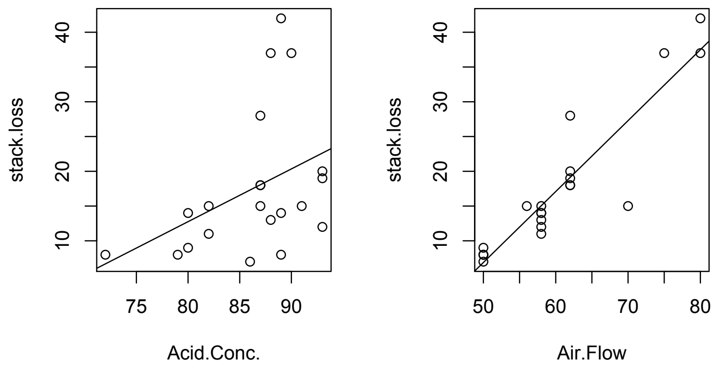
MATH223 2022 Solutions - Week 7

1. There is a weak positive association between `stack.loss` and `Acid.Conc.`, and quite a strong positive association between `stack.loss` and `Air.Flow`.

```
            Air.Flow Water.Temp Acid.Conc. stack.loss
Air.Flow   1.0000000  0.7818523  0.5001429  0.9196635
Water.Temp 0.7818523  1.0000000  0.3909395  0.8755044
Acid.Conc. 0.5001429  0.3909395  1.0000000  0.3998296
stack.loss 0.9196635  0.8755044  0.3998296  1.0000000
```

The scatterplot of `stack.loss` versis `Acid.Con.` shows 3 outliers near the top of the plot, a long way above the fitted least squares line. The scatterplot of `stack.loss` versis `Air.Flow` shows that the majority of points are close to the fitted least squares line.



3. (a) i. Note that A is contained within B.

$$\begin{aligned}
P(B) &= 1 - P(B^c) = 1 - 0.6 = 0.4 \\
P(A \cap B) &= P(A) = 0.2 \\
P(A \cup B) &= P(B) = 0.4 \\
P(A|B) &= \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.4} = 0.5 \\
P(B|A) &= \frac{P(B \cap A)}{P(A)} = \frac{0.2}{0.2} = 1
\end{aligned}$$

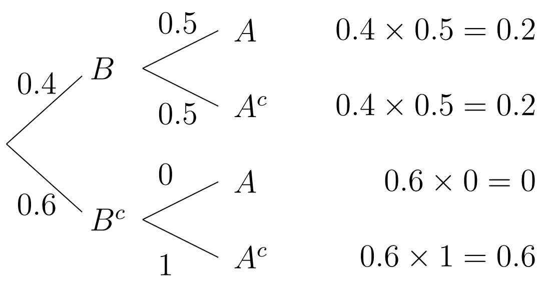
ii. $P(A \cap B^c) = 0$, so A and B^c are disjoint (mutually exclusive).

iii. $P(A \cap B^c) = 0 \neq P(A)P(B^c) = 0.2 \times 0.6$, so A and B^c are *not* independent.

iv. The two-way table of probabilities is

	B	B^c	total
A	0.2	0	0.2
A^c	0.2	0.6	0.8
	0.4	0.6	1

v. Filling in the previous values, and remembering that probabilities sum to 1 at each branching point, the tree diagram is as follows:



(b) i.

$$
\begin{aligned}
P(B) &= 1 - P(B^c) = 1 - (0.4 + 0.3) = 0.3 \\
P(A \cap B) &= P(B) - P(B \cap A^c) = 0.3 - 0.2 = 0.1 \\
P(A \cup B) &= 1 - P(A^c \cap B^c) = 1 - 0.3 = 0.7 \\
P(A|B) &= \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.3} = \frac{1}{3} \\
P(B|A) &= \frac{P(B \cap A)}{P(A)} = \frac{0.1}{0.4 + 0.1} = \frac{1}{5}
\end{aligned}
$$

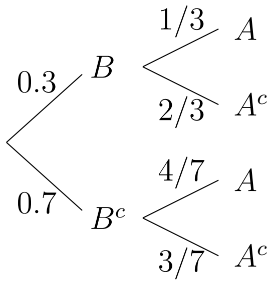
ii. $P(A \cap B^c) = 0.4 > 0$, so A and B^c are not disjoint.

iii. $P(A \cap B^c) = 0.4 \neq P(A)P(B^c) = 0.5 \times 0.7 = 0.35$, so A and B^c are not independent.

iv. The two-way table of probabilities is

	B	B^c	total
A	0.1	0.4	0.5
A^c	0.2	0.3	0.5
	0.3	0.7	1

v. The tree diagram is



$$0.3 \times 1/3 = 0.1$$
$$0.3 \times 2/3 = 0.2$$
$$0.7 \times 4/7 = 0.4$$
$$0.7 \times 3/7 = 0.3$$

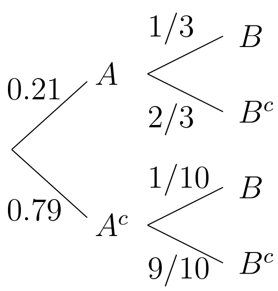
4. (a) Let M and T denote the events that a potential customer sees the magazine advertisement, or sees the television advertisement.

Two-way table:

	T	T^c	total
M	0.01	0.01	0.02
M^c	0.19	0.79	0.98
total	0.2	0.8	1

$$P(M \cup T) = 0.01 + 0.01 + 0.19 = 0.21$$

(b) Let A denote the event that a potential customer sees at least one form of advertisement, and B be the event that a potential customer buys the product.
From (a), $P(A) = 0.21$.
The tree diagram is



$$0.21 \times \frac{1}{3} = 0.07$$
$$0.79 \times \frac{1}{10} = 0.079$$

Probability that a randomly selected potential customer buys the product

$$= 0.07 + 0.079 = 0.149$$