Data Structures and Algorithms for competitive programming

Eoin Davey

November 2, 2018

Contents			RectInHist	8
BFS Dijkstras Trees MST LCA Centroid Decomposition SCC Tarjans AP & Bridges Network Flow Edmond Karp Max Flow	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	6	Maths Miller Rabin	8 8
2 Data Structures Fenwick Tree UFDS Sparse Table Segment Tree	$\begin{array}{cccccccccccccccccccccccccccccccccccc$			
3 Geometry Convex Hull				
4 Strings Suffix Array				
5 Algorithms Inversion Count				

1 Graphs

Traversal

\mathbf{BFS}

```
int dist[MXN];
   vector < int > adjList [MX.N];
3
   int main(){
 5
        for(int i = 0; i < MX_N; i++)
 6
            dist[i]=INF;
        queue<int> q;
 8
        q.push(0);
9
        dist[0] = 0;
10
        while (!q.empty()) {
            int u = q.front(); q.pop();
11
12
            int d = dist[u];
            for(int i : adjList[u]){
13
                if (dist [i]==INF) {
14
                     dist[i]=d+1;
15
16
                    q.push(i);
17
18
19
20
        return 0;
   Dijkstras
   struct path {
2
        int u.d:
3
        path(int _u, int _d) : u(_u), d(_d) {}
        path(){}
        bool operator < (const path& p) const {
 6
            return d > p.d;
 8
    };
    for (int i =0; i < N; ++i)
        dist[i] = INF;
10
11
   dist[S] = 0;
   priority_queue<path> q;
   q. push (path(S,0));
   while (!q.empty()) {
15
        path p = q.top(); q.pop();
16
        u = p.u, d = p.d;
17
        if(dist[u] < d)
18
            continue:
19
        for(auto v : adjList[u]){
20
            nd = d + v.second;
21
            if(nd < dist[v.first])
22
                dist[v.first] = nd;
23
                q.push(path(v.first,nd));
24
```

```
25
26 }
    Trees
   MST
1
   struct edge {
        int x, y, w;
3
        bool operator < (edge e) const {
4
            return w < e.w;
5
6
    };
8
   int main(){
9
        vector<edge> eList; //Input
        for(int i = 0; i < N; i++)// Set up UFDS
10
            p[i]=i;
11
12
        vector<ii> treeList;
13
        sort(eList.begin(),eList.end());
14
        int cost = 0;
15
        int sz=N;
16
        int u,v,w;
17
        for (const auto &i : eList) {
18
            v=i.x; u=i.y; w=i.w;
19
            if (!connected(u,v)){
20
                join (u,v);
21
                treeList.push\_back(\{min(u,v),max(u,v)\});
22
                sz --;
23
                cost+=w;
24
25
26
        if(sz!=1)
27
            puts("Impossible");
28
   LCA
    *H/u is first visit of u
    *E[x] is vertex at time x
    *L/x/ is depth at time x
5
6
   void vis(int u, int d){
       H[u] = vind;
8
       E[vind] = u;
9
        L[vind++] = d;
10
        for(auto i : adjList[u]){
11
            if(H[i]!=-1)
12
                continue;
13
            vis(i,d+1);
14
            E[vind] = u;
15
            L[vind++] = d;
16
17 }
```

```
18
   int LCA(int u, int v){
20
        \mathbf{if}(H[u] > H[v])
21
            int t = u:
22
            u = v;
23
            v = t;
24
25
        //run some range min query on L
26
        //between H[u] and H[v]
27
        int ind = rmq(H[u], H[v]);
28
        return E[ind];
29
30
31
   int dist(int u, int v){
32
        int a = H[u];
33
        int b = H[v];
34
        int ind = LCA(u, v);
35
        return abs(L[H[ind]]-L[a])
36
            + abs(L[H[ind]]-L[b]);
37
   Centroid Decomposition
   void fill_sz(int u, int p){
2
        sz[u] = 1;
3
        for(int v : adjList[u]) {
            if (v==p || mkd[v])
                continue;
5
 6
            fill_sz(v,u);
            sz[u]+=sz[v];
8
9
10
   int get_centroid(int u, int n, int p){
11
12
        for(int v : adjList[u]){
            if (v==p || mkd[v])
13
14
                continue;
15
            if(sz[v] > n/2)
16
                return get_centroid(v, n, u);
17
18
        return u;
19
20
21
   int decomp(int u){
22
        fill_sz(u, -1);
23
        int cent = get_centroid(u, sz[u], -1);
24
        mkd[cent] = true;
25
        for(int v : adjList[cent]){
26
            if (mkd [v])
27
                continue;
28
            int r = decomp(v);
29
            centP[r] = cent;
30
31
        return cent;
32
```

