MATH255: Mathematics for Computing
Tutorial Sheet Week 2 - Autumn 2023

Note. Question 2 is you Tutorial Preparation exercise for this week. It must be completed and handed in on Moodle as a pdf before the start of your tutorial.

1. Determine whether the following are statements. If you find one that is, determine its truth value.
   (a) If $x = 3$, then $x < 2$.
   (b) If $x = 0$ or $x = 1$, then $x^2 = x$.
   (c) $x^2 = x$ only if $x = 0$ or $x = 1$.
   (d) There exists a natural number x such that $x^2 = \frac{x}{2}$.
   (e) $x < -1$.
   (f) $xy = 5 \to (x = 1, y = 5) \vee (x = 5, y = 1)$.
   (g) For $x, y \in \mathbb{R}$, $xy = 0 \to (x = 0 \vee y = 0)$.
   (h) There is a unique even prime number.
   (i) This statement is false.

2. Let p and q be statements.
   (a) Write truth tables for $(\sim p \vee q) \wedge q$ and $(\sim p \wedge q) \vee q$. What do you notice? Based on this result, we hypothesise that we can interchange $\vee$ and $\wedge$ in a statement without affecting the truth table.
   (b) Write truth tables for $(\sim p \vee q) \wedge p$ and $(\sim p \wedge q) \vee p$. What do you think of the hypothesis now?

3. Practice the quick method; do not use truth tables. Determine which of the following statements are tautologies.
   (a) $\sim (p \to q) \vee (q \to p)$
   (b) $(p \wedge q) \to [\sim r \vee (p \to q)]$

4. Use a truth table to prove the distributive law $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$.

5. Let p, q and r be statements. Using the laws seen in lecture, prove the following.
   (a) $\sim (p \to q) \equiv (p \wedge \sim q)$.
   (b) $\big((p \wedge \sim q) \to r\big) \equiv \big(p \to (q \vee r)\big)$
   (c) $p \to (q \vee p)$ is a tautology.
   (d) $\sim [(p \wedge q) \to \big(\sim r \vee (p \to q)\big)]$ is a fallacy.

6. In each case, decide whether the proposition is T or F. Give reasons.
   (a) If x is a positive integer and $x^2 \leq 3$ then $x = 1$.
   (b) $\big(\sim (x > 1) \vee \sim (y \leq 0)\big) \leftrightarrow \sim \big((x \leq 1) \wedge (y > 0)\big)$

7. Rewrite the following logical expressions using $\vee$ and $\wedge$ as the only connectives.
   (a) $\sim (x > 1) \to \sim (y \leq 0)$
   (b) $(y \leq 0) \to (x > 1)$