Topic: Measuring Uncertainty with Probability

Independence

School of Mathematics and Applied Statistics

UNIVERSITY OF WOLLONGONG AUSTRALIA

Introduction: Two scenarios

For certain pairs of events, the occurrence of one of them may or may not change the probability of the occurrence of the other.

1. **Coin Tosses:** Flip a coin twice and think about the event of getting tails for Toss 1 and the event of Tails for Toss 2.

   A B

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If the probability that A occurs is not affected by whether or not B occurs, we say that A and B are **independent** events.

Independence

Definition

Two events A and B are said to be **independent** if and only if

$$P(A \cap B) = P(A)P(B)$$

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Two events A and B are said to be **independent** if and only if

$$P(A \cap B) = \boxed{P(A)P(B)}$$

or

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A).\cancel{P(B)}}{\cancel{P(B)}}$$

This means that

- $\boxed{P(A|B) = P(A)}$ and $\boxed{P(B|A) = P(B)}$
- Conditioning upon B does not alter the probability of A, and vice-versa.
- The knowledge that B has occurred gives no information as to whether A is more or less likely to occur, and vice-versa.

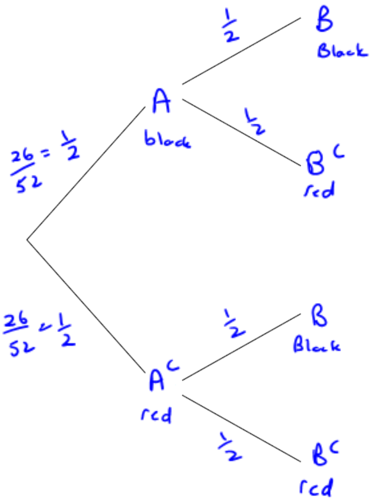
Independence - Exercise

Exercise

1. A card is drawn from a pack of 52 cards. The card is returned, the pack is reshuffled, and a second card is drawn. Let $A = \{\text{first card is black}\}$, $B = \{\text{second card is black}\}$.
   a. Draw a tree diagram.
   b. Are A and B independent?

2. Repeat Q1 if the first card is not returned to the pack before the 2nd card is drawn.

Exercise 1



$P(A \cap B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.$ #

$P(A^c \cap B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ #

$P(B) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Exercise 1 cont.

Are A & B independent ?

Check : $P(A \cap B) = P(A) \cdot P(B)$.

$LHS = P(A \cap B)$

$= \frac{1}{4}$

$RHS = P(A) \cdot P(B)$.

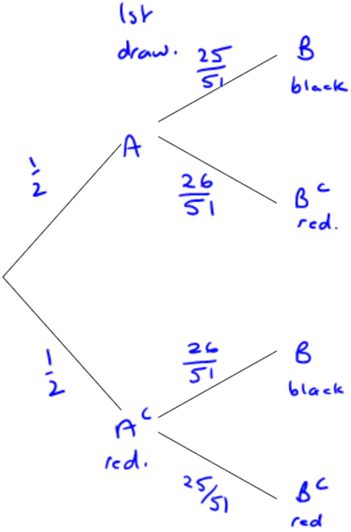
$= \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$.

$LHS = RHS \Rightarrow$ A & B are independent events.

$P(B) = \frac{1}{2}$.

Exercise 2



$$P(A\cap B) = \frac{1}{2}\times\frac{25}{51} = \frac{25}{102}. \quad *$$

$$+$$

$$P(A^c\cap B) = \frac{1}{2}\times\frac{26}{51} = \frac{26}{102} \quad *$$

$$P(B) = \frac{51}{102}$$

Exercise 2 cont.

$$LHS = P(A \cap B)$$

$$= \frac{25}{102}.$$

$$RHS = P(A) . P(B)$$

$$= \frac{1}{2} \times \frac{51}{102}$$

$$= \frac{51}{204}.$$

$$= \frac{1}{4}.$$

$LHS \neq RHS \Rightarrow$ A & B are NOT indep t

$\Rightarrow$ dependent.

Dependence

If two events are not independent, they are **dependent**.

Sometimes it is useful to think of this as

- mathematical
- or theoretical,
- or population
- or model-based

independence.

This contrasts with how statisticians use data from samples to investigate, and make inferences about,

- dependence
- independence

in populations.