \rightarrow \mathbb{N}$

# Hash functions for strings

A first attempt: fold

**Algorithm:** int h(string K)

```
s ← length(K);
sum ← 0;
for i ← 0 to s − 1 do
    sum ← sum + K[i];
return abs(sum)%m; // abs = overflow and %
```

Fair/bad distribution depending on $m$ and $K$

- Suppose that $length(K) = 10$ (in average)
- Let $K$ have only capital letters
- Since $A = 65$ and $Z = 90$, $sum \in [650..900]$
- If $m \leq 100$, it is a fair distribution
- If $m \geq 1000$, it is a bad distribution

# Hash functions for strings

A better approach: sfold:

**Algorithm:** int h(string K)

```
intLength ← length(K)/4;
sum ← 0;
for i ← 0 to intLength − 1 do
    sub ← substring(K, i ∗ 4, (i ∗ 4) + 4);   // initial and final positions
    mult ← 1;
    for j ← 0 to 3 do
        sum ← sum + sub[j] ∗ mult;
        mult ← mult ∗ 256;
sub ← substring(K, intLength ∗ 4);            // initial position to the end
mult ← 1;
s ← length(sub);
for j ← 0 to s − 1 do
    sum ← sum + sub[j] ∗ mult;
    mult ← mult ∗ 256;
return abs(sum)%m; // abs = overflow and %
```

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# Hash functions for strings

**Algorithm:** int h(string K)

```
intLength ← length(K)/4;
sum ← 0;
for i ← 0 to intLength − 1 do
    sub ← substring(K, i ∗ 4, (i ∗ 4) + 4);
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    for j ← 0 to 3 do
        sum ← sum + sub[j] ∗ mult;
        mult ← mult ∗ 256;
sub ← substring(K, intLength ∗ 4);
mult ← 1;
s ← length(sub);
for j ← 0 to s − 1 do
    sum ← sum + sub[j] ∗ mult;
    mult ← mult ∗ 256;
return abs(sum)%m;
```

Let $K =$ “ALGORITHMS”
Let $m = 1000$

$$fold(K) = 762$$

$$sfold(K) = ?$$

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# Hash functions for strings

**Algorithm:** int h(string K)

```
intLength ← length(K)/4;
sum ← 0;
for i ← 0 to intLength − 1 do
    sub ← substring(K, i ∗ 4, (i ∗ 4) + 4);
    mult ← 1;
    for j ← 0 to 3 do
        sum ← sum + sub[j] ∗ mult;
        mult ← mult ∗ 256;
sub ← substring(K, intLength ∗ 4);
mult ← 1;
s ← length(sub);
for j ← 0 to s − 1 do
    sum ← sum + sub[j] ∗ mult;
    mult ← mult ∗ 256;
return abs(sum)%m;
```

Let $K =$ “ALGORITHMS”
Let $m = 1000$

$fold(K) = 762$

$sfold(K) = ?$

$intLength = 2$
$sum = 0$

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# Hash functions for strings

**Algorithm:** int h(string K)

```
intLength ← length(K)/4;
sum ← 0;
for i ← 0 to intLength − 1 do
    sub ← substring(K, i ∗ 4, (i ∗ 4) + 4);
    mult ← 1;
    for j ← 0 to 3 do
        sum ← sum + sub[j] ∗ mult;
        mult ← mult ∗ 256;
sub ← substring(K, intLength ∗ 4);
mult ← 1;
s ← length(sub);
for j ← 0 to s − 1 do
    sum ← sum + sub[j] ∗ mult;
    mult ← mult ∗ 256;
return abs(sum)%m;
```

Let $K$ = “ALGORITHMS”
Let $m = 1000$

$fold(K) = 762$

$sfold(K) = ?$

$intLength = 2$
$i = 0$
$sub$ = “ALGO”
$sum = 1330072641$
$mult = 0$

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# Hash functions for strings

