DIJKSTRA'S ALGORITHM

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Agenda

Greedy algorithms

Greedy algorithms: constructs a solution through a sequence of steps, each expanding a partially constructed solution

On each step, the choice made must be:

- Feasible: satisfies the problem's constraints
- Locally optimal: best feasible local choice
- Irrevocable: cannot be changed later

Dijkstra's algorithm

Let G be a weighted graph and $v \in V$ (*source*), finds the shortest path from v to all other nodes in V

- Single-source shortest paths

Applications

- Transport planning
- Communication networks
- Social networks
- Robotics
- Pathfinding
- Puzzles
- etc.

Dijkstra's algorithm: cannot be used on weighted graphs with negative weights

Agenda

Dijkstra's algorithm

First, find the closest node to v (itself)

On the i-th step:

- Knows the $(i-1)$-th closest nodes to v (they form a tree)
- Since there are no negative weights, the next closest one is adjacent to one of the $i-1$ closest nodes to v
- After choosing the $i-th$ closest node (w), updates the possible shortest paths to yet unchosen nodes (u) if $d_w + weight(w, u) < d_u$

Dijkstra's algorithm

Algorithm: void Dijkstra(Graph G, int s, int[] D)

```
for i ← 0 to n(G) − 1 do
    D[i] ← ∞ ; P[i] ← −;
    setMark(G, i, UNVISITED);
H[1] ← (s, s, 0) ; D[s] ← 0;
for i ← 0 to n(G) − 1 do
    repeat
        (p, v) ← removemin(H);
        if v = NULL then return;
    until getMark(G, v) = UNVISITED;
    setMark(G, v, VISITED) ; P[v] ← p;
    w ← first(G, v);
    while w < n(G) do
        if getMark(G, w) ≠ VISITED ∧
          D[w] > D[v] + weight(G, v, w) then
            D[w] ← D[v] + weight(G, v, w);
            insert(H, (v, w, D[w]));
        w ← next(G, v, w);
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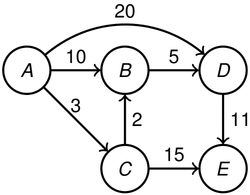
Let $s = A$



	A	B	C	D	E
Mark	$\times$	$\times$	$\times$	$\times$	$\times$
Distance	0	∞	∞	∞	∞
Parent	−	−	−	−	−

(A,A,0)

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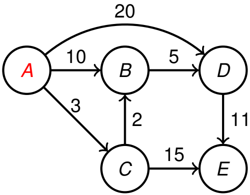

	A	B	C	D	E
Mark	✓	×	×	×	×
Distance	0	∞	∞	∞	∞
Parent	A	–	–	–	–

—

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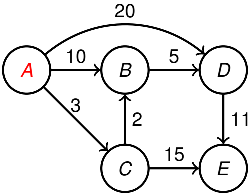

	A	B	C	D	E
Mark	✓	×	×	×	×
Distance	0	10	3	20	∞
Parent	A	−	−	−	−

(A,C,3)

(A,B,10) (A,D,20)

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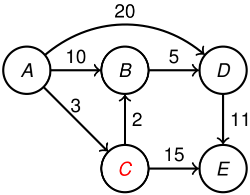
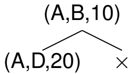


	A	B	C	D	E
Mark	✓	×	✓	×	×
Distance	0	10	3	20	∞
Parent	A	−	A	−	−



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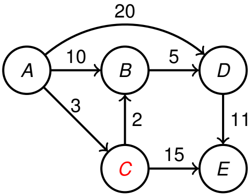
