# Kvantiniai skaičiavimai

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## Tiesinės algebros elementai

**1.** Ar duoteji trys vektoriai yra tiesiškai nepriklausomi? Bet kuriuo atveju pagrįskite atsakymą konkrečiais skaičiavimais:

$$1) \qquad \overrightarrow{v_1} = \begin{bmatrix} -4-5i \\ 4-i \\ -5-3i \\ 4-3i \end{bmatrix}, \quad \overrightarrow{v_2} = \begin{bmatrix} -3+2i \\ -5-5i \\ -5+4i \\ 2+3i \end{bmatrix}, \quad \overrightarrow{v_3} = \begin{bmatrix} 2+i \\ 3+i \\ -2-5i \\ 1-2i \end{bmatrix}$$

$$A = \begin{bmatrix} -4-5i & -3+2i & 2+i \\ 4-i & -5-5i & 3+i \\ -5-3i & -5+4i & -2-5i \\ 4-3i & 2+3i & 1-2i \end{bmatrix}$$

$$\begin{bmatrix} 4-i & -5-5i & 3+i \\ -4-5i & -3+2i & 2+i \\ -5-3i & -5+4i & -2-5i \\ 4-3i & 2+3i & 1-2i \end{bmatrix} \xrightarrow{R_2+R_1,\ 4R_3+5R_1,\ R_4-R_1} \begin{bmatrix} 4-i & -5-5i & 3+i \\ -6i & -8-3i & 5+2i \\ -17i & -45-9i & 7-15i \\ -2i & 7+8i & -2-3i \end{bmatrix}$$

$$\begin{bmatrix} 4-i & -5-5i & 3+i \\ -2i & 7+8i & -2-3i \\ -6i & -8-3i & 5+2i \\ -17i & -45-9i & 7-15i \end{bmatrix} \xrightarrow{(-1)R_3+3R_2,\ (-2)R_4+17R_2} \begin{bmatrix} 4-i & -5-5i & 3+i \\ -2i & 7+8i & -2-3i \\ 0 & 29+27i & -11-11i \\ 0 & 209+154i & -48-21i \end{bmatrix}$$

$$\begin{bmatrix} 4-i & -5-5i & 3+i \\ -2i & 7+8i & -2-3i \\ 0 & 29+27i & -11-11i \\ 0 & 209+154i & -48-21i \end{bmatrix} \xrightarrow{a_2\leftrightarrow a_3,\ R_3:(-11),\ R_4:3} \begin{bmatrix} 4-i & 3+i & -5-5i \\ -2i & -2-3i & 7+8i \\ 0 & 1+i & -\frac{29}{11}-\frac{27}{11}i \\ 0 & -16-7i & \frac{209}{3}+\frac{154}{3}i \end{bmatrix}$$

$$\begin{bmatrix} 4-i & 3+i & -5-5i \\ -2i & -2-3i & 7+8i \\ 0 & 1+i & -\frac{29}{11}-\frac{27}{11}i \\ 0 & -16-7i & \frac{209}{3}+\frac{154}{3}i \end{bmatrix} \xrightarrow{R_4+16R_3} \begin{bmatrix} 4-i & 3+i & -5-5i \\ -2i & -2-3i & 7+8i \\ 0 & 1+i & -\frac{29}{11}-\frac{27}{11}i \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \mathrm{Rang}(A) = 3$$

Atsakymas: Tiesiškai nepriklausomi, nes rangas lygus 3

**2.** Raskite $B^2 + 3A^{\dagger}C^{-2} + (A^{-1})^{\dagger}$(pateikdami detalius skaičiavimus), jeigu:

1) $$A = \begin{bmatrix} -5+6i & -6+5i \\ 7-6i & -4+3i \end{bmatrix}, \quad B = \begin{bmatrix} 6+5i & 5-4i \\ 1-i & -8-3i \end{bmatrix}, \quad C = \begin{bmatrix} 3-i & -3-5i \\ 3+i & 1-3i \end{bmatrix}$$