**Algorithm:** int h(string K)

```
intLength ← length(K)/4;
sum ← 0;
for i ← 0 to intLength − 1 do
    sub ← substring(K, i ∗ 4, (i ∗ 4) + 4);
    mult ← 1;
    for j ← 0 to 3 do
        sum ← sum + sub[j] ∗ mult;
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s ← length(sub);
for j ← 0 to s − 1 do
    sum ← sum + sub[j] ∗ mult;
    mult ← mult ∗ 256;
return abs(sum)%m;
```

Let $K =$ "ALGORITHMS"
Let $m = 1000$

$fold(K) = 762$

$sfold(K) = ?$

$intLength = 2$
$i = 1$
$sub =$ "RITH"
$sum = -1751411309$
$mult = 0$

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# Hash functions for strings

**Algorithm:** int h(string K)

```
intLength ← length(K)/4;
sum ← 0;
for i ← 0 to intLength − 1 do
    sub ← substring(K, i ∗ 4, (i ∗ 4) + 4);
    mult ← 1;
    for j ← 0 to 3 do
        sum ← sum + sub[j] ∗ mult;
        mult ← mult ∗ 256;
sub ← substring(K, intLength ∗ 4);
mult ← 1;
s ← length(sub);
for j ← 0 to s − 1 do
    sum ← sum + sub[j] ∗ mult;
    mult ← mult ∗ 256;
return abs(sum)%m;
```

Let $K$ = “ALGORITHMS”
Let $m = 1000$

$fold(K) = 762$

$sfold(K) = ?$

$intLength = 2$
$i = 2$
$sub$ = “RITH”
$sum = -1751411309$
$mult = 0$

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# Hash functions for strings

**Algorithm:** int h(string K)

```
intLength ← length(K)/4;
sum ← 0;
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sub ← substring(K, intLength ∗ 4);
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s ← length(sub);
for j ← 0 to s − 1 do
    sum ← sum + sub[j] ∗ mult;
    mult ← mult ∗ 256;
return abs(sum)%m;
```

Let $K =$ “ALGORITHMS”
Let $m = 1000$

$fold(K) = 762$

$sfold(K) = ?$

$intLength = 2$
$sub =$ “MS”
$s = 2$
$sum = -1751411309$
$mult = 1$

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# Hash functions for strings

**Algorithm:** int h(string K)

```
intLength ← length(K)/4;
sum ← 0;
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    sub ← substring(K, i ∗ 4, (i ∗ 4) + 4);
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    for j ← 0 to 3 do
        sum ← sum + sub[j] ∗ mult;
        mult ← mult ∗ 256;
sub ← substring(K, intLength ∗ 4);
mult ← 1;
s ← length(sub);
for j ← 0 to s − 1 do
    sum ← sum + sub[j] ∗ mult;
    mult ← mult ∗ 256;
return abs(sum)%m;
```

Let $K =$ "ALGORITHMS"
Let $m = 1000$

$fold(K) = 762$

$sfold(K) = ?$

$intLength = 2$
$sub =$ "MS"
$s = 2$
$j = 0$
$sum = -1751411232$
$mult = 256$

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# Hash functions for strings

**Algorithm:** int h(string K)

```
intLength ← length(K)/4;
sum ← 0;
for i ← 0 to intLength − 1 do
    sub ← substring(K, i ∗ 4, (i ∗ 4) + 4);
    mult ← 1;
    for j ← 0 to 3 do
        sum ← sum + sub[j] ∗ mult;
        mult ← mult ∗ 256;
sub ← substring(K, intLength ∗ 4);
mult ← 1;
s ← length(sub);
for j ← 0 to s − 1 do
    sum ← sum + sub[j] ∗ mult;
    mult ← mult ∗ 256;
return abs(sum)%m;
```

Let $K$ = "ALGORITHMS"
Let $m = 1000$

$fold(K) = 762$

$sfold(K) = ?$

$intLength = 2$
$sub$ = "MS"
$s = 2$
$j = 1$
$sum = -1751389984$
$mult = 65536$

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# Hash functions for strings

