Kvantiniai skaičiavimai

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Kompleksiniai skaičiai

1. Apskaičiuokite reiškinių reikšmes. Būtinai parodykite tarpinius skaičiavimus:

$$1) \quad (2-2i)^{-2} + \frac{6-3i}{2-2i} + \frac{6+3i}{1+i}$$

Sprendimas:

$$\frac{1}{(2-2i)^2} + \frac{(6-3i)\cdot(1+i)}{2(1-i)\cdot(1+i)} + \frac{(6+3i)\cdot 2(1-i)}{(1+i)\cdot 2(1-i)} \tag{1}$$

$$\frac{1}{4-8i+4i^2} + \frac{6+3i-3i^2}{2-2i^2} + \frac{12-6i-6i^2}{2-2i^2} \tag{2}$$

$$\frac{1\cdot(-i)}{-8i\cdot(-i)} + \frac{9+3i}{4} + \frac{18-6i}{4} \tag{3}$$

$$\frac{i}{8} + \frac{27-3i}{4} \tag{4}$$

$$\frac{i}{8} + \frac{(27-3i)\cdot 2}{4\cdot 2} \tag{5}$$

$$\frac{i}{8} + \frac{54-6i}{8} \tag{6}$$

$$\frac{54-5i}{8} \tag{7}$$

$$\frac{54}{8} + \frac{5}{8}i \tag{8}$$

Atsakymas:

$$\frac{27}{4} + \frac{5}{8}i$$

2. Tarkime, kad

$$u_1 = 3 + 3i,$$
$$w = 2 - 2i,$$
$$z = 4 + 2i.$$

Apskaičiuokite reiškinių reikšmes. Būtinai parodykite tarpinius skaičiavimus:

$$1) \quad |u| + \overline{w} + \frac{z}{|z+1|}$$

Sprendimas:

$$|3+3i| + \overline{2-2i} + \frac{4+2i}{|4+2i+1|} \tag{1}$$

$$|3+3i| + \overline{2-2i} + \frac{4+2i}{|5+2i|} \tag{2}$$

$$\sqrt{3^2+3^2} + (2+2i) + \frac{4+2i}{\sqrt{5^2+2^2}} \tag{3}$$

$$3\sqrt{2} + 2 + 2i + \frac{4+2i}{\sqrt{29}} \tag{4}$$

$$3\sqrt{2} + 2 + 2i + \frac{(4+2i)\cdot\sqrt{29}}{\sqrt{29}\cdot\sqrt{29}} \tag{5}$$

$$3\sqrt{2} + 2 + 2i + \frac{4\sqrt{29} + 2\sqrt{29}i}{29} \tag{6}$$

$$3\sqrt{2} + 2 + 2i + \frac{4\sqrt{29}}{29} + \frac{2\sqrt{29}i}{29} \tag{7}$$

Atsakymas:

$$(3\sqrt{2} + 2 + \frac{4\sqrt{29}}{29}) + i(2 + \frac{2\sqrt{29}}{29})$$

3. Užrašykite kompleksinius skaičius trigonometrinėje formoje ($\rho e^{i\theta}$). Būtinai parodykite tarpinius skaičiavimus:

$$1) -\frac{50}{2} + \frac{50\sqrt{3}}{2}i$$

Sprendimas:

$$-25 + 25\sqrt{3}i \tag{1}$$

$$\rho = |z| = \sqrt{x^2 + y^2} = \sqrt{(-25)^2 + (25\sqrt{3})^2} = 50 \tag{2}$$

$$\cos\theta = \frac{x}{|z|} = \frac{-25}{50} = -\frac{1}{2} \tag{3}$$

$$\theta = \arccos(-\frac{1}{2}) = \frac{2\pi}{3} \tag{4}$$

Atsakymas:

$$50e^{i\frac{2\pi}{3}}$$

4. Išspręskite lygtis (detaliai pateikdami sprendimą):

$$1) \quad 2x^2 - 2x + 10 = 0$$

Sprendimas:

$$2x^2 - 2x + 10 = 0 \quad | : 2 \tag{1}$$

$$x^2 - x + 5 = 0 \tag{2}$$

$$D = (-1)^2 - 4 \cdot 1 \cdot 5 = -19 \tag{3}$$

$$\sqrt{D} = \sqrt{-19} = \sqrt{19} \cdot \sqrt{-1} = \sqrt{19}i \tag{4}$$

$$x_{1,2} = \frac{1 \pm \sqrt{19}i}{2} \tag{5}$$

Atsakymas:

$$x_1 = \frac{1}{2} + \frac{\sqrt{19}}{2}i, \quad x_2 = \frac{1}{2} - \frac{\sqrt{19}}{2}i$$

5. Raskite skaičiaus z visas n - tosios šaknies reikšmes (Jūs turite pateikti tikslias reikšmes (su radikalais)):

$$1) \quad n = 6, \quad z = -128;$$

Sprendimas:

$$\rho = |z| = |-128| = 128 \tag{1}$$

$$cos\theta = \frac{x}{|z|} = \frac{-128}{128} = -1 \tag{2}$$

$$\theta = \arccos(-1) = \pi \tag{3}$$

Bendroji n-tosios šaknies ($n = 6$) formulė:

$$w_k = \sqrt[6]{128}\, e^{i\frac{\pi+2k\pi}{6}} = 2^{7/6}\left(\cos\frac{\pi+2k\pi}{6} + i\sin\frac{\pi+2k\pi}{6}\right), \quad k = 0, 1, \ldots, 5.$$

Visos šaknys:

$$w_0 = 2^{7/6}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = 2^{7/6}\left(\tfrac{\sqrt{3}}{2} + i\tfrac{1}{2}\right) = 2^{1/6}(\sqrt{3} + i),$$

$$w_1 = 2^{7/6}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = i\, 2^{7/6},$$

$$w_2 = 2^{7/6}\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) = 2^{7/6}\left(-\tfrac{\sqrt{3}}{2} + i\tfrac{1}{2}\right) = 2^{1/6}(-\sqrt{3} + i),$$

$$w_3 = 2^{7/6}\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right) = 2^{7/6}\left(-\tfrac{\sqrt{3}}{2} - i\tfrac{1}{2}\right) = 2^{1/6}(-\sqrt{3} - i),$$

$$w_4 = 2^{7/6}\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right) = -i\, 2^{7/6},$$

$$w_5 = 2^{7/6}\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right) = 2^{7/6}\left(\tfrac{\sqrt{3}}{2} - i\tfrac{1}{2}\right) = 2^{1/6}(\sqrt{3} - i).$$

6. Nubraižykite sritį kompleksinėje plokštumoje, kurią atitinka visi skaičiai z, kurie tenkina sąlygas:

$$1) \quad 1 \leq |z-5| \leq 10, \quad \Re(z) > 1;$$

Sprendimas:

$$-10 \leq z - 5 \leq -1 \quad | + 5 \tag{1}$$

$$-5 \leq z \leq 4 \tag{2}$$

$$1 \leq z - 5 \leq 10 \quad | + 5 \tag{3}$$

$$6 \leq z \leq 15 \tag{4}$$

Sritis: $1 \leq |z-5| \leq 10, \ \Re(z) > 1$