

Improved performance
through GPU shared memory

Recall: Matrix multiplication

$$\begin{bmatrix} A_{1,1} & A_{2,1} & A_{3,1} & A_{4,1} \\ A_{1,2} & A_{2,2} & A_{3,2} & A_{4,2} \\ A_{1,3} & A_{2,3} & A_{3,3} & A_{4,3} \\ A_{1,4} & A_{2,4} & A_{3,4} & A_{4,4} \end{bmatrix} \times \begin{bmatrix} B_{1,1} & B_{2,1} & B_{3,1} & B_{4,1} \\ B_{1,2} & B_{2,2} & B_{3,2} & B_{4,2} \\ B_{1,3} & B_{2,3} & B_{3,3} & B_{4,3} \\ B_{1,4} & B_{2,4} & B_{3,4} & B_{4,4} \end{bmatrix} = \begin{bmatrix} C_{1,1} & C_{2,1} & C_{3,1} & C_{4,1} \\ C_{1,2} & C_{2,2} & C_{3,2} & C_{4,2} \\ C_{1,3} & C_{2,3} & C_{3,3} & C_{4,3} \\ C_{1,4} & C_{2,4} & C_{3,4} & C_{4,4} \end{bmatrix}$$

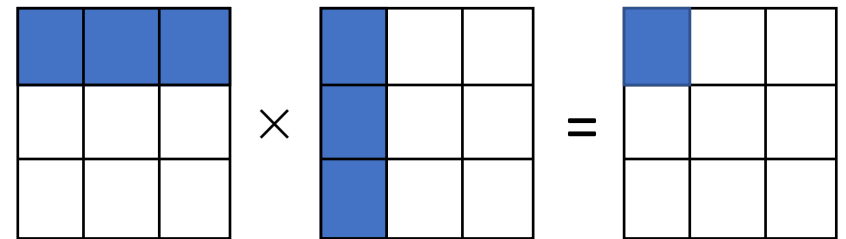
Each element of the product is

Basic CUDA implementation

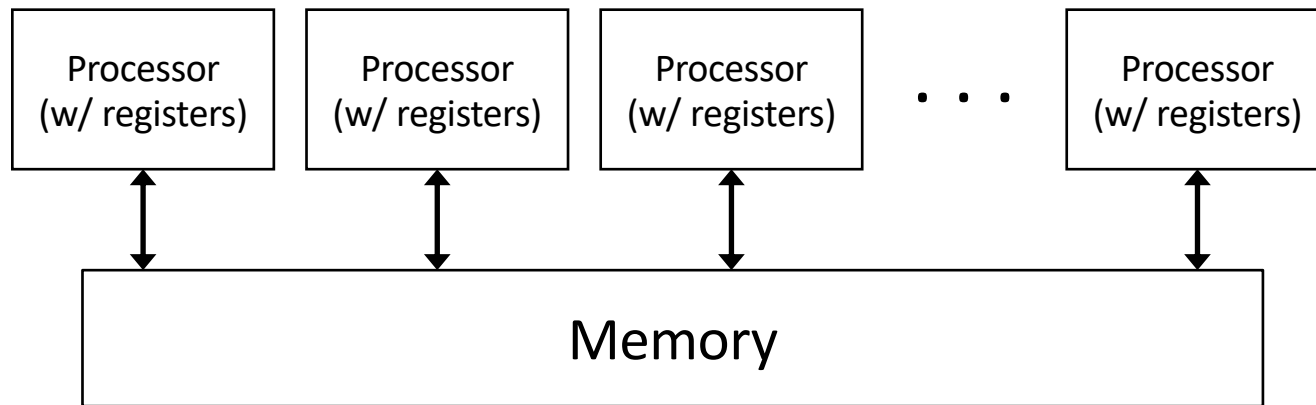
```
__global__ void kernel(float* Md, float* Nd, float* Pd, int width) {  
    int row = blockIdx.y*TILE_WIDTH + threadIdx.y; //calculate indices of element  
    int col = blockIdx.x*TILE_WIDTH + threadIdx.x;  
  
    if(row >= width || col >= width)           //check that indices are in bounds  
        return;  
  
    float tmp = 0; //local variable in which to accumulate the answer  
    for(int k=0; k < width; ++k)  
        tmp += Md[row*width + k] * Nd[k*width+col];  
    Pd[row*width+col] = tmp;  
}
```