**Algorithm:** int h(string K)

```
intLength ← length(K)/4;
sum ← 0;
for i ← 0 to intLength − 1 do
    sub ← substring(K, i ∗ 4, (i ∗ 4) + 4);
    mult ← 1;
    for j ← 0 to 3 do
        sum ← sum + sub[j] ∗ mult;
        mult ← mult ∗ 256;
sub ← substring(K, intLength ∗ 4);
mult ← 1;
s ← length(sub);
for j ← 0 to s − 1 do
    sum ← sum + sub[j] ∗ mult;
    mult ← mult ∗ 256;
return abs(sum)%m;
```

Let $K =$ “ALGORITHMS”

Let $m = 1000$

$fold(K) = 762$

$sfold(K) = ?$

$intLength = 2$

$sub =$ “MS”

$s = 2$

$j = 2$

$sum = -1751389984$

$mult = 65536$

# Hash functions for strings

**Algorithm:** int h(string K)

```
intLength ← length(K)/4;
sum ← 0;
for i ← 0 to intLength − 1 do
    sub ← substring(K, i ∗ 4, (i ∗ 4) + 4);
    mult ← 1;
    for j ← 0 to 3 do
        sum ← sum + sub[j] ∗ mult;
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sub ← substring(K, intLength ∗ 4);
mult ← 1;
s ← length(sub);
for j ← 0 to s − 1 do
    sum ← sum + sub[j] ∗ mult;
    mult ← mult ∗ 256;
return abs(sum)%m;
```

Let $K$ = “ALGORITHMS”

Let $m = 1000$

$$fold(K) = 762$$

$$sfold(K) = 984$$

# Hashing: collisions

Collisions

- If $m <| Key |$, they will occur
- If $m \geq | Key |$, they still might occur

Collision resolution mechanisms

- Open hashing (separate chaining)
- Closed hashing (open addressing)

Perfect hashing

- No collisions at all
- *Key* is known and available beforehand
- Example: data access on a read-only CD

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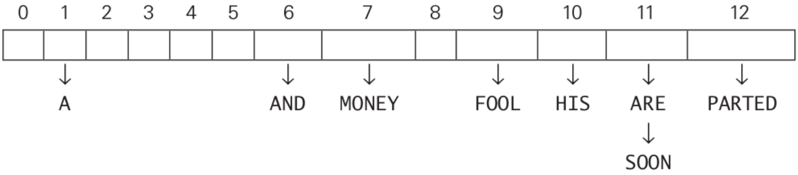
# Open hashing[3]

Each hash address is associated with a linked list

Example (considering $A = 1$ and fold):

| keys | A | FOOL | AND | HIS | MONEY | ARE | SOON | PARTED |
|---|---|---|---|---|---|---|---|---|
| hash addresses | 1 | 9 | 6 | 10 | 7 | 11 | 11 | 12 |




[3] Source: A. Levitin. Introduction to the Design and Analysis of Algorithms. 2011.

# Open hashing

Insertion:

1. Compute $h(K) = j$
2. Insert in the $j$-th linked list
   - Sorted list: better search, worse insertion
   - Unsorted: worse search, better insertion

Search:

1. Compute $h(K) = j$
2. If the $j$-th linked list is empty, element not found
3. Otherwise, search the linked list

Deletion:

- Compute $h(K) = j$
- Remove from the $j$-th linked list

# Dictionary ADT implemented using open hashing

Composite type (Dictionary):

```
int m;                               // size of the hash table
int cnt;                  // number of elements in the dictionary
List[] H;                        // hash table as an array of lists
h : Key → 0..m − 1;                              // hash function
```

**Algorithm:** Dictionary create_dict(int size, h : Key → 0..m-1)

```
d.m ← size ; d.cnt ← 0;
d.H ← new List[size];
for i ← 0 to size − 1 do
    // a list of Entry, which combines Key and E
    d.H[i] ← create_list();
d.h ← h;
return d;
```

