Topic: Measuring Uncertainty with Probability

Law of Total Probability - Exercise

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Probability Rules - Summary

If A and B represent any two events then,

Complement	$P(\text{not } A) = P(A^c) = 1 - P(A)$
Additive	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Conditional	$P(A\|B) = \frac{P(A \cap B)}{P(B)}$
Multiplicative	$P(A \cap B) = P(A\|B)P(B)$

* **Law of Total Probability** $P(A) = \sum_{i=1}^{k} P(A \cap B_i) = \sum_{i=1}^{k} P(A|B_i)P(B_i)$

Exercise: Law of Total Probability

Exercise: In a certain factory, Machines 1, 2, and 3 are all producing springs of the same length. Machines 1, 2, and 3 produce 1%, 4% and 2% defective springs, respectively. Of the total production of springs in the factory, Machine 1 produces 30%, Machine 2 produces ~~25~~ 25%, and Machine 3 produces 45%.

If one spring is selected at random from the total springs produced in a given day, determine the probability that it is defective.

Let D be the event that the spring is defective.
Let M_i be the event that a spring is produced Machine ($i = 1, 2, 3$).

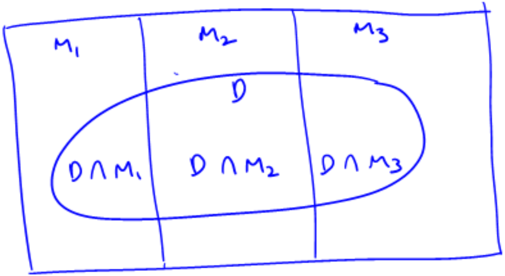
M1 M2 M3.

Ref: From Hogg, McKeon, Craig, (2013) Introduction to Mathematical Statistics. p28 Ex 1.4.8

Exercise cont.: Diagram

- Draw a Venn diagram to show the partitioning of S: How is the sample space partitioned in this context?

3 machines



Exercise cont.: Given information

- Write down known information using correct notation.

Machines 1, 2, and 3 produce 1%, 4% and 2% defective springs, respectively. Machine 1 produces 30%, Machine 2 produces 25%, and Machine 3 produces 45%.

$$P(M_1) = 0.30 \qquad P(D|M_1) = 0.01$$

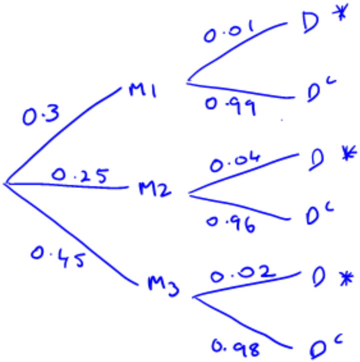
$$P(M_2) = 0.25 \qquad P(D|M_2) = 0.04$$

$$P(M_3) = 0.45 \qquad P(D|M_3) = 0.02$$

$$1.0$$

Exercise cont.: Tree diagram

- Draw a tree diagram showing all probabilities.



$P(M_1 \cap D) = P(M_1) \times P(D \mid M_1)$

$P(M_2 \cap D)$

$P(M_3 \cap D)$.

Exercise cont.: Apply the Law of Total Probability

- If one spring is selected at random from the total springs produced in a given day, determine the probability that it is defective.

$$P(D) = P(D \cap M_1) + P(D \cap M_2) + P(D \cap M_3) \quad \#$$

$$= P(D \mid M_1) \cdot P(M_1) + P(D \mid M_2) \cdot P(M_2) + P(D \mid M_3) \cdot P(M_3).$$

$$= (0.01 \times 0.3) + (0.04 \times 0.25) + (0.02 \times 0.45)$$

$$= 0.003 + 0.01 + 0.009 \quad \#$$

$$= 0.022$$

2.2%.

Exercise cont.: Apply the Law of Total Probability

$P(M_1 \cap D) = 0.003 \quad + \quad P(M_1 \cap D^c) = 0.3 \times 0.99$

$= 0.297$

$P(M_2 \cap D) = 0.01 \quad + \quad P(M_2 \cap D^c) = 0.25 \times 0.96$

$= 0.24.$

$P(M_3 \cap D) = 0.009 \quad + \quad P(M_3 \cap D^c) = 0.45 \times 0.98$

$= \underline{0.441}.$

$P(M_i).$

0.3

0.25

0.45

Exercise cont.: Apply the Law of Total Probability

- Complete the Venn diagram showing showing all probabilities.