Sprendimas:

$$B^2 = \begin{bmatrix} 6+5i & 5-4i \\ 1-i & -8-3i \end{bmatrix} \cdot \begin{bmatrix} 6+5i & 5-4i \\ 1-i & -8-3i \end{bmatrix} =$$

$$= \begin{bmatrix} (6+5i)^2 + (5-4i)(1-i) & (6+5i)(5-4i) + (5-4i)(-8-3i) \\ (1-i)(6+5i) + (-8-3i)(1-i) & (1-i)(5-4i) + (-8-3i)^2 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 12+51i & -2+18i \\ -4i & 56-39i \end{bmatrix}$$

$$(A^{-1})^{\dagger} = (A^{\dagger})^{-1}$$

$$\overline{A} = \begin{bmatrix} -5-6i & -6-5i \\ 7+6i & -4-3i \end{bmatrix}$$

$$A^{\dagger} = \begin{bmatrix} -5-6i & 7+6i \\ -6-5i & -4-3i \end{bmatrix}$$

$$det(A^{\dagger}) = (-5-6i)(-4-3i) - (7+6i)(-6-5i) = 14 - 110i$$

$$adj(A^{\dagger}) = \begin{bmatrix} -4-3i & -7-6i \\ 6+5i & -5-6i \end{bmatrix}$$

$$(A^{\dagger})^{-1} = \frac{1}{14-110i} \begin{bmatrix} -4-3i & -7-6i \\ 6+5i & -5-6i \end{bmatrix}$$

$$(C^{-2}) = (C^{-1})^2$$

$$det(C) = (3-i)(1-3i) - (-3-5i)(3+i) = 4+8i$$

$$C^{-1} = \frac{1}{det(C)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{4+8i} \begin{bmatrix} 1-3i & 3+5i \\ -3-i & 3-i \end{bmatrix}$$

$$C^{-2} = \frac{1}{-48+64i} \begin{bmatrix} -12-24i & 32+8i \\ -16+8i & 4-24i \end{bmatrix}$$

$$B^2 + 3A^{\dagger}C^{-2} + (A^{-1})^{\dagger} = \begin{bmatrix} 12+51i & -2+18i \\ -4i & 56-39i \end{bmatrix} + 3 \begin{bmatrix} -5-6i & 7+6i \\ -6-5i & -4-3i \end{bmatrix} \cdot$$

$$\cdot \frac{1}{-48+64i}\begin{bmatrix} -12-24i & 32+8i \\ -16+8i & 4-24i \end{bmatrix} + \frac{1}{14-110i}\begin{bmatrix} -4-3i & -7-6i \\ 6+5i & -5-6i \end{bmatrix}$$

Atsakymas:

$$\frac{1}{614800}\begin{bmatrix} 13570040+33728420i & -8966424+15118268i \\ 3481060-6203720i & 35491164-17867048i \end{bmatrix}$$

**3.** Užrašykite vektorių $\overrightarrow{v_1}$ bazėje $\{\overrightarrow{v_2}, \overrightarrow{v_3}, \overrightarrow{v_4}\}$.(Pateikite detalius skaičiavimus):

1) $$\overrightarrow{v_1} = \begin{bmatrix} -2-4i \\ 3-3i \\ -4+3i \end{bmatrix}, \quad \overrightarrow{v_2} = \begin{bmatrix} 4+4i \\ 3-i \\ 3+i \end{bmatrix}, \quad \overrightarrow{v_3} = \begin{bmatrix} -5+2i \\ -2+i \\ 3+4i \end{bmatrix}, \quad \overrightarrow{v_4} = \begin{bmatrix} 4-5i \\ -3-5i \\ -5-i \end{bmatrix}$$