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# Dictionary ADT implemented using open hashing

Some assumptions

- The lists are not sorted (always inserting in the end)
- If there is an entry in $d$ with key $k$, nothing is done

**Algorithm:** void insert(Dictionary d, Key k, E e)

**if** $find(d, k) = NULL$ **then**
    $pos \leftarrow d.h(k)$; // $h$ is the hash function
    $l \leftarrow d.H[pos]$; // $H$ is the hash table
    $entry \leftarrow create\_entry(k, e)$;
    $append(l, entry)$;

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# Open hashing: temporal efficiency

Insertion (unsorted): $\Theta(1)$

Search: depends on the load factor $\alpha = \frac{n}{m}$

- $S \approx 1 + \frac{\alpha}{2}$ and $U = \alpha$, where
  $S$ = succesful, $U$ = unsuccessful,
  $n$ = number of elements in the hash table
- When $\alpha \approx 1$, then in $\Theta(1)$ (in average)

Deletion: similar to the temporal efficiency of searching

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# Closed hashing

All elements are stored in the hash table itself (requirement: $m \geq n$)

Strategies for defining an **offset** in the presence of collisions:

- Linear probing
- Pseudo-random probing
- Quadratic probing
- Double hashing

The strategy should ensure that every hash address can be reached when solving a collision

# Closed hashing: linear probing[4]

Insertion: check the cell following the one where the collision occurs

- Probe function (**offset**): $p(K, i) = i$
- Hash table is seen as a circular array
- Example (considering $A = 1$ and fold):

| keys | A | FOOL | AND | HIS | MONEY | ARE | SOON | PARTED |
|---|---|---|---|---|---|---|---|---|
| hash addresses | 1 | 9 | 6 | 10 | 7 | 11 | 11 | 12 |

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | | | | | | | | | | | |
| | A | | | | | | | | FOOL | | | |
| | A | | | | | AND | | | FOOL | | | |
| | A | | | | | AND | | | FOOL | HIS | | |
| | A | | | | | AND | MONEY | | FOOL | HIS | | |
| | A | | | | | AND | MONEY | | FOOL | HIS | ARE | |
| | A | | | | | AND | MONEY | | FOOL | HIS | ARE | SOON |
| PARTED | A | | | | | AND | MONEY | | FOOL | HIS | ARE | SOON |

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[4] Source: A. Levitin. Introduction to the Design and Analysis of Algorithms. 2011.

# Closed hashing: linear probing

Search:

1. Compute $h(K) = j$
2. If $H[j]$ is empty, element not found
3. If $H[j] = K$, element found
4. Otherwise, check the following position (go back to step 2)
   - Attention to a hash table where $m = n$

Example (considering $A = 1$ and fold):

- $h$(“LIT”) $= 2$, as $H[2]$ is empty, “LIT” is not in the table
- $h$(“KID´´) $= 11$, search finishes just after comparing $K$ with “ARE”, “SOON”, “PARTED”, and “A”

# Closed hashing: linear probing

Deletion: need to use a special symbol

- Update insertion and search to consider this symbol

Analysis of temporal efficiency is more complicated:

- $S \approx \frac{1}{2}(1 + \frac{1}{1-\alpha})$
- $U \approx \frac{1}{2}(1 + \frac{1}{(1-\alpha)^2})$

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# Closed hashing: linear probing[5]

Evolution of $S$ and $U$ based on $\alpha$

| $\alpha$ | $\frac{1}{2}(1+\frac{1}{1-\alpha})$ | $\frac{1}{2}(1+\frac{1}{(1-\alpha)^2})$ |
|---|---|---|
| 50% | 1.5 | 2.5 |
| 75% | 2.5 | 8.5 |
| 90% | 5.5 | 50.5 |

When $\alpha \approx 1$, linear probing deteriorates (primary clustering)

- Higher probability of adding an element to a cluster
- Higher probability of coalescing two clusters

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[5] Source: A. Levitin. Introduction to the Design and Analysis of Algorithms. 2011.

