Tutorial Sheet Week 3.

4.

a) $\forall \varepsilon > 0, \exists x \neq 0$ s.t. $|x| < \varepsilon$

$\sim(\forall \varepsilon > 0, \exists x \neq 0$ s.t $|x| < \varepsilon)$

$\equiv \exists \varepsilon > 0, \forall x \neq 0, \sim(|x| < \varepsilon)$

$\equiv \exists \varepsilon > 0, \forall x \neq 0, |x| \geq \varepsilon$

The negation is false. For instance, $x = 1$, $\varepsilon = 3$

$\rightarrow |1| < \varepsilon = 3$

$\rightarrow$ False

b) $\exists y \in R$ s.t $\forall x \in R, y < x^2$

$\sim(\exists y \in R$ s.t $\forall x \in R, y < x^2)$

$\equiv \forall y \in R, \exists x \in R, y \geq x^2$

Negation of statement is false. For instance, $y = \frac{1}{2}$, $x = 2$

$\frac{1}{2} \leq (2)^2 = 4$

$\rightarrow$ False

c) $\forall x, y \in \mathbb{R},\ x < y \to x < \frac{x+y}{2} < y$

$\sim\left(\forall x, y \in \mathbb{R},\ x < y \to x < \frac{x+y}{2} < y\right)$

$\equiv \exists x, y \in \mathbb{R} \ni \sim\left(x < y \cancel{\to} \to x < \frac{x+y}{2} < y\right)$

$\equiv \exists x, y \in \mathbb{R} \ni x < y \wedge \sim\left(x < \frac{x+y}{2} < y\right)$

$\equiv \exists x, y \in \mathbb{R} \ni x < y \wedge \left(\frac{x+y}{2} \leq x \vee \frac{x+y}{2} \geq y\right)$

$\equiv \exists x, y \in \mathbb{R} \ni x < y \wedge (y \leq x \vee x \geq y)$

The negation of the statement is false.

~~We will try to use proof prove by contradiction~~

PROOF

$x < y \wedge (y \leq x \vee x \geq y)$

$\equiv [(x < y) \wedge (y \leq x)] \vee [(x < y) \wedge x \geq y]$

$\equiv \quad F \quad \vee \quad F$

$\equiv \quad F$