

Data Structures and Algorithms for competitive programming

Eoin Davey

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1 Graphs

Traversal

BFS

```
1  int dist[MXN];
2  vector<int> adjList[MXN];
3
4  int main(){
5      for(int i = 0; i < MXN; i++)
6          dist[i]=INF;
7      queue<int> q;
8      q.push(0);
9      dist[0] = 0;
10     while(!q.empty()){
11         int u = q.front(); q.pop();
12         int d = dist[u];
13         for(int i : adjList[u]){
14             if(dist[i]==INF){
15                 dist[i]=d+1;
16                 q.push(i);
17             }
18         }
19     }
20     return 0;
21 }
```

Dijkstras

```
1  struct path {
2      int u,d;
3      bool operator < (const path& p) const {
4          return d > p.d;
5      }
6  };
7  for(int i =0; i < N; ++i)
8      dist[i] = INF;
9  dist[S] = 0;
10 priority_queue<path> q;
11 q.push(path{S,0});
12 while(!q.empty()){
13     path p = q.top(); q.pop();
14     u = p.u,d = p.d;
15     if(dist[u] < d)
16         continue;
17     for(auto v : adjList[u]){
18         nd = d + v.second;
19         if(nd < dist[v.first]){
20             dist[v.first] = nd;
21             q.push(path{v.first ,nd});
22         }
23     }
24 }
```

Trees

MST

```
1  struct edge {
2      int x,y,w;
3      bool operator < (edge e) const {
4          return w < e.w;
5      }
6  };
7
8  int main(){
9      vector<edge> eList; //Input
10     for(int i = 0; i < N; i++)// Set up UFDS
11         p[i]=i;
12     vector<ii> treeList;
13     sort(eList.begin(),eList.end());
14     int cost = 0;
15     int sz=N;
16     int u,v,w;
17     for(const auto &i : eList){
18         v=i.x; u=i.y; w=i.w;
19         if(!connected(u,v)){
20             join(u,v);
21             treeList.push_back({min(u,v),max(u,v)});
22             sz--;
23             cost+=w;
24         }
25     }
26     if(sz!=1)
27         puts("Impossible");
28 }
```

LCA

```
1  /*
2   * H[u] is first visit of u
3   * E[x] is vertex at time x
4   * L[x] is depth at time x
5   */
6  void vis(int u, int d){
7      H[u]=vind;
8      E[vind] = u;
9      L[vind++] = d;
10     for(auto i : adjList[u]){
11         if(H[i]!=-1)
12             continue;
13         vis(i,d+1);
14         E[vind] = u;
15         L[vind++] = d;
16     }
17 }
18
19 int LCA(int u, int v){
20     if(H[u] > H[v]){
```

```

21     int t = u;
22     u = v;
23     v = t;
24 }
25 //run some range min query on L
26 //between H[u] and H[v]
27 int ind = rmq(H[u], H[v]);
28 return E[ind];
29 }
30
31 int dist(int u, int v){
32     int a = H[u];
33     int b = H[v];
34     int ind = LCA(u, v);
35     return abs(L[H[ind]] - L[a])
36         + abs(L[H[ind]] - L[b]);
37 }

```

Centroid Decomposition

```

1 void fill_sz(int u, int p){
2     sz[u] = 1;
3     for(int v : adjList[u]){
4         if(v==p || mkd[v])
5             continue;
6         fill_sz(v, u);
7         sz[u] += sz[v];
8     }
9 }
10
11 int get_centroid(int u, int n, int p){
12     for(int v : adjList[u]){
13         if(v==p || mkd[v])
14             continue;
15         if(sz[v] > n/2)
16             return get_centroid(v, n, u);
17     }
18     return u;
19 }
20
21 int decomp(int u){
22     fill_sz(u, -1);
23     int cent = get_centroid(u, sz[u], -1);
24     mkd[cent] = true;
25     for(int v : adjList[cent]){
26         if(mkd[v])
27             continue;
28         int r = decomp(v);
29         centP[r] = cent;
30     }
31     return cent;
32 }

