Topic: Measuring Uncertainty with Probability

Probability - Tree Diagram Exercise

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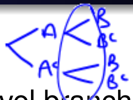
Tree Diagrams



Recall:

- Conditional probabilities correspond to second (or higher) level branches in a **tree diagram**.

- **Multiply** probabilities of all branches along a **path** to find the probability of a single outcome (using the multiplicative law of probability):

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

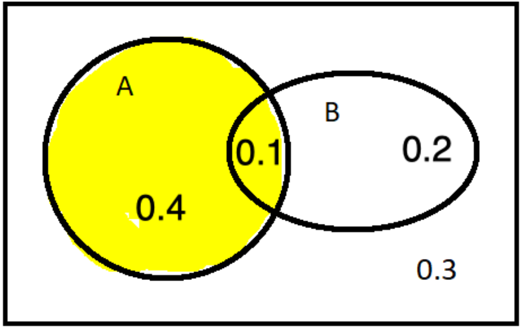
$$P(A \cap B) = P(A) \times \underline{P(B|A)}$$

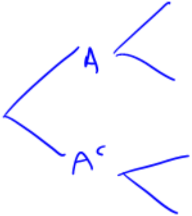
- **Sum** probabilities of all paths leading to an **event** to find its probability. The paths represent mutually exclusive outcomes.

$$P(B) = P(A \cap B) + P(A^c \cap B).$$

Exercise: Tree diagrams from Venn Diagrams

Example: Use the given Venn diagram to build a tree diagram and calculate all conditional probabilities branching on A first.

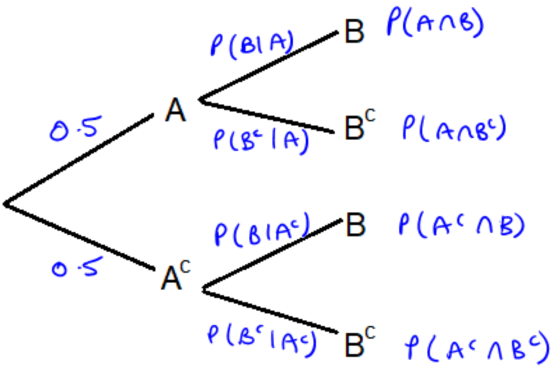




$P(A) = 0.4 + 0.1$
$= 0.5$

Exercise: Step 1

Step 1: Draw the tree diagram and fill in $P(A)$, $P(A^c)$ on first set of branches.

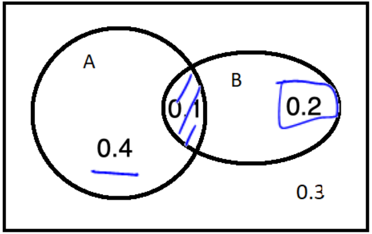


$P(A) = 0.5$

$P(A^c) = 1 - 0.5 = 0.5$

Exercise: Step 2

Step 2: The probabilities on the second set of branches are the conditional probabilities. But first we need the intersection probabilities. Use the Venn diagram:



$$P(A \cap B) = 0.1 = P(B \cap A)$$

$$P(A \cap B^c) = 0.4$$

$$P(A^c \cap B) = 0.2$$

$$P(A^c \cap B^c) = 0.3$$

Exercise: Step 3

Step 3: Calculate the conditional probabilities:

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.1}{0.5} = 0.2$$

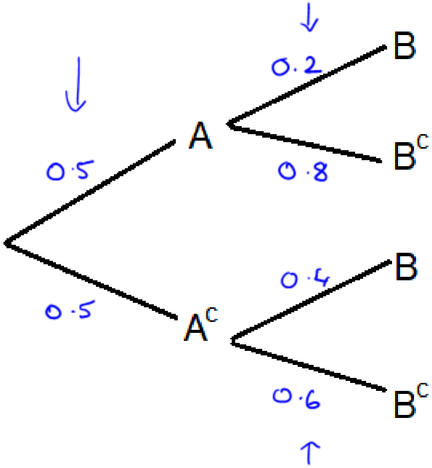
$$P(B^c|A) = \frac{P(B^c \cap A)}{P(A)} = \frac{0.4}{0.5} = 0.8 \checkmark$$

$$P(B|A^c) = \frac{P(B \cap A^c)}{P(A^c)} = \frac{0.2}{0.5} = 0.4 \checkmark$$

$$P(B^c|A^c) = \frac{P(B^c \cap A^c)}{P(A^c)} = \frac{0.3}{0.5} = 0.6. \checkmark$$

Exercise: Step 4

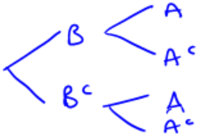
Step 4: Now complete the tree diagram.



Challenge

Try the following:

1. Repeat this process, branching on B first.



2. Consider how to get from the tree diagram to the corresponding two-way table.

	B	B^c	
A			
A^c			