Tutorial Sheet Week 4.

2.

a) $\sum_{i=1}^{5} (2i-5) = (2.1-5) + (2.2-5) + (2.3-5) + (2.4-5) + (2.5-5)$

$= -3 + -1 + 1 + 3 + 5$

$= 5.$

$\rightarrow \sum_{i=1}^{5} (2i-5) = 5.$

b) $\sum_{j=-2}^{2} 2^j = 2^{-2} + 2^{-1} + 2^0 + 2^1 + 2^2$

$= \frac{1}{4} + \frac{1}{2} + 1 + 2 + 4$

$= \frac{31}{4}$

$\rightarrow \sum_{j=-2}^{2} = \frac{31}{4}$

c) $\sum_{k=0}^{3} \frac{k!}{2} = \frac{0!}{2} + \frac{1!}{2} + \frac{2!}{2} + \frac{3!}{2}$

$= \frac{1}{2} + \frac{1}{2} + 1 + 3.$

$= 5$

$\rightarrow \sum_{k=0}^{3} = 5.$

d) $\sum_{t=0}^{99} \frac{(-1)^t}{3}$

We can't just ~~set~~ replace t from 0 to 99 because that's too long and waste time.

If t is even $\to t = 2k \to (-1)^t = (-1)^{2k} = (-1^k)^2 = 1$

If t is odd $\to t = 2p+1 \to (-1)^t = (-1)^{2k+1}$
$= (-1^k)^2 \cdot -1$
$= -1$

From 0 to 99, there are 50 even and 50 odd numbers
~~So we can write~~ So there will be 50s $\frac{1}{3}$ and 50s $\frac{-1}{3}$

$$\to \sum_{t=0}^{99} \frac{(-1)^t}{3} = 50 \cdot \frac{1}{3} + \frac{-1}{3} \cdot 50$$
$$= 0.$$

$$\to \sum_{t=0}^{99} \frac{(-1)^t}{3} = 0.$$

3.

~~$2 + 6 + 10 + \dots + (4n-2) = \sum_{k=1}^{n} 4k-2$~~

~~PROVE $2+6+10+\dots+(4n-2) = 2n^2 \ \forall n \in \mathbb{N}$~~
~~BY INDUCTION.~~

~~① $n = 1 \to 2 = 2 \cdot 1^2 = 2$ ✓~~

~~② ~~let $k \in \mathbb{N}$~~~~

~~let $2+6+10+\dots+(4k-2) = 2k^2 \ \forall k \in \mathbb{N}$~~

~~PROVE $2+6+10+\dots+[4(k+1)-2] = 2(k+1)^2$~~

~~LHS: $2+6+10+\dots+[4k+4-2]$~~

~~$= 2+6+10+\dots+(4k+2)$~~

REMOVED

3.

$$2+6+10+\dots+(4n-2)=\sum_{k=1}^{n}(4k-2)$$

PROVE BY INDUCTION. $2+6+10+\dots+(4n-2)=2n^2$

$\forall n \in \mathbb{N}$.

(1) $n=1 \rightarrow 2 = 2.1^2 = 2$ ✓

(2) let $\cancel{k}$ $2+6+10+\dots+(4k-2)=2k^2,\ k \in \mathbb{N}$.

PROVE $\cancel{k}$ FOR $k+1$

$\rightarrow 2+6+10+\dots+(4k-2)+[4(k+1)-2] = 2(k+1)^2$

LHS. $2+6+10+\dots+(4k-2)+[4k+4-2]$

$= 2+6+10+\dots+(4k-2)+(4k+2)$

$= 2k^2+4k+2$

$= 2(k^2+2k+1)$

$= 2(k+1)^2 =$ RHS. for $k+1$

$\therefore 2+6+10+\dots+(4n-2)=2n^2.\ \forall n \in \mathbb{N}$