```

SCC Tarjans

```

1 stack<int> scc;
2 int dfsCounter=1;
3 int sccIdx=1;
4 map<int, int> sccMap;
5
6 void tarjans(int u){
7     scc.push(u);
8     vis[u]=true;
9
10    dfs_low[u]=dfs_num[u]=dfsCounter++;
11
12    for(int i = 0; i < adjList[u].size(); i++){
13        int v = adjList[u][i];
14        if(dfs_num[v]==0){
15            tarjans(v);
16            dfs_low[u]=min(dfs_low[u], dfs_low[v]);
17        } else if(vis[v]){
18            dfs_low[u]=min(dfs_low[u], dfs_num[v]);
19        }
20    }
21    if(dfs_low[u]==dfs_num[u]){
22        while(1){
23            int v = scc.top(); scc.pop();
24            sccMap[v]=sccIdx;
25            vis[v]=false;
26            if(v==u)
27                break;
28        }
29        sccIdx++;
30    }
31 }

```

AP & Bridges

```

1 int dfs(int u, int p){
2     dfs_num[u] = dfs_low[u] = ++dfs_counter;
3     for(auto v : adjList[u]){
4         if(dfs_num[v]==0){
5             dfs(v, u);
6             if(dfs_low[v] >= dfs_num[u]){
7                 articulation[u]=true;
8             }
9             if(dfs_low[v] > dfs_num[u])
10                bridge = true;
11            dfs_low[u] = min(dfs_low[u], dfs_low[v]);
12        } else if(v!=p)
13            dfs_low[u] = min(dfs_low[u], dfs_num[v]);
14    }
15 }

```

Network Flow

Edmond Karp Max Flow

```

1 void aug(int u, int minE){
2     if(u==S){ f=minE; return; }
3     if(p[u]!=u){
4         aug(p[u], min(minE, res[p[u]][u]));
5         res[p[u]][u]-=f;
6         res[u][p[u]]+=f;
7     }
8 }
9
10 int main(){
11     int mf=0;
12     for(;;){
13         f=0;//Global
14         for(int i = 0; i < N; i++){
15             dist[i]=INF, p[i]==i;
16             dist[S]=0;
17             queue<int> q; q.push(S);
18             while(!q.empty()){
19                 int u = q.front(); q.pop();
20                 if(u==T) break;
21                 for(int i = 0; i < N; i++){
22                     if(res[u][i] > 0 && dist[i]==INF)
23                         dist[i]=dist[u]+1, p[i]=u, q.push(i);
24                 }
25                 aug(T, INF);
26                 if(f==0) break;
27                 mf+=f;
28             }
29             vector<ii> used;
30             for(int i = 0; i < N; i++){
31                 for(int j = 0; j < N; j++){
32                     if(graph[i][j] > 0 && res[i][j] < graph[i][j])
33                         used.push_back(make_pair(i, j));
34             }

```

Ford Fulkerson Max Flow

```

1 int ff(int u, int minE){
2     if(u==T)
3         return minE;
4     vis[u]=true;
5     for(auto i : adjList[u]){
6         if(!vis[i] && res[u][i] > 0){
7             if(int f = ff(i, min(minE, res[u][i]))){
8                 res[u][i] -= f;
9                 res[i][u] += f;
10                return f;
11            }
12        }
13    }
14    return 0;
15 }
16
17 int main(){
18     int mf = 0;

```

```

19     while(1){
20         memset(vis, 0, sizeof(vis));
21         int f = ff(S, INF);
22         if(f==0)
23             break;
24         mf+=f;
25     }
26     printf("%d\n", mf);
27 }

```

2 Data Structures

Fenwick Tree

```
1 int tree[MXN];
2 int N;
3 int lsOne(int i){ return i&(-i); }
4 void update(int k,int v){
5     for(; k<MXN; k+=lsOne(k))
6         tree[k]+=v;
7 }
8 int query(int k){
9     int cnt=0;
10    for(; k; k-=lsOne(k)){
11        cnt+=tree[k];
12    }
13    return cnt;
14 }
```

UFDS

```
1 int find(int u){ return p[u] = (p[u] == u ? u : find(p[u])); }
2
3 inline void join(int a, int b){
4     pa = find(a);
5     pb = find(b);
6     if(pa!=pb){
7         if(rank[pa] < rank[pb]){
8             ni = pb;
9             pb = pa;
10            pa = ni;
11        }
12        p[pb] = pa;
13        if(rank[pa]==rank[pb])
14            rank[pa]++;
15    }
16 }
```

