MATH255 Autumn 2023 Computer Lab - Week 11

Note: Question 2(a) is your Lab Preparation exercise for this week. Your answers must be submitted as a pdf document before the start of your lab.

Key Results from Lectures

- $f(x) = \binom{n}{x} p^x q^{n-x}, \mu = np, \sigma^2 = npq,$ `dbinom, pbinom` — Binomial Distribution
- $f(x) = q^{x-1}p, \mu = \frac{1}{p},$ `dgeom, pgeom` — Geometric Distribution
- $f(x) = \frac{\mu^x}{x!} e^{-\mu}, \mu = \sigma^2 = \lambda t,$ `dpois, ppois` — Poisson distribution
- $F(x) = P(X \leq x) = \sum_{k=0}^{x} f(x)$ — Cumulative Distribution
- $E(X) = \sum_x x f(x)$, $\text{Var}(X) = E[(X-\mu)^2] = E(X^2) - \mu^2$ — Expected Value, Variance

1. It is known that 80% of people who purchase pet insurance are women. If 9 pet insurance owners are randomly selected, find the probability that

   (a) 6 of them are women;

   (b) at least 6 of them are women.

   What is the expected value of the distribution? Plot the distribution's bar graph in R.

2. For $x \in \{-2, -1, 0, 1, 2\}$, consider the function $f(x) = \frac{1}{4}(x-1)^2$.

   (a) What is the expected value of f? Does this value make sense? Why or why not?

   (b) Find the proper factor $\left(\text{not } \frac{1}{4}\right)$ such that f becomes a probability distribution and recalculate the expected value. Does this value make sense?

3. You are playing a game with 4 different playing cards: a jack, a queen, a king and an ace. The cards are shuffled and one card is dealt face down. If you guess the card, you win three times your bet, but you do not recover your initial bet. If not, you lose your bet.

   (a) Plot the distribution function of the first instance of a win over 10 tries.

   (b) Plot the cdf.

   (c) How many times do you expect to play before getting your first win? If this happens, and you bet 1 dollar each time, what will be your net win/loss?

4. A certain lecturer makes typographical errors on lecture slides according to a Poisson process with rate $\lambda = 0.2$ errors per slide.

   (a) What is the expected total number X of errors in a lecture with 20 slides?

   (b) What is the standard deviation of X?

   (c) What is the probability of exactly 3 errors in a lecture with 20 slides?

   (d) What is the probability of at least 3 errors in a lecture with 20 slides?

   (e) Let Y be the number of slides with no errors in a lecture with 20 slides. What type of distribution does Y follow?

   (f) What are the mean and standard deviation of Y?

   (g) What is the probability that exactly 18 slides have no errors?

   (h) What is the probability that no more than 2 slides contain errors?