SCC Tarjans

```
stack<int> scc:
    int dfsCounter=1;
    int sccIdx=1:
    map<int, int> sccMap;
5
    void tarjans(int u){
         scc.push(u);
8
         vis [u]=true;
9
10
         dfs_low[u] = dfs_num[u] = dfsCounter++;
11
12
         for (int i = 0; i < adjList[u].size(); i++)
13
             int v = adjList[u][i];
14
             if(dfs_num[v]==0)
15
                  tarjans (v);
                  dfs_low[u]=min(dfs_low[u],dfs_low[v]);
16
17
             } else if(vis[v]){
                  dfs_low[u]=min(dfs_low[u],dfs_num[v]);
18
19
20
21
         \mathbf{if}(dfs_low[u] = dfs_num[u])
22
             while(1){
23
                  int v = scc.top(); scc.pop();
24
                  \operatorname{sccMap}[v] = \operatorname{sccId}x;
25
                  vis[v] = false;
26
                  i f ( v==u )
27
                      break;
28
29
             sccIdx++;
30
31
```

AP & Bridges

```
int dfs(int u,int p){
2
        dfs_num[u] = dfs_low[u] = ++dfs_counter;
3
        for(auto v : adjList[u]){
4
            if(dfs_num[v]==0)
                dfs(v,u);
                if(dfs_low[v]) = dfs_num[u])
                    articulation [u]=true;
8
9
                if(dfs_low[v] > dfs_num[u])
10
                    bridge = true;
11
                dfs_low[u] = min(dfs_low[u], dfs_low[v]);
12
            else if(v!=p)
13
                dfs_low[u] = min(dfs_low[u], dfs_num[v]);
14
15 }
```

Network Flow

Edmond Karp Max Flow

```
1 void aug(int u, int minE) {
        if(u==S){ f=minE; return; }
3
        if(p[u]!=u){
             \operatorname{aug}(p[u], \min(\min E, \operatorname{res}[p[u]][u]));
 4
             res[p[u]][u]-=f;
 6
             res[u][p[u]]+=f;
 8
   int main(){
11
        int mf=0;
12
        for (;;) {
             f=0;//Global
13
14
             for (int i = 0; i < N; i++)
15
                  dist[i]=INF, p[i]==i;
16
             dist[S]=0;
             queue<int> q; q.push(S);
17
             while (!q.empty()) {
18
19
                 int u = q. front(); q. pop();
20
                 if(u=T) break;
21
                 for (int i = 0; i < N; i++)
                      if(res[u][i] > 0 \&\& dist[i] == INF)
22
                           \operatorname{dist}[i] = \operatorname{dist}[u] + 1, p[i] = u, q.\operatorname{push}(i);
23
24
25
             aug(T, INF);
26
             if(f==0) break;
27
             mf+=f;
28
        }
29
        vector<ii> used;
        for (int i = 0; i < N; i++)
30
31
             for (int j = 0; j < N; j++)
                 if (graph [i][j] > 0 && res[i][j] < graph [i][j])
32
33
                      used.push_back(make_pair(i,j));
34
    Ford Fulkerson Max Flow
 1 int ff(int u, int minE){
 2
        i f ( u==T)
 3
             return minE;
        vis [u]=true;
 4
 5
        for(auto i : adjList[u]){
             if(!vis[i] \&\& res[u][i] > 0)
                  if(int f = ff(i, min(minE, res[u][i])))
                      res[u][i] -= f;
                      res[i][u] += f;
10
                      return f;
11
12
13
14
        return 0;
```

```
15
16
17
   int main(){
        int mf = 0;
18
19
        while(1){
20
            memset(vis,0,sizeof(vis));
21
            int f = ff(S, INF);
22
            if(f==0)
23
                break;
24
            mf+=f;
25
26
        printf("%d\n",mf);
27 }
```

