Topic: Measuring Uncertainty with Probability

Concepts of Probability

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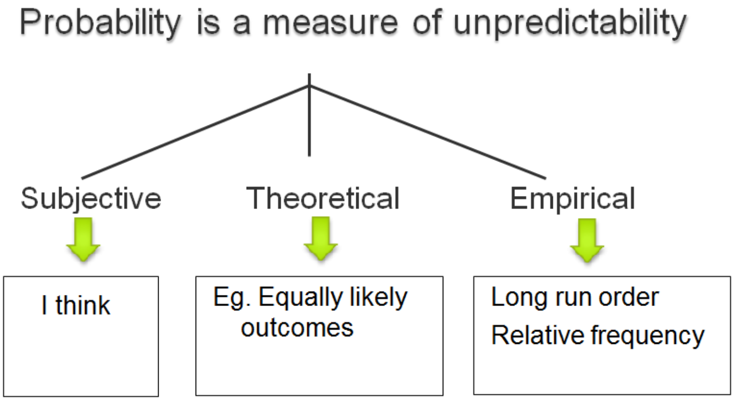
How do we measure uncertainty?



Measuring Uncertainty with Probability

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- **Theoretical** eg equally likely

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- **Empirical**
  - use observed outcomes to determine or estimate probability
  - repeat experiment indefinitely long run frequency

$$P(\text{event}) = \frac{\text{Number of times event occurs}}{\text{Total no. of times experiment in repeated}}$$

Theoretical Probability - Activity

When I toss this die, what is the probability of . . .

$S = \{1, 2, 3, 4, 5, 6\}$

1. Getting a 1? $P(1) = \frac{1}{6}$

2. Getting a 2 or a 3? (event)

   $P(2 \text{ or } 3) = \frac{2}{6} = \frac{1}{3}$

3. Getting an even number **or** a no. greater than 4?

   $\{2, 4, 6\}$ $\{5, 6\}$

   $E_1 = \{2, 4, 5, 6\}$

   $P(E_1) = \frac{4}{6}$

4. Getting an even number **and** a no. greater than 4?

   $\{2, 4, 6\}$ $\{5, 6\}$

   $E_2 = \{6\}$

   $P(E_2) = \frac{1}{6}$

Theoretical Probability - Activity

What thinking gave you those answers? **or**
What have you assumed in your calculations?

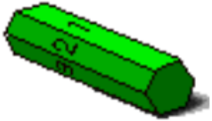
- fair die
- equally likely outcomes

Theoretical Probability - Activity

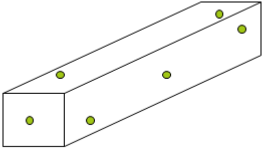
What thinking gave you those answers? or
What have you assumed in your calculations?

Could you use the same thinking for these dice? If not, what could you do?

What is the probability of getting a 1 when this die is tossed? $\frac{1}{6}$



Or this one? ?



Equally Likely Outcomes

- The most common experiments for which equally likely outcomes can be assumed are related to gambling.
- This definition of probability is rarely applicable in experiments involving natural phenomena.
- It would seem to be applicable, for example, to the day of the week (Monday, Tuesday, ..., Sunday) on which a wild animal is born.
- However, for humans, and perhaps even for domesticated animals, various effects such as medical intervention (Caesareans, induced births,...) makes births more likely on any weekday (Monday to Friday) than on Saturday or Sunday.

Equally likely outcomes cont.

When all possible outcomes are equally likely,

$$P(E) = \frac{n(E)}{n(S)}$$

where $n(E) =$ no. of outcomes in E; and $n(S) =$ no. of outcomes in S

Example 1: A coin is tossed twice, the sequence of heads (H) and tails (T) is recorded.

- Sample Space $S = \{HH, HT, TH, TT\}$
- Let $E =$ denote the event "same result for both tosses".

Then $E = \{\overset{2}{HH, TT}\}$ and $P(E) = \frac{n(E)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

Empirical - Relative Frequency

Outcomes are **not always** equally likely: consider

- an unbalanced coin; or
- a loaded die; or
- the proportion of emails received each day of the week.

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Relative frequency is used to estimate the **theoretical** probability.