Sparse Table

```
1 inline int rmq(int u, int v){
2     if(u > v)
3         return -2000000000;
4     int k=(int) floor(log2((double)(v-u+1)));
5     if(r[mtable[u][k]] >
6        r[mtable[v-(1<<k)+1][k]])
7         return mtable[u][k];
8     return mtable[v-(1<<k) + 1][k];
9 }
10
11 for(int i = 0; i < N; i++)
12     mtable[i][0] = i;
13 for(int j = 1; (1 << j) <= N; j++)
```

```
14     for(int i = 0; i + (1<<j)-1 < N; ++i)
15         if(r[mtable[i][j-1]]
16            >r[mtable[i+(1<<(j-1))][j-1]])
17             mtable[i][j]= mtable[i][j-1];
18     else
19         mtable[i][j]=mtable[i+(1<<(j-1))][j-1];
```

Segment Tree

```
1 int tree[MXN*4 + 2];
2 int a[MXN];
3 int N;
4
5 void construct(int p, int L, int R){
6     if(L==R){
7         tree[p] = a[L];
8         return;
9     }
10    if(R<L)
11        return;
12    int md = (L+R)/2;
13    construct(2*p,L,md);
14    construct(2*p+1,md+1,R);
15    tree[p] = min(tree[2*p], tree[2*p+1]);
16 }
17
18 void update(int p, int L, int R, int ind,int v){
19     if(L==R){
20         a[ind] = v;
21         tree[p] = v;
22         return;
23     }
24    int md = (L+R)/2;
25    if(ind <= md)
26        update(2*p,L,md,ind,v);
27    else
28        update(2*p+1,md+1,R,ind,v);
29    tree[p] = min(tree[2*p], tree[2*p+1]);
30 }
31
32 int rmq(int p, int L, int R, int l, int r){
33     if(r < L || l > R)
34         return INF;
35     if(l>=L && r<=R)
36         return tree[p];
37     int md = (l+r)/2;
38     return min(rmq(2*p,L,R,l,md),rmq(2*p+1,L,R,md+1,r));
39 }
```

3 Geometry

Convex Hull

```
1 int main(){
2     for(int i = 0; i < N; i++){
3         perm[i]=i;
4     }
5     sort(perm,perm+N,
6         [](int a, int b){
7             const point &pa = V[a];
8             const point &pb = V[b];
9             if(real(pa)!=real(pb))
10                return real(pa) < real(pb);
11            return imag(pa) < imag(pb);
12        });
13     vector<int> L; vector<int> U;
14     for(int i = 0; i < N;){
15         int t = L.size();
16         if(t >= 2 && !ccw(V[L[t-2]],V[L[t-1]],V[perm[i]]))
17             L.pop_back();
18         else
19             L.push_back(perm[i++]);
20     }
21     for(int i = N-1; i >=0;){
22         int t = U.size();
23         if(t >= 2 && !ccw(V[U[t-2]],V[U[t-1]],V[perm[i]]))
24             U.pop_back();
25         else
26             U.push_back(perm[i--]);
27     }
28     vector<int> hull;
29     for(int i = 0; i < L.size()-1; ++i)
30         hull.push_back(L[i]);
31     for(int i = 0; i < U.size()-1; ++i)
32         hull.push_back(U[i]);
33     return 0;
34 }
```