Sprendimas:

$$\overrightarrow{v_1} = a\overrightarrow{v_2} + b\overrightarrow{v_3} + c\overrightarrow{v_4}$$

$$\left[\begin{array}{ccc|c} 4+4i & -5+2i & 4-5i & -2-4i \\ 3-i & -2+i & -3-5i & 3-3i \\ 3+i & 3+4i & -5-i & -4+3i \end{array}\right] \xrightarrow{R_1\cdot\frac{1-i}{8}} \left[\begin{array}{ccc|c} 1 & \frac{-3+7i}{8} & \frac{-1-9i}{8} & \frac{-3-i}{4} \\ 3-i & -2+i & -3-5i & 3-3i \\ 3+i & 3+4i & -5-i & -4+3i \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & \frac{-3+7i}{8} & \frac{-1-9i}{8} & \frac{-3-i}{4} \\ 3-i & -2+i & -3-5i & 3-3i \\ 3+i & 3+4i & -5-i & -4+3i \end{array}\right] \xrightarrow{R_2-R_1(3-i),\quad R_3-R_1(3+i)} \left[\begin{array}{ccc|c} 1 & \frac{-3+7i}{8} & \frac{-1-9i}{8} & \frac{-3-i}{4} \\ 0 & \frac{-7-8i}{4} & \frac{-6-7i}{4} & \frac{11-6i}{2} \\ 0 & \frac{20+7i}{4} & \frac{-23+10i}{4} & \frac{-4+9i}{2} \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & \frac{-3+7i}{8} & \frac{-1-9i}{8} & \frac{-3-i}{4} \\ 0 & \frac{-7-8i}{4} & \frac{-6-7i}{4} & \frac{11-6i}{2} \\ 0 & \frac{20+7i}{4} & \frac{-23+10i}{4} & \frac{-4+9i}{2} \end{array}\right] \xrightarrow{R_2\cdot 4,\quad R_3\cdot 4} \left[\begin{array}{ccc|c} 1 & \frac{-3+7i}{8} & \frac{-1-9i}{8} & \frac{-3-i}{4} \\ 0 & -7-8i & -6-7i & 22-12i \\ 0 & 20+7i & -23+10i & -8+18i \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & \frac{-3+7i}{8} & \frac{-1-9i}{8} & \frac{-3-i}{4} \\ 0 & -7-8i & -6-7i & 22-12i \\ 0 & 20+7i & -23+10i & -8+18i \end{array}\right] \xrightarrow{R_2:(-7-8i)} \left[\begin{array}{ccc|c} 1 & \frac{-3+7i}{8} & \frac{-1-9i}{8} & \frac{-3-i}{4} \\ 0 & 1 & \frac{98+i}{113} & \frac{-58+260i}{113} \\ 0 & 20+7i & -23+10i & -8+18i \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & \frac{-3+7i}{8} & \frac{-1-9i}{8} & \frac{-3-i}{4} \\ 0 & 1 & \frac{98+i}{113} & \frac{-58+260i}{113} \\ 0 & 20+7i & -23+10i & -8+18i \end{array}\right] \xrightarrow{R_3-R_2\cdot(20+7i))} \left[\begin{array}{ccc|c} 1 & \frac{-3+7i}{8} & \frac{-1-9i}{8} & \frac{-3-i}{4} \\ 0 & 1 & \frac{98+i}{113} & \frac{-58+260i}{113} \\ 0 & 0 & \frac{-4552+424i}{113} & \frac{2076-2760i}{113} \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & \frac{-3+7i}{8} & \frac{-1-9i}{8} & \frac{-3-i}{4} \\ 0 & 1 & \frac{98+i}{113} & \frac{-58+260i}{113} \\ 0 & 0 & \frac{-4552+424i}{113} & \frac{2076-2760i}{113} \end{array}\right] \xrightarrow{R_3\cdot 113} \left[\begin{array}{ccc|c} 1 & \frac{-3+7i}{8} & \frac{-1-9i}{8} & \frac{-3-i}{4} \\ 0 & 1 & \frac{98+i}{113} & \frac{-58+260i}{113} \\ 0 & 0 & -4552+424i & 2076-2760i \end{array}\right]$$

Atsakymas:

$$c = \frac{3(173-230i)}{2(-569+53i)}, \quad b = \frac{-58+260i}{113} - \frac{98+i}{113}c, \quad a = \frac{-3-i}{4} - \frac{-3+7i}{8}b - \frac{-1-9i}{8}c$$

