Tutorial Sheet Week 5

Question 3.

We shall have the following relation: On R, $R = \{(x,y) : |x-y| \leq 1\}$

PROVE: R is not an equivalence relation

To prove any relation is not an equivalence relation, we need to prove it does not satisfy one of these three requirements: reflexive, symmetric and transitive

- Reflexive.

$\forall x \in R$, does $(x,x) \in R$ (relation)?

→ YES because it satisfies $|x - x| \leq 1$

$\Leftrightarrow |0| \leq 1$

$\Leftrightarrow 0 \leq 1$ (Always true)

- Symmetric ✓, $(x,y) \in R$ (relation)

$\forall x, y \in R$ ✓ does $(y,x) \in R$ (relation)?

→ YES

Because: we already have (x,y) in the relation, which satisfies $|x-y| \leq 1$. Even if we were to replace (y,x) into $|x-y|$, it will return a value ≤ 1 as (x,y) did. This is due to the absolute value function

Ex $(1,0) \rightarrow |1-0| = |1| = 1 \leq 1$ ✓

$(0,1) \rightarrow |0-1| = |-1| = 1 \leq 1$ ✓

- Transitive.

$\forall x, y, z \in R$

$(x,y); (y,z) \in R$ (relation) $\Rightarrow (x,z) \in R$?

Ex $(1,0)$ and $(0,-1) \in R$ (relation)

→ $(1,-1) \in R$?

→ NO. Because $|1-(-1)| = |2| \notin 1$ ✗

→ NOT Transitive

∴ This is not an equivalence relation