2 Data Structures

13 for (int j = 1; (1 << j) <= N; j++)

Fenwick Tree

```
1 int tree [MX_N];
2 int N;
  int lsOne(int i) \{ return i \& (-i); \}
   void update(int k,int v){
        for (; k<MX_N; k+=lsOne(k))
6
            tree[k]+=v;
7
   int query(int k){
8
        int cnt=0;
10
        for (; k; k-=lsOne(k)) {
11
            cnt+=tree[k];
12
13
        return cnt;
14 }
   UFDS
1 int find(int u) { return p[u] = (p[u] = u ? u : find(p[u])); }
3
   inline void join (int a, int b) {
        pa = find(a);
        pb = find(b);
6
        if (pa!=pb) {
            if(rank[pa] < rank[pb])
8
                ni = pb;
9
                pb = pa;
10
                pa = ni;
11
12
            p[pb] = pa;
            if (rank [pa]==rank [pb])
13
14
                rank[pa]++;
15
16
   Sparse Table
   inline int rmq(int u, int v){
2
        if(u > v)
3
            return -20000000000;
4
        int k=(int) floor(log2((double)(v-u+1)));
5
        if (r [mtable [u] [k]] >
6
                r [ mtable [ v-(1 << k) + 1 ] [k] ] )
            return mtable[u][k];
8
        return mtable [v-(1 << k) + 1][k];
9
10
   for (int i = 0; i < N; i++)
11
        mtable[i][0] = i;
```

```
14
        for (int i = 0; i + (1 << j) - 1 < N; ++i)
15
             if (r [mtable [i] [j−1]]
16
                     >r [ mtable [ i+(1<<(j-1)) ] [ j-1] ])
17
                 mtable[i][j] = mtable[i][j-1];
18
             else
19
                 mtable[i][j]=mtable[i+(1<<(j-1))][j-1];
    Segment Tree
1 int tree [MX_N*4];
   int a [MX.N];
3
   int N;
    void construct (int p, int L, int R) {
6
        if (L=R) {
7
             tree[p] = a[L];
8
             return:
9
        if (R<L)
10
11
             return;
12
        int md = (L+R)/2;
13
        construct (2*p,L,md);
14
        construct(2*p+1,md+1,R);
15
        tree[p] = min(tree[2*p], tree[2*p+1]);
16
17
18
    void update(int p, int L, int R, int ind, int v){
19
        if (L=R) {
20
             a[ind] = v;
21
             tree[p] = v;
22
             return:
23
24
        int md = (L+R)/2;
25
        if (ind \ll md)
26
             update(2*p,L,md,ind,v);
27
28
             update(2*p+1,md+1,R,ind,v);
        tree[p] = min(tree[2*p], tree[2*p+1]);
29
30
31
32
    int rmq(int p, int L, int R, int l, int r){
        if(r < L \mid \mid l > R)
33
34
             return INF;
35
        if(l>=L && r<=R)
36
             return tree[p];
37
        int md = (l+r)/2;
38
        return \min(\text{rmq}(2*p, L, R, l, md), \text{rmq}(2*p+1, L, R, md+1, r));
39 }
```