**4.** Iš bazės iš ankstesnio uždavinio gaukite ortonormuotą bazę naudodami Gramo-Šmidto procesą `https://en.wikipedia.org/wiki/Gram%E2%80%93Schmidt_process` ir normuodami gautus vektorius. Detaliai aprašykite kiekvieną žingsnį.

$$proj_u(v) = \frac{\langle v, u \rangle}{\langle u, u \rangle} u, \quad u_1 = v_1$$

$$u_k = v_k - \sum j = 1k - 1 proj_{u_j}(v_k)$$

$$\langle x, y \rangle = \sum_k x_k \overline{y_k}$$

Sprendimas:

$$u_1 = \begin{bmatrix} -2-4i \\ 3-3i \\ -4+3i \end{bmatrix}$$

$$u_2 = \begin{bmatrix} 4+4i \\ 3-i \\ 3+i \end{bmatrix} - \frac{\langle \begin{bmatrix} 4+4i \\ 3-i \\ 3+i \end{bmatrix}, \begin{bmatrix} -2-4i \\ 3-3i \\ -4+3i \end{bmatrix} \rangle}{\langle \begin{bmatrix} -2-4i \\ 3-3i \\ -4+3i \end{bmatrix}, \begin{bmatrix} -2-4i \\ 3-3i \\ -4+3i \end{bmatrix} \rangle} \begin{bmatrix} -2-4i \\ 3-3i \\ -4+3i \end{bmatrix}$$

$$\langle u_1, u_1 \rangle = (-2-4i)\overline{(-2-4i)} + (3-3i)\overline{(3-3i)} + (-4+3i)\overline{(-4+3i)} = 63$$

$$\langle v_2, u_1 \rangle = (4+4i)\overline{(-2-4i)} + (3-i)\overline{(3-3i)} + (3+i)\overline{(-4+3i)} = -21 + i$$

$$\frac{\langle v_2, u_1 \rangle}{\langle u_1, u_1 \rangle} = \frac{-21+i}{63} = -\frac{1}{3} + \frac{i}{63}$$

$$proj_{u_1}(v_2) = (-\frac{1}{3} + \frac{i}{63}) \cdot \begin{bmatrix} -2-4i \\ 3-3i \\ -4+3i \end{bmatrix} = \begin{bmatrix} \frac{46}{63} + \frac{82}{63}i \\ -\frac{20}{21} + \frac{22}{21}i \\ \frac{9}{7} - \frac{67}{63}i \end{bmatrix}$$

$$u_2 = v_2 - proj_{u_1}(v_2) = \begin{bmatrix} 4+4i \\ 3-i \\ 3+i \end{bmatrix} - \begin{bmatrix} \frac{46}{63} + \frac{82}{63}i \\ -\frac{20}{21} + \frac{22}{21}i \\ \frac{9}{7} - \frac{67}{63}i \end{bmatrix} = \begin{bmatrix} \frac{206+170i}{63} \\ \frac{249-129i}{63} \\ \frac{108+130i}{63} \end{bmatrix}$$

$$\frac{\langle v_3, u_1 \rangle}{\langle u_1, u_1 \rangle} = -\frac{1}{9} - \frac{52}{63}i$$

$$\frac{\langle v_3, u_2 \rangle}{\langle u_2, u_2 \rangle} = -\frac{473}{2834} + \frac{1295}{2834}i$$

$$u_3 = v_3 - \frac{\langle v_3, u_1\rangle}{\langle u_1, u_1\rangle} u_1 - \frac{\langle v_3, u_2\rangle}{\langle u_2, u_2\rangle} u_2 = \begin{bmatrix} -5+2i \\ -2+i \\ 3+4i \end{bmatrix} - (-\frac{1}{9} - \frac{52}{63} i) \cdot \begin{bmatrix} -2-4i \\ 3-3i \\ -4+3i \end{bmatrix} - (-\frac{473}{2834} + \frac{1295}{2834} i) \cdot \begin{bmatrix} 4+4i \\ 3-i \\ 3+i \end{bmatrix}$$

