1) Sejam:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \end{bmatrix} \qquad C = \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} \qquad E = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} -2 & 0 & 1 \\ 3 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 2 & -1 \end{bmatrix} \qquad F = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 2 \\ -2 & 0 & -2 \end{bmatrix}$$

Calcule:

a) A + B
b) A . C
c) B . C
d) C . D
e) D . A
f) D . B
g) -A
h) -D
i) E + F
j) E - F
k) 2E - F
l) E - 3F
m) E . F
n) F . A
o) -E
p) -F

a) $\begin{bmatrix} -1 & 2 & 4 \\ 5 & 1 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1.(-1)+2.2+3.4 \\ \cancel{2.(-1)}+\cancel{1.2}+4.(-1) \end{bmatrix} = \begin{bmatrix} 15 \\ -4 \end{bmatrix}_{2\times1}$

c) $\begin{bmatrix} (-2).(-1)+\cancel{0.2}+1.4 \\ 3.(-1)+\cancel{0.2}+1.4 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}_{2\times1}$

d) 3×2 $\begin{bmatrix} (-1).2 & (-1).(-1) \\ 2.2 & 2.(-1) \\ 4.2 & 4.(-1) \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 4 & -2 \\ 8 & -4 \end{bmatrix}$

e) $\begin{bmatrix} \cancel{2.1}+\cancel{(-1).2} & 2.2+(-1).1 & 2.(3)+(-1).(-1) \end{bmatrix} = \begin{bmatrix} 0 & 3 & 7 \end{bmatrix}_{1\times3}$

f) $\begin{bmatrix} 2.(-2)+(-1).3 & 0 & 2.1+(-1).1 \end{bmatrix} = \begin{bmatrix} -7 & 0 & 1 \end{bmatrix}_{1\times3}$

g) $-A = \begin{bmatrix} -1 & -2 & -3 \\ -2 & -1 & 1 \end{bmatrix}$

h) $-D = \begin{bmatrix} -2 & 1 \end{bmatrix}$

i) $\begin{bmatrix} 2 & 4 & 3 \\ 7 & 6 & 8 \\ 5 & 8 & 7 \end{bmatrix}$

j) $\begin{bmatrix} 0 & 0 & 3 \\ 1 & 4 & 4 \\ 9 & 8 & 11 \end{bmatrix}$

k) $\begin{bmatrix} 1 & 2 & 6 \\ 5 & 9 & 10 \\ 16 & 16 & 20 \end{bmatrix}$

l) $\begin{bmatrix} -2 & -4 & 3 \\ -5 & 2 & 0 \\ 13 & 8 & 15 \end{bmatrix}$

m) $\begin{bmatrix} 1.1+2.3+3.(-2) & 1.2+2.1+3.\cancel{0} & 1.\cancel{0}+2.2+3.(-2) \\ 4.1+5.3+6.(-2) & 4.2+5.1+6.\cancel{0} & 4.\cancel{0}+5.2+6.(-2) \\ 7.1+8.3+9.(-2) & 7.2+8.1+9.\cancel{0} & 7.\cancel{0}+8.2+9.(-2) \end{bmatrix} = \begin{bmatrix} 1 & 4 & -2 \\ 7 & 13 & -2 \\ 13 & 22 & -2 \end{bmatrix}$

n) $F = 3 \times 3 \quad A = 2 \times 3$

não definido

o) $\begin{bmatrix} -1 & -2 & -3 \\ -4 & -5 & -6 \\ -7 & -8 & -9 \end{bmatrix}$

p) $\begin{bmatrix} -1 & -2 & 0 \\ -3 & -1 & -2 \\ 2 & 0 & 2 \end{bmatrix}$

2) Dadas

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ 4 & -3 & -1 \end{bmatrix} B = \begin{bmatrix} 1 & 4 & 1 & 0 \\ 2 & 1 & 1 & 1 \\ 1 & -2 & 1 & 2 \end{bmatrix} C = \begin{bmatrix} 2 & 1 & -1 & -2 \\ 3 & -2 & -1 & -1 \\ 2 & -5 & -1 & 0 \end{bmatrix} \tag{1}$$

$3 \times 3 \qquad 3 \times 4 \qquad 3 \times 4$

mostre que $AB = AC$.

$$AC = \begin{bmatrix} 1.2+(-3).3+2.2 & 1.1+(-3).(-2)+2.(-5) & -1+3-2 & -2+3 \\ 2.2+1.3+(-3).2 & 2.1+1.(-2)+(-3).(-5) & -2-1+3 & -4-1 \\ 4.2+(-3).3+(-1).2 & 4.1+(-3).(-2)+(-1).(-5) & -4+3+1 & -8+3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -3 & -3 & 0 & 1 \\ 1 & 15 & 0 & -5 \\ -3 & 15 & 0 & -5 \end{bmatrix} \; 3 \times 4$$

$$AB = \begin{bmatrix} 1-6+2 & 4-3-4 & 1-3+2 & -3+4 \\ 2+2-3 & 8+1+6 & 2+1-3 & 1-6 \\ 4-6-1 & 16-3+2 & 4-3-1 & -3-2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -3 & 0 & 1 \\ 1 & 15 & 0 & -5 \\ -3 & 15 & 0 & -5 \end{bmatrix} \Rightarrow \text{Portanto } AB = AC$$

3) Explique por que em geral, $(A+B)^2 \neq A^2 + 2AB + B^2$ e $(A+B)(A-B) \neq (A^2 - B^2)$.