3 Geometry

Convex Hull

```
int main() {
2
        for (int i = 0; i < N; i++){
3
            perm[i]=i;
4
5
        sort (perm, perm+N,
6
                 [](int a, int b){
                     const point &pa = V[a];
                     const point &pb = V[b];
9
                     if (real (pa)!=real (pb))
                         return real(pa) < real(pb);
10
11
                     return imag(pa) < imag(pb);
12
13
        vector<int> L; vector<int> U;
14
        for (int i = 0; i < N;) {
15
            int t = L. size();
16
            if(t \ge 2 \&\& ! ccw(V[L[t-2]],V[L[t-1]],V[perm[i]]))
17
                 L. pop_back();
18
            else
19
                 L. push_back(perm[i++]);
20
21
        for (int i = N-1; i >=0;) {
22
            int t = U. size();
23
            if(t \ge 2 \&\& ! ccw(V[U[t-2]],V[U[t-1]],V[perm[i]]))
24
                 U.pop_back();
25
            else
26
                 U. push_back (perm [i --]);
27
28
        vector<int> hull;
29
        for (int i = 0; i < L. size() -1; ++i)
            hull.push_back(L[i]);
30
31
        for (int i = 0; i < U. size() -1; ++i)
32
             hull.push_back(U[i]);
33
        return 0;
34
```

Geometry Axioms

```
typedef complex<double> pt;
   typedef complex<double> vec;
   typedef vector <pt> pgon;
   typedef struct { pt p,q; } lseg;
   double cross (const vec& a, const vec &b) {
6
        return x(a)*y(b)-y(a)*x(b);
7
    //cross\ product\ of\ (b-a)\ and\ (c-b), 0 is collinear
   int orientation (const pt& a,
10
            const pt& b, const pt& c){
        double v = cross(b-a, c-b);
11
12
        if (abs (v-0.0) < EPS)
13
            return 0;
```

```
14
        return v > 0 ? 1 : 2;
15 }
16
    //Line segment intersection
17
    bool intersects (const lseg& a, const lseg& b) {
18
        if(a.q = b.p | | b.q = a.p)
19
            return false;
20
        if (orientation (a.p,a.q,b.p)
21
                != orientation (a.p, a.q, b.q)
22
                && orientation(b.p,b.q,a.p)
23
                 != orientation(b.p,b.q,a.q))
24
            return true;
25
        return false;
26
27
    //Area of polygon
28
    double area (const pgon& p) {
29
        double area = 0.0;
30
        for(int i = 1; i < p.size(); ++i)
31
            area+=cross(p[i-1],p[i]);
32
        return abs(area)/2.0;
33
    //If a \rightarrow b \rightarrow c is a counterclockwise turn
34
35
    double ccw(const point& a, const point& b,
36
            const point& c){
37
        if (a==b | b==c | a==c)
38
            return false;
39
        point relA = b-a:
40
        point relC = b-c;
41
        return cross(relA, relC) >= 0.0:
42
43
    //Returns if point p is in the polygon poly
    bool in Poly (const pgon& poly, const pt& p) {
45
        for (int i = 0; i < poly.size()-1; i++){
46
             if (! ccw ( poly [ i ] , p , poly [ i +1]))
47
                 return false;
48
49
        return true;
50
    //Distance from p to line (a,b)
51
    double distToLine(const pt& p, const pt& a,
53
            const pt &b){
54
        vec ap = p-a;
55
        vec ap = b-a;
56
        double u = dot(ap, ab)/dot(ab, ab);
57
        //Ignore for non-line segment
58
        if (u < 0.0) //Closer to a
59
            return abs(a-p);
60
        if(u > 1.0) //Closer to b
61
            return abs(b-p);
62
        pt c = a+ab*u; // This is the point
63
        return abs(c-p);
64 }
```