Geometry Axioms

```
1 typedef complex<double> pt;
2 typedef complex<double> vec;
3 typedef vector<pt> pgon;
4 typedef struct { pt p,q; } lseg;
5 struct circ{ pt c; double r; };
6 struct rect{ pt p,q;}; //  $X(p) \leq X(q)$  and  $Y(p) \leq Y(q)$ 
7
8 double cross(const vec& a, const vec &b){
9     return x(a)*y(b)-y(a)*x(b);
10 }
11 //cross product of (b-a) and (c-b), 0 is collinear
12 int orientation(const pt& a,
13     const pt& b, const pt& c){
```

```
14     double v = cross(b-a,c-b);
15     if(abs(v-0.0)<EPS)
16         return 0;
17     return v > 0 ? 1 : 2;
18 }
19 //Line segment intersection
20 bool intersects(const lseg& a, const lseg& b){
21     if(a.q == b.p || b.q == a.p)
22         return false;
23     if(orientation(a.p,a.q,b.p)
24         !=orientation(a.p,a.q,b.q)
25         && orientation(b.p,b.q,a.p)
26         != orientation(b.p,b.q,a.q))
27         return true;
28     return false;
29 }
30 //Area of polygon
31 double area(const pgon& p){
32     double area = 0.0;
33     for(int i = 1; i < p.size(); ++i)
34         area+=cross(p[i-1],p[i]);
35     return abs(area)/2.0;
36 }
37 //If a->b->c is a counterclockwise turn
38 double ccw(const point& a, const point& b,
39     const point& c){
40     if(a==b || b==c || a==c)
41         return false;
42     point relA = b-a;
43     point relC = b-c;
44     return cross(relA,relC) >= 0.0;
45 }
46 //Returns if point p is in the polygon poly
47 bool inPoly(const pgon& poly, const pt& p){
48     for(int i = 0; i < poly.size()-1; i++){
49         if(!ccw(poly[i],p,poly[i+1]))
50             return false;
51     }
52     return true;
53 }
54 //Distance from p to line (a,b)
55 double distToLine(const pt& p, const pt& a,
56     const pt &b){
57     vec ap = p-a;
58     vec ab = b-a;
59     double u = dot(ap,ab)/dot(ab,ab);
60     //Ignore for non-line segment
61     if(u < 0.0) //Closer to a
62         return abs(a-p);
63     if(u > 1.0) //Closer to b
64         return abs(b-p);
65     pt c = a+ab*u; // This is the point
66     return abs(c-p);
67 }
68 //intersection pts of two circles
```

```

69 vector<pt> insct(const circ& a, const circ& b){
70     vector<pt> o;
71     double dist = abs(a.c - b.c);
72     if(dist > a.r + b.r)
73         return o; //none, don't touch
74     if(abs(a.r-b.r) > dist)
75         return o; //none, inside
76     if(abs(dist - (a.r + b.r)) < EPS){ // one intersect
77         pt p = a.c + (a.r/dist)*(b.c-a.c);
78         o.pb(p);
79         return o;
80     }
81     double delta = (sq(dist) + (a.r-b.r)*(a.r+b.r))/(2.0*dist);
82     pt cent = a.c + (delta/dist)*(b.c-a.c);
83
84     double h = sqrt(sq(a.r) - sq(delta));
85
86     pt dVec = (b.c - a.c)/dist;
87     o.pb(cent + h*pt(0,1)*dVec);
88     o.pb(cent + h*pt(0,-1)*dVec);
89     return o;
90 }
91
92 // intersection of two rectangles, sets none to true if no overlap
93 rect overlap(const rect& a, const rect& b, bool& none){
94     rect r;
95     if(X(a.p) > X(b.q) || Y(a.p) > Y(b.q)
96        || X(b.p) > X(a.q) || Y(b.p) > Y(a.q)){
97         none=true;
98         return r;
99     }
100     r.p = {max(X(a.p),X(b.p)), max(Y(a.p),Y(b.p))};
101     r.q = {min(X(a.q),X(b.q)), min(Y(a.q),Y(b.q))};
102     return r;
103 }

```

4 Strings

Suffix Array

```

1 void countingSort(int k){
2     int i, sum, maxi=max(300,N);
3     memset(c,0,sizeof(c));
4     for(i = 0; i < N; i++)
5         c[i+k < N ? RA[i+k] : 0]++;
6     for(i=sum=0; i < maxi; i++){
7         int t = c[i];
8         c[i]=sum;
9         sum+=t;
10    }
11    for(i = 0; i < N; i++)
12        tempSA[c[SA[i]+k < N
13                ? RA[SA[i]+k] : 0]++] = SA[i];
14    for(i=0; i < N; i++)
15        SA[i]=tempSA[i];
16 }
17
18 int main(){
19     for(int i = 0; i < N; i++)
20         SA[i]=i, RA[i]=input[i];
21     int r;
22     for(int k = 1; k < N; k <= 1){
23         countingSort(k);
24         countingSort(0);
25         tempRA[SA[0]]=r=0;
26         for(int i = 1; i < N; i++){
27             tempRA[SA[i]]
28                 =(RA[SA[i]]==RA[SA[i-1]]
29                  && RA[SA[i]+k]==RA[SA[i-1]+k]
30                  ? r:++r);
31         }
32         for(int i = 0; i < N; i++)
33             RA[i]=tempRA[i];
34     }
35     return 0;
36 }

```

Trie

```

1 struct node {
2     node * children[26];
3     int count;
4     node(){
5         memset(children,0,sizeof(children));
6         count=0;
7     }
8 };
9
10 void insert(node* nd, char *s){
11     if(*s){

