Topic: Measuring Uncertainty with Probability

Conditional Probability - Exercise

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Conditional Probability

Recall:

Definition: The probability of an event (A) occurring when it is known that some event (B) has already occurred is called a **conditional probability**.

- The **conditional** probability of event A given that event B has occurred is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Exercise: Conditional Probability

Exercise: Let L be the event that bus leaves on time and A the event that bus arrives on time. Assume that the probability that a bus leaves on time is 0.80, the probability that it arrives on time is 0.75, and the probability that it leaves on time and arrives on time is 0.72.

1. Construct a Venn diagram to represent this information
2. Fill the following table:

	A	A^c	Total
L			
L^c			
Total			

3. Then find the probability that the bus:
   (a) arrives on time given it left on time?
   (b) arrives on time given that it did not leave on time?
   (c) left on time given it arrives on time?
   (d) arrives on times or leaves on time?

Exercise: Conditional Probability Solution

Exercise: Let L be the event that bus leaves on time and A the event that bus arrives on time. Assume that the probability that a bus leaves on time is 0.80, the probability that it arrives on time is 0.75, and the probability that it leaves on time and arrives on time is 0.72.

1. Venn diagram ✓

L A

$P(L) = 0.80$

$P(A) = 0.75$

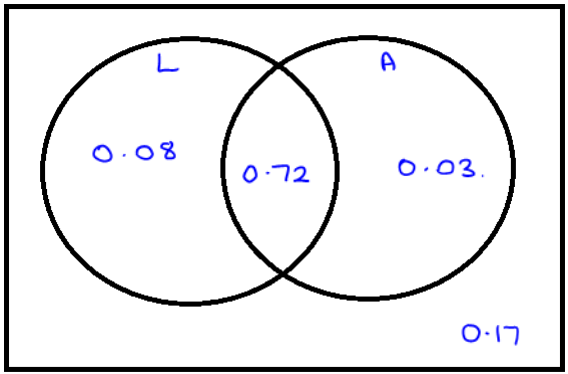
$P(L \cap A) = 0.72$.

$0.8 - 0.72 = 0.08$

$0.75 - 0.72 = 0.03$

$P(L^c \cap A^c) = 1 - (0.08 + 0.72 + 0.03)$

$= 0.17$



✓ Total = 1

Exercise: Conditional Probability Solution cont.

2. The two-way table: ✓

$P(L) = 0.8$

$P(A) = 0.75$

$P(L \cap A) = 0.72$

$P(L^c \cap A) = 0.75 - 0.72 = 0.03$

$P(L \cap A^c) = 0.08$

$P(L^c) = 0.20$

$P(A^c) = 0.25$

$P(L^c \cap A^c) = 0.17$

	A	A^c	Total
L	0.72	0.08	0.80
L^c	*0.03	0.17	0.20
Total	0.75	0.25	1.0

Exercise: Conditional Probability Solution cont.

3. The probability that the bus:
   (a) arrives on time given it left on time?

$$P(A \mid L) = \frac{P(A \cap L)}{P(L)} = \frac{0.72}{0.80} = 0.9$$

   (b) arrives on time given that it did not leave on time?

$$P(A \mid L^c) = \frac{P(A \cap L^c)}{P(L^c)} = \frac{0.03}{0.20} = \frac{3}{20} = 0.15$$

   (c) left on time given it arrives on time?

$$P(L \mid A) = \frac{P(L \cap A)}{P(A)} = \frac{0.72}{0.75} = 0.96$$

Exercise: Conditional Probability Solution cont.

3. The probability that the bus:
   (d) arrives on times or leaves on time?

$$
\begin{aligned}
P(A \text{ or } L) &= P(A \cup L) \\
&= P(A) + P(L) - P(A \cap L) \\
&= 0.75 + 0.80 - 0.72 \\
&= 0.83 .
\end{aligned}
$$