1. Classifique os conjuntos do $\mathbb{R}^n$ abaixo como LI ou LD.

(a) $\{(2,-1),(3,5)\}$
(b) $\{(1,0),(-1,1),(3,5)\}$
(c) $\{(1,2,-1),(2,4,-2),(1,3,0)\}$
(d) $\{(1,-1,-2),(2,1,1),(0,3,5)\}$
(e) $\{(1,2,-1),(1,0,0),(0,1,2),(3,-1,2)\}$
(f) $\{(1,1,2,4),(1,-1,-4,2),(0,-1,-3,1),(2,1,1,5)\}$

a) $a_1(2,-1) + a_2(3,5) = (0,0)$

$$\begin{cases} 2a_1 + 3a_2 = 0 \\ -a_1 + 5a_2 = 0 \quad (\times 2) \end{cases}$$

$$\begin{cases} 2a_1 + 3a_2 = 0 \\ -2a_1 + 10a_2 = 0 \end{cases}$$

$13a_2 = 0 \rightsquigarrow \boxed{a_2 = 0}$

$\rightsquigarrow 2a_1 + 0 = 0$

$\boxed{a_1 = 0}$

$\boxed{LI}$

b) $a(1,0) + b(-1,1) + c(3,5) = (0,0)$

$$\begin{cases} a - b + 3c = 0 \\ b + 5c = 0 \end{cases} \rightsquigarrow \boxed{c = -\frac{b}{5}}$$

$\boxed{b = -5c}$

$a + 5c + 3c = 0$

$\boxed{a = -8c}$

$c = 5$

$b = -25$

$a = -40$

$(-40,0) + (25,-25) + (15,25) = (0,0)$

$\boxed{LD}$

c) $a(1,2,-1) + b(2,4,-2) + c(1,3,0) = (0,0,0)$

$$\begin{cases} a + 2b + c = 0 \rightsquigarrow -\cancel{2b} + \cancel{2b} + c = 0 \rightsquigarrow \boxed{c = 0} \\ 2a + 4b + 3c = 0 \rightsquigarrow 2(-2b) + 4b + 3\cancel{c}0 = 0 \rightsquigarrow \boxed{0 = 0} \\ -a - 2b = 0 \rightsquigarrow \boxed{a = -2b} \end{cases}$$

$\boxed{LD}$

d) $a(1,-1,-2)+b(2,1,1)+c(0,3,5)=(0,0,0)$

$$\begin{cases} a+2b=0 \rightsquigarrow \boxed{a=-2b} \\ -a+b+3c=0 \rightsquigarrow 2b+b+3c=0 \rightsquigarrow \cancel{3}c=-\cancel{3}b \quad \boxed{c=-b} \\ -2a+b+5c=0 \rightsquigarrow -2(-2b)+b+5c=0 \end{cases}$$

$$4b+b+5c=0$$

$$\cancel{5}b=-\cancel{5}c$$

$$\boxed{b=-c}$$

$$\boxed{LD}$$

e) $a(1,2,-1)+b(1,0,0)+c(3,-1,2)=(0,0,0)$

$$\begin{cases} a+b+3c=0 \rightsquigarrow \boxed{b=0} \\ 2a-c=0 \rightsquigarrow a=\frac{c}{2} \rightsquigarrow \boxed{a=0} \\ -a+2c=0 \rightsquigarrow -\frac{c}{2}+2c=0 \ (x2) \end{cases}$$

$$-c+4c=0 \rightsquigarrow \boxed{c=0}$$

$$\boxed{LI}$$

f) $a(1,1,2,4)+b(1,-1,-4,2)+c(0,-1,-3,1)+d(2,1,1,5)=(0,0,0,0)$

$$\begin{cases} a+b+2d=0 \rightsquigarrow a=-b-2d \\ a-b-c+d=0 \rightsquigarrow -b-2d-b-c+d=0 \rightsquigarrow -2b-d-c=0 \rightsquigarrow c=-2b-d \\ 2a-4b-3c+d=0 \rightsquigarrow 2(-b-2d)-4b-3(-2b-d)+d=0 \\ 4a+2b+c+5d=0 \qquad -\cancel{2b}-\cancel{4d}-\cancel{4b}+\cancel{6b}+\cancel{3d}+\cancel{d}=0 \end{cases}$$

$$0=0$$

$$\boxed{LD}$$

2. Determine o valor de k para que o conjunto $\{(-1,0,2),(1,1,1),(k,-2,0)\}$ seja LI.

$$\begin{cases} -a+b+ck=0 \rightsquigarrow c+2c+ck=0 \rightsquigarrow 3c+ck=0 \\ b-2c=0 \rightsquigarrow b=2c \\ 2a+b=0 \rightsquigarrow a=-c \end{cases}$$

$$\cancel{c}k=-3\cancel{c}$$

$$\boxed{k=-3}$$

3. Para quais valores de k o conjunto $\beta = \{(1,k),(k,4)\}$ é base do $\mathbb{R}^2$?

β tem que ser LI

$$\begin{cases} a+bk=0 \rightsquigarrow a=-bk \\ ak+4b=0 \rightsquigarrow (-bk)\cdot k+4b=0 \end{cases}$$

$$-bk^2+4b=0$$

$$\cancel{b}k^2=4\cancel{b}$$

$$k=\underline{\pm}2$$

$$\overset{+}{\rightsquigarrow} \begin{cases} a+2b=0 \\ 2a+4b=0 \end{cases} \quad \boxed{0=0}$$

$$\overset{-}{\rightsquigarrow} \begin{cases} a-2b=0 \\ -2a+4b=0 \end{cases} \quad \boxed{0=0}$$

$k=1$

$$\begin{cases} a-b=0 \rightsquigarrow a=b \\ a+4b=0 \rightsquigarrow b=0 \end{cases}$$

$$(x,y)=a(1,1)+b(1,4)$$

$$\begin{cases} a+b=x \rightsquigarrow a=x-b \rightsquigarrow a=x-\left(\frac{y-x}{3}\right) \\ a+4b=y \rightsquigarrow x-b+4b=y \end{cases}$$

$$x+3b=y$$

$$b=\frac{y-x}{3}$$

$$a=x-\frac{y+x}{3}$$

$$a=\frac{3x}{3}-\frac{y+x}{3}$$

$$a=\frac{4x-y}{3}$$