Tiling

- Divide matrix into “tiles” (submatrices)
- A tile of the output matrix depends on a row and a column of tiles respectively from the input matrices
- Reduce memory demand by computing the elements of an output tile together

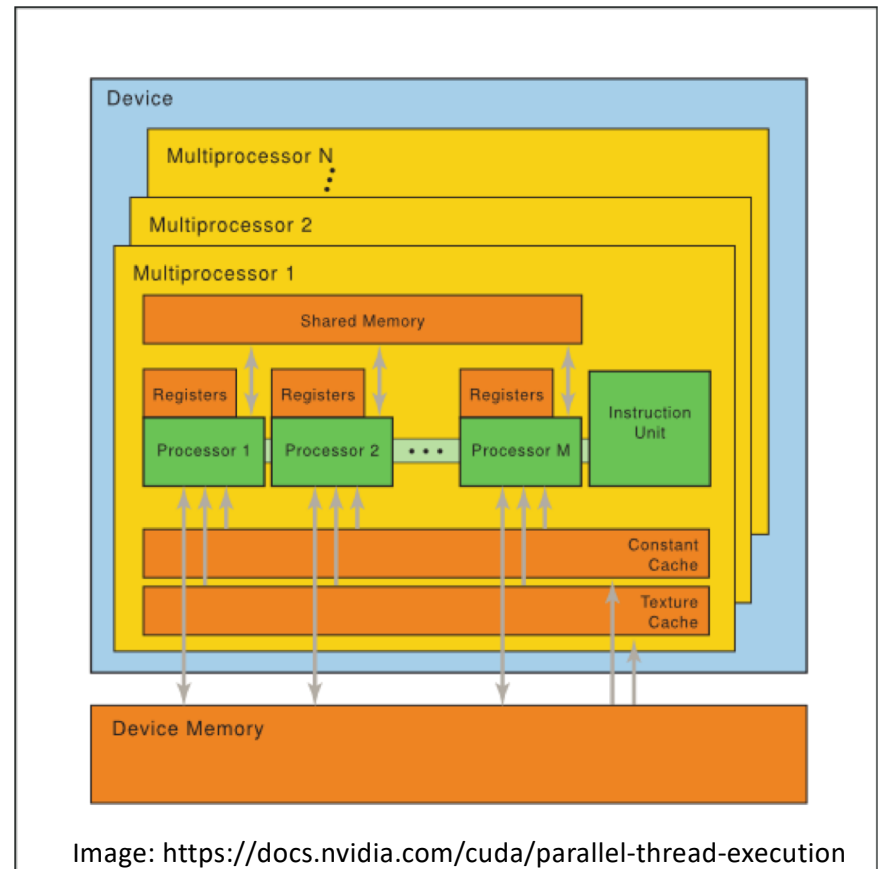


Memory in CUDA so far



CUDA memory types

- Processors split between multiprocessors (each runs 1 block)
- Multiple types of memory
- Registers are per processor
- Shared memory is per multiprocessor
- Multiprocessors also have caches for constant and texture memory



Idea of shared memory implementation

- Use shared memory as programmer managed cache
 - Each block computes a tile of the output matrix
 - Each thread of the block is responsible for one element of the tile
 - Load one tile of each input matrix (1 element per thread), each thread computes their contribution to the output, and then move on to next tiles

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- Between steps, need
`__syncthreads();` //blocks until all threads in the block reach the call

Writing the tiled kernel

```
__global__ void tiledkernel(float* Md, float* Nd, float* Pd, int width) {  
    __shared__ float Mds[TILE_WIDTH][TILE_WIDTH];  
    __shared__ float Nds[TILE_WIDTH][TILE_WIDTH];  
    ...  
}
```