# Closed hashing: pseudo-random probing

Pseudo-random probing:
$p(K, i) = Perm[i-1]$,
where $Perm$ is a permutation of $1..m-1$

Example ($m = 8$)

- $h(k) = k - (8 * \lfloor k/8 \rfloor)$
- $Perm = \{2, 6, 7, 3, 1, 4, 5\}$
- Values: 2, 4, 8, 16, 32, -12

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| | | 2 | | | | | |
| | | 2 | | 4 | | | |
| 8 | | 2 | | 4 | | | |
| 8 | | 2 | | 4 | | 16 | |

$h(2) = 2$
$h(4) = 4$
$h(8) = 0$
$h(16) = 0$

- $p(k, 1) = Perm[0] = 2$
  $(h(16) + p(k, 1)) \bmod 8 = 2$
- $p(k, 2) = Perm[1] = 6$
  $(h(16) + p(k, 2)) \bmod 8 = 6$

# Closed hashing: pseudo-random probing

Pseudo-random probing:
$p(K, i) = Perm[i-1]$,
where *Perm* is a permutation of $1..m-1$

Example ($m = 8$)

- $h(k) = k - (8 * \lfloor k/8 \rfloor)$
- $Perm = \{2, 6, 7, 3, 1, 4, 5\}$
- Values: 2, 4, 8, 16, 32, -12

| **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** |
|---|---|---|---|---|---|---|---|
| 8 | | 2 | | 4 | | 16 | 32 |

$h(32) = 0$

- $p(k, 1) = Perm[0] = 2$
  $(h(32) + p(k, 1)) \bmod 8 = 2$
- $p(k, 2) = Perm[1] = 6$
  $(h(32) + p(k, 2)) \bmod 8 = 6$
- $p(k, 3) = Perm[2] = 7$
  $(h(32) + p(k, 3)) \bmod 8 = 7$

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# Closed hashing: pseudo-random probing

Pseudo-random probing:
$p(K, i) = Perm[i - 1]$,
where *Perm* is a permutation of $1..m - 1$

Example ($m = 8$)

- $h(k) = k - (8 * \lfloor k/8 \rfloor)$
- $Perm = \{2, 6, 7, 3, 1, 4, 5\}$
- Values: 2, 4, 8, 16, 32, -12

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 8 | | 2 | -12 | 4 | | 16 | 32 |

$h(-12) = 4$

- $p(k, 1) = Perm[0] = 2$
  $(h(-12) + p(k, 1)) \bmod 8 = 6$
- $p(k, 2) = Perm[1] = 6$
  $(h(-12) + p(k, 2)) \bmod 8 = 2$
- $p(k, 3) = Perm[2] = 7$
  $(h(-12) + p(k, 3)) \bmod 8 = 3$

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# Closed hashing: pseudo-random probing

Pseudo-random probing:
$p(K, i) = Perm[i - 1]$,
where *Perm* is a permutation of $1..m - 1$

Example ($m = 8$)

- $h(k) = k - (8 * \lfloor k/8 \rfloor)$
- $Perm = \{2, 6, 7, 3, 1, 4, 5\}$
- Values: 2, 4, 8, 16

| **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** |
|---|---|---|---|---|---|---|---|
| 8 | | 2 | | 4 | | 16 | |

$h(2) = 2$
$h(4) = 4$
$h(8) = 0$
$h(16) = 0$

- $p(k, 1) = Perm[0] = 2$
  $(h(16) + p(k, 1)) \bmod 8 = 2$
- $p(k, 2) = Perm[1] = 6$
  $(h(16) + p(k, 2)) \bmod 8 = 6$

# Dictionary ADT implemented using closed hashing

Composite type (Dictionary):

int $m$; // size of the hash table
int $cnt$; // number of elements in the dictionary
Entry[] $H$; // hash table as an array of Entry
int[] $Perm$; // permutation of 1..m-1
$h : Key \to 0..m-1$; // hash function

