Topic: Measuring Uncertainty with Probability

Venn Diagrams

School of Mathematics and Applied Statistics

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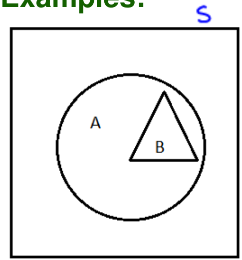
Introduction to Venn Diagrams

A **Venn diagram** represents events as subregions of a larger region representing the entire sample space.
It is a convenient way to represent the relationship between sets.

Examples:



B is a subset of A

$B \subset A$

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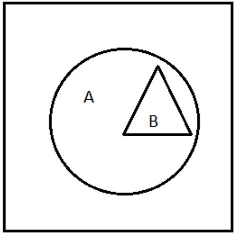

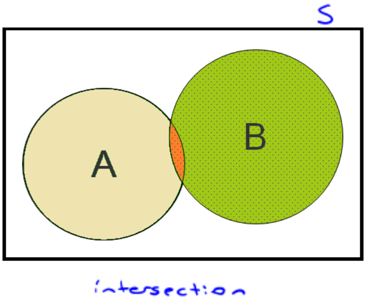

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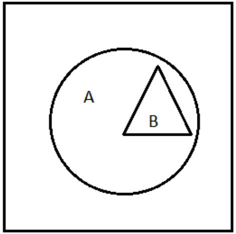
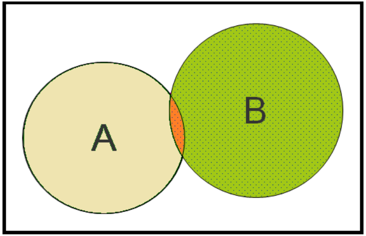
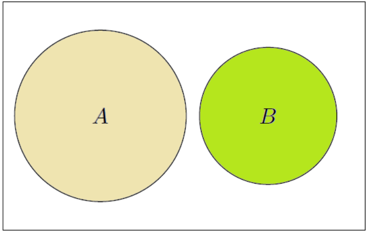




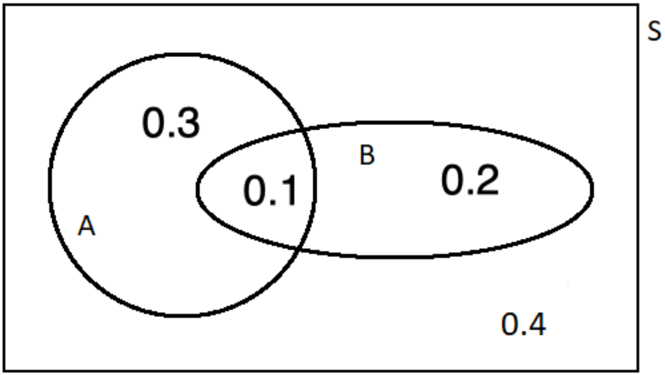
intersection



no intersection

Introduction to Venn Diagrams

Probabilities are represented as **areas** (not necessarily drawn to scale).
Numerical values (counts) can also be shown.

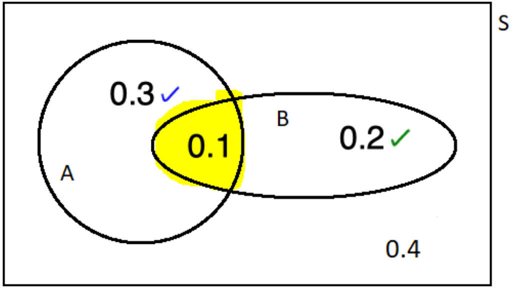


$0.3 + 0.1 + 0.2 + 0.4$
$= 1.0$

Total area $= 1$

Venn Diagrams & Two-way Tables

A **Venn diagram** and a **two-way table** are two different ways to represent the same information:



	B	not B	Total
A	0.1	0.3	0.4
not A	0.2	0.4	0.6
Total	0.3	0.7	1.0

marginal totals

marginal totals

$P(A) = 0\cdot3 + 0\cdot1 = 0\cdot4$

$P(B) = 0\cdot2 + 0\cdot1 = 0\cdot3$

Total area $= 1$

The Complement (Not)

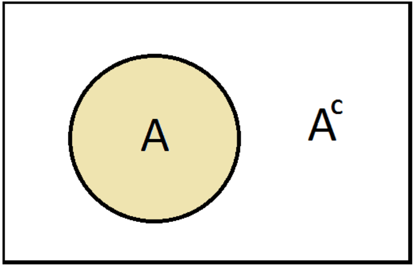
The complement of an event is the event not occurring.

Different notations for the complement of A include: $A^c = A' = \bar{A}$

$$P(A \text{ occurs}) + P(A \text{ does not occur}) = P(A) + P(A^c) = 1$$

$$P(\textbf{ not } A) = P(A^c) = 1 - P(A)$$

S



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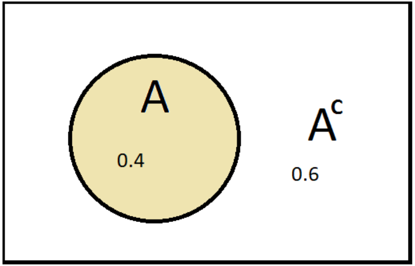
$$P(A \text{ occurs}) + P(A \text{ does not occur}) = P(A) + P(A^c) = 1$$

$$P(\textbf{ not } A) = P(A^c) = 1 - P(A) \quad = 1 - 0.4$$

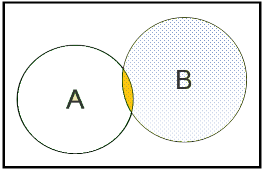
$$= 0.6.$$



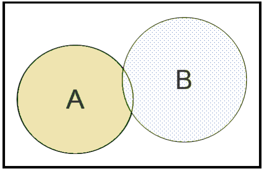
Venn diagram with two events

A Venn diagram with 2 events is subdivided into 4 regions:

