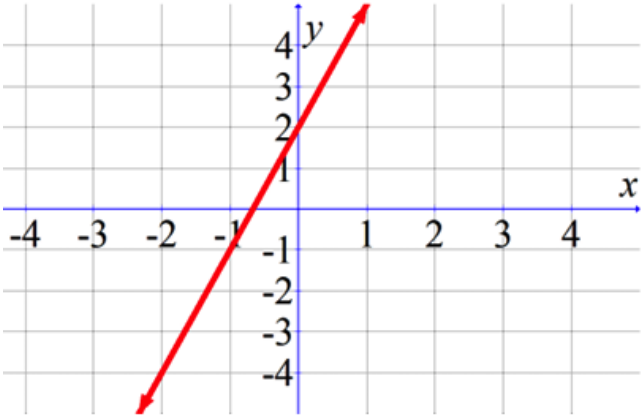
MATH255: Mathematics for Computing
Tutorial Sheet Week 7 - Autumn 2023

Note. Question 2 is your Tutorial Preparation exercise for this week. It must be completed and handed in on Moodle as a pdf before the start of your tutorial.

1. Find the equation of the line in the graph below by finding two points that lie on the line and calculating the slope. Write the slope-intercept form of the equation.



2. Find the equation of the line perpendicular to $y = 2x - 3$ that passes through $(4, 0)$. Graph both lines on the same set of axes and observe that they are indeed perpendicular. Note: perpendicular lines have *negative reciprocal slopes*.

3. Plot the general form of a quartic function that has four roots and a negative leading coefficient.

4. Graph $-2x^3 + 3x^2 + 23x - 12$.

   (a) One factor is $(x + 3)$; find the remaining quadratic factor by long division.

   (b) Further factorise the quadratic piece you found in (a) by using the quadratic formula.

   (c) Find and graph the roots, then use your knowledge of the sign of the leading coefficient to graph the curve.

5. Algae growing on a pond multiply at a rate of 100% per day (i.e. the population doubles every day). On Day 35, half the pond is covered with the algae. On what day will the entire pond be covered?

6. You invest \$1000 in a company stock, whose value is expected to increase at 15% per year. Predict the value of your investment at the end of (a) 5 years and (b) 10 years.

7. The radioactive decay of an element is described by

$$A_f = A_0 e^{-kt},$$

   where A_0 and A_f are the initial and final amounts of the material, respectively, t is the total number of days, $k = \ln 2/T_{1/2}$ and $T_{1/2}$ is the half-life of the material, i.e. the number of days it takes for half of the material to decay.

   (a) Iodine-131 has a half-life of 8 days. If you have a sample of 200 grams, how much of it will remain after 32 days?

   (b) Challenge. It takes 35 days for a 512-gram sample of unknown Element X to decay to a final amount of 4 grams. What is the half-life of Element X?

8. Graph $f(x) = x^2, x \in [-2, 2]$ twice, on two separate sets of axes.

   (a) On one graph, divide the given interval into 4 equal subintervals and draw a midpoint rectangle on each. Midpoint rectangle means that you use the function value at the midpoint of each subinterval as the height of the rectangle. Approximate the area under the curve of the function by adding up the areas of the 4 rectangles.

   (b) On the other graph, do the same with 8 rectangles.

   (c) Use the integral formula for monomials to find the exact area under the curve of the function.

   (d) Find out how accurate your two approximations were by calculating $E = \frac{|approx - true|}{true}$ to get a percentage error.