Q1

Write the value for the covariance Q of the noise added to the observation function, knowing that the parameter `bearing_std` is its standard deviation.

A1

Value of `bearing_std` is 0.35, thus $Q = 0.35^2 = 0.1225$

Q2

Write the equation for the covariance R_t of the noise added to the transition function, and their corresponding numeric values for the initial robot command $u = [\delta_{rot1}, \delta_{trans}, \delta_{rot2}]^\top = [0, 10, 0]^\top$.

A2

According to lecture 3, odometry model, R calculated as $R = VMV^\top$, where V is the Jacobian matrix of the transition function with derriviative according to control input u and M is covariance value:

$$M = \begin{bmatrix} \alpha_1\delta_{rot1}^2 + \alpha_2\delta_{trans}^2 & 0 & 0 \\ 0 & \alpha_3\delta_{trans}^2 + \alpha_4(\delta_{rot1}^2 + \delta_{rot2}^2) & 0 \\ 0 & 0 & \alpha_1\delta_{rot2}^2 + \alpha_2\delta_{trans}^2 \end{bmatrix}$$

Nummerically:

$$M = \begin{bmatrix} 0.05^2 * 0^2 + 0.001^2 * 10^2 & 0 & 0 \\ 0 & 0.05^2 * 10^2 + 0.01^2(0^2 + 0^2) & 0 \\ 0 & 0 & 0.05^2 * 0^2 + 0.001^2 * 10^2 \end{bmatrix}$$

$$M = \begin{bmatrix} 0.05^2 & 0 & 0 \\ 0 & 0.05^2 & 0 \\ 0 & 0 & 0.05^2 \end{bmatrix} = 0.05^2 \times \mathrm{I}$$

Now, we need V

Q3

Derive the equations for the Jacobians G_t, V_t and H_t, and evaluate them at the initial mean state $\mu_1 = [x, y, \theta]^\top = [180, 50, 0]^\top$.

A3

From same lecture we can take the Jacobian G_t:

$$G_t = \left.\frac{\partial g(x_{t-1}, u_t, \varepsilon_t)}{\partial x_{t-1}}\right|_{\mu_{t-1}, \varepsilon_t = 0} = \begin{array}{c} \begin{bmatrix} 1 & 0 & -\delta_{trans}\sin(\theta + \delta_{rot1}) \\ 0 & 1 & \delta_{trans}\cos(\theta + \delta_{rot1}) \\ 0 & 0 & 1 \end{bmatrix} \\ \begin{array}{ccc} \partial/\partial x & \partial/\partial y & \partial/\partial \theta \end{array} \end{array}$$

$$G_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix}$$

Jacobian V_t is:

$$V_t = \left.\frac{\partial g(x_{t-1}, u_t, \varepsilon_t)}{\partial u_t}\right|_{\mu_{t-1}, \varepsilon_t = 0} = \begin{bmatrix} -\delta_{trans} \cdot \sin(\theta + \delta_{rot1}) & \cos(\theta + \delta_{rot1}) & 0 \\ \delta_{trans} \cdot \cos(\theta + \delta_{rot1}) & \sin(\theta + \delta_{rot1}) & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$V_1 = \begin{bmatrix} 0 & 1 & 0 \\ 10 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Returning to previous question, let's calculate R:

$$R = VMV^\top = \begin{bmatrix} 0 & 1 & 0 \\ 10 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.05^2 & 0 & 0 \\ 0 & 0.05^2 & 0 \\ 0 & 0 & 0.05^2 \end{bmatrix} \times \begin{bmatrix} 0 & 10 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.05^2 & 0 & 0 \\ 0 & 0.5^2 & 0.1 \times 0.5^2 \\ 0.05^2 & 0.1 \times 0.5^2 & 0.05^2 \end{bmatrix}$$

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