1 $A \cap B$ intersection

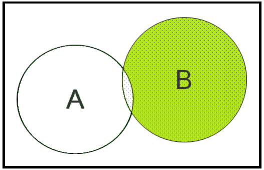


2 $A \cap B^c$

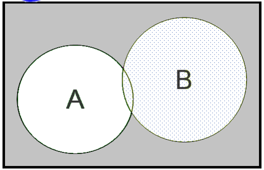


1 + 2 = P(A)

3 $A^c \cap B$



4 $A^c \cap B^c$



3 + 4 = P(A^c)

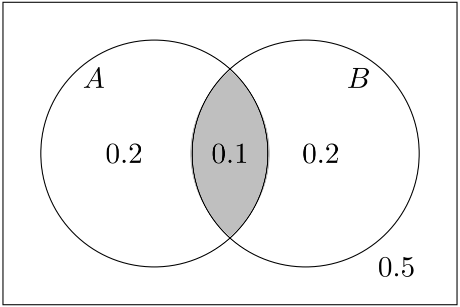
1 + 3 = P(B)

2 + 4 = P(B^c)

The probabilities of these 4 events sum to 1.

Intersection (And)

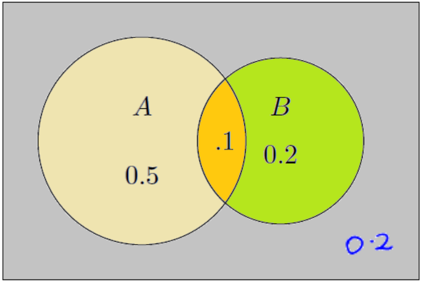
The **intersection** $A \cap B$ is the event that both A **and** B occur.
Another example:



$$P(A \cap B) = 0.1$$

Union (Or)

The **union** $A \cup B$ consists of outcomes that are in A **or** B (or both).



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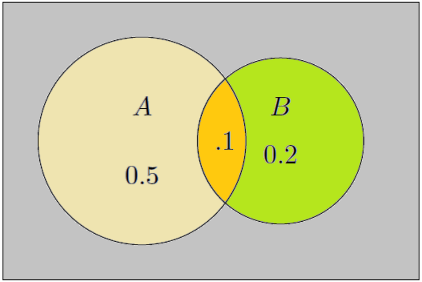


$P(A) = 0.6$
$P(B) = 0.3$
$P(A \cap B) = 0.1$

Additive Law of Probability:

The probability of the union of two events A and B is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

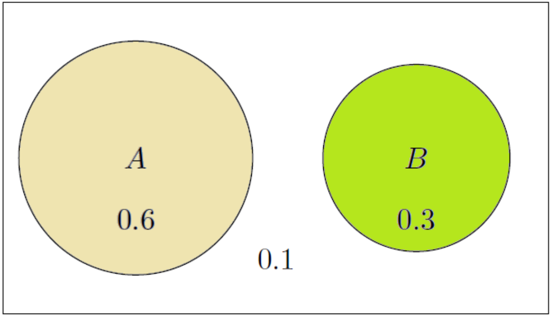
$= 0.6 + 0.3 - 0.1$

$= 0.8$

Disjoint Events

If $A \cap B = \varnothing$ (i.e. no intersection), then the two events are said to be **mutually exclusive** or **disjoint**.

For disjoint (mutually exclusive) events, the previous result can be simplified since $P(A \cap B) = 0$:



$$P(A \cup B) = P(A) + P(B)$$

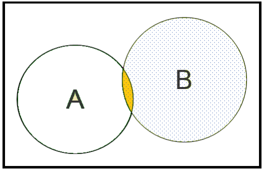
$= 0.6 + 0.3$

$= 0.9$

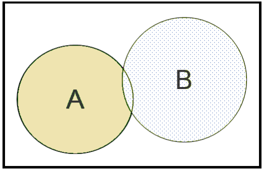
Summary - Venn Diagrams

If we have a 2 $\times$ 2 contingency table we can present the probabilities for outcomes as such:

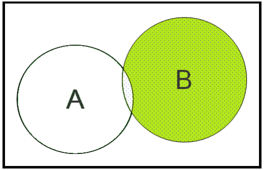
$A \cap B$



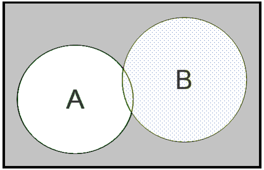
$A \cap B^c$



$A^c \cap B$



$A^c \cap B^c$



	B	not B	Total
A	$A \cap B$	$A \cap B^c$	$P(A)$
not A	$A^c \cap B$	$A^c \cap B^c$	$P(A^c)$
Total	$P(B)$	$P(B^c)$	1.0

Summary

In this lecture segment we have looked at Venn Diagrams as a convenient way to represent relationships between sets and applied set notation:

- Complement: $P(A^c) = 1 - P(A)$
- Intersection: $P(A \cap B)$
- Union: $P(A \cup B)$
- Disjoint events: $P(A \cap B) = 0$

Venn Diagrams can be extended to represent relationships between 3 or more events.

Reference: Wackerley D.D., Mendenhall W. & Scheaffer R.L. [WMS] (2008) “Mathematical Statistics with Applications”, 7th ed. Duxbury, Belmont . (Library: 519.5/40).