

Topic: Measuring Uncertainty with Probability

Introduction, Language and Notation

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How do we measure uncertainty?

Probability is a measure of **uncertainty or likelihood**.

The probability of an event **certain** to happen is set to 1.

In **everyday life**, it is often expressed as a percentage:

- There is a 50-50 chance of getting a tail when a fair coin is tossed.
- The weather forecast states: *Today there is an 80% chance of rain.*
- There is a 20% chance that the train to Sydney from Wollongong is likely to arrive late.

What is probability?

Probability is used to quantify **unpredictability** and describe it precisely.

The probability of an event is a number between 0 and 1 indicating how **likely** it is that the event will occur when an 'experiment' is carried out.

What is probability?

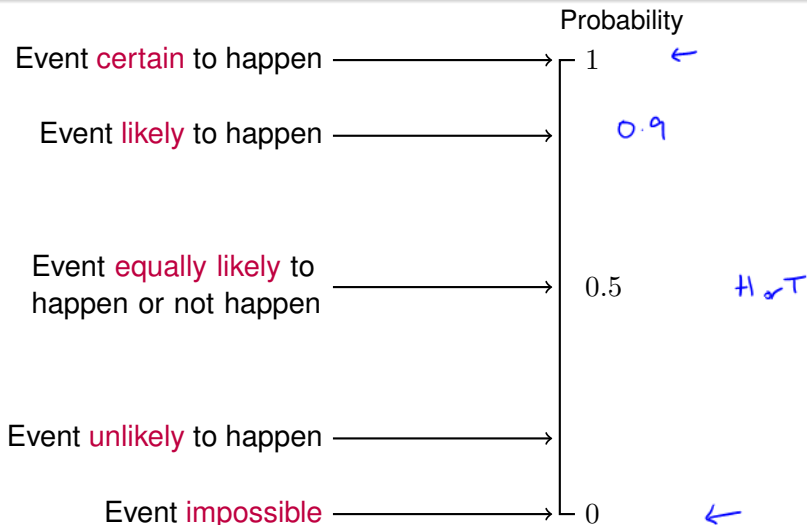
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A probability model describes the uncertainty in an experiment by assigning probabilities to the possible outcomes.

- Weather forecast models
- Artificial intelligence uses statistics to **make independent learning decisions** where a decision is an outcome that has highest probability.

Probability Scale



Language and Notation

- **Random** phenomenon: cannot be predicted with certainty in advance
- **Experiment**: the observation of any phenomenon that is uncertain
- **Outcome** is a single observed result of random phenomenon
 - is a result of an experiment which cannot be reduced to simpler results
Example: getting a Head or Tail on the toss of a coin
- The **sample space** S is the set of all possible outcomes or sample points
 - S may be finite, countably infinite, or uncountably infinite.
 - A **discrete sample space** contains a finite or countable number of distinct sample points
 - Example: $S = \{1, 2, 3, 4, 5, 6\}$ is set of all outcomes for a throw of a die
 - $P(S) = 1$, as the sample space includes all possibilities.

Language and Notation: Events

- The subsets of S are called **events**.
 - E is a subset of the sample space S : $E \subseteq S$
 - Events are collection of outcomes, including both S and \emptyset (the null or empty set).
 - An event in a discrete sample space S is a collection of sample points, any subset of S
 - e.g. in the die experiment the event 'getting an even number' is the collection of outcomes $\{2, 4, 6\}$
- **Null event** $\{ \}$ or \emptyset
 - The empty set (no outcomes) is an event which can **never** occur. e.g. even and odd: \emptyset
 - $P(\emptyset) = 0$, as \emptyset contains no possibilities.

$$P(\emptyset) = 0$$

Language and Notation

- **Intersection** of events: $P(A \cap B)$
 - The event that A and B both occur.
e.g. 1 die rolled: $P(\text{even} \text{ and greater than 4})$

$$A := \{2, 4, 6\}$$

$$B := \{5, 6\}$$

$$E_1 = \{6\}$$

- **Union** of events: $P(A \cup B)$
 - e.g. 1 die rolled: $P(\text{even} \text{ or greater than 4})$

$$E_2 = \{2, 4, 5, 6\}$$

- **Disjoint events**
 - have no outcomes in common
 - If $A \cap B = \emptyset$, the events A and B are said to be disjoint or mutually exclusive;
i.e. they cannot occur simultaneously.
 - 1 die rolled: $P(\text{even} \text{ and odd}) = 0$

$$\{2, 4, 6\} \quad \{1, 3, 5\}$$



Probability Axioms

- 1 The **probability** of each individual outcome is a number between 0 (“can’t happen”) and 1 (“certain to happen”).
i.e. For any event E : $0 \leq P(E) \leq 1$ \leq 'less than or equal to'
- 2 **Total probability** of all outcomes = 1
i.e. $P(S) = 1$ where S represents the sample space
- 3 The probability $P(E)$ of an event E is obtained by adding probabilities of **disjoint** outcomes in E .
i.e. $P(\underset{3}{E_1} \text{ or } \underset{4}{E_2} \text{ happens}) = P(\overset{\text{union}}{E_1 \cup E_2}) = P(E_1) + P(E_2)$

From these basic rules of probability (the axioms) other properties of probabilities can be derived.

Summary

In this lecture segment we have considered:

- Probability - as a measure of uncertainty which is used to quantify unpredictability.
- Language and notation
- Basic probability laws or axioms

Reference: Wackerley D.D., Mendenhall W. & Scheaffer R.L. [WMS] (2008) "Mathematical Statistics with Applications", 7th ed. Duxbury, Belmont . (Library: 519.5/40).