**Algorithm:** Dictionary create_dict(int size, h : Key $\to$ 0..size-1)

$d.m \leftarrow size$ ; $d.cnt \leftarrow 0$;
$d.H \leftarrow new\ Entry[size]$;
$d.Perm \leftarrow create\_permutation(1..size-1)$;
$d.h \leftarrow h$;
**return** $d$;

# Dictionary ADT implemented using closed hashing

Some assumptions

- $d$ is implemented as a hash table with $m$ positions
- If there is an entry in $d$ with key $k$, nothing is done

**Algorithm:** void insert(Dictionary d, Key k, E e)

```
if size(d) < d.m ∧ find(d, k) = NULL then
    pos ← d.h(k);                                   // h is the hash function
    if d.H[pos] ≠ NULL ∧ d.H[pos] ≠ deleted then
        i ← 0;
        repeat
            i ← i + 1;
            offset ← d.Perm[i − 1];
            newPos ← (pos + offset)%d.m;
        until d.H[newPos] = NULL ∨ d.H[newPos] = deleted;
        pos ← newPos;
    entry ← create_entry(k, e);
    d.H[pos] ← entry;
    d.cnt = d.cnt + 1;
```

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# Closed hashing: quadratic probing

Quadratic probing:
$p(K, i) = c_1 i^2 + c_2 i + c3$

Example ($m = 8$):

- $h(k) = k - (8 * \lfloor k/8 \rfloor)$
- $p(k, i) = \frac{i^2+i}{2}$:
- Values: 2, 4, 8, 16, 32, -12

$h(2) = 2$
$h(4) = 4$
$h(8) = 0$

| **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** |
|---|---|---|---|---|---|---|---|
| | | 2 | | | | | |
| | | 2 | | 4 | | | |
| 8 | | 2 | | 4 | | | |

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# Closed hashing: quadratic probing

Quadratic probing:
$p(K, i) = c_1 i^2 + c_2 i + c3$

Example ($m = 8$):

- $h(k) = k - (8 * \lfloor k/8 \rfloor)$
- $p(k, i) = \frac{i^2+i}{2}$:
- Values: 2, 4, 8, 16, 32, e -12

| **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** |
|---|---|---|---|---|---|---|---|
| 8 | 16 | 2 | | 4 | | | |
| 8 | 16 | 2 | 32 | 4 | | | |
| 8 | 16 | 2 | 32 | 4 | -12 | | |

$h(16) = 0$

- $p(k, 1) = \frac{1^2+1}{2} = 1$
  $(h(16) + p(k, 1)) \bmod 8 = 1$

$h(32) = 0$

- $p(k, 1) = \frac{1^2+1}{2} = 1$
  $(h(32) + p(k, 1)) \bmod 8 = 1$
- $p(k, 2) = \frac{2^2+2}{2} = 3$
  $(h(32) + p(k, 2)) \bmod 8 = 3$

$h(-12) = 4$

- $p(k, 1) = \frac{1^2+1}{2} = 1$
  $(h(-12) + p(k, 1)) \bmod 8 = 5$

# Closed hashing: pseudo-random | quadratic probing

We might still have secondary clustering

- If $h(k_1) = h(k_2)$, then $k_1$ and $k_2$ share the same probe sequence
- Rationale: $p$ ignores the value of $K$

Alternative: double hashing

# Closed hashing: double hashing

A new hash function $s(K)$ is used to determine the probe sequence

- $p(K, i) = i * s(K)$

Temporal efficiency

- Also deteriorates when $\alpha \approx 1$
- Alternative: resize the hash table and perform rehashing

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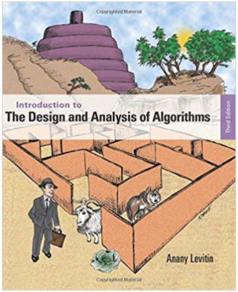
# Agenda

# Bibliography





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# HASH TABLES

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