1. (a) $A \times B = \{(1,x),(1,y),(2,x),(2,y),(3,x),(3,y)\}$, $B \times A = \{(x,1),(x,2),(x,3),(y,1),(y,2),(y,3)\}$, so $|A \times B| = |B \times A| = 6$.
Since $(1,x) \in A \times B$ and $(1,x) \notin B \times A$, we have $A \not\subseteq B$ and $A \times B \neq B \times A$. Since $(x,1) \in B \times A$ and $(x,1) \notin A \times B$, we have $B \times A \not\subseteq A \times B$.

   (b) The only one from the list that is in R is $(1,y)$.

2. (a) The only potential problem here is that the denominator could be zero, so find those points.

$$2x^2 - 5x + 3 = 0 \Leftrightarrow (2x-3)(x-1) = 0 \Leftrightarrow x = \frac{3}{2} \vee x = 1$$
$$\therefore \ \operatorname{dom} f = \mathbb{R} \setminus \left\{1, \frac{3}{2}\right\} = (-\infty,1) \cup \left(1, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$$

   (b) There is no problem with the cube root, but both square roots must have nonnegative entries.

$$1 - x^2 \geq 0 \Leftrightarrow (1-x)(1+x) \geq 0 \Leftrightarrow x \in [-1,1]$$
$$-x \geq 0 \Leftrightarrow x \leq 0 \Leftrightarrow x \in (-\infty, 0]$$
$$\therefore \ \operatorname{dom} g = [-1, 0]$$

   (c) Both numerator and denominator have restrictions.

$$x + 5 > 0 \Leftrightarrow x > -5 \Leftrightarrow x \in (-5, \infty)$$
$$x^2 + 3x - 28 > 0 \Leftrightarrow (x+7)(x-4) > 0 \Leftrightarrow x < -7 \vee x > 4 \Leftrightarrow x \in (-\infty, -7) \cup (4, \infty)$$
$$\therefore \ \operatorname{dom} h = (4, \infty)$$

3. The relation is reflexive and symmetric (prove it if you like), but not transitive. For instance, let $x = 1, y = 2, z = 3$. Then

$$|x-y| = |1-2| = 1 \leq 1,$$
$$|y-z| = |2-3| = 1 \leq 1,$$
$$|x-z| = |1-3| = 2 \not\leq 1.$$

So we have an example of $(x,y),(y,z) \in R$ and $(x,z) \notin R$. Therefore, R is not an equivalence relation.

4. Reflexive: let $(a,b) \in \mathbb{R}^2$. Then

$$2a - b = 2a - b \Rightarrow (a,b)R(a,b).$$

Thus, R is reflexive.

Symmetric: Let $(a,b)R(c,d)$. Then

$$2a - b = 2c - d \Rightarrow 2c - d = 2a - b \Rightarrow (c,d)R(a,b)$$

Thus, R is symmetric.

Transitive: let $(a,b)R(c,d)$ and $(c,d)R(e,f)$. Then

$$2a - b = 2c - d \text{ and } 2c - d = 2e - f \Rightarrow 2a - b = 2e - f \Rightarrow (a,b)R(e,f).$$

Thus, R is transitive. Therefore, R is an equivalence relation.

The equivalence class $[(1,2)]$ is the set of all elements that are related to $(1,2)$, so

$$(a,b)R(1,2) \Rightarrow 2a - b = 2 \cdot 1 - 2 \Rightarrow b = 2a \Rightarrow [(1,2)] = \{(a, 2a) : a \in \mathbb{R}\}.$$

For example, three elements of $[(1,2)]$ are $(2,4)$, $\left(-\frac{1}{3}, -\frac{2}{3}\right)$ and $(0,0)$.

5. (a) Injective: let $f(x_1) = f(x_2)$. Then

$$x_1^2 + 1 = x_2^2 + 1 \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = x_2, (\text{since } -x_2 \notin \operatorname{dom} f)$$

Therefore, f is injective.

Surjective: let $y = 5 \in \operatorname{ran} f$. Then

$$5 = x^2 + 1 \Leftrightarrow x^2 = 4 \Leftrightarrow x = \pm 2 \notin \operatorname{dom} f.$$

Therefore, f is not surjective.

(b) Surjective: let $y \in [0, \infty)$. Then

$$y = x^4 \Leftrightarrow x = \pm\sqrt[4]{y},$$

both of which exist since $y \geq 0$, and both of which are in $\operatorname{dom} f = \mathbb{R}$. Therefore, f is surjective.

Injective: let $x_1 = 1, x_2 = -1$. Then

$$f(x_1) = 1^4 = 1, f(x_2) = (-1)^4 = 1.$$

Therefore, f is not injective.

6. (a) Injective: let $f(x_1) = f(x_2)$. Then

$$x_1^3 = x_2^3 \Rightarrow \sqrt[3]{x_1^3} = \sqrt[3]{x_2^3} \Rightarrow x_1 = x_2.$$

Hence, f is injective.

Surjective: let $y \in \mathbb{R}$. Then

$$y = x^3 \Leftrightarrow x = \sqrt[3]{y} \in \mathbb{R}.$$

Hence, f is surjective. So f is bijective and the inverse function f^{-1} exists. From the surjectivity argument, $f^{-1}(x) = x^{\frac{1}{3}}$.

(b)

$$f(g(x)) = \frac{g(x)}{1 - g(x)} = \frac{\frac{x}{x+1}}{1 - \frac{x}{x+1}} = \frac{\frac{x}{x+1}}{\frac{x+1}{x+1} - \frac{x}{x+1}} = \frac{\frac{x}{x+1}}{\frac{x+1-x}{x+1}} = \frac{\frac{x}{x+1}}{\frac{1}{x+1}} = \frac{x}{x+1}\frac{x+1}{1} = x$$

$$g(f(x)) = \frac{f(x)}{f(x) + 1} = \frac{\frac{x}{1-x}}{\frac{x}{1-x} + 1} = \frac{\frac{x}{1-x}}{\frac{x}{1-x} + \frac{1-x}{1-x}} = \frac{\frac{x}{1-x}}{\frac{x+1-x}{1-x}} = \frac{\frac{x}{1-x}}{\frac{1}{1-x}} = \frac{x}{1-x}\frac{1-x}{1} = x$$

Since we are given that f is bijective and we found that $f(g(x)) = g(f(x)) = x$, then g is the inverse function of f.