- the estimate is calculated from available data
- or by experiment

A principle concern of Statistics is

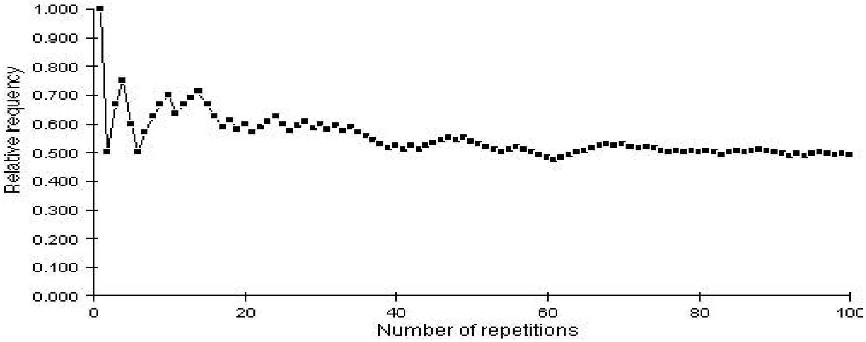
- estimating probabilities from samples...
- and using those estimates to make inferences about populations

Relative Frequency

- If we repeat the experiment indefinitely, the relative frequency of the random event from all past repetitions will **initially fluctuate markedly** but will tend to settle down or **stabilise** at a narrow band of values.
- The more times this experiment is repeated, the closer will the relative frequencies be to a particular value which we define to be the probability of the event.

Relative Frequency

- This is an example of the phenomenon known as the **Law of Large Numbers.**



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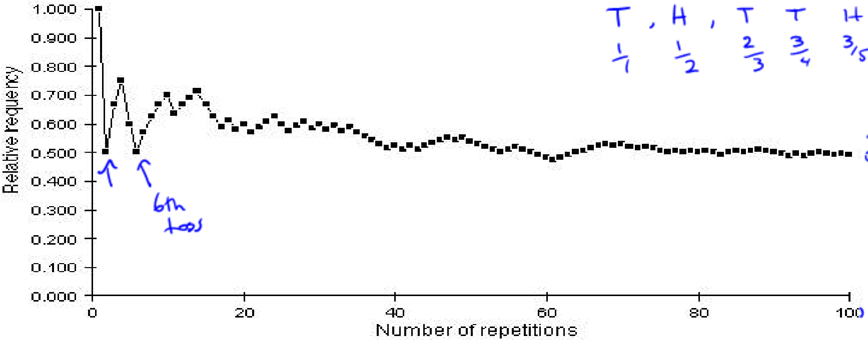

T, H, T T H H
1/1 1/2 2/3 3/4 3/5 3/6

100 tosses
≈ 50/100

≈ 0.5

- Outcomes are **not always** equally likely; sometimes probabilities are estimated as **long-run relative frequencies**.

Example 2

The grades obtained by 500 students completing a certain subject over a number of years are as follows:

F	PC (TF)	P	C	D	HD	Total
75	25	175	100	75	50	500

1. Estimate $P(C)$, $P(D)$, and $P(HD)$

   $P(C) = \frac{100}{500} = 0.2$ $\quad P(D) = \frac{75}{500} = 0.15$ $\quad P(HD) = \frac{50}{500} = 0.1$

2. Let E denote the event that a randomly selected student obtains at least a credit. Estimate $P(E)$.

   $P(E) = P(C \text{ or } D \text{ or } HD)$
   $= P(C) + P(D) + P(HD) = 0.2 + 0.15 + 0.1$
   $= 0.45$

3. Are the events C, D and HD disjoint?

   Yes.

Summary - Concepts of Probability

In this lecture segment we have looked at three ways that probability is measured:

- Subjective - depends on the person and information they draw on
- Theoretical - may have equally likely outcomes or not
- Empirical - relative frequencies are estimates of probabilities

Probability theory is the foundation used in **inferential statistics.**

Reference: Wackerley D.D., Mendenhall W. & Scheaffer R.L. [WMS] (2008) “Mathematical Statistics with Applications”, 7th ed. Duxbury, Belmont . (Library: 519.5/40).