Topic: Measuring Uncertainty with Probability

Conditional Probability

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Conditional Probability

Definition: The probability of an event (A) occurring when it is known that some event (B) has already occurred is called a **conditional probability**.

- The **conditional** probability of event A given that event B has occurred is

$$P(A|B) = \frac{P(A \overset{\text{and}}{\cap} B)}{P(B)}$$

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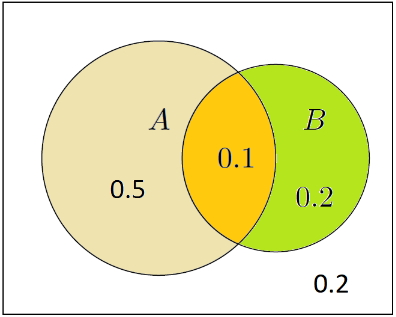
✓

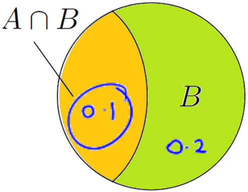
$P(A|B)$, can only be applied if $P(B) \neq 0$

Note: All probabilities can be considered as conditional probabilities, since $P(A)$ is really shorthand of $P(A|S)$

Conditional Probability $P(A|B)$

In terms of Venn diagrams, all of the sample space lying outside B is discarded, and B becomes the new sample space.





$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.1}{0.2 + 0.1} = \frac{0.1}{0.3}$$

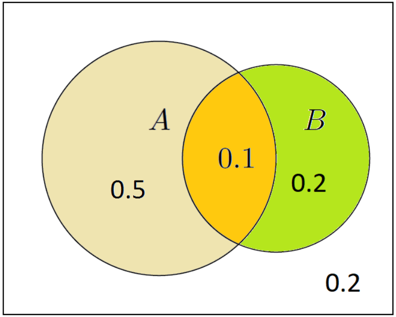
$$= \frac{1}{3}.$$

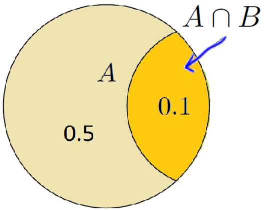
What about $P(B|A)$?

We just reverse A with B in the formula so that:
conditional probability of event B given that event A has occurred is

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.1}{0.5 + 0.1} = \frac{1}{6}$$

$(\neq P(A|B).$



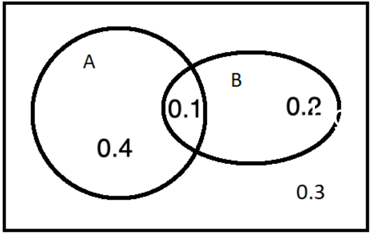


$P(B \cap A) = P(A \cap B)$

Conditional probability using a two-way table

To find a conditional probability using a two-way table, divide the intersection value by the appropriate marginal total.

Example:



	B	B'	Total
A	0.1	0.4	0.5
A'	0.2	0.3	0.5
Total	0.3	0.7	1

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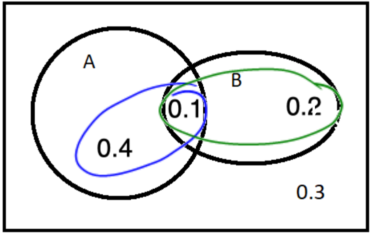

P(A∩B)

	B	B'	Total
A	0.1	0.4	$\boxed{0.5}$
A'	0.2	0.3	0.5
Total	$\boxed{0.3}$	0.7	1

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.3} = \frac{1}{3}$$

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Multiplicative Law of Probability

Sometimes, it is more convenient to start with information on the conditional probability and use it to find the joint probability (intersection).
We can just rearrange the rule for conditional probability:

$$P(A \text{ given } B) = \boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

Multiplicative Law of Probability

The probability of the intersection of two events A and B is

$$\underline{P(A \cap B)} = P(B) \times P(A|B)$$

or $$P(B \cap A) = P(A) \times P(B|A)$$

Example continued

Example: If $P(A) = 0.5$, $P(B) = 0.3$, and $P(A|B) = 1/3$, determine $P(A \cap B)$

$$
\begin{aligned}
P(A \cap B) &= P(B) \times P(A|B) \\
&= 0.3 \times \frac{1}{3} \\
&= \frac{\cancel{3}}{10} \times \frac{1}{\cancel{3}} \\
&= \frac{1}{10} \\
&= \underline{0.1}
\end{aligned}
$$

$P(B \cap A) =$

$\underline{0.1}$

Summary

In this lecture segment we have looked at **Conditional probabilities**:

$$P(A \text{ given } B) = P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- Multiplicative Law:

$$P(A \cap B) = P(B) \times P(A|B)$$

Conditional probabilities can also be determined using **two-way tables**.

Reference: Wackerley D.D., Mendenhall W. & Scheaffer R.L. [WMS] (2008) “Mathematical Statistics with Applications”, 7th ed. Duxbury, Belmont . (Library: 519.5/40).