

MATH255: Mathematics for Computing

Tutorial Sheet Week 5 - Autumn 2023

Note. Question 3 is your Tutorial Preparation exercises for this week. It must be completed and handed in on Moodle as a pdf before the start of your tutorial.

1. Let $A = \{1, 2, 3\}, B = \{x, y\}$.
 - (a) Which is bigger, $|A \times B|$ or $|B \times A|$? Is any of the following true: $A \times B \subseteq B \times A, B \times A \subseteq A \times B, A \times B = B \times A$?
 - (b) Define relation $R : A \rightarrow B, R = \{(a, y) : a \in A\}$. Which of the following are in R ?
 $(y, 1), (1, y), (x, 2), (2, x)$
2. On \mathbb{R} , find the largest domain possible for the following functions.
 - (a) $f(x) = \frac{1}{2x^2 - 5x + 3}$
 - (b) $g(x) = \sqrt{1 - x^2} + \sqrt[3]{x + 2} + \sqrt{-x}$
 - (c) $h(x) = \frac{\ln(x+5)}{\sqrt{x^2 + 3x - 28}}$
3. On \mathbb{R} , let relation R be defined $xRy \Leftrightarrow |x - y| \leq 1$. Prove that R is not an equivalence relation.
4. On \mathbb{R}^2 , let the relation R be defined $(a, b)R(c, d) \Leftrightarrow 2a - b = 2c - d$. Prove that R is an equivalence relation. Write 3 elements of $[(1, 2)]$.
5.
 - (a) Let $f : [0, 1) \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 + 1$. Show that f is one-to-one but not onto.
 - (b) Let $f : \mathbb{R} \rightarrow [0, \infty)$ be defined by $f(x) = x^4$. Show that f is onto but not one-to-one.
6.
 - (a) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) := x^3$ for $x \in \mathbb{R}$. Can the inverse function be defined? If so, what is it?
 - (b) Let $f : (0, 1) \rightarrow (0, \infty)$ be defined by $f(x) := \frac{x}{1-x}$ for $x \in (0, 1)$. The function f is bijective; show that $g : (0, \infty) \rightarrow (0, 1)$, defined by $g(x) := \frac{x}{x+1}$ is the inverse of f by demonstrating that $f(g(x)) = x$ and $g(f(x)) = x$.