O produto notável ou quadrado do binomio é uma operação fechada para os número reais e o professor quando explicou matrizes disse que agora as operações eram definidas de outra forma e que apenas algumas coisas seguiriam como já aprendemos na álgebra normalmente.
Mas é simples de enxergar. Suponha duas matrizes quadradas A e B, tal que:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \Rightarrow (A+B)^2 = \left(\begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}\right)^2 \Rightarrow \begin{bmatrix} 36 & 64 \\ 100 & 144 \end{bmatrix}$$

Utilizando as mesmas matrizes e aplicando o quadrado do binomio o resultado seria

$$= \begin{bmatrix} 48 & 68 \\ 108 & 144 \end{bmatrix}$$

Logo $(A+B)^2$!= $A^2+2AB+B^2$

Para esta resolução, mantenha o uso das matrizes A e B citadas a cima.
Realizando (A-B)(A+B)

$$A-B = \begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix} \quad A+B = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix} \Rightarrow \begin{bmatrix} -64 & -80 \\ -64 & -80 \end{bmatrix}$$

Realizando (A^2-B^2)

$$A^2 = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} \quad B^2 = \begin{bmatrix} 67 & 78 \\ 91 & 106 \end{bmatrix} \Rightarrow \begin{bmatrix} -60 & -68 \\ -76 & -84 \end{bmatrix}$$

Logo (A-B)(A+B) != (A^2-B^2)

4) **Seja:**

$$A = \begin{bmatrix} 2 & x^2 \\ 2x-1 & 0 \end{bmatrix}$$ **Se $A' = A$, então $x = ?$**

$$A' = \begin{bmatrix} 2 & 2x-1 \\ x^2 & 0 \end{bmatrix} \rightsquigarrow x^2 = 2x - 1 \Rightarrow \boxed{x^2 - 2x + 1 = 0}$$

$$\Delta = 4 - 4 \Rightarrow 0 \rightsquigarrow 1 \text{ raiz} \in \mathbb{R}$$

$$x = \frac{2 \pm \sqrt{0}}{2} \Rightarrow \boxed{x = 1}$$

5) **Ache x, y, w, z se**

$$\begin{bmatrix} x & y \\ z & w \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$x \cdot 2 + y \cdot 3 = 1 \qquad x \cdot 3 + y \cdot 4 = 0$$

$$z \cdot 2 + w \cdot 3 = 0 \qquad z \cdot 3 + w \cdot 4 = 1$$

$$2x + 3y = 1$$

$$x = \frac{1 - 3y}{2}$$

$$3 \cdot \left(\frac{1 - 3y}{2}\right) + 4y = 0 \Rightarrow \frac{3 - y}{2} = 0$$

$$\frac{3 - 9y}{2} + \frac{8y}{2} = 0$$

$$\boxed{y = 3}$$

$x = \frac{1-3\cdot 3}{2} \rightsquigarrow \boxed{x = -4}$

$2z + 3w = 0$

$z = -\frac{3w}{2}$

$3z + 4w = 1$

$3\cdot\left(-\frac{3w}{2}\right) + 4w = 1$

$-\frac{9w}{2} + \frac{8w}{2} = 1$

$\Rightarrow -\frac{w}{2} = 1$

$\boxed{w = -2}$

$z = \frac{(-3)\cdot(-2)}{2}$

$\boxed{z = 3}$

6) Determinar x,y,z e w tal que a equação matricial:

$$2\begin{bmatrix} x & y \\ z & -w \end{bmatrix} - \begin{bmatrix} 3 & x-y \\ z+w & 6+y \end{bmatrix} = \begin{bmatrix} x+y & 5 \\ 2w & 2w-z \end{bmatrix}$$

seja verdadeira.

