Topic: Measuring Uncertainty with Probability

Introduction, Language and Notation

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How do we measure uncertainty?

Probability is a measure of **uncertainty or likelihood**.

The probability of an event **certain** to happen is set to 1.

In everyday life, it is often expressed as a percentage:

- There is a 50-50 chance of getting a tail when a fair coin is tossed.
- The weather forecast states: *Today there is an 80% chance of rain.*
- There is a 20% chance that the train to Sydney from Wollongong is likely to arrive late.

What is probability?

Probability is used to quantify **unpredictability** and describe it precisely.

The probability of an event is a number between 0 and 1
indicating how likely it is that the event will occur
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A probability model describes the uncertainty in an experiment by assigning probabilities to the possible outcomes.

- Weather forecast models
- Artificial intelligence uses statistics to make independent learning decisions where a decision is an outcome that has highest probability.

Probability Scale

	Probability	
Event certain to happen →	1	←
Event likely to happen →	0.9	
Event equally likely to happen or not happen →	0.5	H or T
Event unlikely to happen →		
Event impossible →	0	←

Language and Notation

- **Random** phenomenon: cannot be predicted with certainty in advance
- **Experiment**: the observation of any phenomenon that is uncertain
- **Outcome** is a single observed result of random phenomenon
  - is a result of an experiment which cannot be reduced to simpler results
    Example: getting a Head or Tail on the toss of a coin
- The **sample space** S is the set of all possible outcomes or sample points
  - S may be finite, countably infinite, or uncountably infinite.
  - A **discrete sample space** contains a finite or countable number of distinct sample points
  - Example: $S = \{1, 2, 3, 4, 5, 6\}$ is set of all outcomes for a throw of a die
  - $P(S) = 1$, as the sample space includes all possibilities.

Language and Notation: Events

- The subsets of S are called **events.**
  - E is a subset of the sample space $S : E \subseteq S$
  - Events are collection of outcomes, including both S and $\varnothing$ (the null or empty set).
  - An event in a discrete sample space S is a collection of sample points, any subset of S
  - e.g. in the die experiment the event 'getting an even number' is the collection of outcomes $\{2, 4, 6\}$
- **Null event** $\{\ \}$ or $\varnothing$
  - The empty set (no outcomes) is an event which can **never** occur. e.g. even and odd: $\varnothing$
  - $P(\varnothing) = 0$, as $\varnothing$ contains no possibilities.

$P(7) = 0$

Language and Notation

- **Intersection** of events: $P(A \cap B)$
  - The event that A **and** B both occur.
    e.g. 1 die rolled: P(even **and** greater than 4)

$A := \{2, 4, 6\}$
$B := \{5, 6\}$
$E_1 = \{6\}$

- **Union** of events: $P(A \cup B)$
  - e.g. 1 die rolled: P(even **or** greater than 4)

$E_2 = \{2, 4, 5, 6\}$

- **Disjoint events**
  - have no outcomes in common
  - If $A \cap B = \varnothing$, the events A and B are said to be **disjoint** or mutually exclusive; i.e. they cannot occur simultaneously.
  - 1 die rolled: P(even **and** odd)=0

$\{2, 4, 6\}$ $\{1, 3, 5\}$

Probability Axioms

1. The **probability** of each individual outcome is a number between 0 ("can't happen") and 1 ("certain to happen").
   i.e. For any event E: $0 \leq P(E) \leq 1$

   $\leq$ 'less than or equal to'

2. **Total probability** of all outcomes $= 1$
   i.e. $P(S) = 1$ where S represents the sample space

3. The probability $P(E)$ of an event E is obtained by **adding** probabilities of **disjoint** outcomes in E.
   union
   i.e. $P(E_1$ or E_2 happens$) = P(E_1 \cup E_2) = P(E_1) + P(E_2)$
   3 4

From these basic rules of probability (the axioms) other properties of probabilities can be derived.

Summary

In this lecture segment we have considered:

- Probability - as a measure of uncertainty which is used to quantify unpredictability.
- Language and notation
- Basic probability laws or axioms

Reference: Wackerley D.D., Mendenhall W. & Scheaffer R.L. [WMS] (2008) “Mathematical Statistics with Applications”, 7th ed. Duxbury, Belmont . (Library: 519.5/40).