```

```

12         if(!nd->children[*s-'a'])
13             nd->children[*s-'a']=new node();
14         insert(nd->children[*s-'a'],s+1);
15     }
16     nd->count++;
17 }
18
19 int count(node* nd, char *s){
20     if(*s){
21         if(!nd->children[*s-'a'])
22             return 0;
23         return count(nd->children[*s-'a'],s+1);
24     } else {
25         return nd->count;
26     }
27 }

```

```

37     }
38     return p;
39 }

```

KMP

```

1  vector<int> buildFailure(string s){
2      vector<int> T(n+1,0);
3      T[0]=-1;
4      int j = 0;
5      for(int i = 1; i < s.size();++i){
6          if(s[i]==s[j]){
7              T[i]=T[j];
8              j++;
9          } else{
10             T[i] = j;
11             j = T[j];
12             while(j >= 0 && s[i]!=s[j])
13                 j = T[j];
14             j++;
15         }
16     }
17     T[s.size()] = j;
18     return T;
19 }
20 vector<int> search(string W, string S){
21     auto T=buildFailure(W);
22     vector<int> p;
23     int k = 0;
24     int j = 0;
25     while(j < S.size()){
26         if(W[k]==S[j]){
27             k++; j++;
28             if(k==W.size()){
29                 p.push_back(j-k);
30                 k = T[k];
31             }
32         } else{
33             k = T[k];
34             if(k < 0)
35                 j+=1, k+=1;
36         }

```


5 Algorithms

NlogN LIS

```
1  int ls[MX_N];
2  int L[MX_N];
3  int I[MX_N];
4
5  void nlogn(){
6      for(int i = 1; i < N+1; ++i)
7          I[i]=INF;
8      I[0] = -INF;
9      int mx = 1;
10     for(int i = 0; i < N; ++i){
11         int ind = lower_bound(I, I+N+1, ls[i]) - I;
12         I[ind] = ls[i];
13         L[i] = ind;
14         mx = max(mx, ind);
15     }
16     int prv = INF;
17     vector<int> out;
18     for(int i = N-1; i >= 0; --i){
19         if(ls[i] < prv && L[i]==mx){
20             out.push_back(ls[i]);
21             prv = ls[i];
22             mx--;
23         }
24     }
25 }
```

RectInHist

```
1  int R,C;
2  char board[MX_RC][MX_RC];
3  int h[MX_RC][MX_RC];
4
5  int perim(int l, int w){
6      if(l==0 || w==0)
7          return 0;
8      return 2*l + 2*w;
9  }
10
11 int main(){
12     for(int i = 0; i < R; i++){
13         int run=0;
14         for(int j = 0; j < C; j++){
15             run = (board[i][j]=='.'?run+1:0);
16             h[i][j] = run;
17         }
18     }
19     int mx = 0;
20     for(int j = 0; j < C; j++){
21         stack<int> s;
22         for(int i = 0; i < R; i++){
```

```
23         if(s.empty()
24            || h[i][j]>h[s.top()][j])
25             s.push(i);
26         else if(h[i][j]<h[s.top()][j]){
27             while(!s.empty()
28                &&h[i][j]<h[s.top()][j]){
29                 int l = h[s.top()][j];
30                 s.pop();
31                 int pm = perim(l,
32                    (s.empty()?
33                     i:s.top()-1));
34                 mx = max(mx,pm);
35             }
36             s.push(i);
37         } else if(h[i][j]==h[s.top()][j]){
38             s.pop();
39             s.push(i);
40         }
41     }
42     while(!s.empty()){
43         int l = h[s.top()][j]; s.pop();
44         int pm = perim(l, s.empty()? R : R - s.top()-1);
45         mx = max(mx,pm);
46     }
47 }
48 printf("%d\n",mx-1);
49 }
```

6 Maths

Miller Rabin

```
1 void factor(ll x, ll& e, ll& k){
2     while(x%2LL==0LL){
3         x/=2LL;
4         ++e;
5     }
6     k = x;
7 }
8
9 //increase x for higher certainty, 5 works well
10 bool is_prime(ll n, int x){
11     if(n&2LL==0 || n==1LL)
12         return false;
13     if(n==2 || n==3 || n==5 || n==7)
14         return true;
15     ll e, k;
16     factor(n-1,e,k);
17     while(x-->0){
18         ll a = (rand())%(n-5LL) + 2LL;
19         ll p = mod_exp(a,k,n);
20         if(p==1LL || p==n-1LL)
21             continue;
22         bool all_fail = true;
23         for(int i = 0; i < e-1; ++i){
24             p = mod_exp(p, 2, n);
25             if(p==n-1LL){
26                 all_fail = false;
27                 break;
28             }
29         }
30         if(all_fail)
31             return false;
32     }
33     return true;
34 }
```