Step 1: Allocate the cache to store a tile of each matrix

Writing the tiled kernel

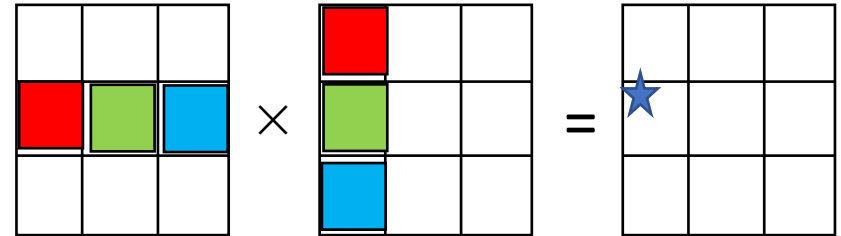
```
__global__ void tiledkernel(float* Md, float* Nd, float* Pd, int width) {  
    ...  
    int row = blockIdx.y * TILE_WIDTH + threadIdx.y;  
    int col = blockIdx.x * TILE_WIDTH + threadIdx.x;  
  
    int num_tiles = (width+TILE_WIDTH-1)/TILE_WIDTH;  
    ...  
}
```

Step 2: Figure out the index for which this thread is responsible
(Block and tile have the same width)
Figure out the width of the matrix in tiles

Step 3: Main loop

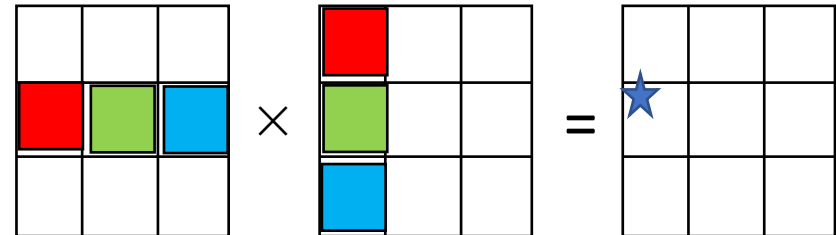
```
float tmp = 0;  
for (int m=0; m < num_tiles; m++) {
```

```
}
```



Step 3: Main loop

```
float tmp = 0;  
for (int m=0; m < num_tiles; m++) {  
    if(/* in bounds */)   
        Mds[threadIdx.y][threadIdx.x] = ...;  
    if(/* in bounds */)   
        Nds[threadIdx.y][threadIdx.x] = ...;  
    __syncthreads();  
}
```



Step 3a: Load cell of
cached submatrices
(then wait)

Step 3: Main loop

```
float tmp = 0;  
for (int m=0; m < num_tiles; m++) {
```

```
    if(/* in bounds */)
```

```
        Mds[threadIdx.y][threadIdx.x] = ...;
```

```
    if(/* in bounds */)
```

```
        Nds[threadIdx.y][threadIdx.x] = ...;
```

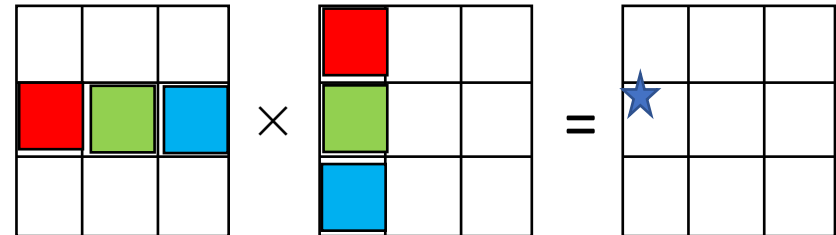
```
    __syncthreads();
```

```
    for(k=0; k < TILE_WIDTH; k++)
```

```
        tmp += ...
```

```
    __syncthreads();
```

```
}
```



Step 3a: Load cell of
cached submatrices
(then wait)