$\begin{bmatrix} 2x & 2y \\ 2z & -2w \end{bmatrix}$

$2x - 3 = x + y$

$2y - (x-y) = 5$

$2z - (z+w) = 2w$

$-2w - (6+y) = 2w - z$

$y = x - 3$

$y = 7 - 3$

$\boxed{y = 4}$

$2\cdot(x-3) - \cancel{x} + \cancel{x} - 3 = 5$

$2x - 6 - 3 = 5$

$\boxed{x = 7}$

$2w = 2z - z - w$

$w = \frac{z}{3}$

$2\left(\frac{z}{3}\right) - z = -2\left(\frac{z}{3}\right) - 6 - 4$

$\frac{2z}{3} - z = -\frac{2z}{3} - 10$

$$\boxed{w = \frac{10}{3}}$$

$$\boxed{z = 10}$$

8) Considere as matrizes:

$$A = \begin{pmatrix} 5 & 4 & 6 & 8 \\ 1 & 2 & 3 & 4 \\ 2 & 0 & -1 & 1 \\ 1 & 1 & 2 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 4 & -1 \\ 2 & 5 & 1 & 3 \\ -2 & -5 & -2 & 5 \end{pmatrix}$$

Calcular o $det(A)$ e $det(B)$, usando o desenvolvimento de Laplace.

$$|A| = \sum_{j=1}^{n} a_{ij} \cdot \Delta_{ij}$$

$$\Delta_{ij} = (-1)^{i+j} \cdot |A_{ij}|$$

A) $i = 1$

$$5 \cdot (-1)^{1+1} \cdot \begin{vmatrix} 2 & 3 & 4 \\ 0 & -1 & 1 \\ 1 & 2 & 1 \end{vmatrix} \rightarrow 1 \quad + 4 \cdot (-1)^{1+2} \cdot \begin{vmatrix} 1 & 3 & 4 \\ 2 & -1 & 1 \\ 1 & 2 & 1 \end{vmatrix} \rightarrow 14$$

$$+ 6 \cdot (-1)^{1+3} \cdot \begin{vmatrix} 1 & 2 & 4 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} \rightarrow 5 \quad + 8 \cdot (-1)^{1+4} \cdot \begin{vmatrix} 1 & 2 & 3 \\ 2 & 0 & -1 \\ 1 & 1 & 2 \end{vmatrix} \rightarrow -3$$

$$5 - 56 + 30 + 24 = \boxed{3}$$

$$\begin{vmatrix} 2 & 3 & 4 \\ 0 & -1 & 1 \\ 1 & 2 & 1 \end{vmatrix} \begin{matrix} 2 & 3 \\ 0 & -1 \\ 1 & 2 \end{matrix}$$

$0 \qquad 1$

$1-0=1$

$$\begin{vmatrix} 1 & 3 & 4 \\ 2 & -1 & 1 \\ 1 & 2 & 1 \end{vmatrix} \begin{matrix} 1 & 3 \\ 2 & -1 \\ 1 & 2 \end{matrix}$$

$4 \qquad 20$

$18-4=14$

$$\begin{vmatrix} 1 & 2 & 4 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} \begin{matrix} 1 & 2 \\ 2 & 0 \\ 1 & 1 \end{matrix}$$

$5 \qquad 10$

$10-5=5$

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 0 & -1 \\ 1 & 1 & 2 \end{vmatrix} \begin{matrix} 1 & 2 \\ 2 & 0 \\ 1 & 1 \end{matrix}$$

$7 \qquad 4$

$4-7=-3$

$$B = \begin{pmatrix} 1 & 3 & 0 & -1 \\ 0 & 1 & 4 & -1 \\ 2 & 5 & 1 & 3 \\ -2 & -5 & -2 & 5 \end{pmatrix} \quad j=1$$

$$1 \cdot (-1)^{1+1} \cdot \underbrace{\begin{vmatrix} 1 & 4 & -1 \\ 5 & 1 & 3 \\ -5 & -2 & 5 \end{vmatrix}}_{} \; 0 = 144 \; + 0 + 2 \cdot (-1)^{1+3} \cdot \begin{vmatrix} 3 & 0 & -1 \\ 1 & 4 & -1 \\ -5 & -2 & 5 \end{vmatrix} \; 36$$

$$+(-2)\cdot(-1)^{1+4}\cdot\begin{vmatrix}3&0&-1\\1&4&-1\\5&1&3\end{vmatrix} \quad 158$$

$$-144+72+116=\boxed{44}$$

$$\begin{vmatrix}1&4&-1\\5&1&3\\-5&-2&5\end{vmatrix}\begin{matrix}1&4\\5&1\\-5&-2\end{matrix}$$

$5-60+10=-45$

$5-6+100$

99

$-45-99=-144$

$$\begin{vmatrix}3&0&-1\\1&4&-1\\-5&-2&5\end{vmatrix}\begin{matrix}3&0\\1&4\\-5&-2\end{matrix}$$

$60+0+2=62$

$6+20=26$

$62-26=36$

$$\begin{vmatrix}3&0&-1\\1&4&-1\\5&1&3\end{vmatrix}\begin{matrix}3&0\\1&4\\5&1\end{matrix}$$

$-20-3=-23$

$36-1=35$

$35+23=58$