4 Strings

Suffix Array

```
void countingSort(int k){
2
        int i, sum, maxi=max(300, N);
 3
        memset(c, 0, sizeof(c));
 4
        for (i = 0; i < N; i++)
 5
            c[i+k < N ? RA[i+k] : 0]++;
        for (i=sum=0; i < maxi; i++)
            int t = c[i];
 8
            c[i]=sum;
9
            sum+=t;
10
11
        for (i = 0; i < N; i++)
            tempSA [c [SA [i]+k < N
12
                 ? RA[SA[i]+k]: 0]++] = SA[i];
13
14
        for (i = 0; i < N; i++)
15
            SA[i] = tempSA[i];
16
17
18
   int main(){
19
        for (int i = 0; i < N; i++)
20
            SA[i]=i,RA[i]=input[i];
21
22
        for (int k = 1; k < N; k <<= 1) {
23
            countingSort(k);
24
            countingSort(0);
25
            tempRA[SA[0]] = r = 0;
26
            for (int i = 1; i < N; i++){
27
                 tempRA[SA[i]]
28
                     =(RA[SA[i]]==RA[SA[i-1]]
29
                     && RA[SA[i]+k] = RA[SA[i-1]+k]
30
                     ? r:++r);
31
32
            for (int i = 0; i < N; i++)
33
                 RA[i] = tempRA[i];
34
35
        return 0;
36
    Trie
   struct node {
        node * children [26];
 3
        int count;
 4
        node(){
 5
            memset(children, 0, sizeof(children));
 6
            count=0;
 8
    };
   void insert(node* nd, char *s){
11
        if (*s) {
```

```
12
              if (!nd->children [*s-'a'])
13
                  nd \rightarrow children [*s-'a'] = new node();
14
              insert (nd \rightarrow children [*s-'a'], s+1);
15
16
         nd \rightarrow count + +;
17
18
19
    int count(node* nd, char *s){
20
         if(*s){
21
              if (!nd->children[*s-'a'])
22
                  return 0:
23
              return count (nd\rightarrowchildren [*s-'a'], s+1);
24
         } else {
25
              return nd->count;
26
27
28
29
    int main(){
30
         node * trie = new node();
         int N; scanf("%d",&N);
31
32
         \mathbf{char} * \mathbf{buff} = \mathbf{new} \mathbf{char} [40];
33
         for (int i = 0; i < N; i++){
34
              scanf("%s", buff);
35
              printf("%d\n", count(trie, buff));
36
              insert (trie, buff);
37
38
         return 0;
39
    KMP
    vector < int > build Failure (string s) {
2
         vector < int > T(n+1,0);
3
         T[0] = -1;
4
         int j = 0;
         for (int i = 1; i < s.size();++i){
              if (s[i]==s[j]) {
7
                  T[i]=T[j];
8
                  i++;
9
              } else{
10
                  T[i] = j;
11
                  j = T[j];
12
                  while (j >= 0 \&\& s[i]! = s[j])
13
                       j = T[j];
14
                  j++;
15
16
17
         T[s.size()] = j;
18
         return T:
19
20
    vector < int > search (string W, string S) {
21
         auto T=buildFailure(W);
22
         vector <int> p;
23
         int k = 0;
24
         int j = 0;
```

```
25
        while(j < S.size()){
26
            if(W[k]==S[j])
27
                k++; j++;
28
                if (k=-W. size ()) {
29
                    p.push_back(j-k);
30
                     k = T[k];
31
32
            }else{
33
                k = T[k];
34
                if(k < 0)
35
                    j+=1, k+=1;
36
37
38
        return p;
39
```

