1. (a) ALL FUNCTION VALUES ARE NONNEGATIVE AND

$0.1+0.4+0.25+0.15+0.1=1$, SO YES IT'S A PDF.

(b) $E(x)=\sum_{i=1}^{5} x_i f(x_i)=1\cdot 0.1+2\cdot 0.4+3\cdot 0.25+4\cdot 0.15+5\cdot 0.1=2.75$

$E(x^2)=\sum_{i=1}^{5} x_i^2 f(x_i)=1(0.1)+4(0.4)+9(0.25)+16(0.15)+25(0.1)=8.85$

(c) $Var(x)=\sum_{i=1}^{5}(x_i-\mu)^2 f(x_i)=(1-2.75)^2(0.1)+(2-2.75)^2(0.4)+$
$(3-2.75)^2(0.25)+(4-2.75)^2(0.15)+(5-2.75)^2(0.1)=1.2875$

$Var(x)=E(x^2)-\mu^2=8.85-2.75^2=1.2875$

(d) NOTICE $\tilde{x}=2x+1$, LINEAR TRANSFORMATION.

$E(\tilde{x})=2E(x)+1=2\cdot 2.75+1=6.5$

$Var(\tilde{x})=2^2\cdot Var(x)=4\cdot 1.2875=5.15$

2. $E(x)=\sum_{i=0}^{3} x_i P(x=x_i)=0\cdot\binom{3}{0}p^0q^3+1\cdot\binom{3}{1}p^1q^2+2\cdot\binom{3}{2}p^2q^1+3\cdot\binom{3}{3}p^3q^0$

$=0\cdot\frac{3!}{0!(3-0)!}q^3+1\cdot\frac{3!}{1!(3-1)!}pq^2+2\cdot\frac{3!}{2!(3-2)!}p^2q+3\cdot\frac{3!}{3!(3-0)!}p^3$

$=0+\frac{6}{2}pq^2+\frac{2\cdot 6}{2}p^2q+\frac{3\cdot 6}{6}p^3=3pq^2+6p^2q+3p^3$

$=3p(q^2+2pq+p^2)=3p(q+p)^2=3p(1-p+p)^2=3p=np.$

3. (a) $P(x=1)=p=\frac{1}{4}$. $P(x=2)=qp=\frac{3}{16}$. $P(x=3)=q^2p=\frac{9}{64}$.

$F(3)=P(1)+P(2)+P(3)=\frac{16+12+9}{64}=\frac{37}{64}$.

(b) $\mu=\frac{1}{p}=4$. $P(x>4)=1-P(x\le 4)=1-F(4)$.

$P(x=4)=q^3p=\frac{27}{256}$. $F(4)=\frac{27+37\cdot 4}{256}=\frac{175}{256}$

$1-F(4)=\boxed{\frac{81}{256}}$

4. $\lambda = \frac{10000}{801150} \approx 0.0125$. $A = 10 \Rightarrow \mu = \lambda \cdot A = 0.125$.

(a) $P(1) = \frac{\mu^1}{1!} e^{-\mu} = \frac{0.125^1}{1!} e^{-0.125} \approx 0.11$

(b) $P(2) = \frac{\mu^2}{2!} e^{-\mu} = \frac{0.125^2}{2} e^{-0.125} \approx 0.007$

5. NOTE $f(x) \geq 0 \ \forall x \in [-\sqrt{b}, \sqrt{b}]$, SO THE ONLY REQUIREMENT IS $\int_{-\infty}^{\infty} f(x)\,dx = 1$. SINCE $\int_{-\infty}^{-\sqrt{b}} f(x)\,dx = \int_{\sqrt{b}}^{\infty} f(x) = 0$, WE HAVE

$$\int_{-\sqrt{b}}^{\sqrt{b}} f(x)\,dx = 1 \Rightarrow \int_{-b^{1/2}}^{b^{1/2}} (-x^2+b)\,dx = 1 \Rightarrow \left(\frac{-1}{3}x^3 + bx\right)\Big|_{-b^{1/2}}^{b^{1/2}} = 1 \Rightarrow$$

$$\left(\frac{-1}{3}b^{3/2} + b^{3/2}\right) - \left(\frac{-1}{3}(-b^{3/2}) - b^{3/2}\right) = 1 \Rightarrow \frac{2}{3}b^{3/2} + \frac{2}{3}b^{3/2} = 1 \Rightarrow b^{3/2} = \frac{3}{4}$$

$$\Rightarrow b^3 = \frac{9}{16} \Rightarrow \boxed{b = \sqrt[3]{\frac{9}{16}}}$$