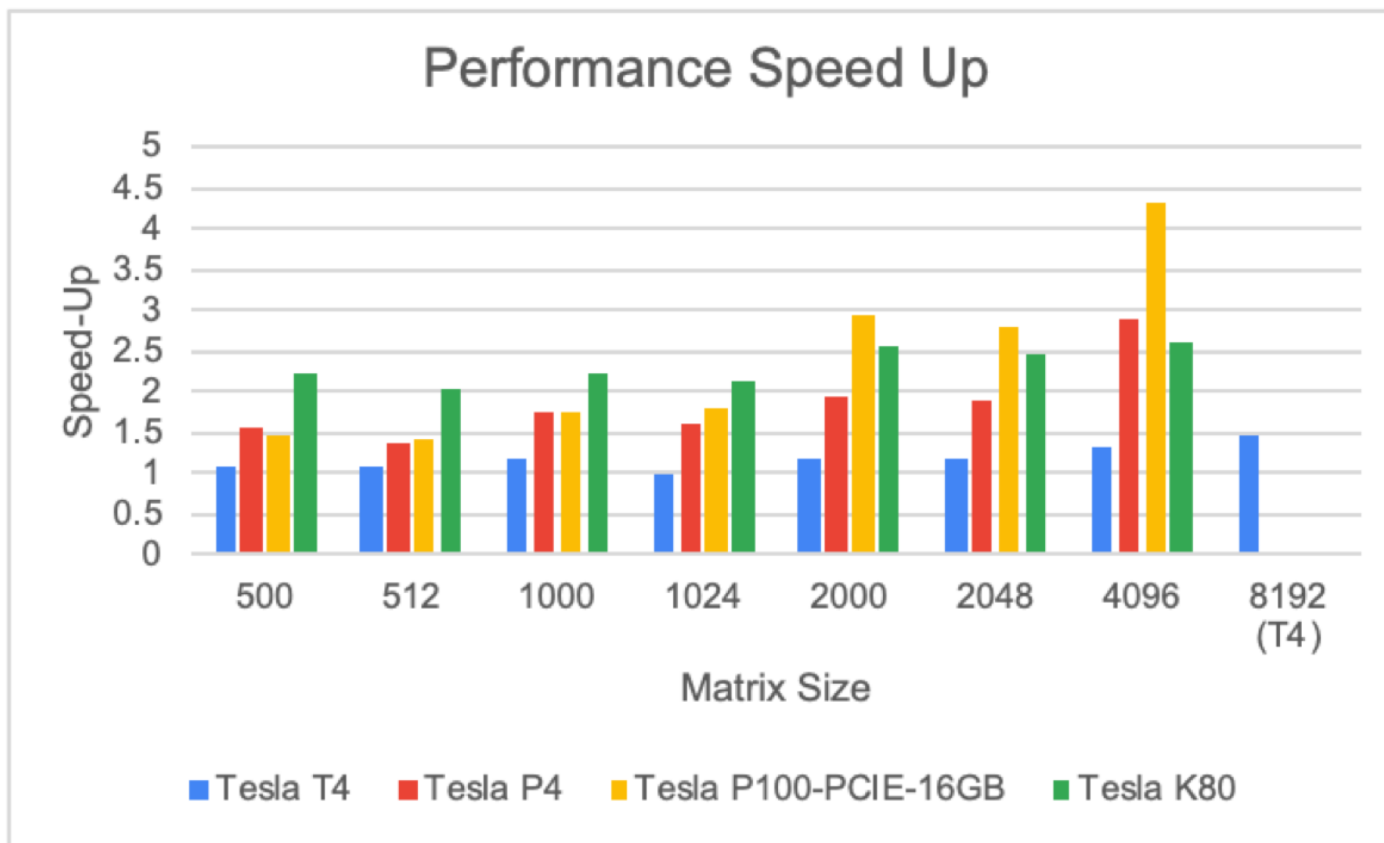
Step 3b: Calculate submatrix
contribution (then wait)

Writing the tiled kernel

```
__global__ void tiledkernel(float* Md, float* Nd, float* Pd, int width) {  
    ...  
    for (int m=0; m < num_tiles; m++) {  
        ...  
    }  
  
    if (row < width && col < width)  
        Pd[row*width+col] = tmp;  
}
```

Step 4: Write the value into the appropriate cell

Performance results



Speed-up = Time for untiled version / time for tiled version