Binomial Coefficients

```
1 ll ncrmem[MXN][MXN];
2
3 ll ncr(int n, int r){
4     if(n==0)
5         return r==0;
6     if(r==0)
7         return 1;
8     if(ncrmem[n][r] != -1)
9         return ncrmem[n][r];
10    return ncrmem[n][r] = ncr(n-1, r-1) + ncr(n-1, r);
11 }
```

```
1 void factor(ll x, ll& e, ll& k){
2     while(x%2LL==0LL){
3         x/=2LL;
4         ++e;
5     }
6     k = x;
7 }
8
9 //increase x for higher certainty, 5 works well
10 bool is_prime(ll n, int x){
11     if(n&2LL==0 || n==1LL)
12         return false;
13     if(n==2 || n==3 || n==5 || n==7)
14         return true;
15     ll e, k;
16     factor(n-1,e,k);
17     while(x-->0){
18         ll a = (rand())%(n-5LL) + 2LL;
19         ll p = mod_exp(a,k,n);
20         if(p==1LL || p==n-1LL)
21             continue;
22         bool all_fail = true;
23         for(int i = 0; i < e-1; ++i){
24             p = mod_exp(p, 2, n);
25             if(p==n-1LL){
26                 all_fail = false;
27                 break;
28             }
29         }
30         if(all_fail)
31             return false;
32     }
33     return true;
34 }
```

Gaussian Elimination

```
1 /*
2  * mat is augmented matrix
3  * e.g 3x + 4y = 2 is [3,4,2]
4  */
5 void gauss(double mat[MXN][MXN+1], double ans[MXN], int n){
6     int i,j,k,l;double t;
7
8     for(j = 0; j < n-1; ++j){
9         l = j;
10        for(i = j+1; i < n; ++i){
11            if(fabs(mat[i][j]) > fabs(mat[l][j]))
12                l=i;
13        }
14        for(k = j; k <= n; ++k)
15            t=mat[j][k],mat[j][k]=mat[l][k],mat[l][k]=t;
16        for(i = j+1; i < n; ++i)
17            for(k = n; k >= j; --k)
18                mat[i][k] -= mat[j][k] * mat[i][j] / mat[j][j];
```

```

19     }
20
21     for (j = n-1; j >= 0; --j){
22         for (t=0.0, k = j+1; k<n; ++k)
23             t += mat[j][k] * ans[k];
24         ans[j] = (mat[j][n] - t) / mat[j][j];
25     }
26 }

```

Ternary Search

```

1 double ternary_search(double l, double r) { //maximises
2     while (r - l > EPS) {
3         double m1 = l + (r - l) / 3;
4         double m2 = r - (r - l) / 3;
5         double f1 = f(m1);
6         double f2 = f(m2);
7         if (f1 < f2)
8             l = m1;
9         else
10            r = m2;
11     }
12     return f(l);
13 }

```

Matrix Exponential

```

1  /* c=a*b */
2  void mu(ll a[][NMAT], ll b[][NMAT], ll c[][NMAT], int _n) {
3      for (int i=0; i<_n; i++)
4          for (int j=0; j<_n; j++) {
5              c[i][j]=0;
6              for (int h=0; h<_n; h++) {
7                  c[i][j] += (a[i][h] * b[h][j]) % mod;
8                  c[i][j] %= mod;
9              }
10         }
11     }
12
13     /*returns ans=mat^b*/
14     void power(ll ans[][NMAT], ll mat[][NMAT], ll b, int _n) {
15         ll tmp[NMAT][NMAT];
16         for (int i=0; i<_n; i++)
17             for (int j = 0; j<_n; j++)
18                 ans[i][j] = i==j;
19
20         while (b) {
21             if (b&1) {
22                 mu(ans, mat, tmp, _n);
23                 for (int i=0; i<_n; i++)
24                     for (int j=0; j<_n; j++)
25                         ans[i][j] = tmp[i][j];
26             }
27             mu(mat, mat, tmp, _n);
28             for (int i=0; i<_n; i++)

```

```

28         for (int j=0; j<_n; j++)
29             mat[i][j] = tmp[i][j];
30         b>>=1;
31     }
32 }

```