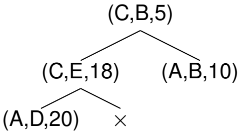


	A	B	C	D	E
Mark	✓	×	✓	×	×
Distance	0	5	3	20	18
Parent	A	–	A	–	–



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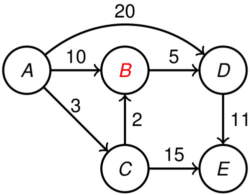
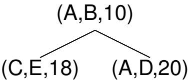


	A	B	C	D	E
Mark	✓	✓	✓	×	×
Distance	0	5	3	20	18
Parent	A	C	A	–	–



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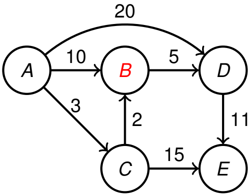
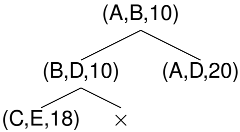


	A	B	C	D	E
Mark	✓	✓	✓	×	×
Distance	0	5	3	10	18
Parent	A	C	A	–	–



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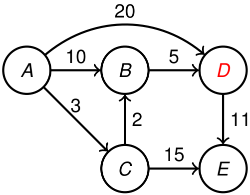
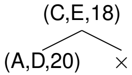


	A	B	C	D	E
Mark	✓	✓	✓	✓	×
Distance	0	5	3	10	18
Parent	A	C	A	B	–



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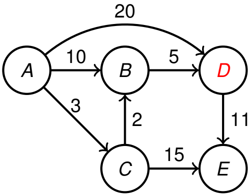
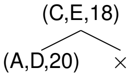


	A	B	C	D	E
Mark	✓	✓	✓	✓	×
Distance	0	5	3	10	18
Parent	A	C	A	B	–



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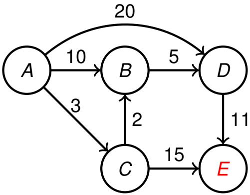

	A	B	C	D	E
Mark	✓	✓	✓	✓	✓
Distance	0	5	3	10	18
Parent	A	C	A	B	C

(A,D,20)

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for $i \leftarrow 0$ **to** $n(G) - 1$ **do**
    $D[i] \leftarrow \infty$; $P[i] \leftarrow -$;
    $setMark(G, i, UNVISITED)$;
$H[1] \leftarrow (s, s, 0)$; $D[s] \leftarrow 0$;
for $i \leftarrow 0$ **to** $n(G) - 1$ **do**
    repeat
        $(p, v) \leftarrow removemin(H)$;
        if $v = NULL$ **then return**;
    until $getMark(G, v) = UNVISITED$;
    $setMark(G, v, VISITED)$; $P[v] \leftarrow p$;
    $w \leftarrow first(G, v)$;
    while $w < n(G)$ **do**
        if $getMark(G, w) \neq VISITED \wedge D[w] > D[v] + weight(G, v, w)$ **then**
            $D[w] \leftarrow D[v] + weight(G, v, w)$;
            $insert(H, (v, w, D[w]))$;
        $w \leftarrow next(G, v, w)$;

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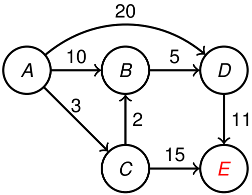

	A	B	C	D	E
Mark	✓	✓	✓	✓	✓
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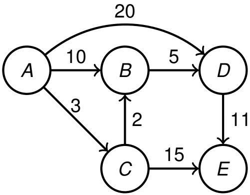

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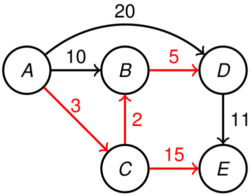

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Distance	0	5	3	10	18
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```
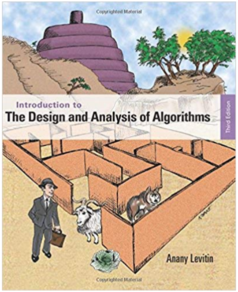

Time efficiency

- Matrix and no heap
  $\Theta(|V|^2 + E|) = \Theta(|V|^2)$, since $|E| \in O(|V|^2)$
  - Better for dense graphs
- List and heap
  $\Theta((|V| + |E|) \log |V|)$
  - Better for sparse graphs

Agenda

Bibliography



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