Atsakymas:

$$u_1 = \begin{bmatrix} -2-4i \\ 3-3i \\ -4+3i \end{bmatrix}, \quad u_2 = \begin{bmatrix} \frac{206+170i}{63} \\ \frac{249-129i}{63} \\ \frac{108+130i}{63} \end{bmatrix}, \quad u_3 = \begin{bmatrix} -\frac{201}{1417} - \frac{1614}{1417} i \\ \frac{756}{1417} + \frac{1410}{1417} i \\ \frac{1854}{1417} + \frac{840}{1417} i \end{bmatrix}$$

Pasitikrinimas:

$$u_1 \cdot u_2 = \frac{-1092-484i}{63} + \frac{1134-360i}{63} + \frac{-42+844i}{63} = \frac{0+0i}{63} = 0$$

$$u_1 \cdot u_3 = \frac{6858-2424i}{1417} + \frac{-1962-6498i}{1417} - \frac{-4896+8922i}{1417} = \frac{0+0i}{1417} = 0$$

$$u_1 \cdot u_3 = \frac{-315789+298314i}{63 \cdot 1417} + \frac{6354-448614i}{63 \cdot 1417} - \frac{309432+150300i}{63 \cdot 1417} = \frac{0+0i}{63 \cdot 1417} = 0$$

$$u_1 \cdot u_2 = 0, \quad u_1 \cdot u_3 = 0, \quad u_2 \cdot u_3 = 0$$

**5.** Tikrinės reikšmės. Raskite matricos A tikrines reikšmes ir tirkrinius vektorius (detaliai pateikdami sprendimo žingsnius):

$$1) \qquad A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -3i & 5 \\ 0 & -2 & i \end{bmatrix}$$

Tikrinės reikšmės tenkina:

$$det(A - \lambda I) = 0$$

$$A - \lambda I = \begin{bmatrix} -1-\lambda & 0 & 0 \\ 0 & -3i-\lambda & 5 \\ 0 & -2 & i-\lambda \end{bmatrix}$$

Sprendimas:

$$\det(A - \lambda I) = (-1-\lambda) \cdot \det \begin{bmatrix} -3i-\lambda & 5 \\ -2 & i-\lambda \end{bmatrix}$$

$$\begin{aligned} &\det \begin{bmatrix} -3i-\lambda & 5 \\ -2 & i-\lambda \end{bmatrix} \\ &= (-3i-\lambda)(i-\lambda) - (5)(-2) \\ &= (-3i)(i) + (-3i)(-\lambda) + (-\lambda)(i) + (-\lambda)(-\lambda) + 10 \\ &= 3 + 3i\lambda - i\lambda + \lambda^2 + 10 \\ &= \lambda^2 + 2i\lambda + 13 \end{aligned}$$

$$det(A - \lambda I) = (-1-\lambda)(\lambda^2 + 2i\lambda + 13) = 0$$

$$\lambda_1 = -1 \quad arba \quad \lambda^2 + 2i\lambda + 13 = 0$$

$$\lambda_{2,3} = \frac{-2i \pm 2\sqrt{14}i}{2} = -i \pm i\sqrt{14}$$

Taigi, tikrinės reikšmės:

$$\lambda_1 = -1, \quad \lambda_2 = -i + i\sqrt{14}, \quad \lambda_2 = -i - i\sqrt{14}$$

Toliau:

$$A - (-1)I = A + I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3i+1 & 5 \\ 0 & -2 & i+1 \end{bmatrix}$$

$$(1-3i)\left(\frac{1+i}{2}\right)x_3+5x_3=0$$

$$\frac{4-2i}{2}x_3+5x_3=(2-i)x_3+5x_3=(7-i)x_3=0$$

Kadangi $7-i\neq 0$, todėl $x_3=0$, bei $x_2=0$, o $x_1$ laisvas.
Taigi:

$$\mathbf{v}_1=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

Toliau:
$k_1=\sqrt{14}-1$, tai $\lambda_2=ik_1$

$$A-\lambda_2 I=\begin{bmatrix}-1-ik_1 & 0 & 0\\ 0 & -3i-ik_1 & 5\\ 0 & -2 & i-ik_1\end{bmatrix}$$

