1. (a) Statement. False.
   (b) Statement. True.
   (c) Statement. False. Be careful, the universe is not declared here. For example, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.
   (d) Statement. False. Solving the equations yields $x = 0$ or $x = \frac{1}{2}$, neither of which is a natural number.
   (e) Not a statement.
   (f) Statement. False. For instance $x = 10, y = \frac{1}{2}$.
   (g) Statement. True.
   (h) Statement. True.
   (i) Not a statement.

2. (a)

p	q	$\sim p$	$\sim p \vee q$	$\sim p \wedge q$	$(\sim p \vee q) \wedge q$	$(\sim p \wedge q) \vee q$
T	T	F	T	F	T	T
T	F	F	F	F	F	F
F	T	T	T	T	T	T
F	F	T	T	F	F	F

The last two columns are identical.

   (b)

p	q	$\sim p$	$\sim p \vee q$	$\sim p \wedge q$	$(\sim p \vee q) \wedge p$	$(\sim p \wedge q) \vee p$
T	T	F	T	F	T	T
T	F	F	F	F	F	T
F	T	T	T	T	F	T
F	F	T	T	F	F	F

The last two columns are not identical; the hypothesis is false.

3. (a)

$$\begin{array}{ccccccccccc}
\sim & (& p & \Rightarrow & q &) & \vee & (& q & \Rightarrow & p &) \\
 & & & & & & \mathrm{F} & & & & & \\
\sim & (& p & \Rightarrow & q &) & \vee & (& q & \Rightarrow & p &) \\
\mathrm{F} & & & & & & & & & \mathrm{F} & & \\
\sim & (& p & \Rightarrow & q &) & \vee & (& q & \Rightarrow & p &) \\
 & & & \mathrm{T} & & & & & \mathrm{T} & & \mathrm{F} & \\
\sim & (& p & \Rightarrow & q &) & \vee & (& q & \Rightarrow & p &) \\
 & & \mathrm{F} & & \mathrm{T} & & & & & & &
\end{array}$$

No contradiction. Therefore, the statement is not a tautology.

   (b)

$$\begin{array}{cccccccccccccccc}
(& p & \wedge & q &) & \Rightarrow & [& \sim & r & \vee & (& p & \to & q &) &] \\
 & & & & & \mathrm{F} & & & & & & & & & & \\
(& p & \wedge & q &) & \Rightarrow & [& \sim & r & \vee & (& p & \to & q &) &] \\
 & & \mathrm{T} & & & & & & & \mathrm{F} & & & & & & \\
(& p & \wedge & q &) & \Rightarrow & [& \sim & r & \vee & (& p & \to & q &) &] \\
 & \mathrm{T} & & \mathrm{T} & & & & \mathrm{F} & & & & & \mathrm{F} & & & \\
(& p & \wedge & q &) & \Rightarrow & [& \sim & r & \vee & (& p & \to & q &) &] \\
 & & & & & & & & \mathrm{T} & & & \mathrm{T} & & \mathrm{T} & &
\end{array}$$

Contradiction. Therefore, the statement is a tautology.

4.

p	q	r	$q \wedge r$	$p \vee q$	$p \vee r$	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F

The last two columns are identical in every row. Therefore, they are equivalent statements.

5. (a)

$$\begin{aligned} \sim (p \;\Rightarrow\; q) &\equiv \sim (\sim p \vee q) && \text{(implication)} \\ &\equiv \sim\sim p \wedge \sim q && \text{(De Morgan)} \\ &\equiv p \wedge \sim q && \text{(double negation)} \end{aligned}$$

(b)

$$\begin{aligned} [(p \wedge \sim q) \rightarrow r] &\equiv \sim (p \wedge \sim q) \vee r && \text{(implication)} \\ &\equiv (\sim p \vee \sim\sim q) \vee r && \text{(De Morgan)} \\ &\equiv \sim p \vee (\sim\sim q \vee r) && \text{(associative)} \\ &\equiv \sim p \vee (q \vee r) && \text{(double negation)} \\ &\equiv p \rightarrow (q \vee r) && \text{(implication)} \end{aligned}$$

(c)

$$\begin{aligned} p \rightarrow (q \vee p) \equiv & \quad \sim p \vee (q \vee p) && \text{(implication)} \\ \equiv & \quad \sim p \vee (p \vee q) && \text{(commutative)} \\ \equiv & \; (\sim p \vee p) \vee q && \text{(associative)} \\ \equiv & \; T \vee q \equiv T \end{aligned}$$

Therefore, the statement is a tautology.

(d)

$$\begin{aligned} & \sim [(p \;\wedge\; q) \;\Rightarrow\; (\sim r \;\vee\; (p \;\Rightarrow\; q))] \\ \equiv & \sim [\sim (p \wedge q) \vee (\sim r \vee (\sim p \vee q))] && \text{(implication)} \\ \equiv & \sim [(\sim p \vee \sim q) \vee (\sim r \vee (\sim p \vee q))] && \text{(De Morgan)} \\ \equiv & \sim (\sim p \vee \sim q) \wedge \sim (\sim r \vee (\sim p \vee q)) && \text{(De Morgan)} \\ \equiv & (\sim\sim p \wedge \sim\sim q) \wedge (\sim\sim r \wedge \sim (\sim p \vee q)) && \text{(De Morgan)} \\ \equiv & (p \wedge q) \wedge (r \wedge \sim (\sim p \vee q)) && \text{(double negation)} \\ \equiv & (p \wedge q) \wedge (r \wedge (\sim\sim p \wedge \sim q)) && \text{(De Morgan)} \\ \equiv & (p \wedge q) \wedge (r \wedge (p \wedge \sim q)) && \text{(double negation)} \\ \equiv & (q \wedge \sim q) \wedge (p \wedge r \wedge p) && \text{(commutative and associative)} \\ \equiv & F \wedge (p \wedge r \wedge p) \equiv F \end{aligned}$$

Therefore, the statement is a fallacy.

6. (a) Let x be a positive integer greater than 1. Then, since 2 is the next smallest integer, we have $x^2 \geq 2^2 = 4$. So every element $x \in \mathbb{N} \setminus \{1\}$ is such that $x^2 > 3$. Therefore, the only possibility is $x = 1$. In that case, $x^2 = 1 \leq 3$, so the statement is true.

   (b)

$$\begin{aligned}
&\big(\sim (x > 1) \lor \ \sim (y \leq 0)\big) \ \leftrightarrow \ \sim \big((x \leq 1) \ \land \ (y > 0)\big) \\
\equiv &((x \leq 1) \lor (y > 0)) \leftrightarrow (\sim (x \leq 1) \lor \sim (y > 0)) \\
\equiv &((x \leq 1) \lor (y > 0)) \leftrightarrow ((x > 1) \lor (y \leq 0))
\end{aligned}$$

   The statement says that $x \leq 1$ or $y > 0$ if and only if $x > 1$ or $y \leq 0$, which is contingent.

7. (a)

$$\begin{aligned}
\sim (x > 1) \ \rightarrow \sim (y \leq 0) &\equiv (x \leq 1) \rightarrow (y > 0) \\
&\equiv \sim (x \leq 1) \lor (y > 0) \\
&\equiv (x > 1) \lor (y > 0)
\end{aligned}$$

   (b)

$$\begin{aligned}
(y \leq 0) \ \rightarrow \ (x > 1) &\equiv \sim (y \leq 0) \lor (x > 1) \\
&\equiv (y > 0) \lor (x > 1)
\end{aligned}$$