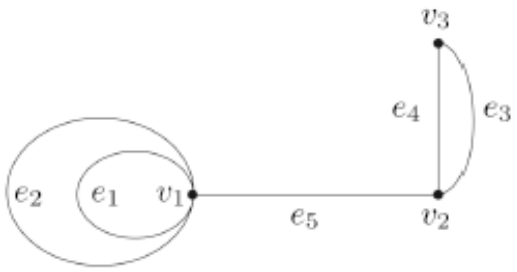
MATH255: Mathematics for Computing
Tutorial Sheet Week 6 - Autumn 2023

Note. Question 8 is your Tutorial Preparation exercise for this week. It must be completed and handed in on Moodle as a pdf before the start of your tutorial (SWS) or Thursday night (Wolllongong).

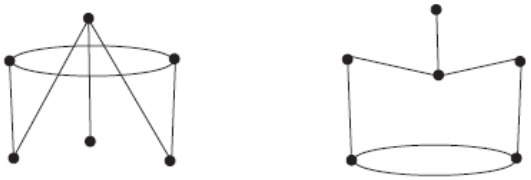
1. Let $G = (V, E)$ be the graph below.



   Draw two different subgraphs of G, H_1 a connected subgraph and H_2 a disconnected subgraph, for which $\delta(H_i) = 6$.

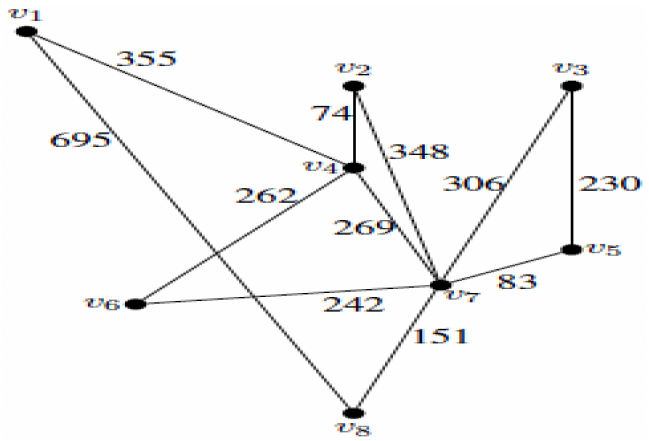
2. Draw a non-simple graph with 4 vertices of respective degrees 1,1,3 and 5.

3. By suitably labelling the vertices and edges of the two graphs below, show that they are isomorphic.

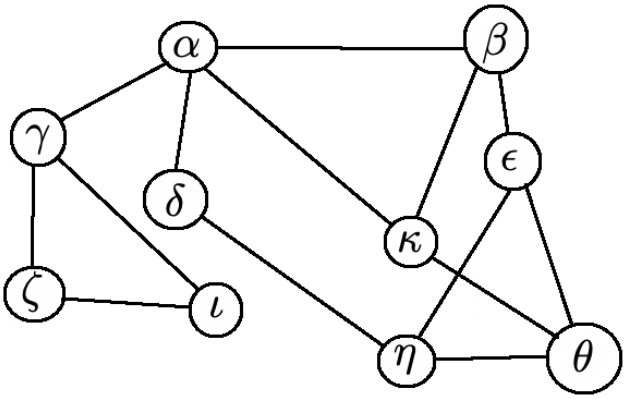


4. (a) How many subgraphs of K_n are there that have all n vertices? (Hint: ideas about power sets are relevant.)
   (b) How many non-isomorphic subgraphs of K_3 that have three vertices are there?

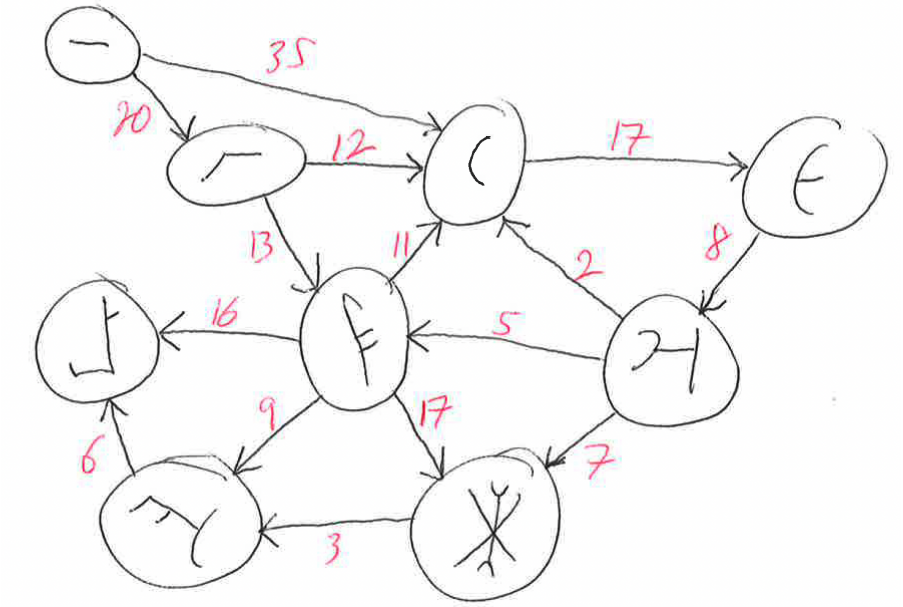
5. Use Kruskal's and Prim's algorithms to find a minimum spanning tree and its weight for the following graph. Did you get the same tree in both instances? Under what circumstances will that happen and when will it not happen?



6. Do not look at the whole graph below yet, only at α and β at the top. You want to find node θ in the graph below, but you only have access to nodes α and β to start, and then any adjacent nodes thereafter. Choose either α or β as your source node, then look at the whole graph and run (a) a breadth-first search and (b) a depth-first search until you find θ. Draw the two resulting search trees.



7. On the following directed graph, use Dijkstra's algorithm to find the shortest path from the top-left node to every other node. Indicate the path and total weight for each. Bonus point: the nodes are the numbers from 0 to 8 in what language?



8. Which of the following four graphs are bipartite? Justify your answers.