$$-2x_2+i(1-k_1)\cdot\frac{i(3+k_1)}{5}x_2=0$$

$$-2x_2+\frac{i^2(1-k_1)(3+k_1)}{5}x_2=-2x_2-\frac{(1-k_1)(3+k_1)}{5}x_2=0$$

$$(1-k_1)(3+k_1)=3+k_1-3k_1-k_1^2=3-2k_1-k_1^2$$

$$k_1^2=(\sqrt{14}-1)^2=14-2\sqrt{14}+1=15-2\sqrt{14}$$

$$3-2k_1-k_1^2=3-2(\sqrt{14}-1)-(15-2\sqrt{14})=3-2\sqrt{14}+2-15+2\sqrt{14}=-10$$

Todėl:

$$-2x_2-\frac{-10}{5}x_2=-2x_2+2x_2=0$$

$x_2$ laisvas. Pasirenkame $x_2=5$:

$$x_3=\frac{i(3+k_1)}{5}\cdot 5=i(3+k_1)=i(3+\sqrt{14}-1)=i(\sqrt{14}+2)$$

Taigi:

$$\mathbf{v}_2=\begin{bmatrix}0\\5\\i(\sqrt{14}+2)\end{bmatrix}$$

Toliau:
Tarkime $k_2=1+\sqrt{14}$, tai $\lambda_3=-ik_2$

$$A - \lambda_3 I = \begin{bmatrix} -1 + ik_2 & 0 & 0 \\ 0 & -3i + ik_2 & 5 \\ 0 & -2 & i + ik_2 \end{bmatrix}$$

$$-2x_2 + i(1 + k_2) \cdot \frac{-i(k_2 - 3)}{5} x_2 = 0$$

$$-2x_2 + \frac{-i^2(1 + k_2)(k_2 - 3)}{5} x_2 = -2x_2 + \frac{(1 + k_2)(k_2 - 3)}{5} x_2 = 0$$

$$(1 + k_2)(k_2 - 3) = (2 + \sqrt{14})(\sqrt{14} - 2) = 14 - 4 = 10$$

Todėl:

$$-2x_2 + \frac{10}{5} x_2 = -2x_2 + 2x_2 = 0$$

$x_2$ laisvas. Pasirenkame $x_2 = 5$:

$$x_3 = \frac{-i(k_2 - 3)}{5} \cdot 5 = -i(k_2 - 3) = -i(1 + \sqrt{14} - 3) = -i(\sqrt{14} - 2)$$

Taigi:

$$\mathbf{v}_3 = \begin{bmatrix} 0 \\ 5 \\ -i(\sqrt{14} - 2) \end{bmatrix}$$

Tikrinės reikšmės:

$$\lambda_1 = -1, \quad \lambda_2 = i(\sqrt{14} - 1), \quad \lambda_3 = -i(1 + \sqrt{14})$$

Tikriniai vektoriai:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0 \\ 5 \\ i(\sqrt{14} + 2) \end{bmatrix}, \quad v_3 = \begin{bmatrix} 0 \\ 5 \\ -i(\sqrt{14} - 2) \end{bmatrix}$$

Patikrinimas:

$$Av_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -3i & 5 \\ 0 & -2 & i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda_1 v_1 = -1 \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

$$Av_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -3i & 5 \\ 0 & -2 & i \end{bmatrix} \begin{bmatrix} 0 \\ 5 \\ i(\sqrt{14} + 2) \end{bmatrix} = \begin{bmatrix} 0 \\ 5i(\sqrt{14} - 1) \\ -12 - \sqrt{14} \end{bmatrix}$$

$$\lambda_2 v_2 = i(\sqrt{14} - 1) \cdot \begin{bmatrix} 0 \\ 5 \\ i(\sqrt{14} + 2) \end{bmatrix} = \begin{bmatrix} 0 \\ 5i(\sqrt{14} - 1) \\ -12 - \sqrt{14} \end{bmatrix}$$

$$Av_3 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -3i & 5 \\ 0 & -2 & i \end{bmatrix} \begin{bmatrix} 0 \\ 5 \\ -i(\sqrt{14} - 2) \end{bmatrix} = \begin{bmatrix} 0 \\ -5i(1 + \sqrt{14}) \\ -12 + \sqrt{14} \end{bmatrix}$$

$$\lambda_3 v_3 = -i(\sqrt{14} + 1) \cdot \begin{bmatrix} 0 \\ 5 \\ -i(\sqrt{14} - 2) \end{bmatrix} = \begin{bmatrix} 0 \\ -5i(1 + \sqrt{14}) \\ -12 + \sqrt{14} \end{bmatrix}$$