Algorithms

Inversion Count

```
1 int N;
   int a [MX.N];
    long long cnt=0;
4
5
    void mergesort (int L, int R) {
        i f (L>=R)
6
7
            return;
8
        int mid = (L+R)/2;
9
        mergesort (L, mid);
        mergesort (mid+1,R);
10
11
        int n[R-L+1];
12
        int i = 0;
13
        int lp = L;
14
        int rp = mid+1;
        while(rp<=R || lp<=mid){
15
16
            if (rp<=R && lp <= mid) {
17
                 if (a[rp]<a[lp]) {
18
                     n[i]=a[rp];
19
                     rp++;
20
                     cnt += ((long long) (mid-lp+1));
21
22
                     n[i]=a[lp], lp++;
23
            } else if (rp <= R) {
24
                 n[i]=a[rp++];
25
            } else {
26
                 n[i]=a[lp++];
27
28
            i++;
29
30
        for (int j = L; j \ll R; j++)
31
            a[j]=n[j-L];
32
```

NlogN LIS

```
1 int ls[MX_N];
2 int L[MX_N];
3
   int I [MX.N];
4
    void nlogn(){
6
        for (int i = 1; i < N+1; ++i)
            I[i]=INF;
7
8
        I[0] = -INF;
9
        int mx = 1;
10
        for (int i = 0; i < N; ++i){
            int ind = lower_bound(I, I+N+1, ls[i]) - I;
11
12
            I[ind] = ls[i];
13
            L[i] = ind;
14
            mx = max(mx, ind);
15
```

```
int prv = INF;
16
17
        vector < int > out;
18
        for (int i = N-1; i >= 0; --- i) {
            if(ls[i] < prv && L[i]==mx){
19
20
                out.push_back(ls[i]);
21
                prv = ls[i];
22
                mx--;
23
24
25
   RectInHist
   int R,C;
   char board[MX_RC][MX_RC];
   int h[MX_RC][MX_RC];
5
   int perim(int 1, int w){
6
        if ( l==0 || w==0)
7
            return 0;
8
        return 2*l + 2*w;
9
10
11
   int main(){
12
        for (int i = 0; i < R; i++){
13
            int run=0;
14
            for (int j = 0; j < C; j++){
                run = (board[i][j] == '. '?run + 1:0);
15
16
                h[i][j] = run;
17
18
19
        int mx = 0;
20
        for (int j = 0; j < C; j++){
21
            stack<int> s;
22
            for (int i = 0; i < R; i++){
23
                 if (s.empty()
24
                         | | h [ i ] [ j] > h [ s . top () ] [ j ] )
25
                     s.push(i);
26
                 else if(h[i][j]<h[s.top()][j]){
27
                     while (!s.empty()
28
                     &&h[i][j]<h[s.top()][j]){
                         int l = h[s.top()][j];
29
30
                         s.pop();
31
                         int pm = perim(1,
32
                             (s.empty()?
33
                              i:i-s.top()-1));
34
                         mx = max(mx,pm);
35
                     }
36
                     s.push(i);
                } else if(h[i][j]==h[s.top()][j]){
37
38
                     s.pop();
39
                     s.push(i);
40
41
42
            while (!s.empty()) {
```

6 Maths

Miller Rabin

```
void factor(ll x, ll& e, ll& k){
       while(x%2LL==0LL){
2
3
            x/=2LL;
4
           ++e;
5
6
       k = x;
7
   //increase x for higher certainty, 5 works well
   bool is_prime(ll n, int x){
10
       if (n&2LL==0 n==1LL)
11
12
            return false;
13
       if (n==2 || n==3 || n==5 || n==7)
14
            return true;
15
       ll e, k;
16
       factor (n-1,e,k);
17
       while (x-->0){
            11 a = (rand())\%(n-5LL) + 2LL;
18
19
            ll p = mod_exp(a,k,n);
20
            if (p==1LL || p==n-1LL)
21
                continue;
22
            bool all_fail = true;
            for (int i = 0; i < e-1; ++i) {
23
24
                p = mod_exp(p, 2, n);
25
                if(p=n-1LL)
                    all_fail = false;
26
27
                    break;
28
29
30
            if (all_fail)
31
                return false;
32
33
       return true;
34 }
```