**6.** Ermito ir unitarinės matricos. Tenzorinė tiesinių erdvių sandauga. Matricai $A$ iš ankstesnės užduoties raskite:

a) matricą $X$, tokią, kad $AX$ būtų nediagonalinė Ermito matrica;

b) matricą $Y$, tokią, kad $AY$ būtų nediagonalinė unitarinė matrica;

c) tenzorines sandaugas $Z \otimes A$ ir $A \otimes Z$, kai matrica $Z$ yra:

1. $Z = \begin{bmatrix} -1 & 2i \\ -2i & 1 \end{bmatrix}$,

**a)** Matrica $H$ yra Ermito, jei $H^\dagger = H$. Turime $AX = H$, kur $H$ Ermito. Tada:

$$(AX)^\dagger = X^\dagger A^\dagger = AX$$

$$A^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -3i & -2 \\ 0 & 5 & i \end{bmatrix}$$

$$A^\dagger = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3i & -2 \\ 0 & 5 & -i \end{bmatrix}$$

$$AA^\dagger = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 34 & i \\ 0 & -i & 5 \end{bmatrix}$$

$X = A^\dagger$, tada $AX = AA^\dagger$ - Ermito, bet reikia patikrinti, ar nediagonalinė.

Yra Ermito, nes įstrižainės elementai realūs, o $(2,3) = i, (3,2) = -i$. Ji nėra diagonalinė, nes yra nuliniai elementai šalia įstrižainės.

**b)** Matrica yra unitarinė, jei $U^\dagger U = I$

$$Jei \quad U = AY, \quad tai \quad U^\dagger U = Y^\dagger A^\dagger AY = I$$

Jei $A$ yra invertuojama, galima pasirinkti $Y = A^{-1}U_0$, kur $U_0$ yra unitarinė matrica. Tada:

$$AY = A(^{-1}U_0) = U_0$$

Kad $AY$ būtų nediagonalinė, $U_0$ turi būti nediagonalinė unitarinė matrica.

$$det(A) = (-1) \cdot det \begin{bmatrix} -3i & 5 \\ -2 & i \end{bmatrix} = -13 \neq 0$$

Vadinasi, $A$ invertuojama.

$$A^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & \frac{i}{13} & \frac{-5}{13} \\ 0 & \frac{2}{13} & \frac{-3i}{13} \end{bmatrix}$$

$$U_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$Y = A^{-1}U_0 = \begin{bmatrix} 0 & 0 & -1 \\ \frac{i}{13} & -\frac{5}{13} & 0 \\ \frac{2}{13} & -\frac{3i}{13} & 0 \end{bmatrix}$$

**c)**

$$Z \otimes A = \begin{bmatrix} -1 & 2i \\ -2i & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 & 0 \\ 0 & -3i & 5 \\ 0 & -2 & i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & -2i & 0 \\ 0 & 3i & -5 & 0 & 6 & 10i \\ 0 & 2 & -i & 0 & -4i & -2 \\ 2i & 0 & 0 & -1 & 0 & 0 \\ 0 & -6 & -10i & 0 & -3i & 5 \\ 0 & 4i & 2 & 0 & -2 & i \end{bmatrix}$$

$$A \otimes Z = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -3i & 5 \\ 0 & -2 & i \end{bmatrix} \cdot \begin{bmatrix} -1 & 2i \\ -2i & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2i & 0 & 0 & 0 & 0 \\ 2i & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3i & -6i & -5 & 10i \\ 0 & 0 & 6 & -3i & -10i & 5 \\ 0 & 0 & 2 & -4i & -i & -2 \\ 0 & 0 & 0 & 4i & -2 & i \end{bmatrix}$$